Light-front quantization methods: from QED to QCD

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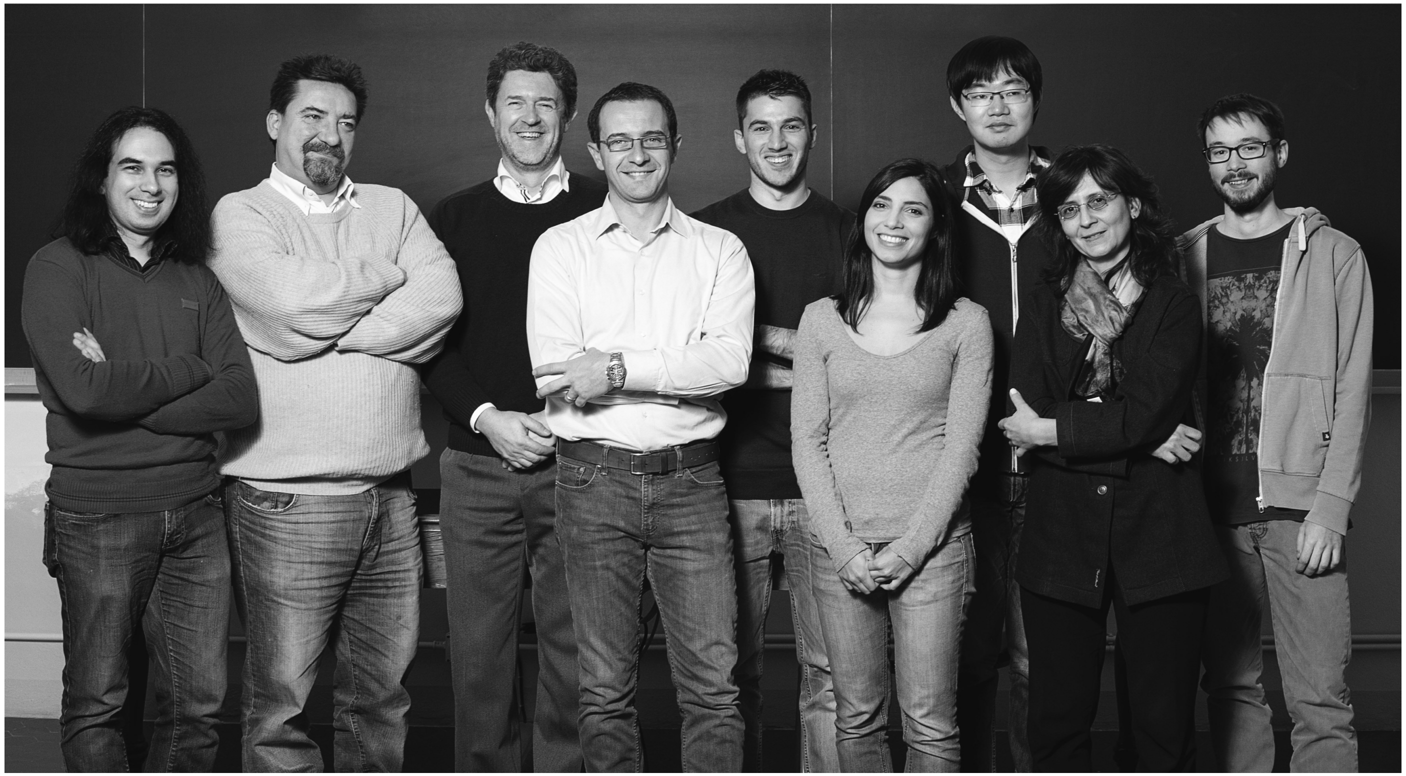
Has | QCD
HADRONICS STRUCTURE AND QCD



http://www.hadronicphysics.it/hasqcd/

Outline

- Light-front quantization in a nutshell
- Applications in QED
    - Wigner distributions and TMDs
    - Photon propagator
- Hint on: QCD energy-momentum tensor
- Conclusions

At École polytechnique!

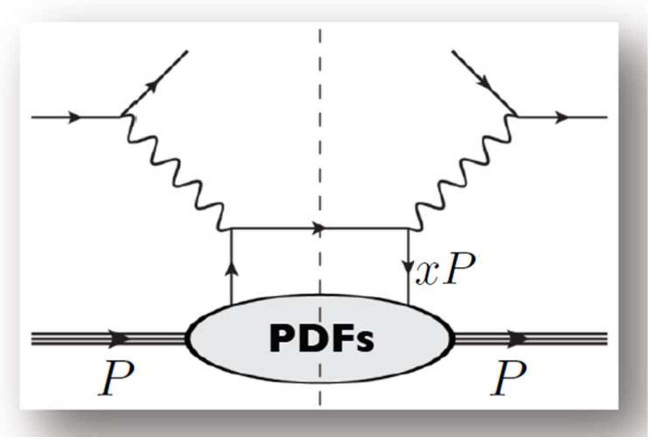
Light-front quantization

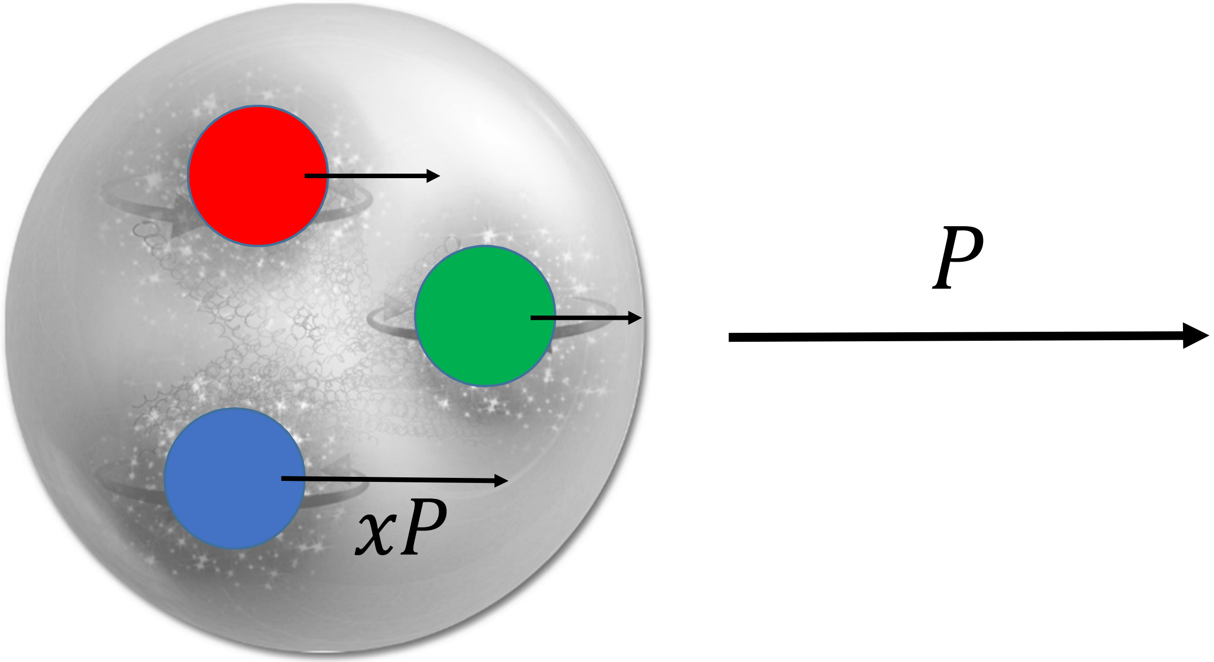
Motivation

GOAL: unreveale the internal structure of nucleons

Motivation

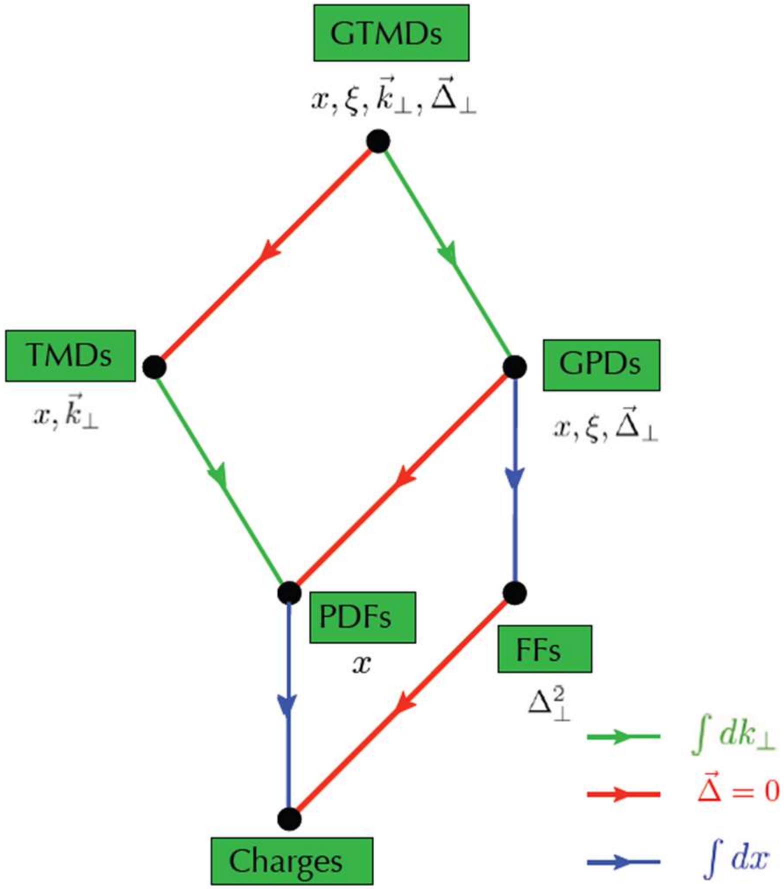
Deep Inealstic Scattering





Parton Distribution Functions (x)

Landscape of parton distributions



Lorcé, Pasquini, Vanderhaeghen (2011)

Infinite momentum frame

Canonical frame

$$x^\mu = (t, x, y, z)$$

z

Infinite momentum frame

Canonical frame

$$x^\mu = (t, x, y, z)$$

Infinite Momentum frame

$$x^\mu = \left(x^+, x^-, \mathbf{x}_\perp\right)$$

$P \to \infty$

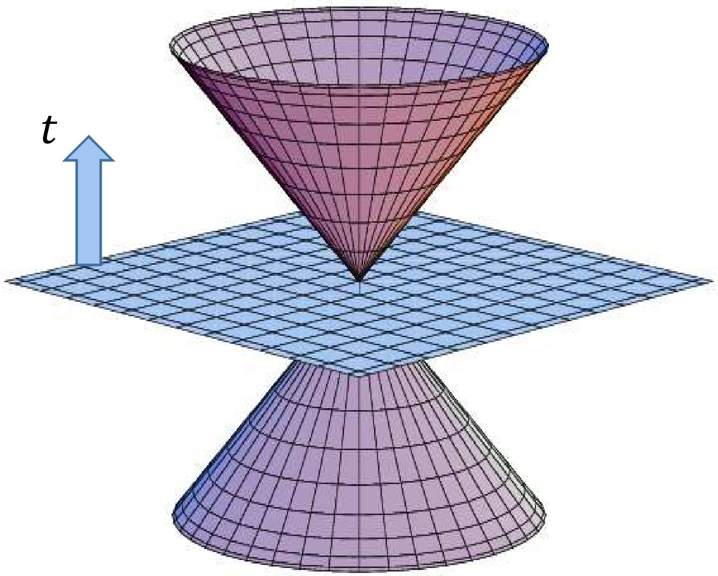
Light-cone coordinates $\quad \mathbf{x}_\perp = (x, y) \qquad x^\pm = \frac{1}{\sqrt{2}}(z \pm t)$

Z

Forms of relativistic dynamics

Instant form

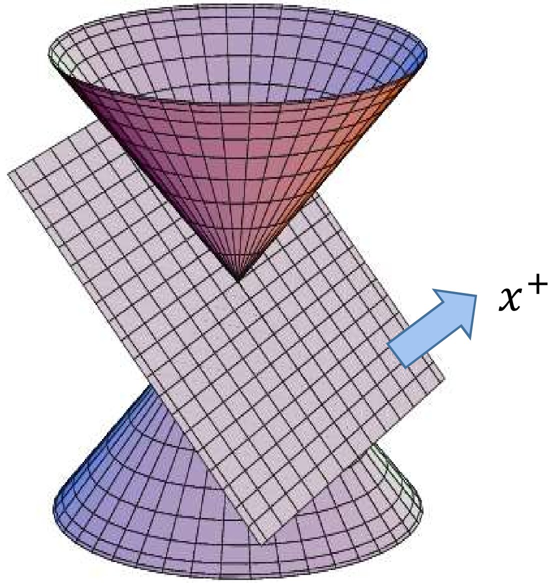
$$x^\mu = (t, x, y, z)$$



$$i\frac{\partial}{\partial t}\Psi(x) = H\Psi(x)$$

Light-front Form

$$x^\mu = \left(x^+, x^-, \mathbf{x}_\perp\right)$$



$$i\frac{\partial}{\partial x^+}\Psi(x) = H\Psi(x)$$

Forms of dynamics

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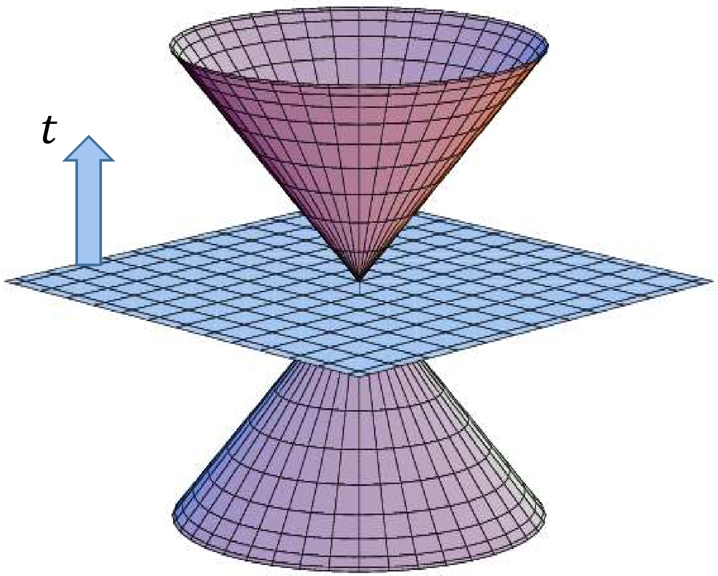

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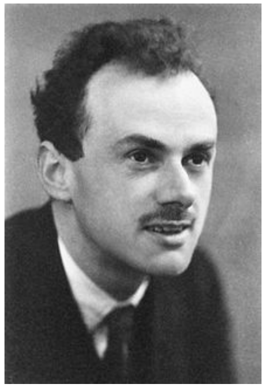
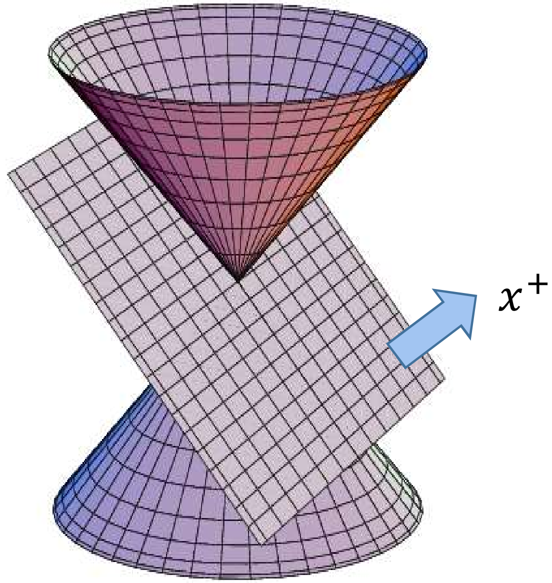


Dirac (1949)

Light-front Form

$$x^\mu = \left(x^+, x^-, \mathbf{x}_\perp\right)$$

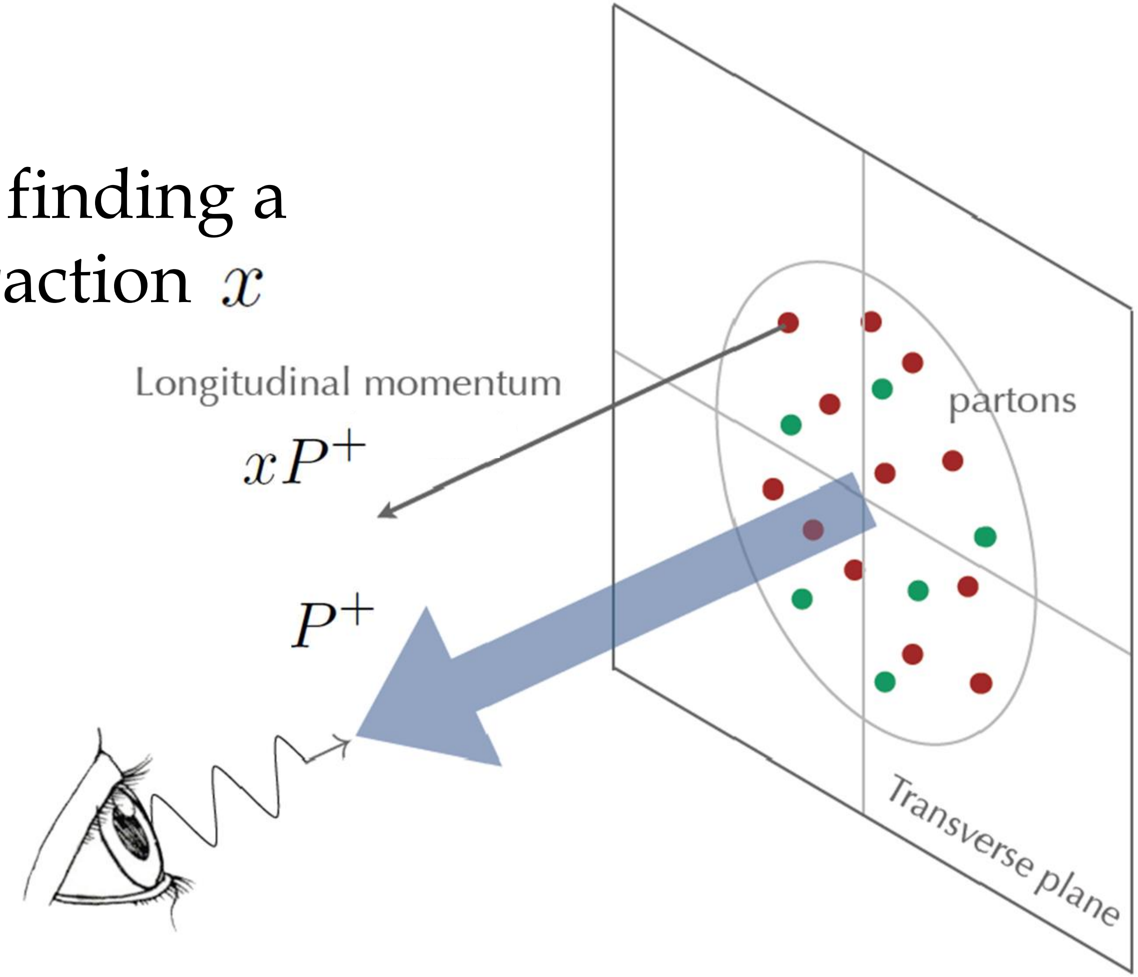


$$i\frac{\partial}{\partial x^+}\Psi(x) = H\Psi(x)$$

Infinite-momentum frame picture

PDFs (x)

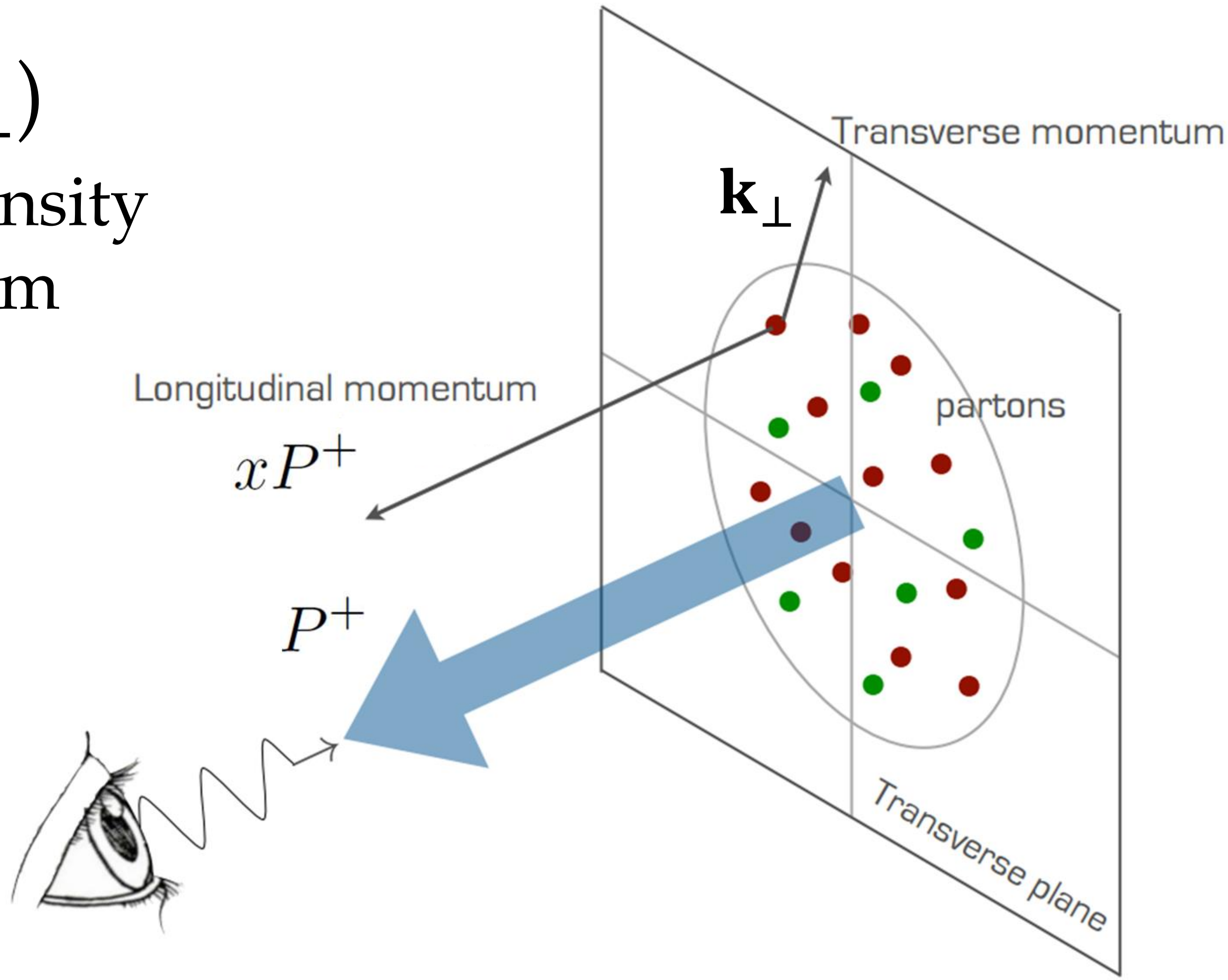
Probability of finding a parton with fraction x



Infinite-momentum frame picture

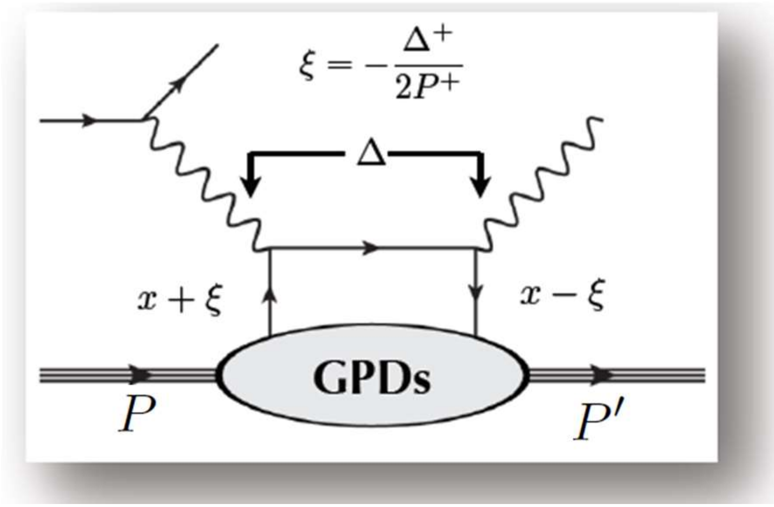
TMDs $(x, \mathbf{k}_\perp)$

Probability density in 3-momentum space



Impact parameter space

Deeply Virtual Compton Scattering



Generalized Parton Distributions $(x, \xi, \mathbf{\Delta}_\perp)$

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik^+z^-} \left\langle P'^+, -\frac{\mathbf{\Delta}_\perp}{2}, \Lambda' \middle| \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{W} \psi\left(\frac{z}{2}\right) \middle| P^+, \frac{\mathbf{\Delta}_\perp}{2}, \Lambda \right\rangle \Bigg|_{(z^+, \mathbf{z}_\perp)=(0, \mathbf{0}_\perp)}$$

Impact parameter space

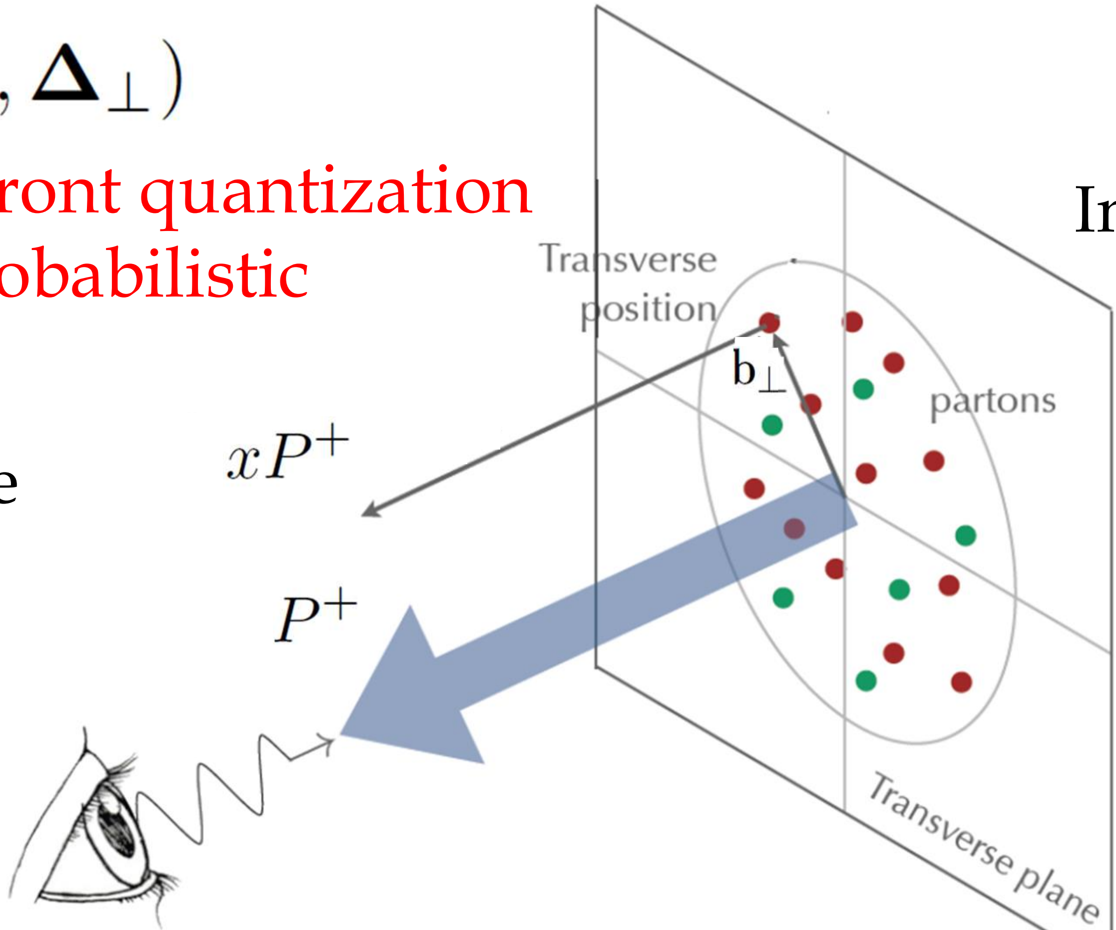
GPDs $(x, \mathbf{\Delta}_\perp)$

Only in light-front quantization they have a probabilistic interpretation!

Drell-Yan frame

$$\Delta^+ = 0$$

Burkardt (2003)



$$\mathbf{\Delta}_\perp \overset{\text{FT}}{\longleftrightarrow} \mathbf{b}_\perp$$

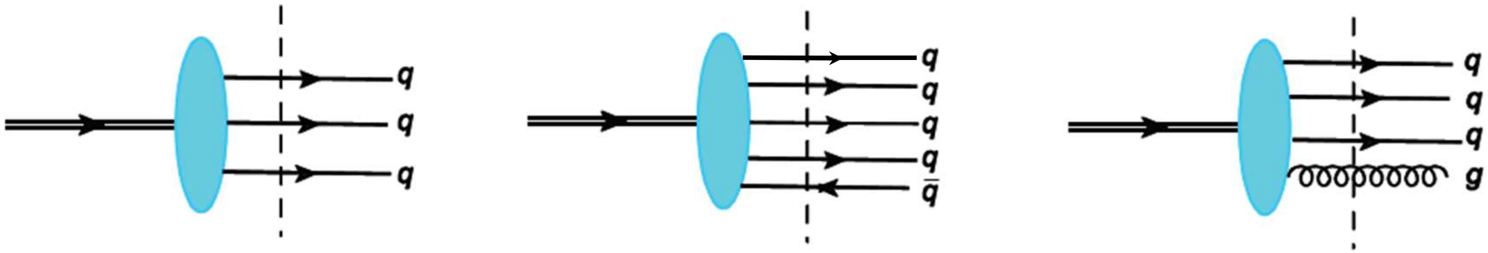
Impact parameter

Light-Front Wave Functions (LFWF)

Fock state expansion of Nucleon state

$$|P\rangle = \sum_{n,[\omega]} \int d[\omega] \boxed{|n[\omega]\rangle} \langle n[\omega]|P\rangle$$

Fock states $\quad |0\rangle \quad |qqq\rangle \quad |3qq\bar{q}\rangle \quad |3qg\rangle \quad \ldots$



Light-Front Wave Functions (LFWF)

$$|P\rangle = \sum_{n,[\omega]} \int d[\omega]\ |n[\omega]\rangle \boxed{\langle n[\omega]|P\rangle}$$

LFWFs $\Psi_n(\omega)$

- Probability of finding n partons in the nucleon $= |\Psi_n|^2$
- Eigeinstates of momentum, parton light-front helicity and total orbital angular momentum
- Model dependent

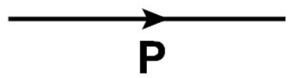
Applications in QED

Based on:

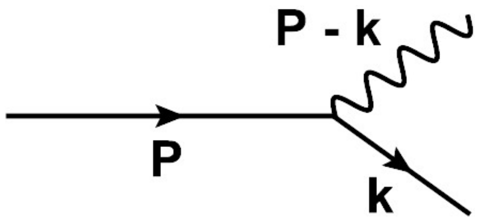
Bacchetta, LM, Pasquini (LM master thesis, 2014)

Bacchetta, LM, Pasquini (2015)

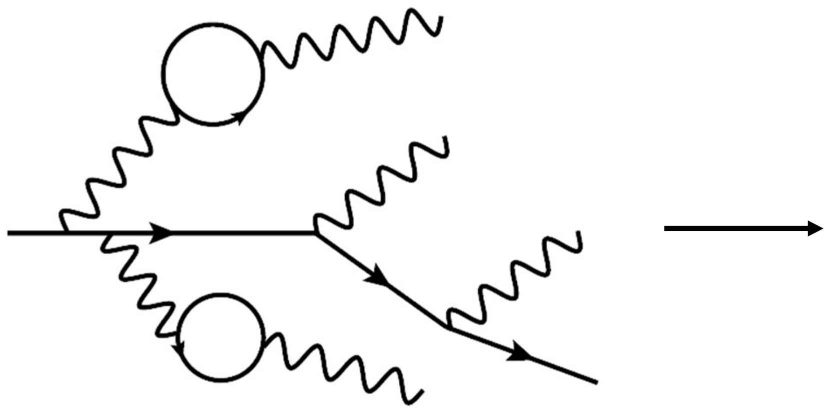
3D Electron



3D Electron



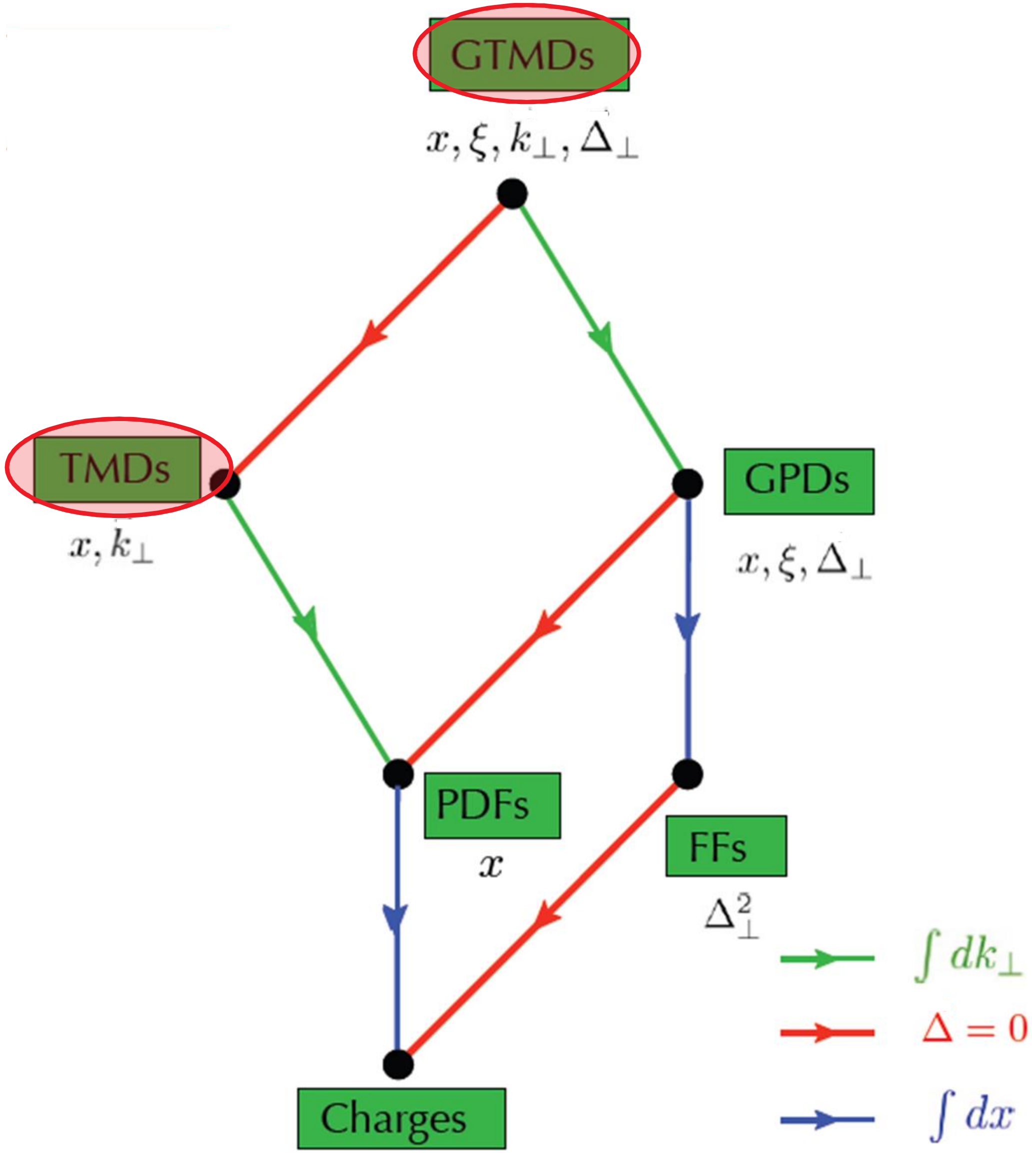
3D Electron



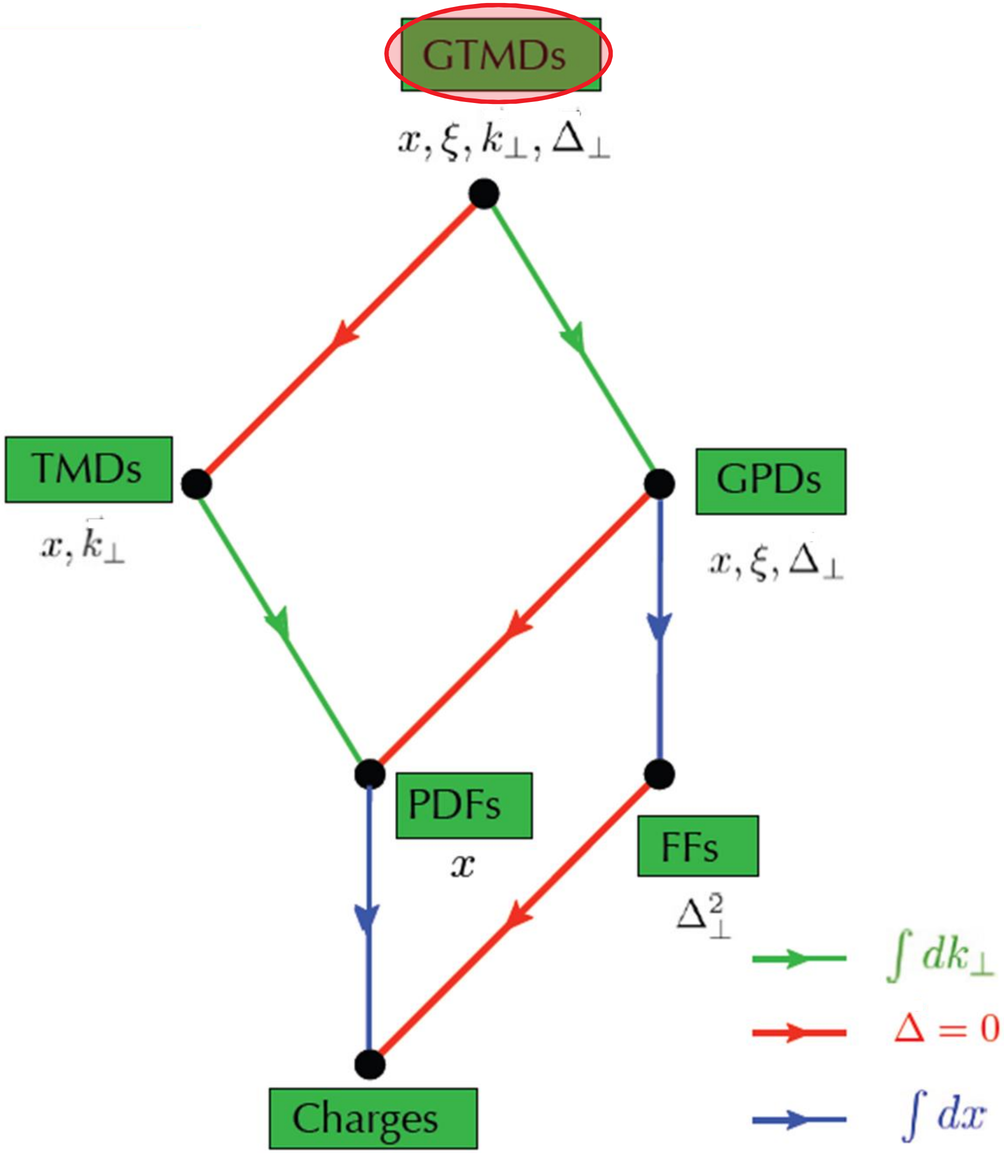
[Brodsky, Hwang, Ma, Schmidth (2000)](#)

[Hoyer, Kurki (2009)](#)

[Miller (2014)](#)



GTMDs
$x, \xi, k_\perp, \Delta_\perp$
TMDs
$x, k_\perp$
GPDs
$x, \xi, \Delta_\perp$
PDFs
x
FFs
$\Delta_\perp^2$
Charges
$\int dk_\perp$
$\Delta = 0$
$\int dx$


GTMDs
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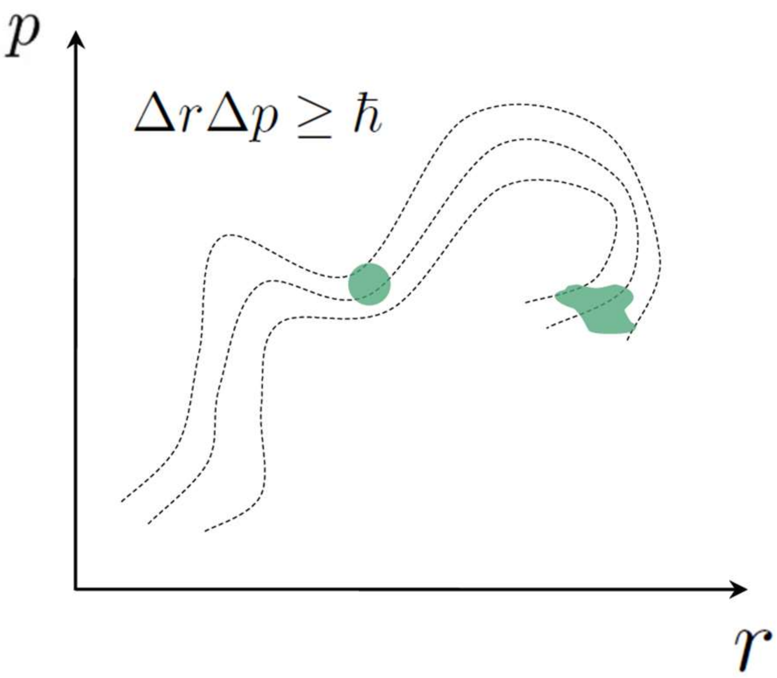
Wigner distributions

Phase space distributions



$$\rho_W(r,p) = \int \frac{dz}{2\pi}\, e^{-ikz}\, \psi^*\left(r - \frac{z}{2}\right)\psi\left(r + \frac{z}{2}\right)$$

$$\xrightarrow{\hbar \to 0} \quad f(r,p)$$

Classical distribution

[Wigner (1932)]

Quasi-probabilistic interpretation

Wigner distributions on the light front

Generalized Transverse-Momentum Dependent Parton Distributions $(x, \mathbf{k}_\perp, \xi, \mathbf{\Delta}_\perp)$

$$\frac{1}{2}\int \frac{dz^- d^2\mathbf{z}_\perp}{(2\pi)^3} e^{ikz} \left\langle P'^+, -\frac{\mathbf{\Delta}_\perp}{2}, \Lambda' \left| \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{W} \psi\left(\frac{z}{2}\right) \right| P^+, \frac{\mathbf{\Delta}_\perp}{2}, \Lambda \right\rangle \Bigg|_{z^+=0}$$

Wigner distributions on the light front

In the Drell-Yan frame $\Delta^+ = 0$

$$\rho^{[\Gamma]}_{\Lambda,\Lambda'}(\mathbf{b}_\perp, x, \mathbf{k}_\perp) = \frac{1}{2}\int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp} \boxed{\left\langle P^+, \frac{\boldsymbol{\Delta}_\perp}{2}, \Lambda' \middle| \hat{W}^{[\Gamma]}(\mathbf{0}, x, \mathbf{k}_\perp) \middle| P^+, -\frac{\boldsymbol{\Delta}_\perp}{2}, \Lambda \right\rangle}$$

$=$ 2-D Fourier transform of $\boxed{\text{GTMD}\ (x, \mathbf{k}_\perp, \xi = 0, \boldsymbol{\Delta}_\perp)}$

Lorcè, Pasquini (2011)

Lorcè, Pasquini, Xiong, Yuan (2012)

Wigner distributions on the light front

In the Drell-Yan frame $\Delta^+ = 0$

$$\rho^{[\Gamma]}_{\Lambda,\Lambda'}(\mathbf{b}_\perp, x, \mathbf{k}_\perp) = \frac{1}{2}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\, e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \boxed{\left\langle P^+, \frac{\mathbf{\Delta}_\perp}{2}, \Lambda' \middle| \hat{W}^{[\Gamma]}(\mathbf{0}, x, \mathbf{k}_\perp) \middle| P^+, -\frac{\mathbf{\Delta}_\perp}{2}, \Lambda \right\rangle}$$

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2+3 dimensional

Wigner distributions on the light front

In the Drell-Yan frame $\Delta^+ = 0$

$$\rho^{[\Gamma]}_{\Lambda,\Lambda'}(\mathbf{b}_\perp, x, \mathbf{k}_\perp) = \frac{1}{2}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\, e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \boxed{\left\langle P^+, \frac{\mathbf{\Delta}_\perp}{2}, \Lambda' \middle| \hat{W}^{[\Gamma]}(\mathbf{0}, x, \mathbf{k}_\perp) \middle| P^+, -\frac{\mathbf{\Delta}_\perp}{2}, \Lambda \right\rangle}$$

$=$ 2-D Fourier transform of $\boxed{\text{GTMD}\,(x, \mathbf{k}_\perp, \xi = 0, \mathbf{\Delta}_\perp)}$

Lorcè, Pasquini (2011)

Lorcè, Pasquini, Xiong, Yuan (2012)

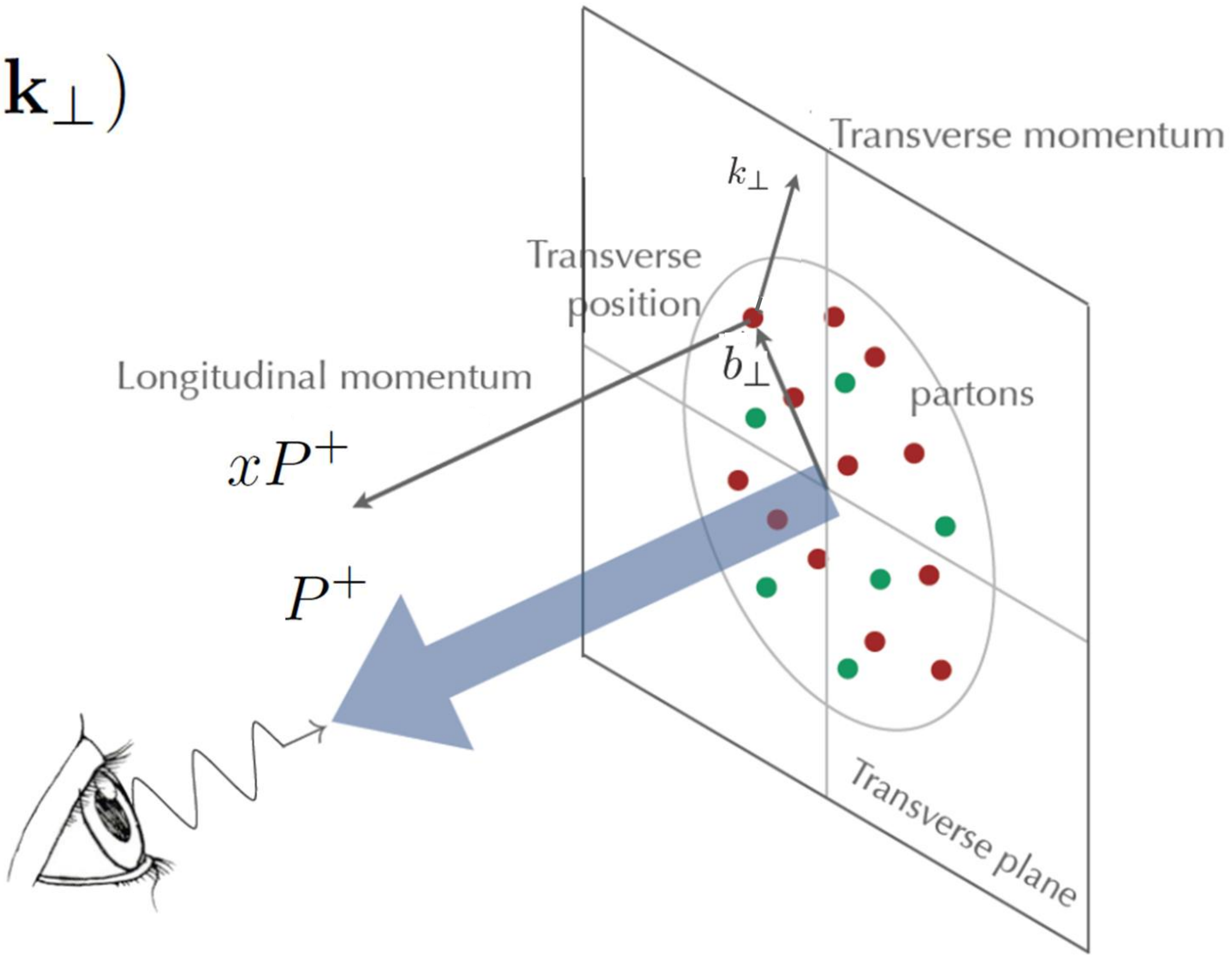
Semi-classical probabilistic interpretation

Infinite-momentum frame picture

GTMDs $(x, \mathbf{\Delta}_\perp, \mathbf{k}_\perp)$

$$\mathbf{\Delta}_\perp \overset{\text{FT}}{\longleftrightarrow} \mathbf{b}_\perp$$

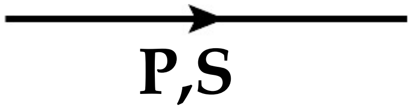
Impact parameter



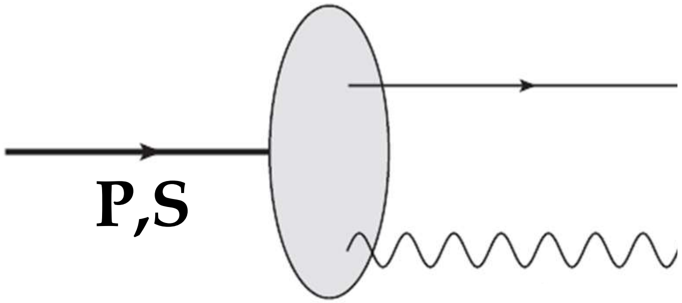
Fock state expansion of the dressed electron

$$|e_D\rangle = |e_1\rangle + |e_2\rangle + \ldots$$

$|e_1\rangle$



$|e_2\rangle$



$$= \sum_{[\omega]} \int d[\omega] \frac{\langle e\gamma|V|e_1\rangle}{P^- - p_e^- - p_\gamma^-} |e\gamma\rangle$$

$$= \int \frac{dx\, d^2\mathbf{k}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \sum_{s,\lambda} \Psi^S_{s,\lambda}(x, \mathbf{k}_\perp)|e\gamma\rangle$$

Exact in QED!

LFWF overlap representation of GTMDs

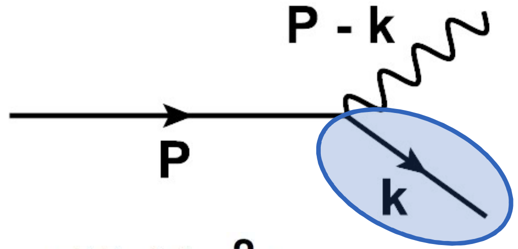
$$W_{S,S'}^{[\gamma^+]} = \int \frac{dz^- d^2\mathbf{k}_\perp}{2(2\pi)^3} e^{ikz} \langle e_D; \mathbf{P}', S' | \bar{\psi}\left(-\frac{z}{2}\right) \gamma^+ \psi\left(\frac{z}{2}\right) | e_D; \mathbf{P}, S \rangle$$

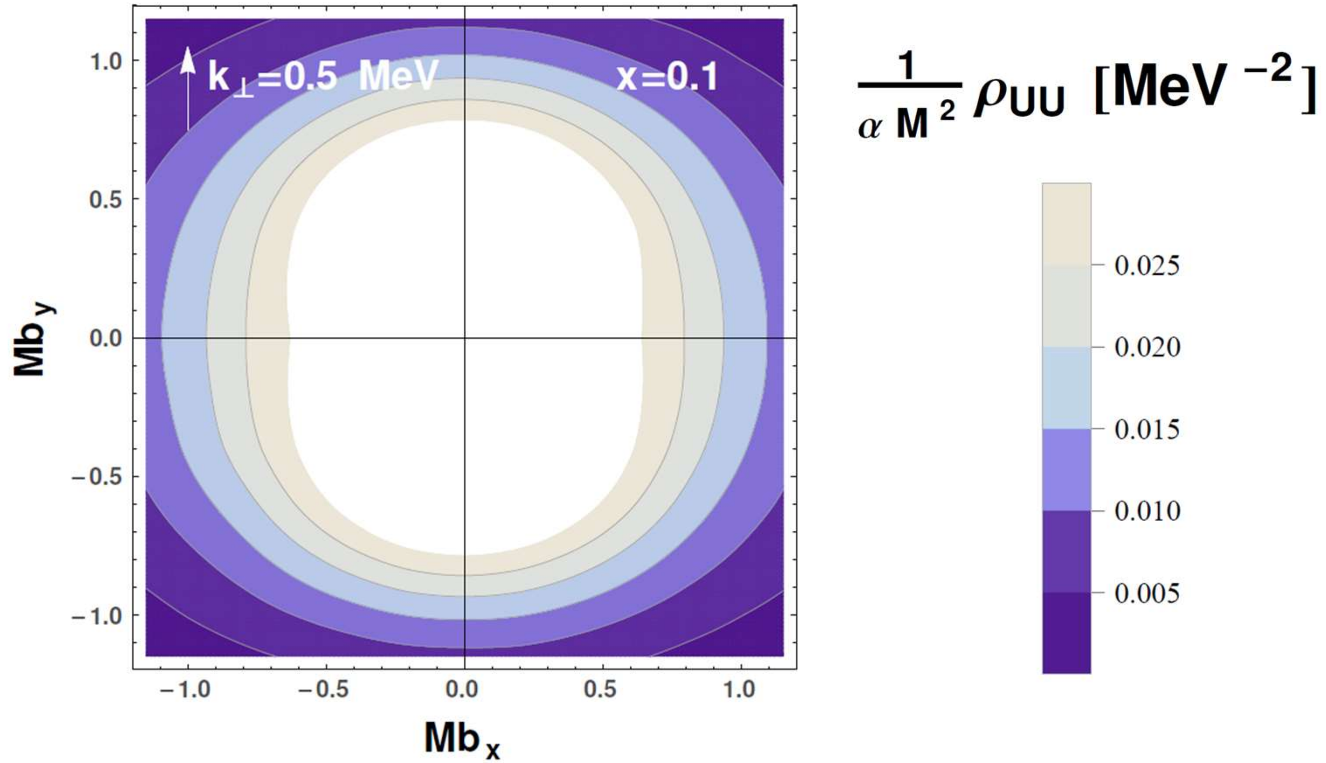
$$|e_D\rangle = |e_1\rangle + |e_2\rangle$$

$$W_{S,S'}^{[\gamma^+]} = \frac{1}{2(2\pi)^3} \sum_{s,\lambda} \Psi_{s,\lambda}^{S'*}\left(x, \mathbf{k}_\perp + (1-x)\frac{\mathbf{\Delta}_\perp}{2}\right) \Psi_{s,\lambda}^{S}\left(x, \mathbf{k}_\perp - (1-x)\frac{\mathbf{\Delta}_\perp}{2}\right)$$

Unpol. electron in unpol. dressed electron

$$\rho_{UU} := \frac{1}{2}\left[\rho_{\uparrow\uparrow}^{[\gamma^+]} + \rho_{\downarrow\downarrow}^{[\gamma^+]}\right] = \mathrm{FT}\left[F_{11}\right]$$





Bacchetta, LM, Pasquini (LM master thesis, 2014)

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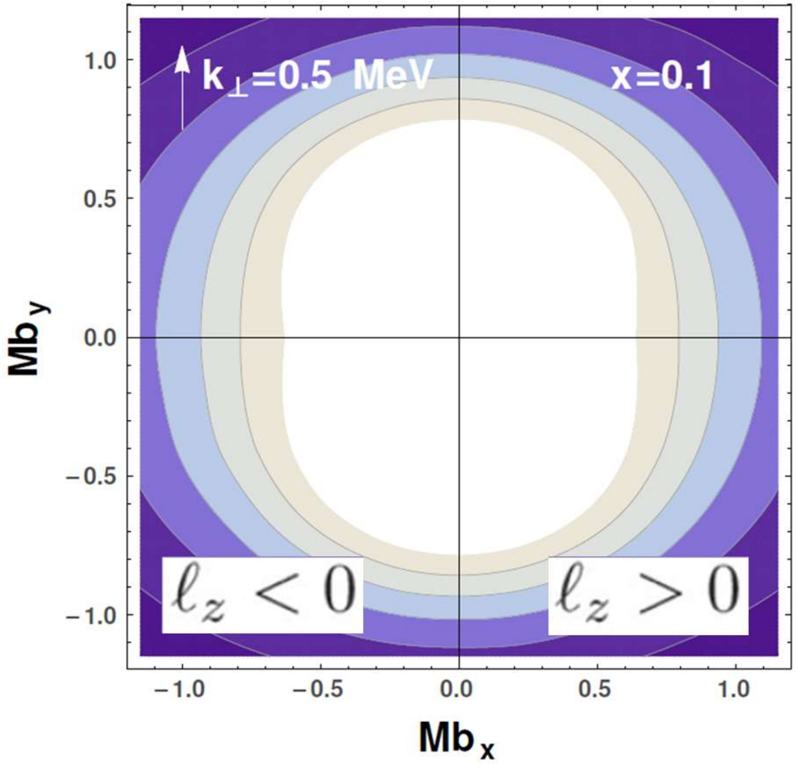


$\mathbf{b}_\perp \perp \mathbf{k}_\perp$ favored

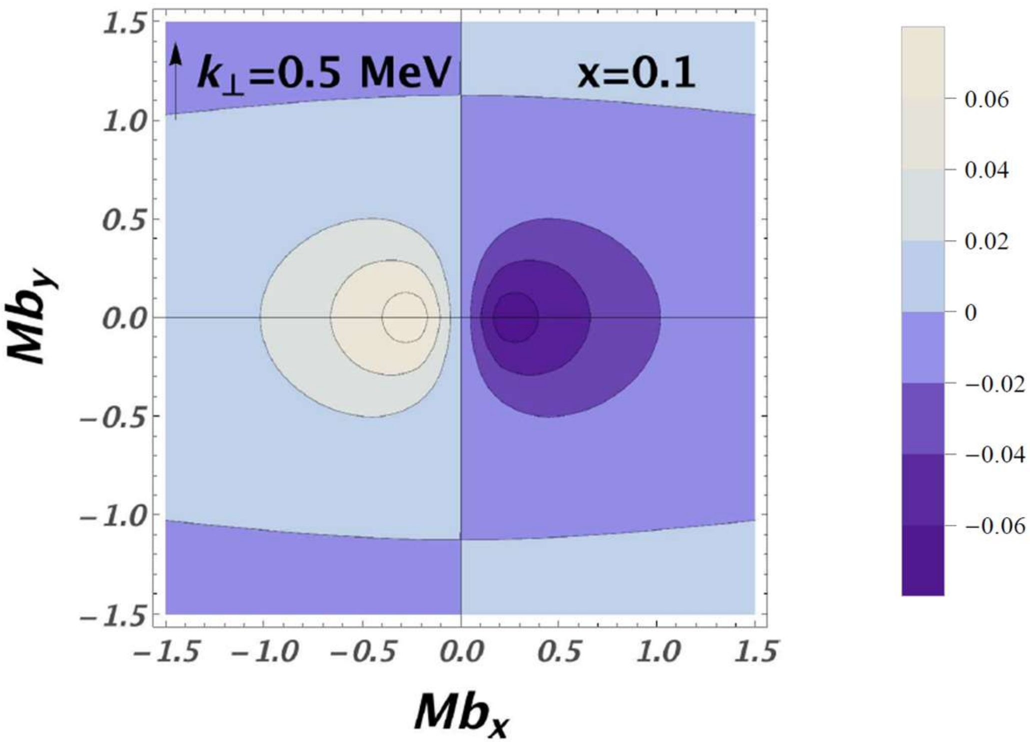
$\mathbf{b}_\perp \parallel \mathbf{k}_\perp$ unfavored

$$l_z^U = \int dx\, d^2\mathbf{b}_\perp d^2\mathbf{k}_\perp \left(\mathbf{b}_\perp \times \mathbf{k}_\perp\right)_z \rho_{UU} = 0$$

Bacchetta, LM, Pasquini (LM master thesis, 2014)

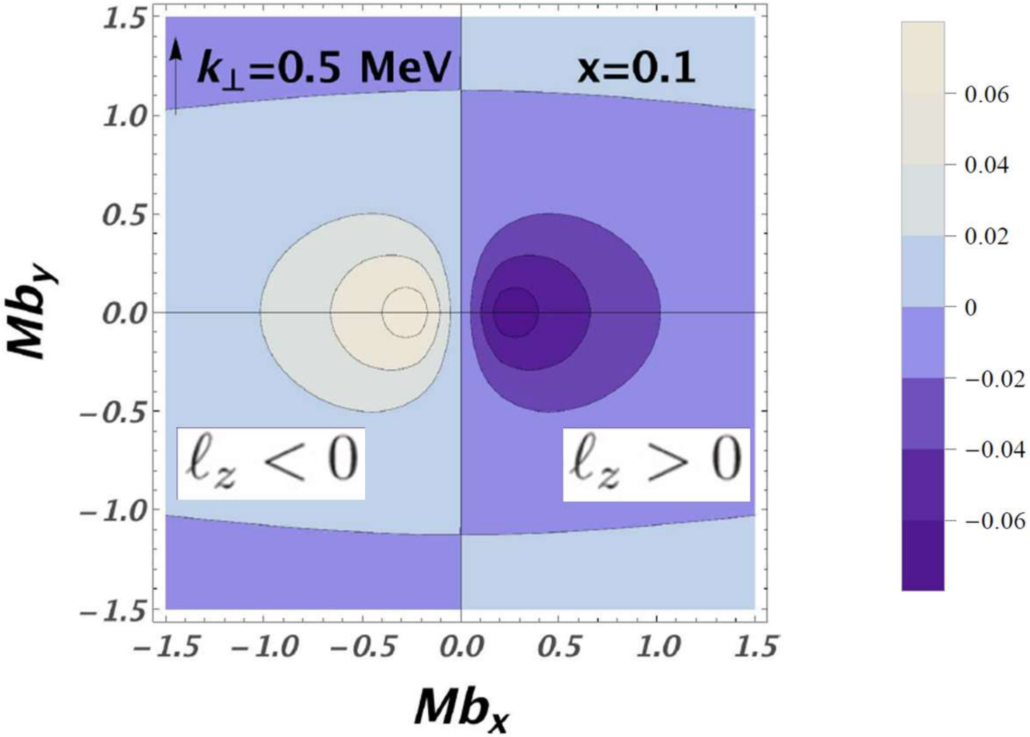
Unpol. electron in long. pol. dressed electron

$$\rho_{LU} := \frac{1}{2}\left[\rho_{\uparrow\uparrow}^{[\gamma^+]} - \rho_{\downarrow\downarrow}^{[\gamma^+]}\right] = -\frac{1}{M^2}\left(\mathbf{k}_\perp \times \frac{\partial}{\partial \mathbf{b}}\right)_z \mathrm{FT}\left[F_{1,4}\right]$$



Unpol. electron in long. pol. dressed electron

$$\rho_{LU} := \frac{1}{2}\left[\rho^{[\gamma^+]}_{\uparrow\uparrow} - \rho^{[\gamma^+]}_{\downarrow\downarrow}\right] = -\frac{1}{M^2}\left(\mathbf{k}_\perp \times \frac{\partial}{\partial \mathbf{b}}\right)_z \mathrm{FT}\left[F_{1,4}\right]$$

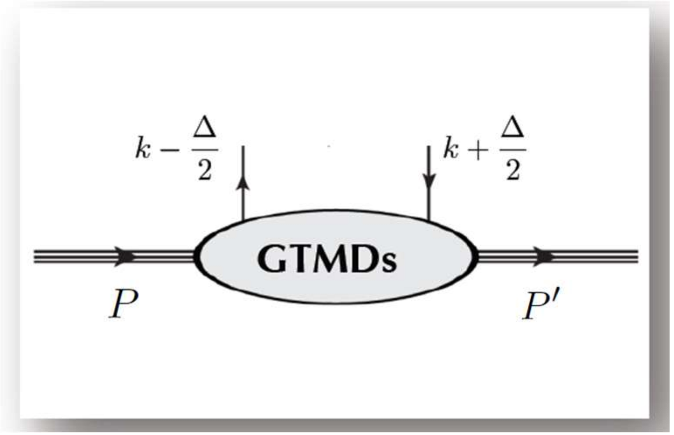


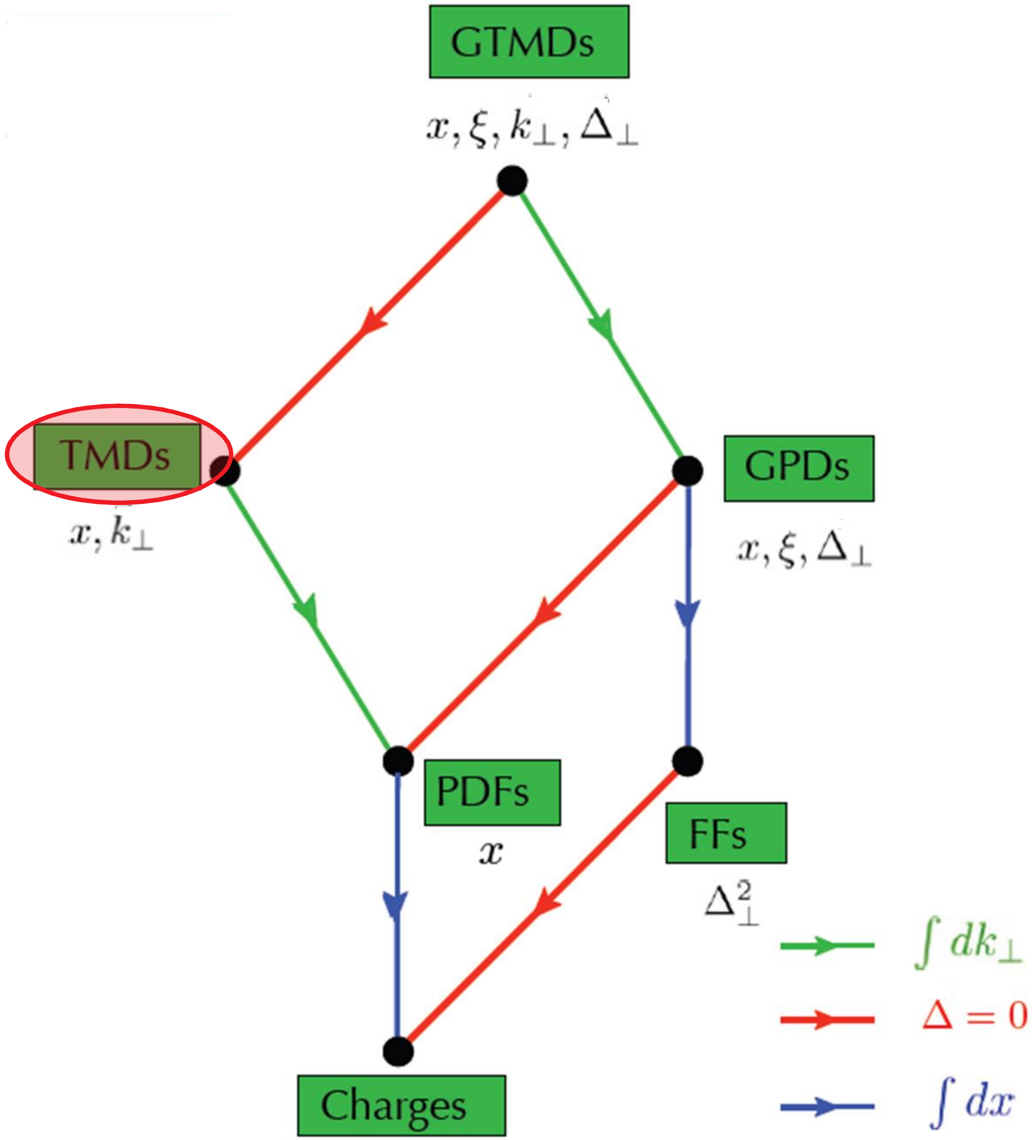
$$l_z^L = \int dx\, d^2\mathbf{b}_\perp d^2\mathbf{k}_\perp \left(\mathbf{b}_\perp \times \mathbf{k}_\perp\right)_z \rho_{LU} \neq 0$$

What we learned about GTMDs in QED

- First model-independent evaluation (α^2 order, $x \neq 1$, $\mathbf{k}_\perp \neq \mathbf{0}_\perp$)
- Generic features of GTMDs recovered
- Mesauring GTMDs: could be done in QED (quantum optics)?

? ? ? ? ? ?

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$\Delta_\perp^2$
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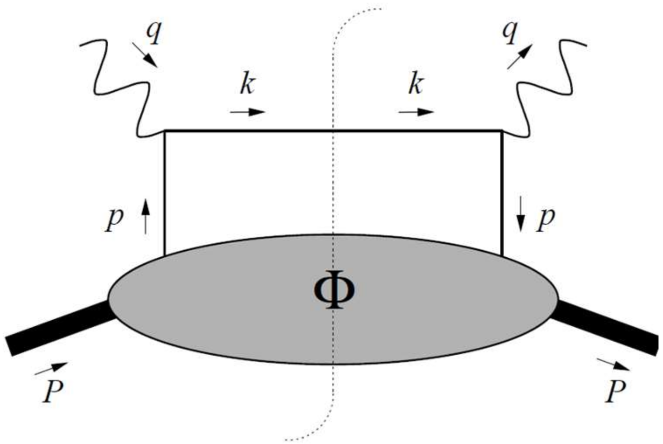
$\Delta = 0$
$\int dx$

Electron TMDs

TMD correlator:

$$\Phi(x, \mathbf{k}_\perp; P, S) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3} e^{ik\cdot\xi} \langle P, S|\bar{\psi}(0)\mathcal{U}_{(0,\xi)}\psi(\xi)|P, S\rangle\Big|_{\xi^+=0}$$



Electron TMDs

TMD correlator:

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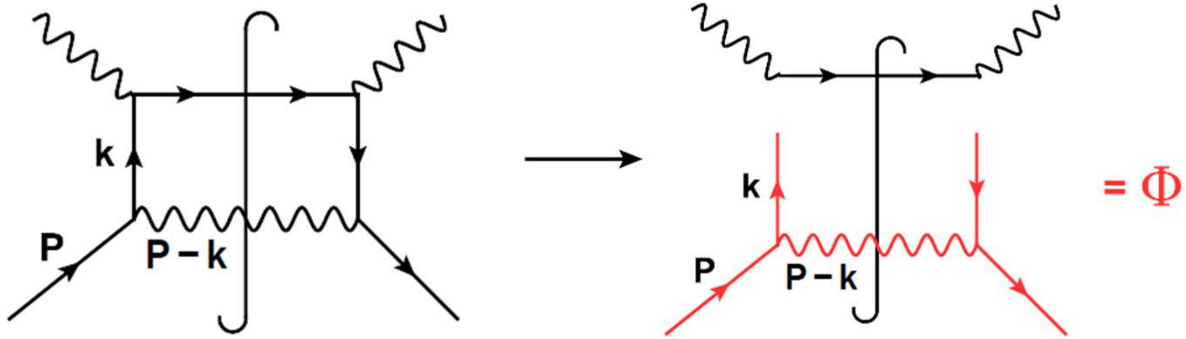
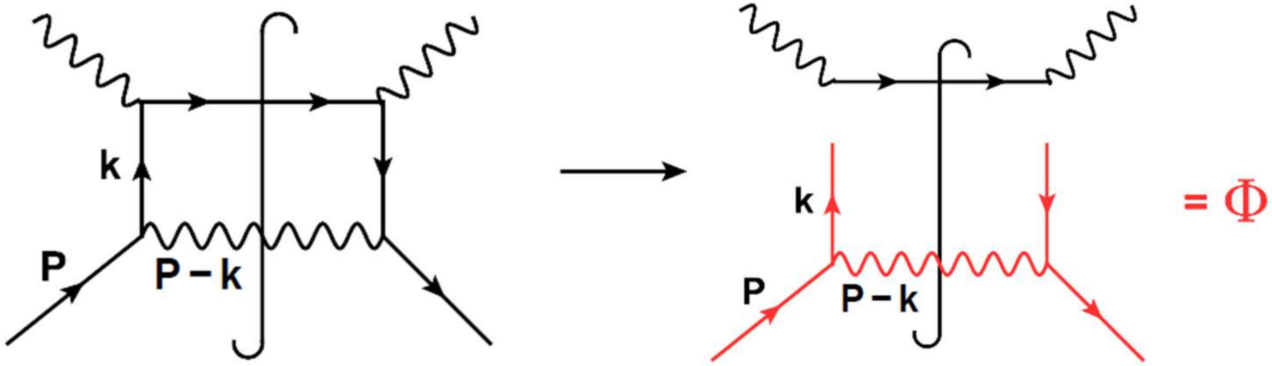


Electron TMDs



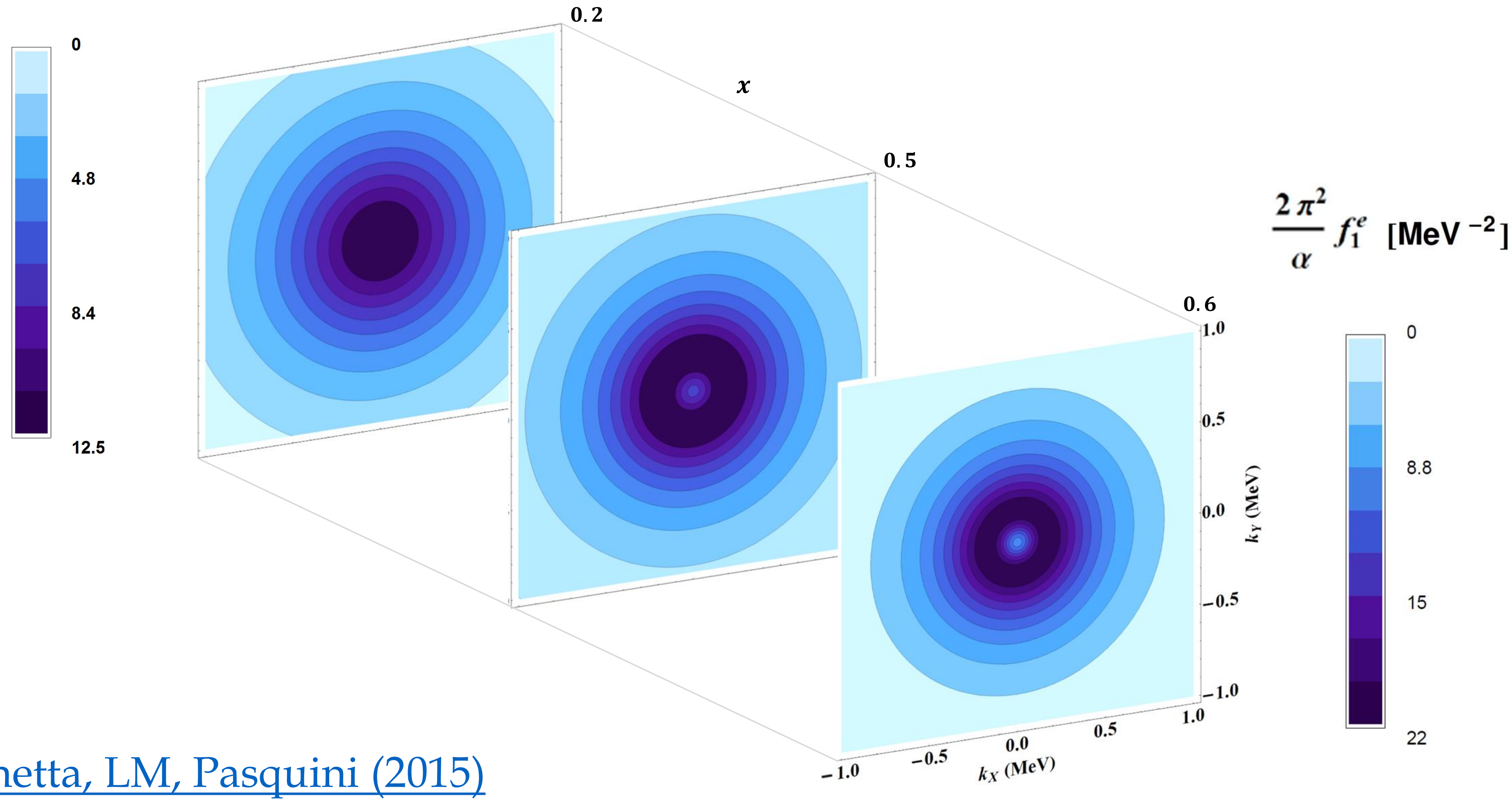
$$\Phi \approx \frac{1}{2}\Big[\not{n}_+ f_1^e - \frac{\epsilon_\perp^{ij} k_\perp^i S_\perp^j}{m} \not{n}_+ f_{1T}^{\perp e} + S_z \gamma^5 \not{n}_+ g_{1L}^e + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{m} \gamma^5 \not{n}_+ g_{1T}^e + \frac{[\not{\mathbf{S}}_\perp, \not{n}_+]}{2} \gamma^5 h_{1T}^e$$

$$+ S_Z \frac{[\not{\mathbf{k}}_\perp, \not{n}_+]}{2m} \gamma^5 h_{1L}^{\perp e} + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp [\not{\mathbf{k}}_\perp, \not{n}_+]}{2m^2} \gamma^5 h_{1T}^{\perp e} + \frac{i[\not{\mathbf{k}}_\perp, \not{n}_+]}{2m} h_1^{\perp e} \Big]$$

T-even T-odd

Meissner, Metz, Goeke (2007)

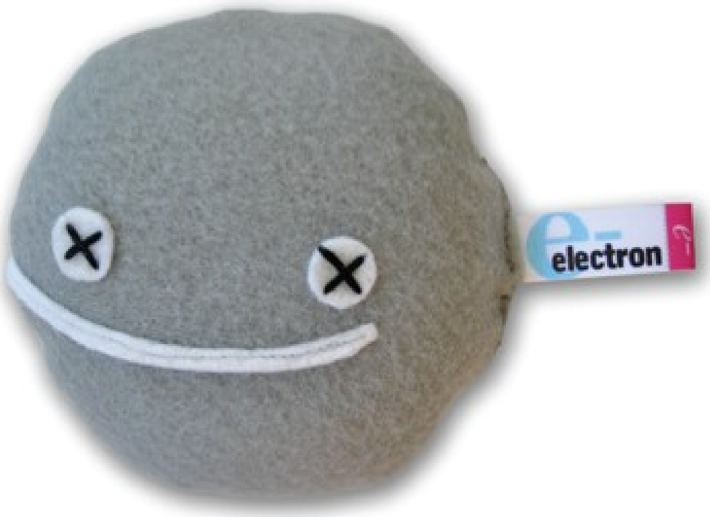
3D-picture for unpol. electron in unpol. dressed electron



Bacchetta, LM, Pasquini (2015)

The electron is ring-shaped!

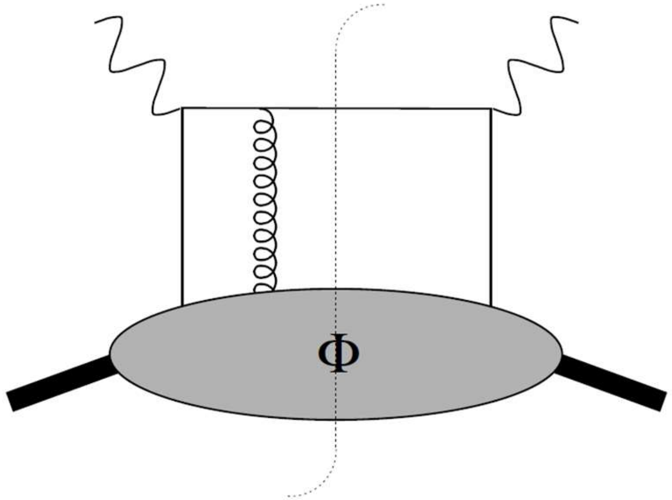
(in momentum space, for large x)



T-odd TMDs in QED

T-odd TMDs

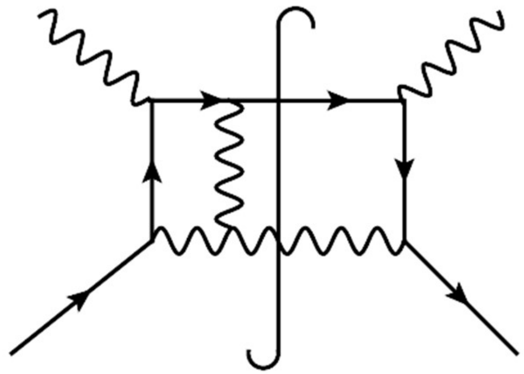
$h_1^\perp(x, \mathbf{k}_\perp^2)$ $\quad$ $f_{1T}^\perp(x, \mathbf{k}_\perp^2)$



T-odd TMDs in QED

T-odd TMDs

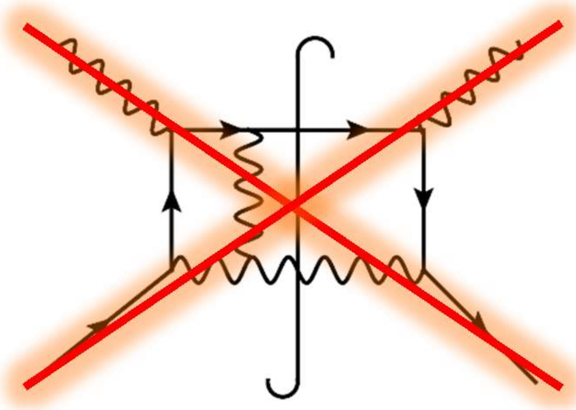
$$h_1^{\perp}(x, \mathbf{k}_\perp^2) \qquad f_{1T}^{\perp}(x, \mathbf{k}_\perp^2)$$



T-odd TMDs in QED

T-odd TMDs are vanishing

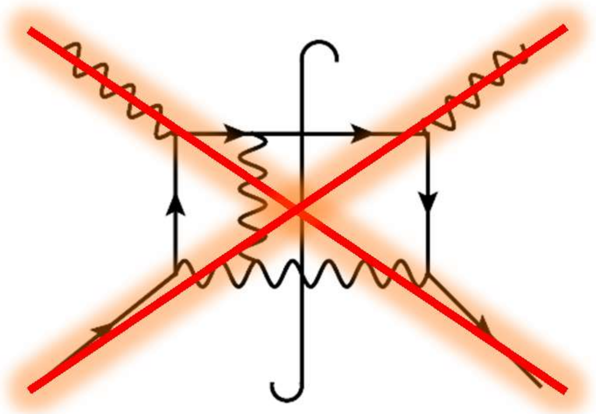
$$h_1^{\perp}(x, \mathbf{k}_{\perp}^2) = 0 \quad f_{1T}^{\perp}(x, \mathbf{k}_{\perp}^2) = 0$$



α^2 order

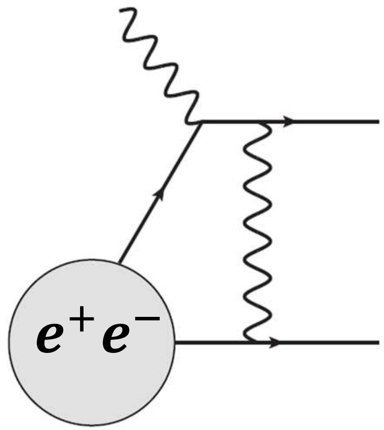
T-odd TMDs in QED

T-odd TMDs are vanishing…?



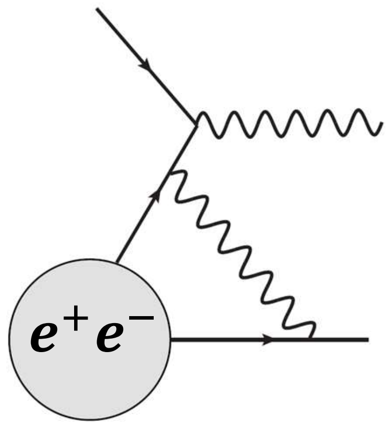
Positron

SIDIS



$= \; -$

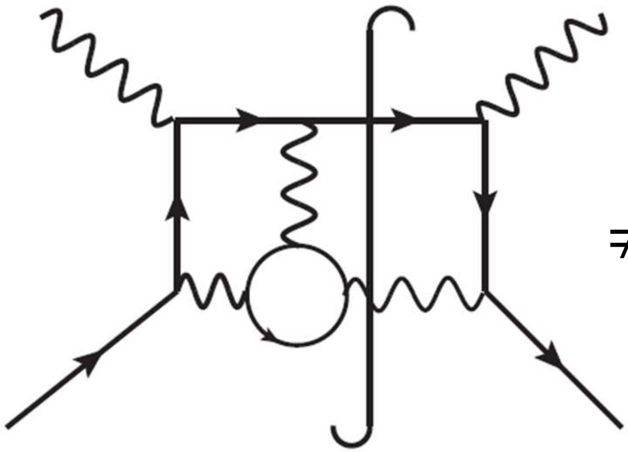
DY



?

T-odd TMDs in QED

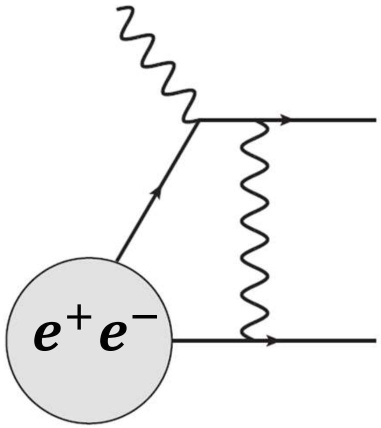
T-odd TMDs are vanishing…?



$\neq 0$?

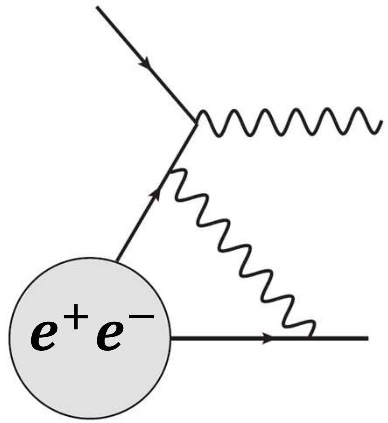
Positron

SIDIS



$= -$

DY



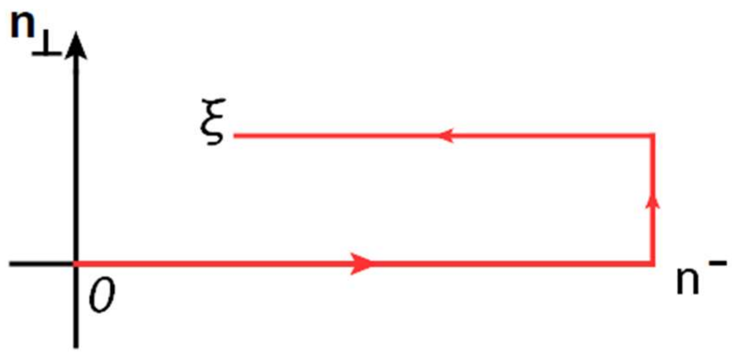
Sivers effect should be a property of a gauge theory, independently of its (non) Abelian nature

Gauge link

$$\Phi(x, \mathbf{k}_\perp; P, S) := \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3} e^{ik\cdot\xi} \langle P, S | \bar{\psi}(0) \mathcal{U}_{(0,\xi)} \psi(\xi) | P, S \rangle \Big|_{\xi^+=0}$$

with

$$\mathcal{U}_{(\xi_1,\xi_2)} := \exp\left[-ie\int_{\xi_1}^{\xi_2} d\eta^\mu A_\mu(\eta)\right]$$

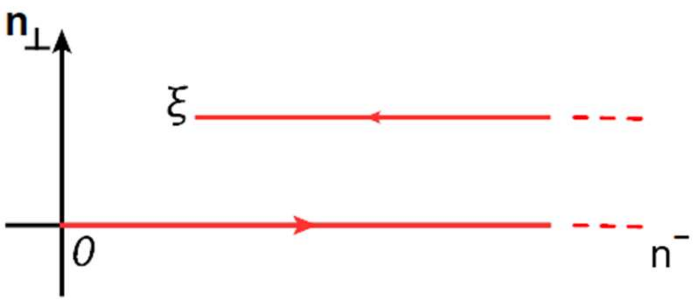


Gauge link

$$\Phi(x, \mathbf{k}_\perp; P, S) := \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3} e^{ik\cdot\xi} \langle P, S|\bar{\psi}(0)\mathcal{U}_{(0,\xi)}\psi(\xi)|P, S\rangle\Big|_{\xi^+=0}$$

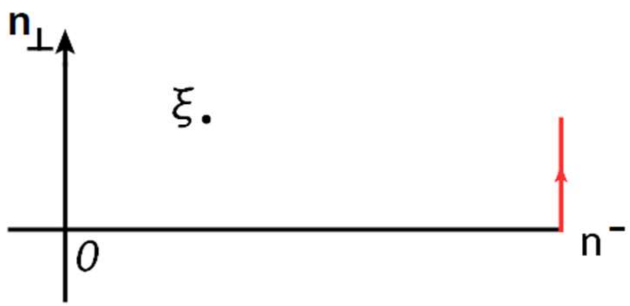
Feynman gauge

$$\mathbf{A}_\perp(\infty) = 0$$



Light-cone gauge

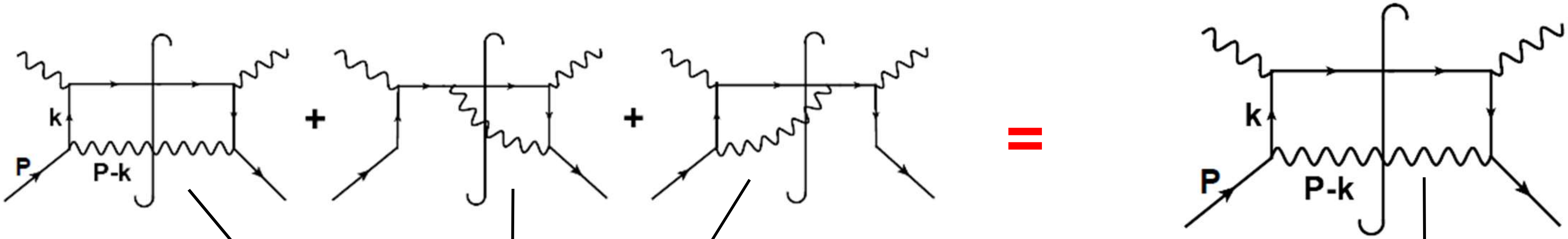
$$A^+ = 0$$



Gauge link

Feynman gauge + gauge link

LC gauge, no gauge link



$$d^{\mu\nu}(p-k) = -g^{\mu\nu}$$

$$d^{\mu\nu}(p-k) = g^{\mu\nu} + \frac{(p-k)^{\nu} n^{\mu}}{(p-k)^{+}} + \frac{(p-k)^{\mu} n^{\nu}}{(p-k)^{+}}$$

What about the transverse gauge link?

Prescription choice for regularization

$$\frac{1}{(p-k)^+} \longrightarrow \begin{cases} \dfrac{1}{(p-k)^+ + i\epsilon} & \text{Retarted} \\[2ex] \dfrac{1}{(p-k)^+ - i\epsilon} & \text{Advanced} \\[2ex] \dfrac{1}{2}\left[\dfrac{1}{(p-k)^+ + i\epsilon} + \dfrac{1}{(p-k)^+ - i\epsilon}\right] & \text{Principal Value} \end{cases}$$

Belitsky, Ji, Yuan (2002)

Prescription choice for regularization

Extra terms from regularization

$$\begin{cases} -\dfrac{ie^2}{(2\pi)^2}\dfrac{1}{\mathbf{k}_\perp^2}\delta(1-x) & \text{Retarted} \\ \dfrac{ie}{(2\pi)^2}\dfrac{1}{\mathbf{k}_\perp^2}\delta(1-x) & \text{Advanced} \\ 0 & \text{Principal Value} \end{cases}$$

Extra terms from transverse gl

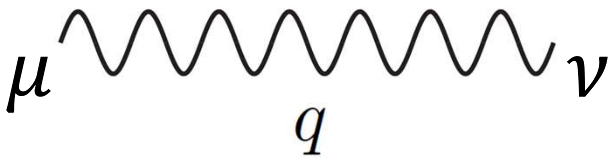
$$\begin{cases} \dfrac{ie^2}{(2\pi)^2}\dfrac{1}{\mathbf{k}_\perp^2}\delta(1-x) \\ -\dfrac{ie}{(2\pi)^2}\dfrac{1}{\mathbf{k}_\perp^2}\delta(1-x) \\ 0 \end{cases}$$

The transverse gauge link makes the evaluation of TMDs prescription-independent in the light-cone gauge

Photon propagator in light-cone gauge

Based on: LM, Pasquini, Xiong, Bacchetta(2016)

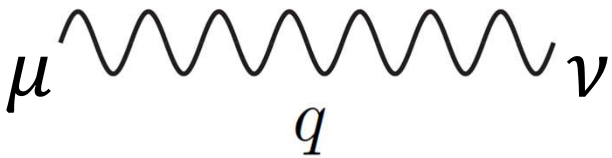
Gauge-field propagator in light-cone gauge



$\mathcal{D}^{\mu\nu}(q)$ with $A^+ = 0$

Gauge-field propagator in light-cone gauge

Which is the correct one?



$$\mathcal{D}^{\mu\nu}(q) = \frac{-i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu n^\nu + q^\nu n^\mu}{q^+} + q^2 \frac{n^\mu n^\nu}{(q^+)^2}\right)$$

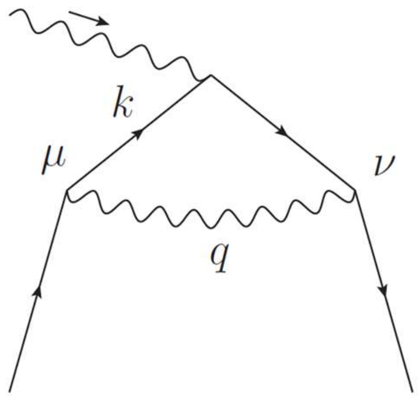
$$\mathcal{D}^{\mu\nu}(q) \quad \text{with} \quad A^+ = 0$$

$$\mathcal{D}^{\mu\nu}(q) = \frac{-i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu n^\nu + q^\nu n^\mu}{q^+}\right)$$

Comparison between TOPT and covariant theory

QED triangle diagram in instant form and in Feynman gauge:

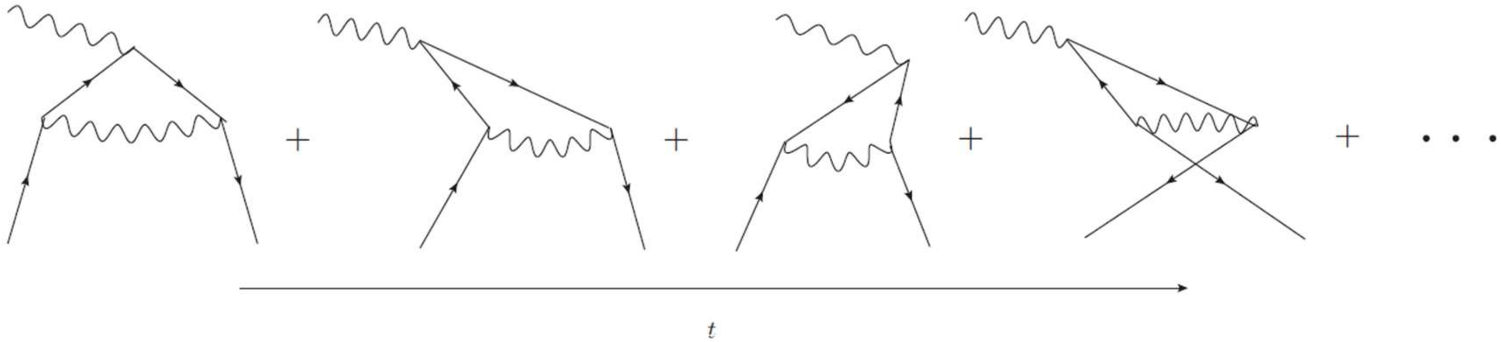
Covariant theory



$$\mathcal{D}^{\mu\nu}(q) = -g^{\mu\nu}$$

$\int dk^0$

TOPT



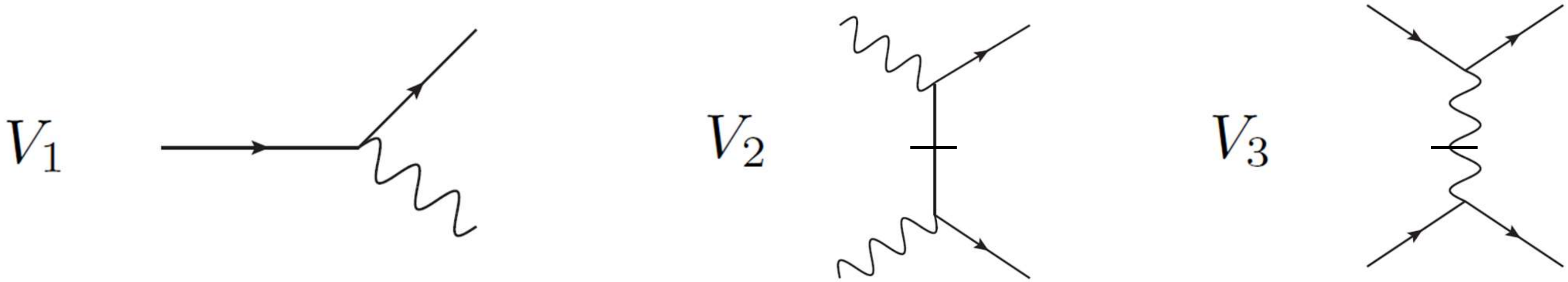
TOPT on the light-front

Diagrams with vacuum fluctuations are vanishing, but...

$$\mathscr{L}_{QED} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$$

Using $A^{+} = 0$ and the equations of motion in light-front coordinates:

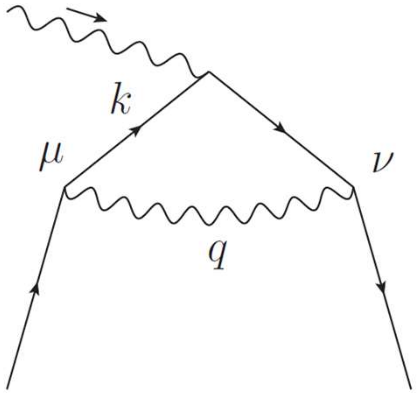
$$H = \int d^2\mathbf{x}_{\perp} dx^{-}\, T^{+-} = H_0 + V_1 + V_2 + V_3$$



Mustaki *et al.* (1991)

TOPT on the light-front

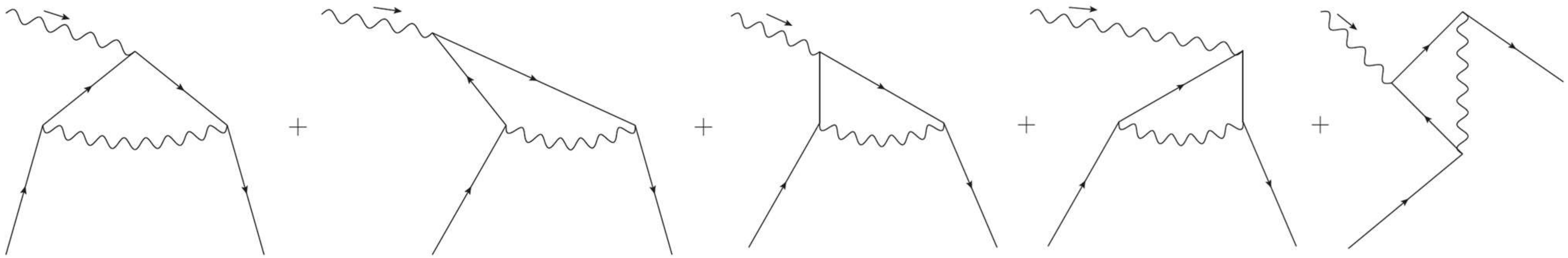
Covariant theory in LC gauge



$$\mathcal{D}^{\mu\nu}(q) = ??$$

$$\int dk^-$$

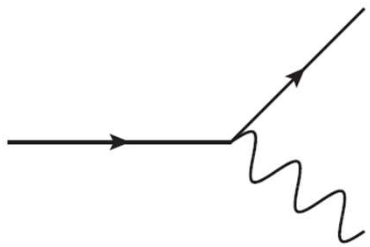
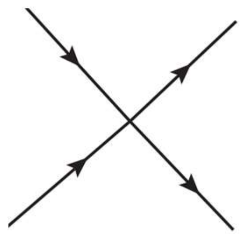
TOPT



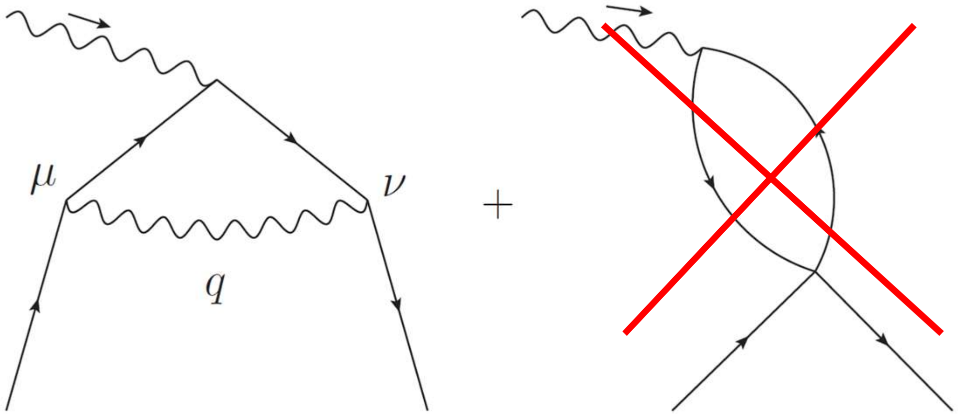
Interaction Hamiltonian in light-cone gauge

If we start with $\mathcal{D}_T^{\mu\nu}(q) = \frac{-i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu n^\nu + q^\nu n^\mu}{q^+} + q^2 \frac{n^\mu n^\nu}{(q^+)^2}\right)$ in covariant theory, the interaction Hamiltonian becomes:

$$H_I = e\bar{\psi}\gamma^\mu A_\mu \psi = V + V_I$$

V  V_I 

Covariant theory for the QED triangle diagram



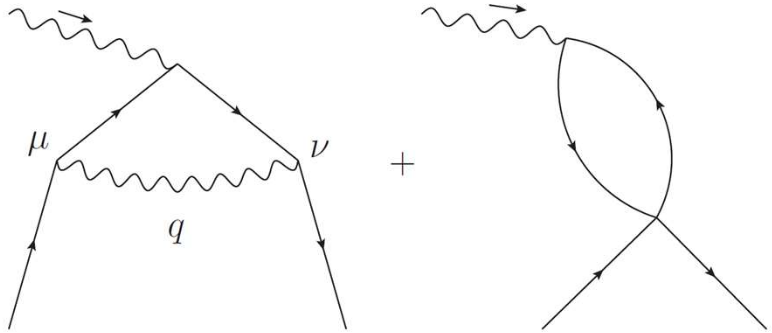
with $$\mathcal{D}^{\mu\nu}(q) = \frac{-i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu n^\nu + q^\nu n^\mu}{q^+} + \cancel{q^2\frac{n^\mu n^\nu}{(q^+)^2}}\right)$$

The cancelation happens in any physical process, as it is for any non-covariant gauge (e.g. Coulomb gauge)

Brodsky, Srivastava (2000)

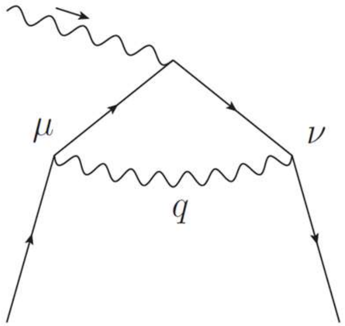
Covariant theory for the QED triangle diagram

Either



$$\mathcal{D}_T^{\mu\nu}(q) = \frac{-i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu n^\nu + q^\nu n^\mu}{q^+} + q^2 \frac{n^\mu n^\nu}{(q^+)^2}\right)$$

Or



$$\mathcal{D}^{\mu\nu}(q) = \frac{-i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu n^\nu + q^\nu n^\mu}{q^+}\right)$$

We can generate the diagrams with instantaneous photons appearing in TOPT, even starting from the two-term propagator.

How does the proof of the equivalence work?

Perfectly fine…!

(ça marche!)

Towards QCD:
the energy momentum tensor

The energy momentum tensor

Energy (mass) density

Momentum density

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

Energy flux

Shear stress; pressure

The energy momentum tensor

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

$$T^{\mu\nu} = \frac{\partial \mathscr{L}}{\partial(\partial_\mu \varphi_r)} \partial^\nu \varphi_r - g^{\mu\nu} \mathscr{L}$$

QCD energy momentum tensor

$$\mathscr{L}_{QCD} \longrightarrow \langle P| T^{q;g}_{\mu\nu} |P'\rangle = \bar{u}' \Big[A(t) \frac{P^\mu P^\nu}{M} + [A(t) + B(t)] \frac{P^{\{\mu} i\sigma^{\nu\}\rho} \Delta_\rho}{4M}$$

$$+ C(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} + \bar{C}(t) M g^{\mu\nu} + D(t) \frac{P^{[\mu} i\sigma^{\nu]\rho} \Delta_\rho}{4M} \Big] u$$

$$t = (P' - P)^2$$

Polyakov (2002)

Polyakov et a. (2007)

Instant form; Breit frame $\Delta^0 = 0$

Us: Light-front form; generic frame

Connection with GPDs

Angular momentum density: $$M^{\alpha\mu\nu} = T^{\alpha\nu}x^{\mu} - T^{\alpha\mu}x^{\nu}$$

Light-cone helicity operator: $$J^3 = \int dx^- d^2\mathbf{x}_\perp M^{+12}(x)$$

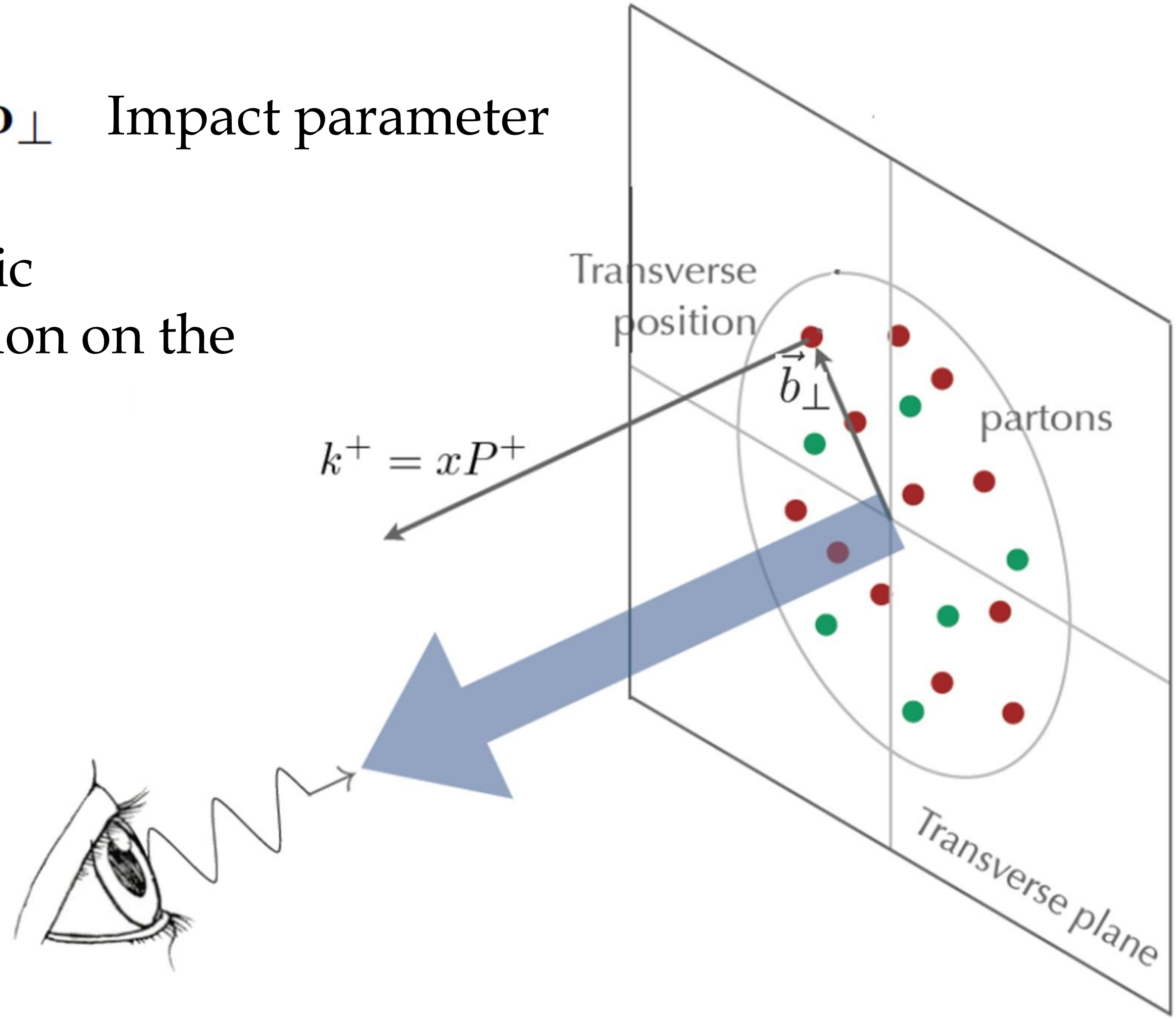
$$\langle J^3 \rangle = \frac{1}{2}\left[A(0) + B(0)\right] \ ,$$

Relations with GPDs:
$$A_q(t) + B_q(t) = \int_{-1}^{1} dx\, x\left[H_q(x,\xi,t) + E_q(x,\xi,t)\right]$$
$$A_g(t) + B_g(t) = \int_{0}^{1} dx\, x\left[H_g(x,\xi,t) + E_g(x,\xi,t)\right]$$

Diehl (2003)

$$\mathbf{\Delta}_\perp \overset{\text{FT}}{\longleftrightarrow} \mathbf{b}_\perp$$ Impact parameter

Probabilistic interpretation on the light-front



Conclusions

Conclusions

- Overview on light-front quantization methods: importance of probabilistic interpretation in the impact paramter space.
- Wigner distributions in QED: exact result, recovering generic properties. Mesaurements?
- TMDs in QED: differences compared to QCD due to Abelian nature; role of transverse gauge link
- Gauge-field propagator in light-cone gauge for the equivalence between covariant theory and TOPT
- Hint on: QCD energy-momentum tensor on the light-front

Light-front vacuum

We take $|0\rangle$ such that $(P^+, \mathbf{P}_\perp)\,|0\rangle = (0, \mathbf{0}_\perp)$

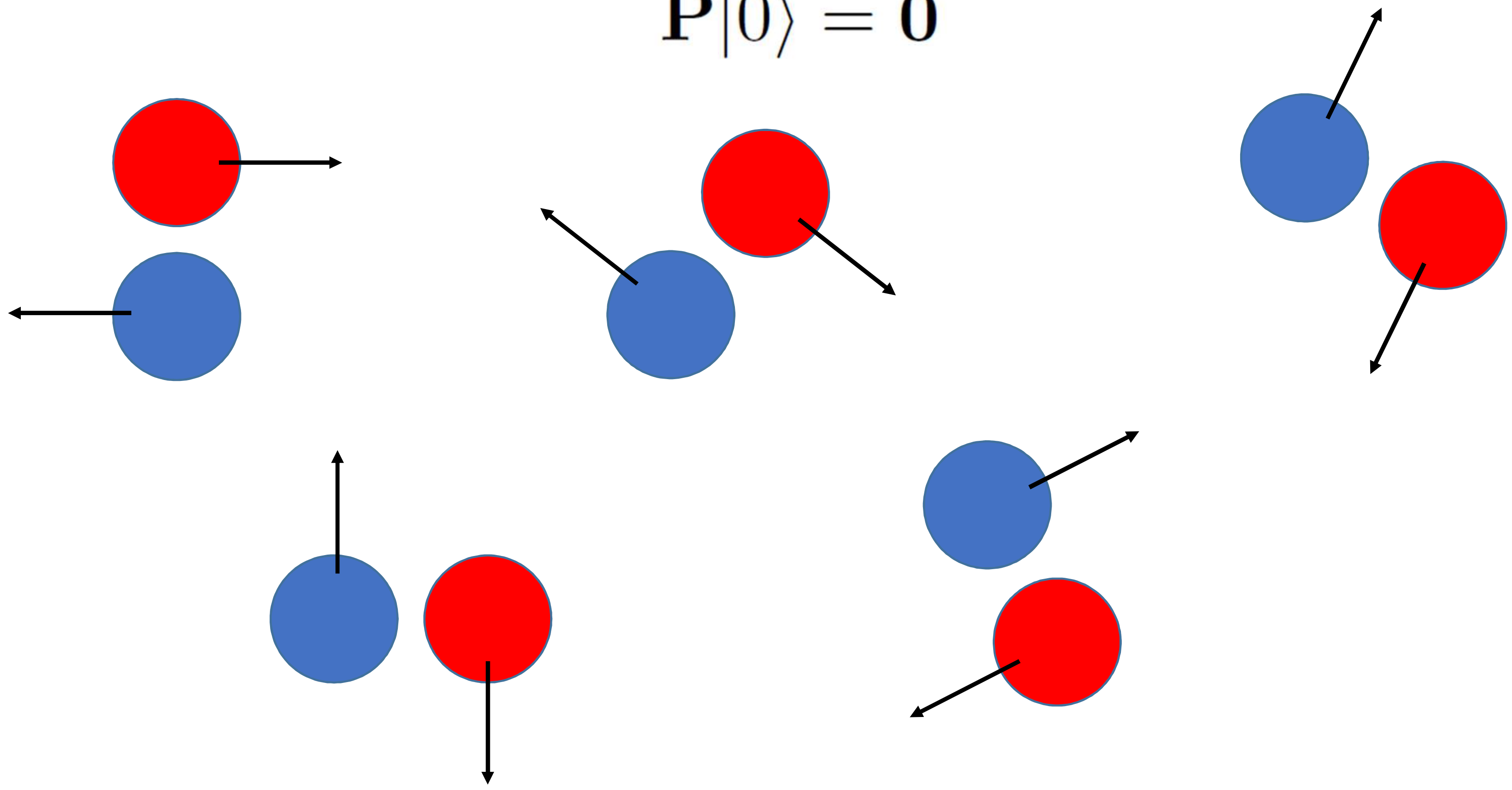
Constraint: $P^+ > 0$

$$\Downarrow$$

The vacuum is empty!

Instant-form vacuum

$$\mathbf{P}|0\rangle = \mathbf{0}$$



Wigner distributions in QFT

$$\hat{W}(\mathbf{r},\mathbf{k}) = \int \frac{dz^- d^2\mathbf{z}_\perp}{(2\pi)^3}\, e^{ikz}\, \bar{\psi}\left(\mathbf{r}-\frac{z}{2}\right)\Gamma\mathcal{W}\,\psi\left(\mathbf{r}+\frac{z}{2}\right)\bigg|_{z^+=0}$$

Wigner distribution in the Breit frame $\Delta^0 = 0$

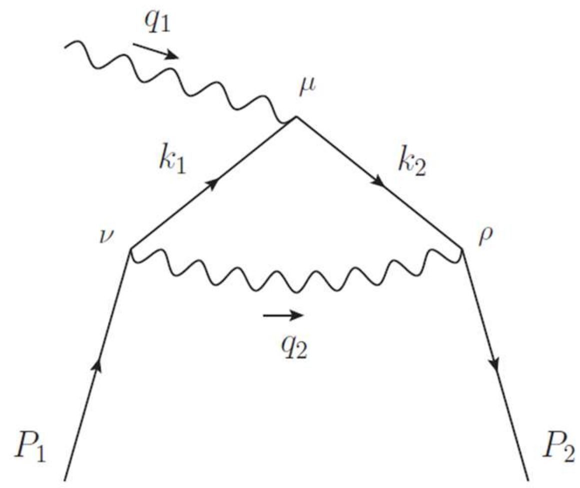
$$\rho_{\Lambda,\Lambda'}(\mathbf{r},\mathbf{k}) = \frac{1}{2}\int \frac{d^3\mathbf{\Delta}}{(2\pi)^3}\, e^{-i\mathbf{\Delta}\cdot\mathbf{r}} \left\langle \frac{\mathbf{\Delta}}{2},\Lambda' \left| \hat{W}(\mathbf{0},\mathbf{k}) \right| -\frac{\mathbf{\Delta}}{2},\Lambda \right\rangle$$

No semi-classical probabilistic interpretation!!

Ji (2003)

Belitsky, Ji, Yuan (2004)

Proof of the equivalence



$$J^{\mu}_{COV} = \frac{e^2}{(2\pi)^4} \int d^4k_1\, \bar{u}(P_2)\, \gamma^{\rho} \mathcal{S}_F(k_2) \gamma^{\mu} \mathcal{S}_F(k_1) \gamma^{\nu}\, \mathcal{D}^{\nu\rho}_T(q_2) u(p_1)$$

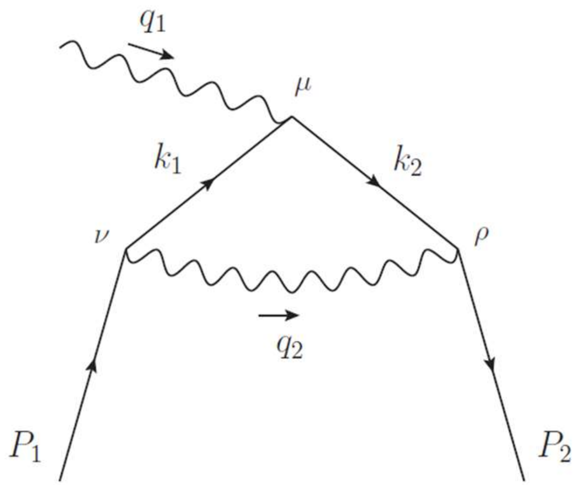
$$k^{\mu}_{\mathrm{on-shell}} \equiv \tilde{k}^{\mu} = \left(k^+, \frac{\mathbf{k}_\perp^2 + m^2}{2k^+}, \mathbf{k}_\perp \right),$$

$$k^{\mu}_{\mathrm{off-shell}} \equiv \hat{k}^{\mu} = \left(0, \frac{k^2 - m^2}{2k^+}, \mathbf{0}_\perp \right) = \frac{k^2 - m^2}{2k^+} n^{\mu}$$

$$\mathcal{S}_F(k) = \frac{i\left(\tilde{\not{k}} + m\right)}{k^2 - m^2 + i\epsilon} + \frac{i\gamma^+}{2k^+} \qquad \mathcal{D}^{\mu\nu}(q) = \frac{-i}{q^2 + i\epsilon}\left(g^{\mu\nu} - \frac{\tilde{q}^{\mu} n^{\nu} + \tilde{q}^{\nu} n^{\mu}}{q^+} \right) + \frac{i n^{\mu} n^{\nu}}{(q^+)^2}$$

Numerators do not depend on k^-

Proof of the equivalence



$$J^{\mu}_{COV} = \frac{e^2}{(2\pi)^4} \int d^4 k_1 \, \bar{u}\,(P_2)\,\gamma^{\rho} \mathcal{S}_F(k_2) \gamma^{\mu} \mathcal{S}_F(k_1) \gamma^{\nu}\, \mathcal{D}_T^{\nu\rho}(q_2) u(p_1)$$

$$k^{\mu}_{\text{on-shell}} \equiv \tilde{k}^{\mu} = \left(k^+, \frac{\mathbf{k}_{\perp}^2 + m^2}{2k^+}, \mathbf{k}_{\perp} \right),$$

$$k^{\mu}_{\text{off-shell}} \equiv \hat{k}^{\mu} = \left(0, \frac{k^2 - m^2}{2k^+}, \mathbf{0}_{\perp} \right) = \frac{k^2 - m^2}{2k^+} n^{\mu}$$

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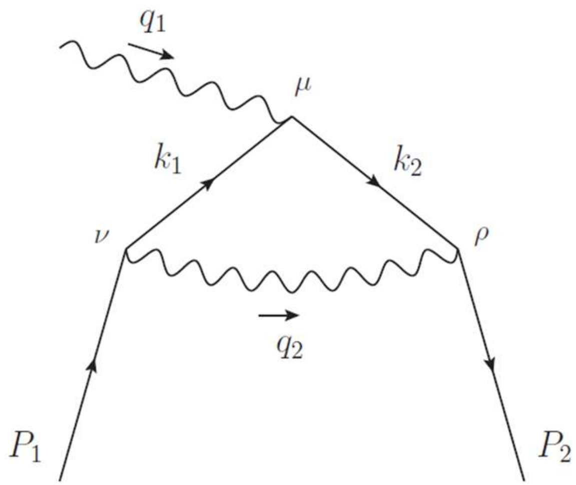
On-shell (propagating) parts

Proof of the equivalence



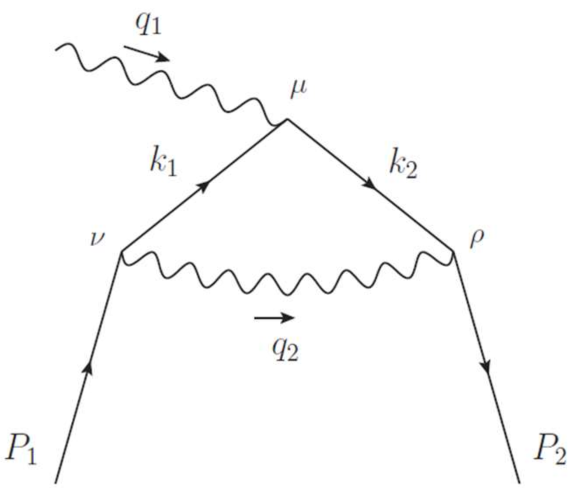
$$J^{\mu}_{COV} = \frac{e^2}{(2\pi)^4} \int d^4 k_1 \, \bar{u}\,(P_2)\,\gamma^{\rho} \mathcal{S}_F(k_2)\gamma^{\mu}\mathcal{S}_F(k_1)\gamma^{\nu}\,\mathcal{D}^{\nu\rho}_T(q_2) u(p_1)$$

$$k^{\mu}_{\text{on-shell}} \equiv \tilde{k}^{\mu} = \left(k^+, \frac{\mathbf{k}_\perp^2 + m^2}{2k^+}, \mathbf{k}_\perp\right),$$

$$k^{\mu}_{\text{off-shell}} \equiv \hat{k}^{\mu} = \left(0, \frac{k^2 - m^2}{2k^+}, \mathbf{0}_\perp\right) = \frac{k^2 - m^2}{2k^+} n^{\mu}$$

$$\mathcal{S}_F\,(k) = \frac{i\left(\tilde{\not{k}} + m\right)}{k^2 - m^2 + i\epsilon} + \frac{i\gamma^+}{2k^+} \qquad \mathcal{D}^{\mu\nu}\,(q) = \frac{-i}{q^2 + i\epsilon}\left(g^{\mu\nu} - \frac{\tilde{q}^{\mu} n^{\nu} + \tilde{q}^{\nu} n^{\mu}}{q^+}\right) + \frac{i n^{\mu} n^{\nu}}{(q^+)^2}$$

Off-shell (non-propagating,
or instantaneous) parts



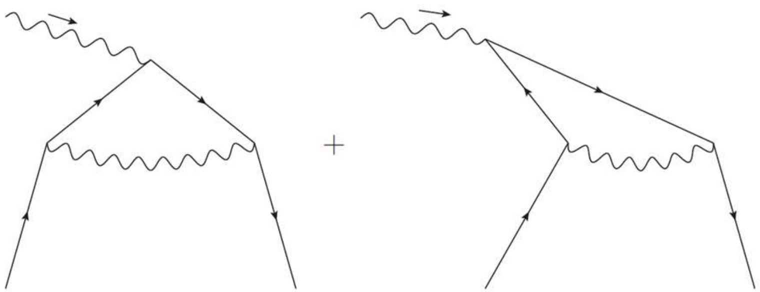
$$J^{\mu}_{COV} = \frac{e^2}{(2\pi)^4}\int d^4k_1\, \bar{u}\left(P_2\right)\gamma^{\rho}\mathcal{S}_F(k_2)\gamma^{\mu}\mathcal{S}_F(k_1)\gamma^{\nu}\,\mathcal{D}_T^{\nu\rho}(q_2)u(p_1)$$

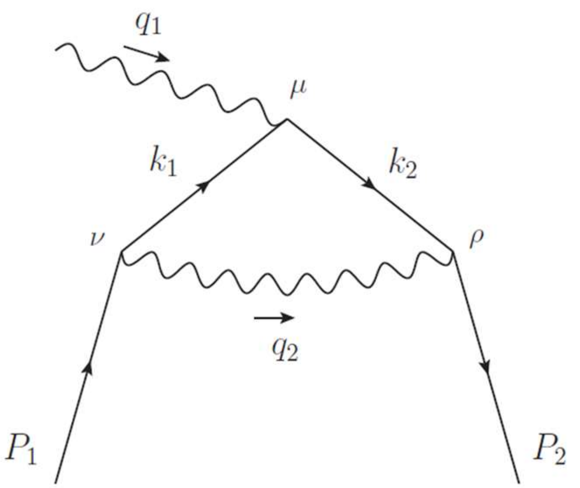
$$k^{\mu}_{\mathrm{on-shell}} \equiv \tilde{k}^{\mu} = \left(k^+, \frac{\mathbf{k}_{\perp}^2 + m^2}{2k^+}, \mathbf{k}_{\perp}\right),$$

$$k^{\mu}_{\mathrm{off-shell}} \equiv \hat{k}^{\mu} = \left(0, \frac{k^2 - m^2}{2k^+}, \mathbf{0}_{\perp}\right) = \frac{k^2 - m^2}{2k^+}n^{\mu}$$

$$\mathcal{S}_F\left(k\right) = \frac{i\left(\tilde{\not{k}} + m\right)}{k^2 - m^2 + i\epsilon} + \frac{i\gamma^+}{2k^+} \qquad \mathcal{D}^{\mu\nu}\left(q\right) = \frac{-i}{q^2 + i\epsilon}\left(g^{\mu\nu} - \frac{\tilde{q}^{\mu}n^{\nu} + \tilde{q}^{\nu}n^{\mu}}{q^+}\right) + \frac{in^{\mu}n^{\nu}}{\left(q^+\right)^2}$$

$$\int dk^-$$





$$J^{\mu}_{COV} = \frac{e^2}{(2\pi)^4} \int d^4k_1 \, \bar{u}\left(P_2\right) \gamma^{\rho} \mathcal{S}_F(k_2) \gamma^{\mu} \mathcal{S}_F(k_1) \gamma^{\nu} \, \mathcal{D}_T^{\nu\rho}(q_2) u(p_1)$$

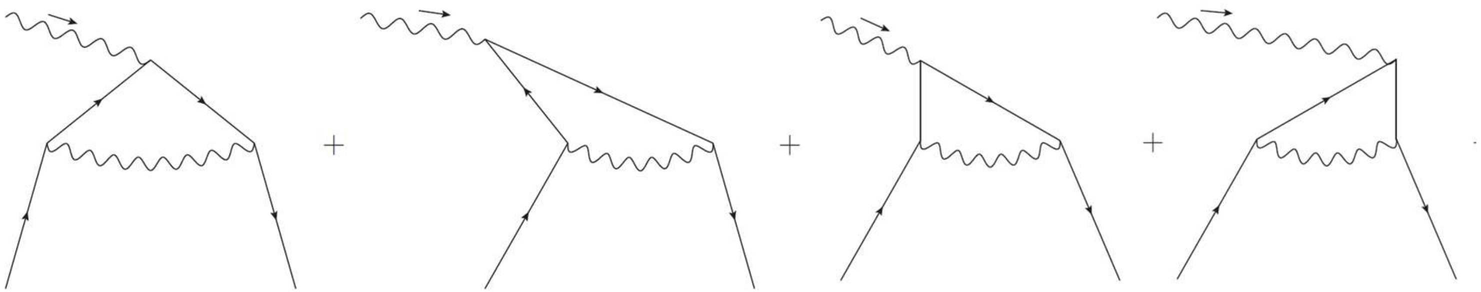
$$k^{\mu}_{\mathrm{on-shell}} \equiv \tilde{k}^{\mu} = \left(k^+, \frac{\mathbf{k}_{\perp}^2 + m^2}{2k^+}, \mathbf{k}_{\perp}\right),$$

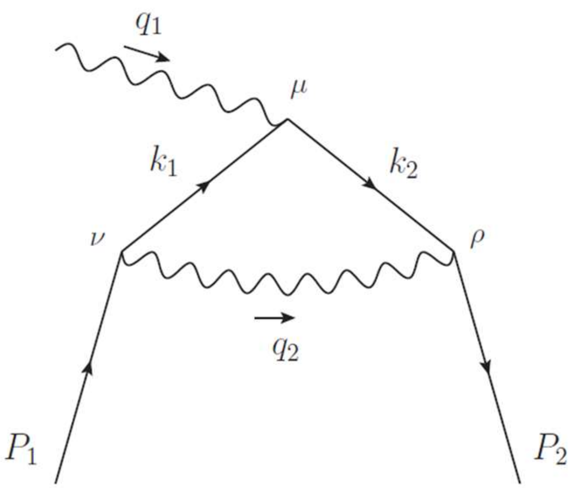
$$k^{\mu}_{\mathrm{off-shell}} \equiv \hat{k}^{\mu} = \left(0, \frac{k^2 - m^2}{2k^+}, \mathbf{0}_{\perp}\right) = \frac{k^2 - m^2}{2k^+} n^{\mu}$$

$$\mathcal{S}_F\left(k\right) = \frac{i\left(\tilde{\not{k}} + m\right)}{k^2 - m^2 + i\epsilon} + \frac{i\gamma^+}{2k^+}$$

$$\mathcal{D}^{\mu\nu}\left(q\right) = \frac{-i}{q^2 + i\epsilon}\left(g^{\mu\nu} - \frac{\tilde{q}^{\mu} n^{\nu} + \tilde{q}^{\nu} n^{\mu}}{q^+}\right) + \frac{i n^{\mu} n^{\nu}}{\left(q^+\right)^2}$$

$$\int dk^-$$





$$J^{\mu}_{COV} = \frac{e^2}{(2\pi)^4} \int d^4k_1 \, \bar{u}\left(P_2\right) \gamma^{\rho} \mathcal{S}_F\left(k_2\right) \gamma^{\mu} \mathcal{S}_F\left(k_1\right) \gamma^{\nu} \, \mathcal{D}^{\nu\rho}_{T}\left(q_2\right) u(p_1)$$

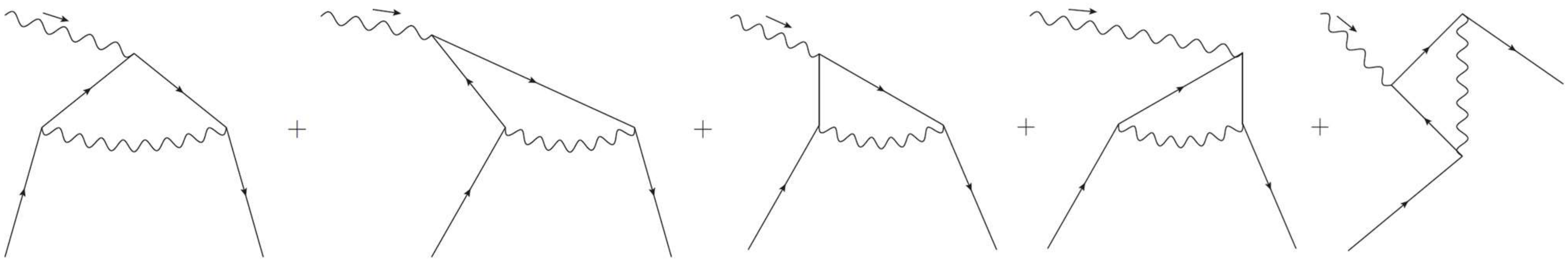
$$k^{\mu}_{\mathrm{on-shell}} \equiv \tilde{k}^{\mu} = \left(k^{+}, \frac{\mathbf{k}_{\perp}^2 + m^2}{2k^{+}}, \mathbf{k}_{\perp}\right),$$

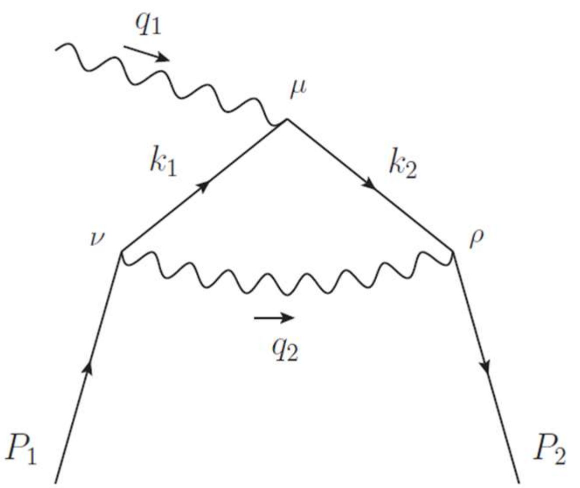
$$k^{\mu}_{\mathrm{off-shell}} \equiv \hat{k}^{\mu} = \left(0, \frac{k^2 - m^2}{2k^{+}}, \mathbf{0}_{\perp}\right) = \frac{k^2 - m^2}{2k^{+}} n^{\mu}$$

$$\mathcal{S}_F\left(k\right) = \frac{i\left(\tilde{\not{k}} + m\right)}{k^2 - m^2 + i\epsilon} + \frac{i\gamma^{+}}{2k^{+}}$$

$$\mathcal{D}^{\mu\nu}\left(q\right) = \frac{-i}{q^2 + i\epsilon}\left(g^{\mu\nu} - \frac{\tilde{q}^{\mu} n^{\nu} + \tilde{q}^{\nu} n^{\mu}}{q^{+}}\right) + \frac{i n^{\mu} n^{\nu}}{\left(q^{+}\right)^2}$$

$$\int dk^{-}$$





$$J^{\mu}_{COV} = \frac{e^2}{(2\pi)^4}\int d^4k_1\, \bar{u}(P_2)\,\gamma^{\rho}\mathcal{S}_F(k_2)\gamma^{\mu}\mathcal{S}_F(k_1)\gamma^{\nu}\,\mathcal{D}^{\nu\rho}_{T}(q_2)u(p_1)$$

$$k^{\mu}_{\mathrm{on-shell}} \equiv \tilde{k}^{\mu} = \left(k^+, \frac{\mathbf{k}_\perp^2 + m^2}{2k^+}, \mathbf{k}_\perp\right),$$

$$k^{\mu}_{\mathrm{off-shell}} \equiv \hat{k}^{\mu} = \left(0, \frac{k^2 - m^2}{2k^+}, \mathbf{0}_\perp\right) = \frac{k^2 - m^2}{2k^+} n^{\mu}$$

$$\mathcal{S}_F(k) = \frac{i\left(\tilde{\not{k}} + m\right)}{k^2 - m^2 + i\epsilon} + \frac{i\gamma^+}{2k^+} \qquad \mathcal{D}^{\mu\nu}(q) = \frac{-i}{q^2 + i\epsilon}\left(g^{\mu\nu} - \frac{\tilde{q}^{\mu}n^{\nu} + \tilde{q}^{\nu}n^{\mu}}{q^+}\right) + \frac{in^{\mu}n^{\nu}}{(q^+)^2}$$

$$\int dk^-$$

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