Diffractive production of dijets in DIS

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Overview

Overview

1. Introduction to high energy QCD
   - Divergences, resummation and factorization
   - Collinear factorization
   - k_t factorization
   - Saturation
2. The shockwave formalism
3. Production of dijets in DDIS
   - Motivation
   - Diffractive DIS
   - Impact Factor for dijet production
4. Conclusion and applications
   - Phenomenology
   - Extensions

Divergences in high energy QCD

UV : $k^2 \to \infty \, (\frac{dk^2}{k^2})$ **Soft** : $E_k \to 0 \, (\frac{dE_k}{E_k})$ **Collinear** : $\theta \to 0 \, (\frac{d\theta}{\theta})$

Rapidity divergence (gauge specific)

In lightlike gauge $(n \cdot A = 0, \; n^2 = 0)$: when $n \cdot k \to 0$

$$d_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k}$$

Divergences and resummation

Typical divergence in dimensional regularization (dimension $D = 4 + 2\varepsilon$) :

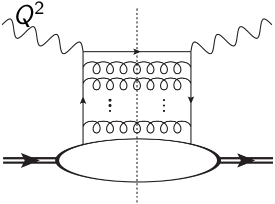
$$\frac{1}{\varepsilon}\left(\frac{Q^2}{\mu^2}\right)^{\varepsilon} = \frac{1}{\varepsilon} + \log\left(\frac{Q^2}{\mu^2}\right) + O(\varepsilon)$$

Terms like $\alpha_s \log(Q^2)$ remain.

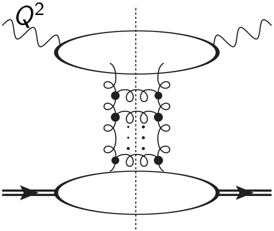
For a large scale Q^2, the log compensates the smallness of α_s.

Resummation of such terms is necessary.

QCD factorization



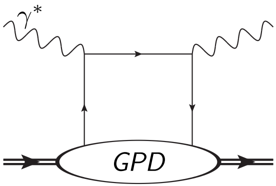
Collinear factorization
DGLAP evolution
Resums $(\alpha_s \log Q^2)^n$



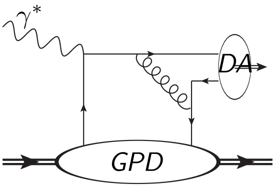
k_t factorization
BFKL evolution
Resums $(\alpha_s \log s)^n$

Collinear factorization

Proven at all orders in α_s for several processes



DVCS

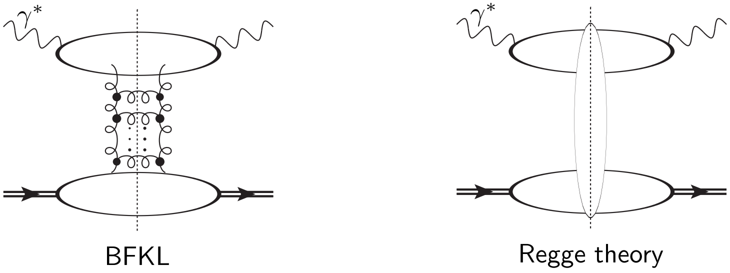


DVMP

Valid for single-scale processes, moderate Bjorken x

k_t factorization

No rigorous proof at all orders in α_s, but
it reproduces the pre-QCD result that at high energy a Pomeron is exchanged



$$\sigma \propto s^{\alpha_P - 1}$$

Valid for processes with two scales, small Bjorken x

Towards saturation

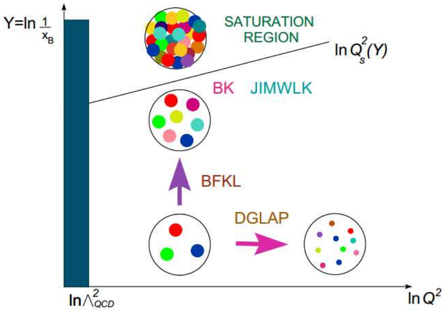
$$\sigma \propto s^{\alpha_P - 1}$$

with

$$\alpha_P - 1 = \frac{4N_c\alpha_s}{\pi}\log(2) > 0$$

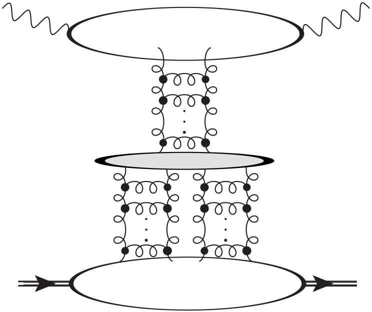
Violation of unitarity through the violation of the Froissart bound

$$\sigma < A\log^2(s)$$



Saturation effects should take place to reduce the growth with s

Saturation in the BFKL formalism



"Fan diagram" [Gribov, Levin, Ryskin]
Resums $\alpha_s \ln Q^2 \ln s$ terms

Resummation of fan diagrams leads to non-linear terms in the evolution equation

Overview

The shockwave approach

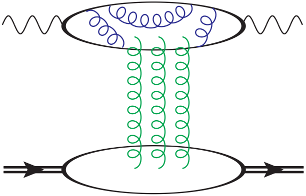
One decomposes the gluon field $\mathcal{A}$ into an internal field and an external field :

$$\mathcal{A}^\mu = A^\mu + b^\mu$$

The internal one contains the gluons with rapidity $p_g^+ > e^\eta p_\gamma^+$ and the external one contains the gluons with rapidity $p_g^+ < e^\eta p_\gamma^+$. One writes :

$$b_\eta^\mu(z) = \delta\left(z^+\right) B_\eta(\vec{z})\, n_2^\mu$$

Intuitively, large boost Λ along the $+$ direction :



$$b^+\left(x^+, x^-, \vec{x}\right) \rightarrow \frac{1}{\Lambda} b^+\left(\Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x}\right)$$

$$b^-\left(x^+, x^-, \vec{x}\right) \rightarrow \Lambda b^-\left(\Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x}\right)$$

$$b^i\left(x^+, x^-, \vec{x}\right) \rightarrow b^i\left(\Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x}\right)$$

Propagator through a shockwave

$$G\left(z_2,\, z_0\right) = -\int d^4 z_1 \theta\left(z_2^+\right)\delta\left(z_1^+\right)\theta\left(-z_0^+\right) G\left(z_2 - z_1\right)\gamma^+ G\left(z_1 - z_0\right) U_1$$

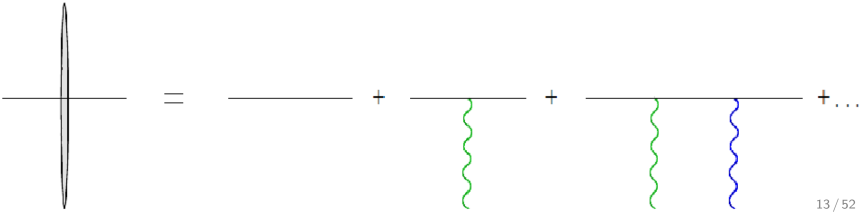
$$G\left(q,\, p\right) = (2\pi)\,\theta\left(p^+\right)\int d^D q_1 \delta\left(q + q_1 - p\right) G\left(q\right)\gamma^+ \tilde{U}_{\vec{q}_1} G\left(p\right)$$

Wilson lines :

$$U_i = U_{\vec{z}_i} = U\left(\vec{z}_i, \eta\right) = P\exp\left[ig\int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) dz_i^+\right]$$

$$U_i = 1 + ig\int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) dz_i^+ + (ig)^2\int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) b_\eta^-\left(z_j^+, \vec{z}_j\right)\theta\left(z_{ji}^+\right) dz_i^+ dz_j^+$$

...



Evolution equation for a color dipole

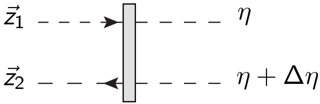
Dipole operator

$$\mathbf{U}_{12} = \frac{1}{N_c}\mathrm{Tr}\left(U_1 U_2^\dagger\right) - 1$$

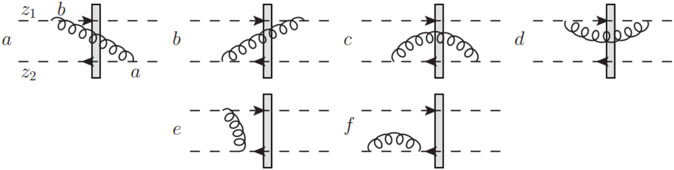
involving Wilson lines

$$U_1 = \ldots \left[1 + igb_\eta(z^+ + \Delta z^+, \vec{z}_1)\Delta z^+\right]\left[1 + igb_\eta(z^+, \vec{z}_1)\Delta z^+\right]\ldots$$

$$U_2 = \ldots \left[1 + igb_{\eta+\Delta\eta}(z^+ + \Delta z^+, \vec{z}_1)\Delta z^+\right]\left[1 + igb_{\eta+\Delta\eta}(z^+, \vec{z}_1)\Delta z^+\right]\ldots$$



Balitsky's hierarchy of equations



B-JIMWLK equation

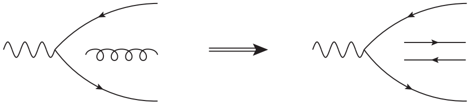
[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

$$\frac{\partial \mathbf{U}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} - \mathbf{U}_{13}\mathbf{U}_{32}\right]$$

$$\frac{\partial \mathbf{U}_{13}\mathbf{U}_{32}}{\partial \eta} = \dots$$

The BK equation

Mean field approximation, or 't Hooft limit $N_c \to 0$ in Balitsky's equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \mathbf{U}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} - \mathbf{U}_{13}\mathbf{U}_{32}\right]$$

Evolves a dipole into a double dipole

Non-linear term : saturation

Overview

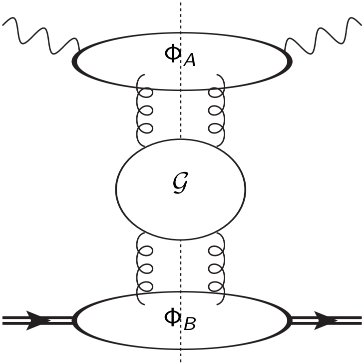
Probing QCD in the Regge limit and towards saturation

What kind of observable?

- Perturbation theory should apply : a hard scale Q^2 is required
- One needs semihard kinematics : $s \gg p_T^2 \gg \Lambda_{QCD}^2$ where all the typical transverse scales p_T are of the same order
- Saturation is reached when $Q^2 \sim Q_s^2 = \left(\frac{A}{x}\right)^{\frac{1}{3}}$: the smaller $x \sim \frac{Q^2}{s}$ is and the heavier the target ion, the easier saturation is reached.
- Typical processes : DIS, Mueller-Navelet double jets, high p_T central jets, ultraperipheral events at the LHC...

Precision tests of BFKL dynamics

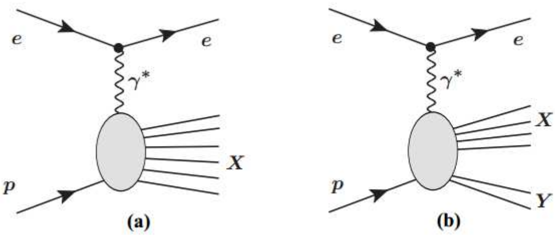
- The BFKL kernel is known at NLL accuracy, resumming $\alpha_s(\alpha_s \log s)^n$ corrections (Lipatov, Fadin ; Camici, Ciafaloni)
- Very few impact factors are known at NLO accuracy
  - $\gamma^* \to \gamma^*$ (Bartels, Colferai, Gieseke, Kyrielis, Qiao; Balitsky, Chirilli)
  - Forward jet production (Bartels, Colferai, Vacca ; Caporale, Ivanov, Murdaca, Papa, Perri ; Chachamis, Hentschinski, Madrigal, Sabio Vera)
  - Inclusive production of a pair of hadrons separated by a large interval of rapidity (Ivanov, Papa)
  - $\gamma_L^* \to \rho_L$ in the forward limit (Ivanov, Kostsky, Papa)



Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a rapidity gap

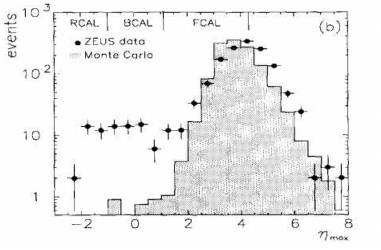


DIS events DDIS events

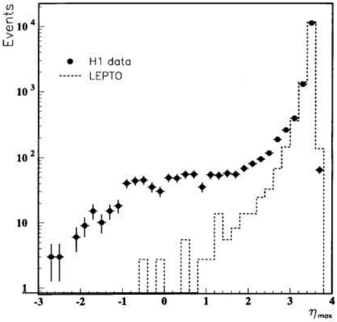
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of events reveal a rapidity gap



ZEUS, 1993



H1,1994

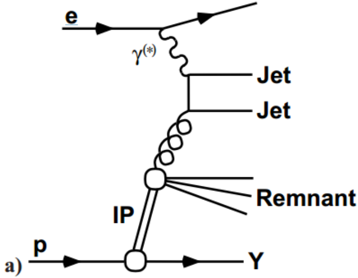
Diffractive DIS

Theoretical approaches for DDIS using pQCD

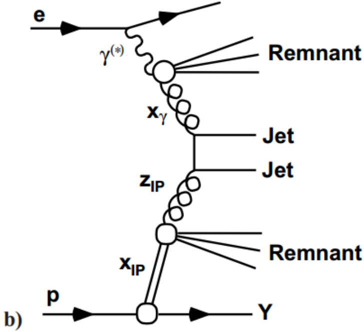
- Collinear factorization approach
  - Relies on QCD factorization theorem, using a hard scale such as the virtuality Q^2 of the incoming photon
  - One needs to introduce a diffractive distribution function for partons *within a pomeron*
- k_T factorization approach for two exchanged gluons
  - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a two-gluon color-singlet state

Theoretical approaches for DDIS using pQCD

Collinear factorization approach



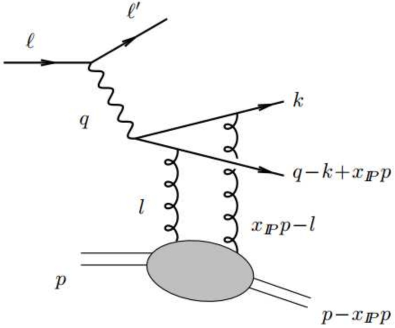
Direct



Resolved

Theoretical approaches for DDIS using pQCD

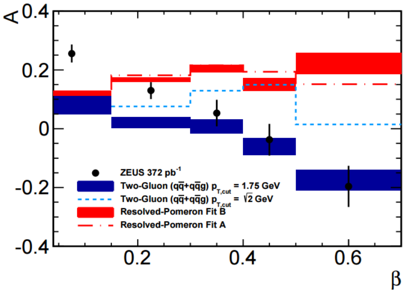
k_T-factorization approach : two gluon exchange



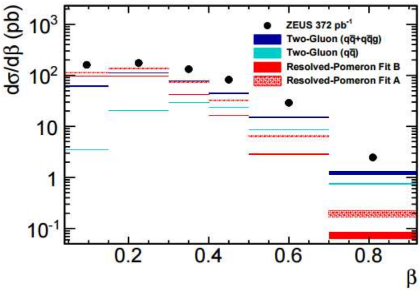
Bartels, Ivanov, Jung, Lotter, Wüsthoff
Braun and Ivanov developed a similar model in collinear factorization

Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data



ZEUS collaboration, 2015



ZEUS collaboration, 2015

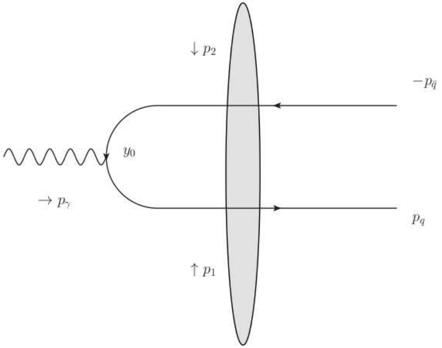
Assumptions

- Regge limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- No approximation for the outgoing gluon, contrary to e.g. :
  - Collinear approximation [Wüsthoff, 1995]
  - Soft approximation [Bartels, Jung, Wüsthoff, 1999]
- Lightcone coordinates $(p^+, p^-, \vec{p})$ and lightcone gauge $n_2 \cdot \mathcal{A} = 0$
- Transverse dimensional regularization $d = 2 + 2\varepsilon$, longitudinal cutoff

$$p_g^+ < \alpha p_\gamma^+$$

- Shockwave (Wilson lines) approach [Balitsky, 1995]

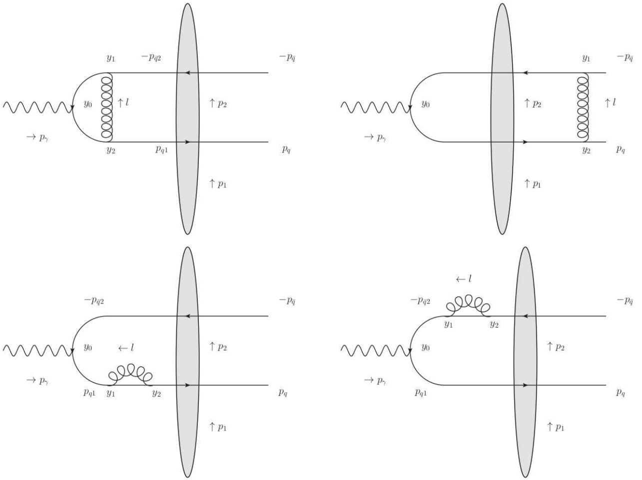
Leading Order



$$\mathcal{A}_0 = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_0^\alpha (\vec{p}_1, \vec{p}_2)\, \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2})\, \tilde{\mathbf{U}}_{12}$$

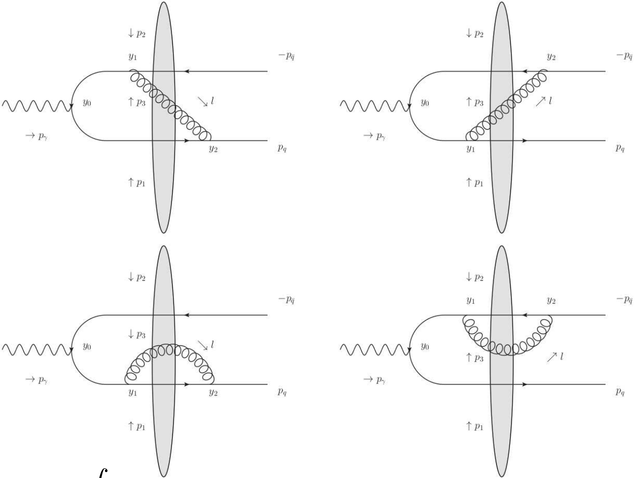
$p_{ij} = p_i - p_j$

First kind of virtual corrections



$$\mathcal{A}_{V1} \propto \varepsilon_\alpha N_c \int d^d\vec{p}_1 d^d\vec{p}_2 \Phi^{\alpha}_{V1}(\vec{p}_1, \vec{p}_2)\, \delta(\vec{p}_{q1}+\vec{p}_{\bar{q}2}) \left(\frac{N_c^2-1}{N_c}\right) \tilde{\mathbf{U}}_{12}$$

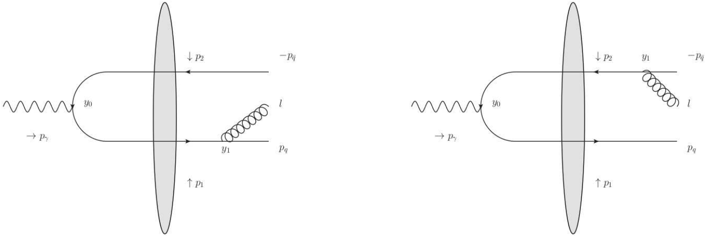
Second kind of virtual corrections



$$\mathcal{A}_{V2} \propto \varepsilon_\alpha N_c \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \Phi^{\alpha}_{V2}(\vec{p}_1,\, \vec{p}_2,\, \vec{p}_3)\delta\left(\vec{p}_{q1}+\vec{p}_{\bar{q}2}-\vec{p}_3\right)$$

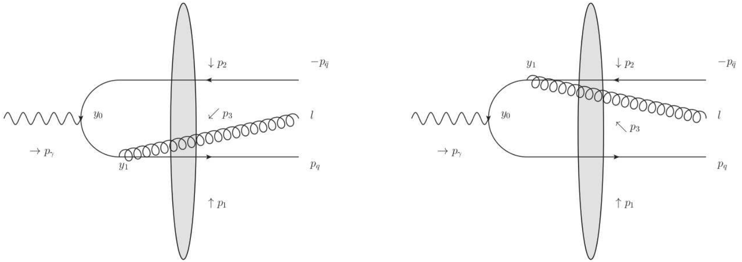
$$\left[\delta\left(\vec{p}_3\right)\left(\frac{N_c^2-1}{N_c}\right)\tilde{\mathbf{U}}_{12} + N_c\left(\tilde{\mathbf{U}}_{13}\tilde{\mathbf{U}}_{32}+\tilde{\mathbf{U}}_{13}+\tilde{\mathbf{U}}_{32}-\tilde{\mathbf{U}}_{12}\right)\right]$$

First kind of real corrections



$$\mathcal{A}_{R1} = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \, \Phi^\alpha_{R1}(\vec{p}_1, \, \vec{p}_2) \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} + \vec{p}_g) \left(\frac{N_c^2 - 1}{N_c}\right) \tilde{\mathbf{U}}_{12}$$

Second kind of real corrections



$$\mathcal{A}_{R2} = \varepsilon_\alpha N_c \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \Phi^\alpha_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3)\delta(\vec{p}_{q1}+\vec{p}_{\bar{q}2}+\vec{p}_{g3})$$
$$\left[\left(\frac{N_c^2-1}{N_c}\right)\tilde{\mathbf{U}}_{12}\delta(\vec{p}_3) + N_c\left(\tilde{\mathbf{U}}_{13}\tilde{\mathbf{U}}_{32}+\tilde{\mathbf{U}}_{13}+\tilde{\mathbf{U}}_{32}-\tilde{\mathbf{U}}_{12}\right)\right]$$

Divergences

- UV divergence $\vec{p}_g^{\,2} \to +\infty$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$$

- Soft divergence $p_g \to 0$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

$$\Phi_{R1}\Phi_{R1}^*$$

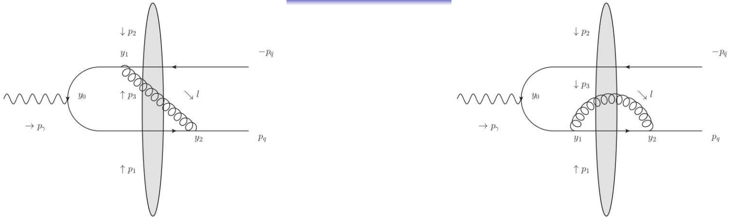
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$

$$\Phi_{R1}\Phi_{R1}^*$$

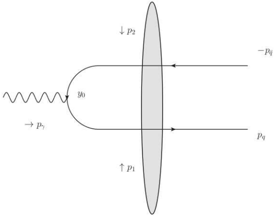
- Rapidity divergence $p_g^+ \to 0$

$$\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$$

Rapidity divergence



Double dipole virtual correction Φ_{V2}



Balitsky's evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Rapidity divergence

B-JIMWLK equation

$$\frac{\partial \tilde{\mathbf{U}}_{12}^{\alpha}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2\right) \left(\tilde{\mathbf{U}}_{13}\tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12}\right)$$

$$\left[2\frac{\left(\vec{k}_1 - \vec{p}_1\right) \cdot \left(\vec{k}_2 - \vec{p}_2\right)}{\left(\vec{k}_1 - \vec{p}_1\right)^2 \left(\vec{k}_2 - \vec{p}_2\right)^2} + \frac{\pi^{\frac{d}{2}} \Gamma\left(1 - \frac{d}{2}\right) \Gamma^2\left(\frac{d}{2}\right)}{\Gamma(d-1)} \left(\frac{\delta\left(\vec{k}_2 - \vec{p}_2\right)}{\left[\left(\vec{k}_1 - \vec{p}_1\right)^2\right]^{1-\frac{d}{2}}} + \frac{\delta\left(\vec{k}_1 - \vec{p}_1\right)}{\left[\left(\vec{k}_2 - \vec{p}_2\right)^2\right]^{1-\frac{d}{2}}} \right) \right]$$

η typical rapidity for the lower impact factor

$$\tilde{\mathbf{U}}_{12}^{\alpha}\Phi_0 \to \Phi_0 \tilde{\mathbf{U}}_{12}^{\eta} + \log\left(\frac{e^{\eta}}{\alpha}\right) \mathcal{K}_{BK} \Phi_0 \left(\tilde{\mathbf{U}}_{13}\tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12}\right)$$

Rapidity divergence

Virtual contribution

$$(\Phi^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{\alpha^2}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

BK contribution

$$(\Phi^{\mu}_{BK})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{\alpha^2}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] \right\}$$

Sum

$$(\Phi'^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

Rapidity divergence

Convolution

$$
\begin{aligned}
(\Phi'^{\mu}_{V2} \otimes \mathbf{UU}) &= \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \left\{ 4\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\} \\
&\times \quad \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3)\left[\tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} - \tilde{\mathbf{U}}_{13}\tilde{\mathbf{U}}_{32}\right]\Phi_0^{\mu}(\vec{p}_1, \vec{p}_2)
\end{aligned}
$$

Rq :

- $\Phi_0(\vec{p}_1, \vec{p}_2)$ only depends on one of the t-channel momenta.
- The double-dipole operators cancels when $\vec{z}_3 = \vec{z}_1$ or $\vec{z}_3 = \vec{z}_2$.

This permits one to show that the convolution cancels the remaining $\frac{1}{\varepsilon}$ divergence.

Then $\tilde{\mathbf{U}}^{\alpha}_{12}\Phi_0 + \Phi_{V2}$ is finite

Divergences

- UV divergence $\vec{p}_g^{\,2} \to +\infty$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$$

- Soft divergence $p_g \to 0$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

$$\Phi_{R1}\Phi_{R1}^*$$

- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+}p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+}p_{\bar{q}}$, $p_g^+ \to 0$

$$\Phi_{R1}\Phi_{R1}^*$$

- Rapidity divergence

Contructing a finite cross section

From partons to jets

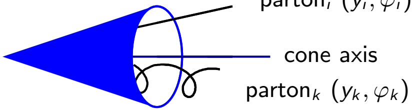
Soft and collinear divergence

Jet cone algorithm

We define a cone width for each pair of particles with momenta p_i and p_k, rapidity difference ΔY_{ik} and relative azimuthal angle $\Delta\phi_{ik}$

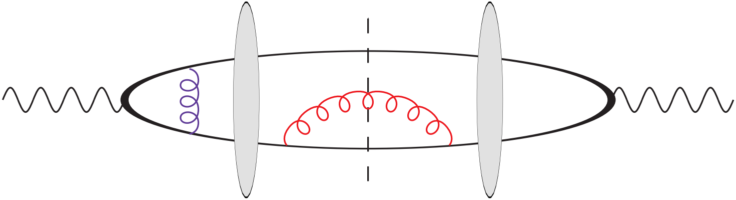
$$(\Delta Y_{ik})^2 + (\Delta\varphi_{ik})^2 = R_{ik}^2$$

If $R_{ik}^2 < R^2$, then the two particles together define a single jet of momentum $p_i + p_k$.



Applying this in the small R^2 limit cancels our soft and collinear divergence.

Remaining divergence



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- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

$$\Phi_{R1}\Phi_{R1}^*$$

Remaining divergence

Soft real emission

$$(\Phi_{R1}\Phi_{R1}^*)_{soft} \propto (\Phi_0\Phi_0^*) \int_{\text{outside the cones}} \left| \frac{p_q^\mu}{(p_q.p_g)} - \frac{p_{\bar{q}}^\mu}{(p_{\bar{q}}.p_g)} \right|^2 \frac{dp_g^+}{p_g^+} \frac{d^d p_g}{(2\pi)^d}$$

Collinear real emission

$$(\Phi_{R1}\Phi_{R1}^*)_{col} \propto (\Phi_0\Phi_0^*)\,(\mathcal{N}_q + \mathcal{N}_{\bar{q}})$$

Where $\mathcal{N}$ is the number of jets in the quark or the antiquark

$$\mathcal{N}_k = \frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2-\frac{d}{2})} \int_{\alpha p_\gamma^+}^{p_{jet}^+} \frac{dp_g^+ dp_k^+}{2p_g^+ 2p_k^+} \int_{\text{in cone k}} \frac{d^d \vec{p}_g\, d^d \vec{p}_k}{(2\pi)^d\, \mu^{d-2}} \frac{\mathrm{Tr}\left(\hat{p}_k \gamma^\mu \hat{p}_{jet} \gamma^\nu\right) d_{\mu\nu}(p_g)}{2p_{jet}^+ \left(p_k^- + p_g^- - p_{jet}^-\right)^2}$$

Those two contributions cancel exactly the virtual divergences (both UV and soft)

Cancellation of divergences

Total divergence

$$(d\sigma_1)_{div} = \alpha_s \frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}} \left(\frac{N_c^2-1}{2N_c}\right) (S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2}) d\sigma_0$$

Virtual contribution

$$\begin{aligned} S_V &= \left[2\ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - 3\right]\left[\ln\left(\frac{x_j x_{\bar{j}} \mu^2}{(x_j \vec{p}_{\bar{j}} - x_{\bar{j}} \vec{p}_j)^2}\right) - \frac{1}{\epsilon}\right] \\ &+ 2i\pi \ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) + \ln^2\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - \frac{\pi^2}{3} + 6 \end{aligned}$$

Real contribution

$$\begin{aligned} S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} &= 2\left[\ln\left(\frac{(x_{\bar{j}} \vec{p}_j - x_j \vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2} \vec{p}_j^{\,2}}\right) \ln\left(\frac{4E^2}{x_{\bar{j}} x_j (p_\gamma^+)^2}\right) + 2I(R,E)\right. \\ &+ 2\ln\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right)\left(\frac{1}{\epsilon} - \ln\left(\frac{x_{\bar{j}} x_j \mu^2}{(x_{\bar{j}} \vec{p}_j - x_j \vec{p}_{\bar{j}})^2}\right)\right) - \ln^2\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right) \\ &+ \left.\frac{3}{2}\ln\left(\frac{16\mu^4}{R^4 \vec{p}_j^{\,2} \vec{p}_{\bar{j}}^{\,2}}\right) - \ln\left(\frac{x_j}{x_{\bar{j}}}\right) \ln\left(\frac{x_j \vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}} \vec{p}_j^{\,2}}\right) - \frac{3}{\epsilon} - \frac{2\pi^2}{3} + 7\right] \end{aligned}$$

Cancellation of divergences

Total divergence

$$
\begin{aligned}
div &= S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} \\
&= 4\left[\frac{1}{2}\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\left(\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right)+\frac{3}{2}\right)\right. \\
&+ \left. I(R,E) + \ln(8) - \frac{1}{2}\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right) + \frac{13-\pi^2}{2}\right]
\end{aligned}
$$

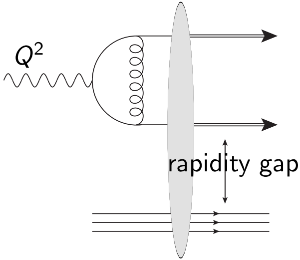
Overview

Conclusions about exclusive dijet production

- We computed the amplitude for the production of an open $q\bar{q}$ pair in DDIS
- Using this result, we constructed a finite expression for the cross section for the exclusive production of dijets
- The remaining part can be expressed as a finite integral, so it can be used straightforwardly for phenomenology
- Any model can be used for the matrix elements of the Wilson line operators (GBW, AAMQS if the target is a proton or an ion
- The target can also be perturbative, involving any impact factor...

Phenomenological applications : exclusive dijet production at NLO accuracy

- HERA data for exclusive dijet production in diffractive DIS can be fitted with our results
- We can also give predictions for the same process in a future electron-ion or electron-proton collider (EIC, LHeC...)
- For $Q^2 = 0$ we can give predictions for ultraperipheral collisions at the LHC



Amplitude for diffractive dijet production

Phenomenological applications : exclusive trijet production at LO accuracy

- HERA data for exclusive trijet production in diffractive DIS can be fitted with our results
- We can also give predictions for the same process in a future electron-ion collider
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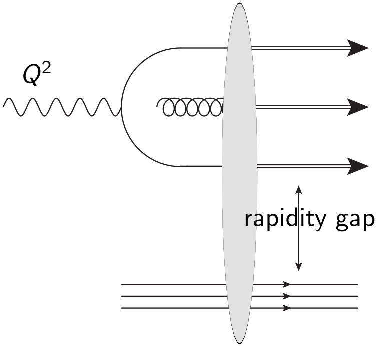
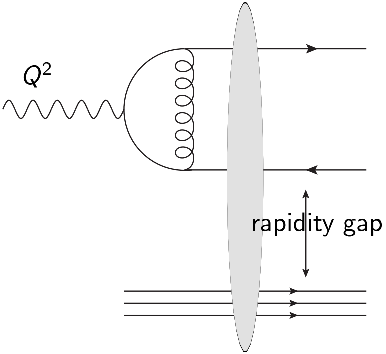


Amplitude for diffractive trijet production

General amplitude

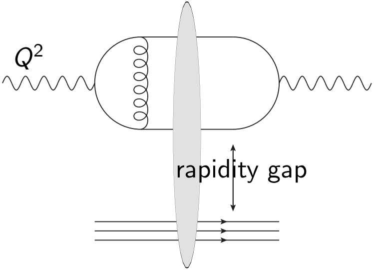
- Most general kinematics
- The hard scale can be Q^2, t, M_X^2 or m^2 in the (future) massive extension of our computation.
- The target can be either a proton or an ion, or another impact factor.
- One can study ultraperipheral collision by tagging the particle which emitted the photon, in the limit $Q^2 \to 0$.



The general amplitude

NLO DIS

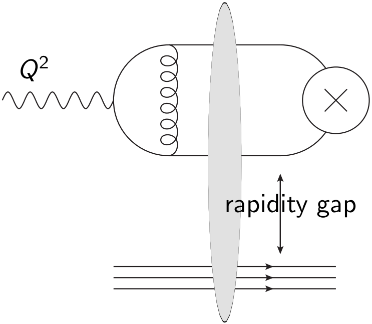
- One can adapt our general amplitude to obtain the NLO expression for (non diffractive) DIS
- Such a result would have to be compared with Balitsky and Chirilli's result, and with an ingoing study by Beuf.



NLO DIS cross section

Diffractive production of a ρ meson

- By forcing the quark and antiquark to be collinear and using the right Fierz projection, one can study ρ production
- Generalization of previous results of Ivanov, Kotsky, Papa to the non-forward case
- This would give a better understanding of the formal transition between BFKL and BK

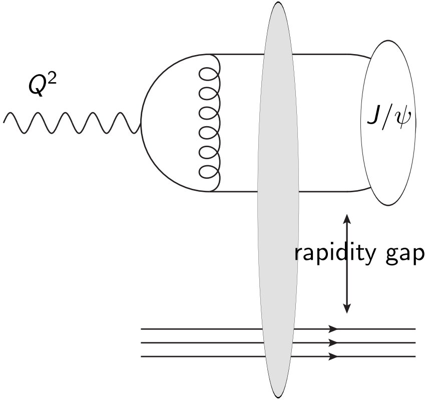


Amplitude for diffractive ρ production

$$\Phi_{BK} \otimes \mathcal{K}_{BK} \otimes \Phi'_{BK} = (\Phi_{BFKL} \otimes \mathcal{O})(\mathcal{O}^{-1} \otimes \mathcal{K}_{BFKL}\mathcal{O})(\mathcal{O}^{-1} \otimes \Phi'_{BFKL})$$

With an added mass

- Open charm production (straightforward)
- Heavy quarkonium production (in the Color Evaporation formalism)



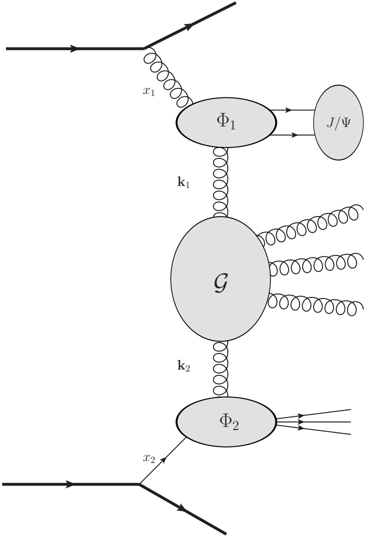
Amplitude for diffractive production of a charmonium

Thank you

Why $J\psi$?

- Numerous J/ψ mesons are produced at LHC
- J/ψ is easy to reconstruct experimentaly through its decay to $\mu^+\mu^-$ pairs
- The mechanism for the production of J/ψ mesons is still to be completely understood (see discussion later), although it was observed more than 40 years ago [E598 collab 1974], [SLAC-SP collab 1974]
- The vast majority of J/ψ theoretical predictions are done in the collinear factorization framework : would k_t factorization give something different?
- We will perform an MN-like analysis, considering a process with a rapidity gap which is large enough to use BFKL dynamics but small enough to be able to detect J/ψ mesons.

An MN-like analysis



$$\frac{\mathrm{d}\sigma}{\mathrm{d}|\mathbf{k}_{J/\psi}|\,\mathrm{d}|\mathbf{k}_{jet}|\,dy_{J/\psi}\,dy_{jet}} = \int d\phi_{J/\psi}\,d\phi_{jet} \int \mathrm{d}^2\mathbf{k}_1\,\mathrm{d}^2\mathbf{k}_2$$

$$\times \int dk_{gluon}\,\Phi_1(\mathbf{k}_{J/\psi}, x_{J/\psi}, -\mathbf{k}_1, k_{gluon})$$

$$\times\,\mathcal{G}(\mathbf{k}_1, \mathbf{k}_2, \hat{s})$$

$$\times\,\Phi_2(\mathbf{k}_{jet}, x_{jet}, \mathbf{k}_2)$$

First computation : NRQCD formalism

The NRQCD formalism

J/ψ production in NRQCD

We will first use the Non Relativistic QCD (NRQCD) formalism [Bodwin, Braaten, Lepage], [Cho, Leibovich].

Basically, one expands the onium wavefunction wrt the velocity of its constituents $v \sim \frac{1}{\log M}$:

$$|\Psi\rangle = O(1)\left|Q\bar{Q}[^3S_1^{(1)}]\right\rangle + O(v)\left|Q\bar{Q}[^3S_1^{(8)}]g\right\rangle + O(v^2)$$

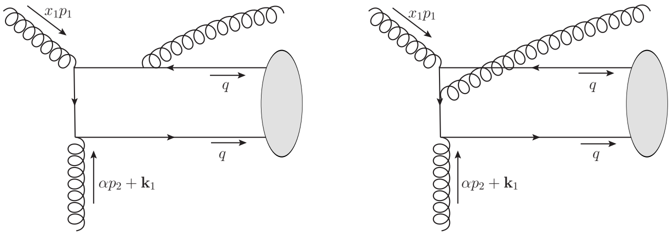
One assumes that all the non-perturbative physics is encoded in this wavefunction.

$\Rightarrow$ One computes the hard part using the usual Feynman diagram methods and convolute it with the wavefunction afterwards.

Charge parity conservation $\rightarrow$ Hard part : $c\bar{c}$ in a color singlet state $+ g$, $c\bar{c}$ in a color octet state.

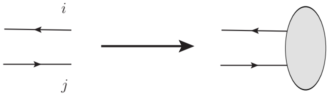
The relative importance of this additional color-octet contribution is still to be determined.

The J/ψ impact factor for color singlet production



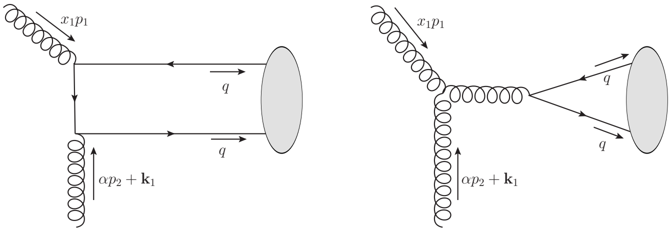
Two examples out of 6 diagrams

Quark-antiquark to J/ψ transition vertex from NRQCD expansion :



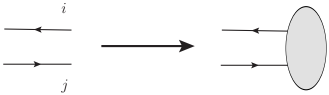
$$v^i_\alpha(q_2)\bar{u}^j_\beta(q_1) \rightarrow \frac{\delta^{ij}}{4N_c}\left(\frac{\langle\mathcal{O}_1\rangle_{J/\psi}}{m}\right)^{\frac{1}{2}}\left[\hat{\varepsilon}^*_{J/\psi}\left(\hat{k}_{J/\psi}+M\right)\right]_{\alpha,\,\beta} \qquad (1)$$

The J/ψ impact factor for color octet production



2 examples out of 3 diagrams

Quark-antiquark to J/ψ transition vertex from NRQCD expansion :



$$\left[v^i_\alpha(q_2)\bar{u}^j_\alpha(q_1)\right]^a \to \frac{t^a_{ij}}{4N_c}\left(\frac{\langle \mathcal{O}_8\rangle_V}{m}\right)^{\frac{1}{2}}\left[\hat{\varepsilon}^*_V\left(\hat{k}_{J/\psi}+M\right)\right]_{\alpha,\,\beta}$$

Second computation
The Color Evaporation Model

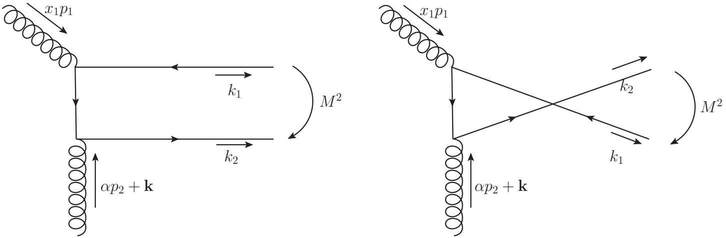
The Color Evaporation Model

Relies on the local duality hypothesis :
A heavy quark pair $Q\bar{Q}$ with an invariant mass below the threshold for the production of a pair of the lightest meson which contains Q will eventually produce a bound $Q\bar{Q}$ pair after a series of randomized soft interactions between its production and its confinement in $\frac{1}{9}$ cases, independently of its color and spin.
It is assumed that the repartition between all the possible charmonium states is universal.
Thus the procedure is the following :

- Compute all the Feynman diagrams for open $Q\bar{Q}$ production
- Sum over all spins and colors
- Integrate over the $Q\bar{Q}$ invariant mass

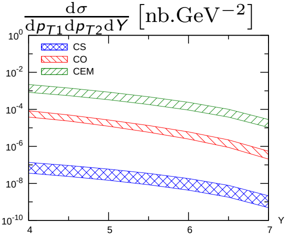
Then, neglecting the contributions from $Q\bar{Q}$ pairs with an invariant mass above the threshold, use :

$$\sigma_{J/\psi} = F_{J/\psi} \int_{4m_c^2}^{4m_D^2} dM^2 \frac{d\sigma_{c\bar{c}}}{dM^2}$$

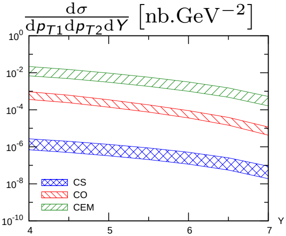


2 examples out of 3 diagrams to compute in the CEM

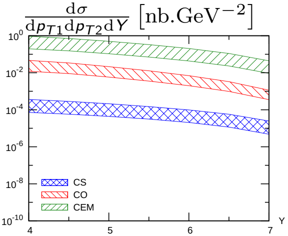
Numerical results



$p_{T1} = 30\,\mathrm{GeV},\ p_{T2} = 30\,\mathrm{GeV}$



$p_{T1} = 20\,\mathrm{GeV},\ p_{T2} = 20\,\mathrm{GeV}$



$p_{T1} = 10\,\mathrm{GeV},\ p_{T2} = 10\,\mathrm{GeV}$

Differential cross sections from both models

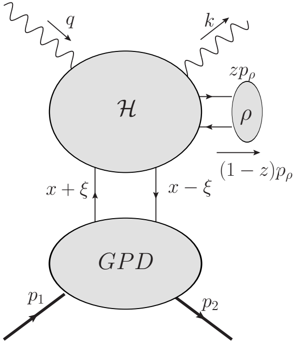
Summary

- The production of Mueller-Navelet jets was successfully described using the BFKL formalism
- We applied the same formalism for the production of a forward J/Ψ meson and a backward jet, using both the NRQCD formalism and the Color Evaporation Model

Final computation

$$\mathcal{A} \propto \int_{-1}^{1} dx \int_{0}^{1} dz \;\; T(x, \xi, z) \times H(x, \xi, t)\Phi_\rho(z) + \cdots$$

- One performs the z integration analytically using an asymptotic DA $\propto z(1-z)$
- One then plugs a GPD model into the formula and performs the integral wrt x numerically.



A model based on the Double Distribution ansatz

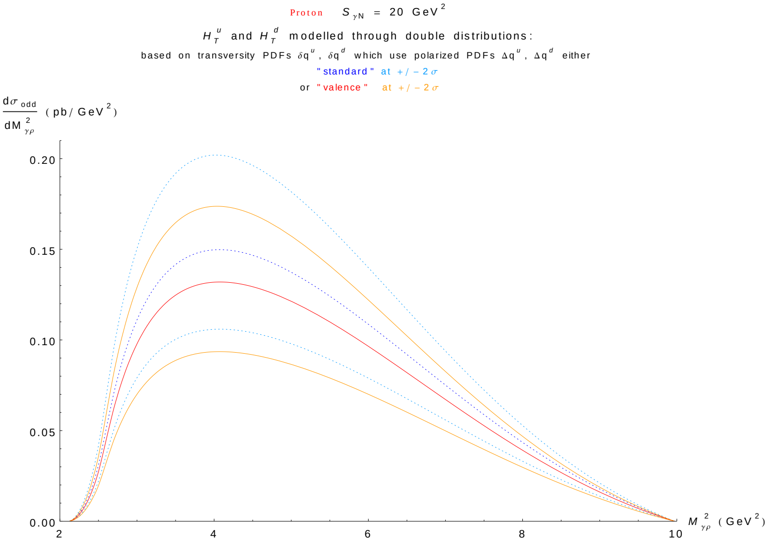
Realistic Parametrization of GPDs

- GPDs can be represented in terms of Double Distributions [Radyushkin]
  based on the Schwinger representation of a toy model for GPDs which has the structure of a triangle diagram in scalar ϕ^3 theory

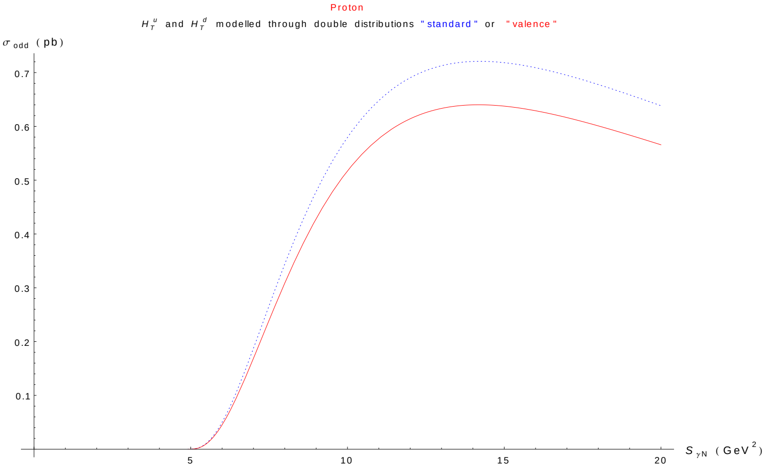
$$H^q(x, \xi, t = 0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\ \delta(\beta + \xi\alpha - x)\, f^q(\beta, \alpha)$$

- ansatz for these Double Distributions [Radyushkin]:
  - $f^q(\beta, \alpha) = \Pi(\beta, \alpha)\, q(\beta)$ in the chiral even case
  - $f_T^q(\beta, \alpha) = \Pi(\beta, \alpha)\, \Delta_T q(\beta)$ in the chiral odd case
  - $q(x)$: PDF (polarized or unpolarized) [MSTW, GRV...]
  - $\Delta_T q(x)$: Chiral odd PDF [Anselmino *et al.*]
  - $\Pi(\beta, \alpha) = \frac{3}{4} \frac{(1-\beta)^2 - \alpha^2}{(1-\beta)^3}$: profile function

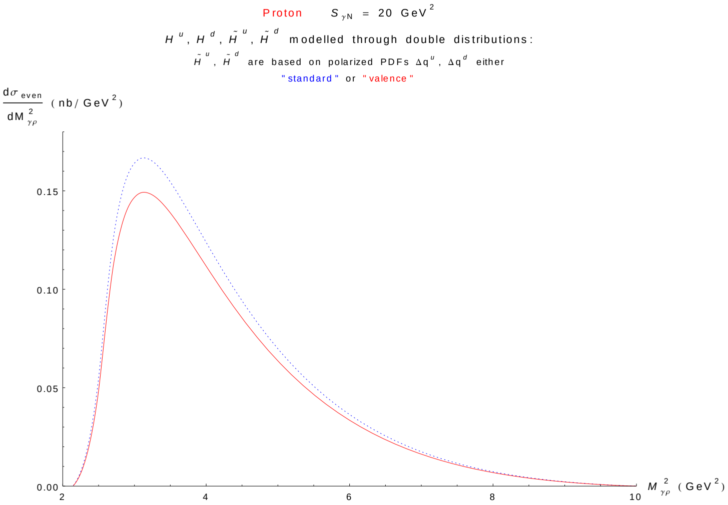
Chiral odd cross section



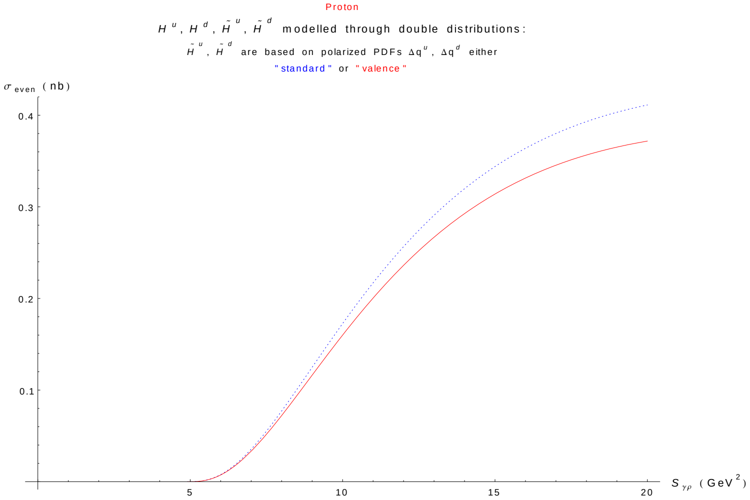
Chiral odd cross section for γp



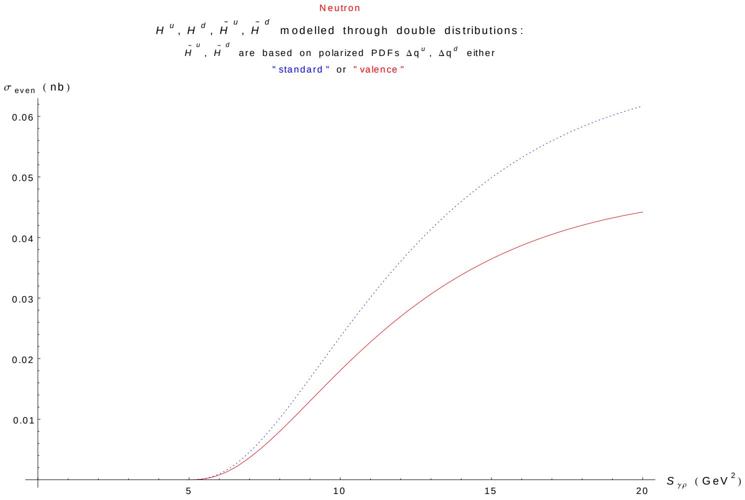
Chiral even cross section



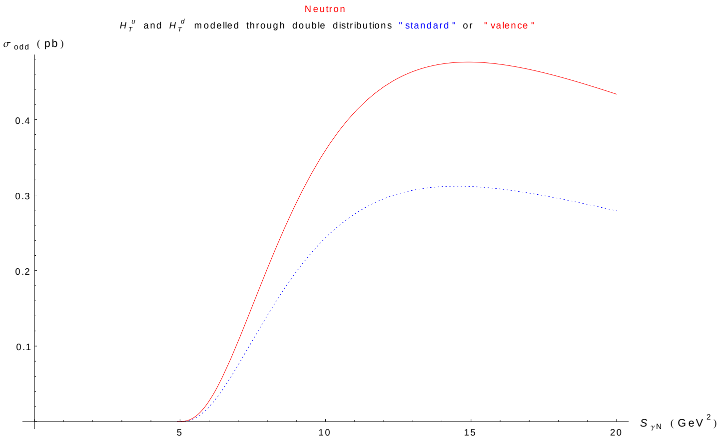
Chiral even cross section



Chiral even cross section for γn



Chiral odd cross section for γn



Counting rates for 100 days

CLAS

Chiral even case : between $4.9 * 10^6$ and $5.4 * 10^6$ events
Chiral odd case : between $1.3 * 10^3$ and $2.7 * 10^3$ events

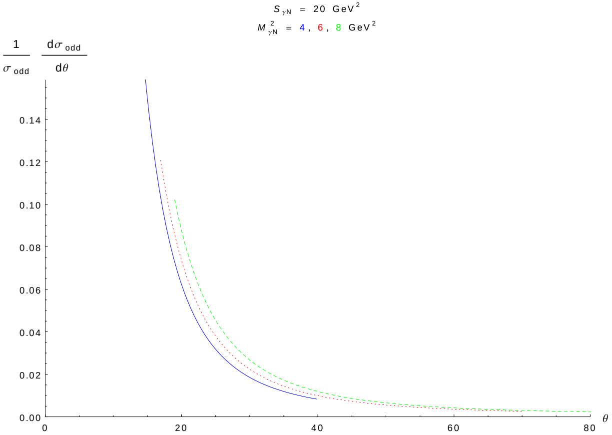
GLUEX

Chiral even case : between $1.6 * 10^5$ and $1.8 * 10^5$ events
Chiral odd case : between 45 and 90 events

CLAS FT

Chiral even case : between $1.6 * 10^5$ and $1.8 * 10^5$ events
Chiral odd case : between 45 and 90 events

Effects of an experimental angular restriction



A $\theta_{\gamma/beam} < 35^\circ$ restriction implies a 33% loss in the chiral even cross section

and a 18% loss in the chiral odd cross section

Conclusion

- This mechanism will give us access to transversity GPDs but also to the usual GPDs by analogy with Timelike Compton Scattering, the $\gamma\rho$ pair playing the role of the γ^*.
- Our result will also be applied to electroproduction ($Q^2 \neq 0$) after adding Bethe-Heitler contributions and interferences.
- Possible measurement in JLAB and in COMPASS