Cours de physique théorique

agréé par **l'École doctorale « Physique en Île-de-France » – ED PIF**

Introduction to *Random Matrix Theory* and its various applications

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Les vendredis 20/11/2015, 27/11, 4/12, 18/12
et le <u>*lundi*</u> *14/12. Tous les cours auront lieu à 10h00.*

Outline of the course

1) Brief historical introduction to RMT: applications.
   - Discussion of basic properties of matrices, different random matrix ensembles, rotationally invariant ensembles such as Gaussian ensembles etc.

2) Gaussian ensembles: derivation of the joint probability distribution of eigenvalues, starting from the joint distribution of matrix entries.

3) Analysis of the spectral properties of eigenvalues: given the joint distribution of eigenvalues, how to calculate various observables such as:
   - Average density of eigenvalues ----Wigner semi-circle law
   - Counting statistics, spacings between eigenvalues etc.
   - Distribution of the extreme (maximum or minimum eigenvalues)

4) Two complementary approaches to study spectral statistics:
   - Large N (for an NxN matrix) method by the Coulomb gas approach: saddle point method
   - Finite N method: for Gaussian unitary ensemble: orthogonal polynomial method (essentially quantum mechanics of free fermions at zero temperature).

5) Tracy-Widom distribution: prob. distribution of the top eigenvalue. Its appearance in a large number of problems, universality and an associated third order phase transition.

6) Perspectives and summary.