Full NLO corrections for DIS structure functions in the dipole factorization formalism

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Outline

- Introduction: dipole factorization for DIS at low x_{Bj}
- One-loop correction to the $\gamma^*_{T,L} \to q\bar{q}$ light-front wave-functions:

  Direct calculation

  G.B., PRD94 (2016)
- DIS at NLO in the dipole factorization

  Example: F_L case

  Cancellation of the UV divergences between the $q\bar{q}$ and $q\bar{q}g$ terms

  G.B., *in preparation*

Introduction

At low x_{Bj}, many DIS observables can be expressed within dipole factorization, including gluon saturation $\rightarrow$ rich phenomenology.

In particular: Dipole amplitude obtained from fits of HERA data for DIS structure functions in the dipole factorization at LO+LL with rcBK
Albacete *et al.*, PRD80 (2009); EPJC71 (2011)
Kuokkanen *et al.*, NPA875 (2012);
Lappi, Mäntysaari, PRD88 (2013)

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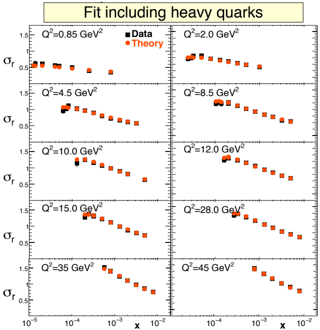
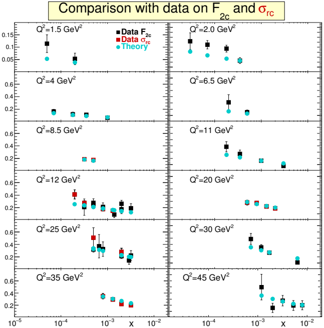
In the last 10 years, many theoretical (including numerical) progresses towards NLO/NLL accuracy for gluon saturation/CGC.

Obviously, DIS structure functions at NLO in the dipole factorization are required to push the fits beyond LO+LL accuracy.

DIS phenomenology at LO+LL

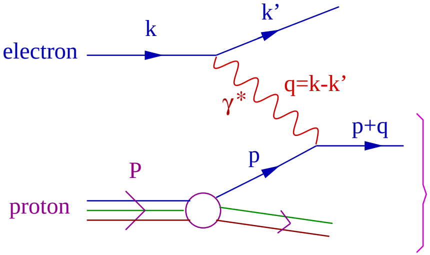




Fits of the reduced DIS cross-section σ_r and its charm contribution σ_{rc} at HERA data with numerical solutions of the running coupling BK equation.

Albacete, Armesto, Milhano, Quiroga, Salgado (2011)
see also: Kuokkanen, Rummukainen, Weigert (2012);
Lappi, Mäntysaari (2013); ...

Kinematics for Deep Inelastic Scattering (DIS)



$$\frac{d\sigma^{ep\to e+X}}{dx_{Bj}\, d^2Q} = \frac{\alpha_{em}}{\pi x_{Bj} Q^2}\left[\left(1-y+\frac{y^2}{2}\right)\sigma_T^{\gamma}(x_{Bj}, Q^2) + (1-y)\,\sigma_L^{\gamma}(x_{Bj}, Q^2)\right]$$

Photon virtuality: $Q^2 \equiv -q^2 > 0$

Bjorken x variable: $x_{Bj} \equiv \frac{Q^2}{2P\cdot q} \in [0,1]$

Inelasticity: $y \equiv \frac{2P\cdot q}{(P+k)^2} \in [0,1]$

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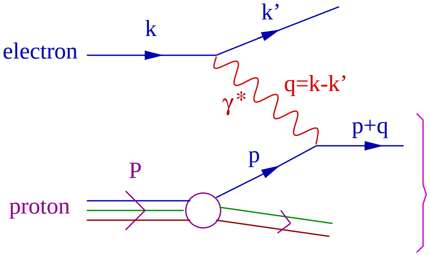

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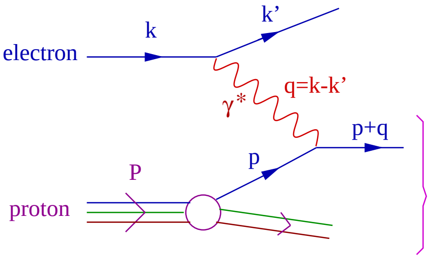

$$\frac{d\sigma^{ep\to e+X}}{dx_{Bj}\,d^2Q} = \frac{\alpha_{em}}{\pi x_{Bj} Q^2}\left[\left(1-y+\frac{y^2}{2}\right)\sigma_T^{\gamma}(x_{Bj},Q^2)+(1-y)\,\sigma_L^{\gamma}(x_{Bj},Q^2)\right]$$

Other equivalent parametrization: structure functions F_i

$$\sigma^{\gamma}_{T,L}(x_{Bj},Q^2) = \frac{(2\pi)^2\alpha_{em}}{Q^2}\,F_{T,L}(x_{Bj},Q^2)$$

$$F_2 = F_T+F_L \quad \text{and} \quad 2x_{Bj}\,F_1 = F_T$$

Eikonal dilute-dense scattering

Recipe for *dilute-dense* processes at high-energy in light-front perturbation theory (LFPT)
following Bjorken, Kogut and Soper (1971):

- Decompose the projectile on a Fock basis at the time $x^+ = 0$, with appropriate light-front wave-functions.
- Each parton n scatters independently on the target via a light-like Wilson line $U_{\mathcal{R}_n}(\mathbf{x}_n)$ through the target:

$$U_{\mathcal{R}_n}(\mathbf{x}_n) = \mathcal{P}_+ \exp\left[ig \int dx^+ \, T^a_{\mathcal{R}_n} \, A^-_a(x^+, \mathbf{x}_n)\right]$$

  with $\mathcal{R}_n = A$, F or $\bar{F}$ for g, q or $\bar{q}$ partons.

- Include final-state evolution of the projectile remnants.

Comments:

1. Final form of the result is general
2. But building blocks can be calculated separately only in LFPT in light-cone gauge $A^+_a = 0$

Dipole factorization for eikonal DIS

Total cross section for photon of polarization λ on the background field target:

$$\sigma_\lambda^\gamma = 2\,\mathrm{Im}\,\mathcal{M}^{\mathrm{fwd}}_{\gamma_\lambda\to\gamma_\lambda} = 2\,\mathrm{Re}\,(-i)\,\mathcal{M}^{\mathrm{fwd}}_{\gamma_\lambda\to\gamma_\lambda}$$

With the forward elastic scattering amplitude defined by:

$$\begin{aligned} &\Big\langle \gamma_\lambda^*(q'^+;\mathbf{q}'=0)_{\mathrm{dressed}}\Big|\Big(\hat{S}_E-\mathbf{1}\Big)\Big|\gamma_\lambda^*(q^+;\mathbf{q}=0)_{\mathrm{dressed}}\Big\rangle \\ = \quad &(2q^+)\,2\pi\delta(q'^+-q^+)\;i\,\mathcal{M}^{\mathrm{fwd}}_{\gamma_\lambda\to\gamma_\lambda} \end{aligned}$$

Note:

$$\begin{aligned} \sigma_L^\gamma &\equiv \sigma_{\lambda=L}^\gamma \\ \sigma_T^\gamma &\equiv \frac{1}{\#\ \mathrm{phys.\ pol.}} \sum_{\lambda\ \mathrm{phys.\ pol.}} \sigma_\lambda^\gamma \end{aligned}$$

Dipole factorization for eikonal DIS

Fock-state decomposition (in mixed space) of an incoming virtual photon:

$$\left|\gamma_\lambda^*(q^+;\mathbf{q}=0)_{\text{dressed}}\right\rangle = \text{Non-QCD Fock states}$$
$$+ \widetilde{\sum_{q_0\bar{q}_1 \text{ F. states}}} 2\pi\delta(k_0^+ + k_1^+ - q^+)\, \mathbf{1}_{\alpha_0\alpha_1}\, \widetilde{\psi}_{\gamma_\lambda\to q_0\bar{q}_1}\; b_0^\dagger d_1^\dagger |0\rangle$$
$$+ \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. states}}} 2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+)\, t^{a_2}_{\alpha_0\alpha_1}\, \widetilde{\psi}_{\gamma_\lambda\to q_0\bar{q}_1 g_2}\; b_0^\dagger d_1^\dagger a_2^\dagger |0\rangle + \cdots$$

Note: the Fock states with no quarks nor gluons cannot contribute to $\mathcal{M}^{\text{fwd}}_{\gamma_\lambda\to\gamma_\lambda}$ and thus to σ^γ_λ

Dipole factorization for eikonal DIS

$$
\begin{aligned}
&(2q^+)\, i\,\mathcal{M}^{\text{fwd}}_{\gamma_\lambda \to \gamma_\lambda} \\
= &\widetilde{\sum_{q_0 \bar{q}_1 \text{ F. states}}} 2\pi\delta(k_0^+ + k_1^+ - q^+) \left|\widetilde{\psi}_{\gamma_\lambda \to q_0 \bar{q}_1}\right|^2 \left[\mathrm{Tr}\left(U_F(\mathbf{x}_0)\, U_F^\dagger(\mathbf{x}_1)\right) - N_c\right] \\
+ &\widetilde{\sum_{q_0 \bar{q}_1 g_2 \text{ F. states}}} 2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+) \left|\widetilde{\psi}_{\gamma_\lambda \to q_0 \bar{q}_1 g_2}\right|^2 \\
&\times \left[\mathrm{Tr}\left(t^{b_2}\, U_F(\mathbf{x}_0)\, t^{a_2}\, U_F^\dagger(\mathbf{x}_1)\right) U_A(\mathbf{x}_2)_{b_2 a_2} - N_c\, C_F\right] + \cdots
\end{aligned}
$$

Dipole factorization for eikonal DIS

⇒ Dipole factorization formula:

$$\sigma_\lambda^\gamma = 2N_c \widetilde{\sum_{q_0\bar{q}_1 \text{ F. states}}} \frac{2\pi\delta(k_0^+ + k_1^+ - q^+)}{2q^+} \left|\widetilde{\psi}_{\gamma_\lambda \to q_0\bar{q}_1}\right|^2 \text{Re}\left[1 - \mathcal{S}_{01}\right]$$
$$+ 2N_c C_F \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. states}}} \frac{2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+)}{2q^+}$$
$$\times \left|\widetilde{\psi}_{\gamma_\lambda \to q_0\bar{q}_1 g_2}\right|^2 \text{Re}\left[1 - \mathcal{S}_{012}^{(3)}\right] + \cdots$$

Dipole operator: $\mathcal{S}_{01} \equiv \frac{1}{N_c} \text{Tr}\left(U_F(\mathbf{x}_0)\, U_F^\dagger(\mathbf{x}_1)\right)$

Tripole operator: $\mathcal{S}_{012}^{(3)} \equiv \frac{1}{N_c\, C_F} \text{Tr}\left(t^b\, U_F(\mathbf{x}_0)\, t^a\, U_F^\dagger(\mathbf{x}_1)\right) U_A(\mathbf{x}_2)_{ba}$

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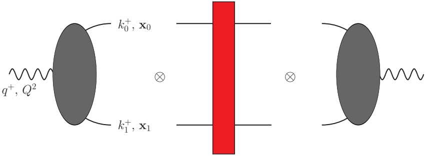

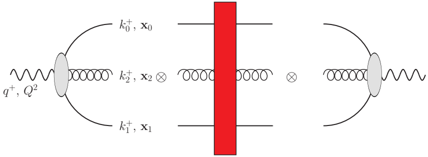

DIS at NLO: previous results

2 independent calculations had been performed earlier for NLO corrections to photon impact factor and/or DIS cross-section:

1. Balitsky, Chirilli, PRD83 (2011); PRD87 (2013)
   Using covariant perturbation theory. Results provided as
   - Current correlator in position space
   - Impact factor for $k_\perp$ factorization $\rightarrow$ Good for BFKL phenomenology

2. G.B., PRD85 (2012)
   Using light-front perturbation theory. Results provided as
   - DIS structure functions in dipole factorization
     $\rightarrow$ Good for gluon saturation phenomenology

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However, in both papers only the $q\bar{q}g$ contribution was calculated explicitly, whereas NLO corrections to the $q\bar{q}$ contribution were guessed. Methods used for that:

1. In Balitsky, Chirilli, PRD83 (2011):
   Matching with older vacuum results. (But not very clear to me.)
2. In G.B., PRD85 (2012):
   Unitary argument. But I realized later that it does not work...

Unitarity sum rule: real photon case

Fock state decomposition of the physical state of an incoming real γ:

$$|\gamma_{\mathrm{dressed}}\rangle = \sqrt{Z_\gamma}\Bigg[a_\gamma^\dagger\,|0\rangle + \sum_{l\bar{l}\ \mathrm{states}} \Psi^\gamma_{l\bar{l}}\ b_l^\dagger\, d_{\bar{l}}^\dagger\,|0\rangle + \sum_{q\bar{q}\ \mathrm{states}} \Psi^\gamma_{q\bar{q}}\ b_q^\dagger\, d_{\bar{q}}^\dagger\,|0\rangle + \sum_{q\bar{q}g\ \mathrm{states}} \Psi^\gamma_{q\bar{q}g}\ b_q^\dagger\, d_{\bar{q}}^\dagger\ a_g^\dagger\,|0\rangle + \cdots\Bigg]$$

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Normalization of both the dressed state and the Fock states implies:

$$\frac{1-Z_\gamma}{Z_\gamma} = \sum_{l\bar{l}\ \text{states}} \left|\Psi^\gamma_{l\bar{l}}\right|^2 + \sum_{q\bar{q}\ \text{states}} \left|\Psi^\gamma_{q\bar{q}}\right|^2 + \sum_{q\bar{q}g\ \text{states}} \left|\Psi^\gamma_{q\bar{q}g}\right|^2 + O(\alpha_{em}\,\alpha_s^2)$$

Perturbative expansion $\Rightarrow$ at each order, one gets a new relation .

Unitarity sum rule: real photon case

In particular, terms of order $\alpha_{em}\,\alpha_s$:

$$\Big(1-Z_\gamma\Big)_{\alpha_{em}\,\alpha_s} = \left(\sum_{q\bar{q}\text{ states}} \left|\Psi^{\gamma}_{q\bar{q}}\right|^2\right)_{\alpha_{em}\,\alpha_s} + \left(\sum_{q\bar{q}g\text{ states}} \left|\Psi^{\gamma}_{q\bar{q}g}\right|^2\right)_{\alpha_{em}\,\alpha_s}$$

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In the previous study (G.B., PRD85 (2012)):

I implicitly assumed that $\Big(1-Z_\gamma\Big)$ received no $\alpha_{em}\,\alpha_s$ contribution, in order to get $\left(\sum_{q\bar{q}\ \mathrm{states}} \left|\Psi^{\gamma}_{q\bar{q}}\right|^2\right)_{\alpha_{em}\,\alpha_s}$ from $\left(\sum_{q\bar{q}g\ \mathrm{states}} \left|\Psi^{\gamma}_{q\bar{q}g}\right|^2\right)_{\alpha_{em}\,\alpha_s}$

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However, there is a non-trivial (and finite) contribution to $\Big(1-Z_\gamma\Big)$ at order $\alpha_{em}\,\alpha_s$.

This discussion is also valid in the virtual (T or L) photon case.

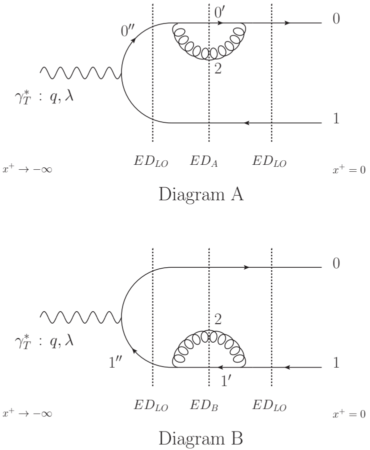
$\Rightarrow$ In this approach, not possible to get the $\left|\Psi^{\gamma}_{q\bar{q}}\right|^2$ at NLO from unitarity!

$\Rightarrow$ One-loop correction to $\Psi^{\gamma}_{q\bar{q}}$ has to be calculated independently

Calculation of the $\gamma_{T,L} \to q\bar{q}$ LF wave-functions at NLO

- Calculation done in Light-front perturbation theory for QCD+QED
- Cut-off k^+_{min} introduced to regulate the small k^+ (soft) divergences
- UV divergences from various tensor integrals, but no UV renormalization at this order.
  $\Rightarrow$ UV divergences (and finite regularization artifacts) have to cancel at cross-section level
  $\Rightarrow$ Use (Conventional) Dimensional Regularization, and pay attention to rational terms in $(D-4)/(D-4)$
- Convenient trick: Tensor reduction of transverse integrals (Passarino-Veltman)
  Allows to organize better the calculation (reduces the number of integrals to calculate and of Dirac structures) and show the cancellation of unphysical divergences already at the integrand level

Diagrams for γ_T and γ_L LFWFs: self-energies



- Straightforward to calculate
- Clearly factors into LO wave-function times Form Factor
- DimReg prevents quadratic UV divergences to appear, only logarithmic ones remain
- Contain not only log but also unphysical $\log^2$ soft divergences

Diagrams for γ_T and γ_L LFWFs: vertex corrections

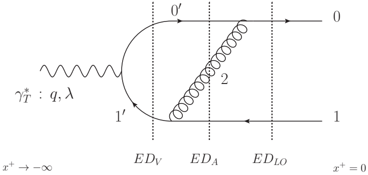


Diagram 1

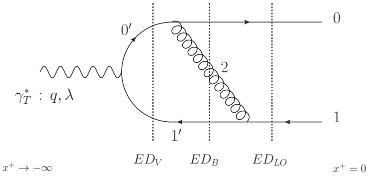


Diagram 2

- By far the hardest to calculate
- Involves various tensor integrals in transverse-momentum as well as various Dirac structures
- Contain unphysical $\log^2$ soft divergences which cancel the ones of the previous graphs.
- In the γ_L case: contain unphysical power-like soft divergences.
- In the γ_T case: even after tensor reduction, still not proportional to the LO LFWF: one extra piece remain. However, it cancels between the diagrams 1 and 2.

Diagrams for γ_T and γ_L LFWFs: vertex corrections

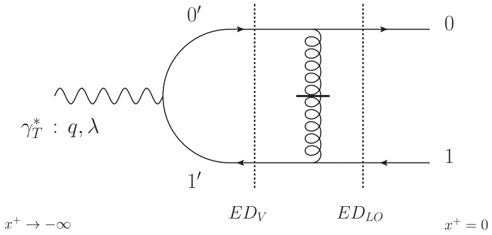
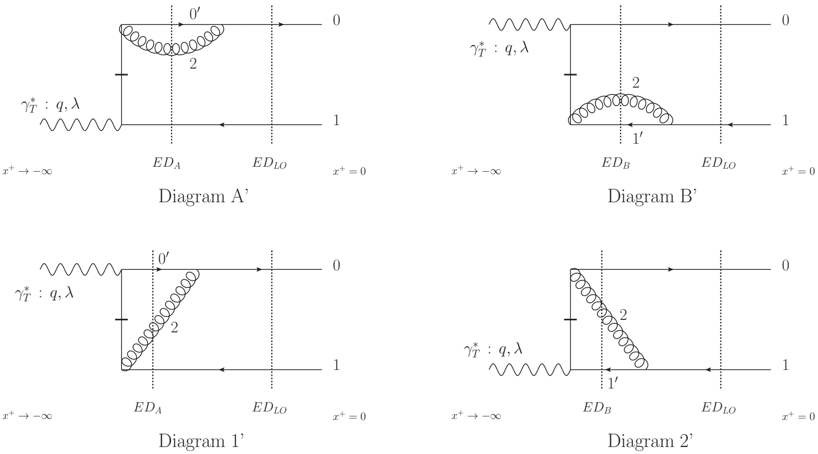


Diagram 3

- In the γ_T case: vanishes due to Lorentz symmetry
- In the γ_L case: non-zero, and cancels the unphysical power-like soft divergences of the other vertex correction graphs.

Diagrams for the $\gamma_T \to q\bar{q}$ LF wave-function only



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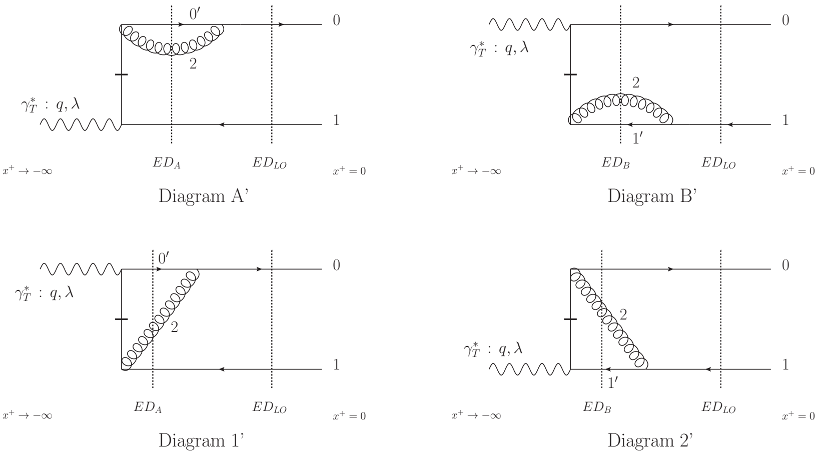

All four vanish due to Lorentz symmetry!

Results for NLO $\gamma_{T,L} \to q\bar{q}$ LFWFs in momentum space

$$\psi_{\gamma^*_{T,L}\to q_0\bar{q}_1} = \left[1 + \left(\frac{\alpha_s\, C_F}{2\pi}\right) \mathcal{V}^{T,L}\right] \psi^{\text{tree}}_{\gamma^*_{T,L}\to q_0\bar{q}_1} + \mathcal{O}(e\,\alpha_s^2)$$

$$\begin{aligned}\mathcal{V}^L &= 2\left[\log\left(\frac{k^+_{\text{min}}}{\sqrt{k^+_0 k^+_1}}\right) + \frac{3}{4}\right]\left[\Gamma\left(2-\frac{D}{2}\right)\left(\frac{\overline{Q}^2}{4\pi\,\mu^2}\right)^{\frac{D}{2}-2} - 2\log\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\overline{Q}^2}\right)\right] \\ &\quad + \frac{1}{2}\left[\log\left(\frac{k^+_0}{k^+_1}\right)\right]^2 - \frac{\pi^2}{6} + 3 + O\left(D-4\right)\end{aligned}$$

$$\mathcal{V}^T = \mathcal{V}^L + 2\left[\log\left(\frac{k^+_{\text{min}}}{\sqrt{k^+_0 k^+_1}}\right) + \frac{3}{4}\right]\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\mathbf{P}^2}\right)\log\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\overline{Q}^2}\right) + O\left(D-4\right)$$

Notations: $\overline{Q}^2 \equiv \frac{k^+_0 k^+_1}{(q^+)^2}\, Q^2$,

and relative transverse momentum: $\mathbf{P} \equiv \mathbf{k}_0 - \frac{k^+_0}{q^+}\,\mathbf{q} = -\mathbf{k}_1 + \frac{k^+_1}{q^+}\,\mathbf{q}$

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$$+\frac{1}{2}\left[\log\left(\frac{k^+_0}{k^+_1}\right)\right]^2 - \frac{\pi^2}{6} + 3 + O\left(D-4\right)$$

$$\mathcal{V}^T = \mathcal{V}^L + 2\left[\log\left(\frac{k^+_{\text{min}}}{\sqrt{k^+_0\,k^+_1}}\right) + \frac{3}{4}\right]\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\mathbf{P}^2}\right)\log\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\overline{Q}^2}\right) + O\left(D-4\right)$$

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Remark: results consistent with the ones of Boussarie, Grabovsky, Szymanowski and Wallon, JHEP11(2016)149

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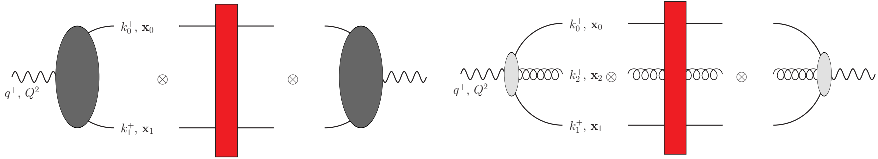
$$\widetilde{\psi}_{\gamma^*_{T,L}\to q_0\bar{q}_1} = \left[1 + \left(\frac{\alpha_s\, C_F}{2\pi}\right)\, \widetilde{\mathcal{V}}^{T,L}\right] \widetilde{\psi}^{\text{tree}}_{\gamma^*_{T,L}\to q_0\bar{q}_1} + \mathcal{O}(e\,\alpha_s^2)$$

$$\begin{aligned}
\widetilde{\mathcal{V}}^T &= \widetilde{\mathcal{V}}^L + O(D-4) \\
&= 2\left[\log\left(\frac{k^+_{\text{min}}}{\sqrt{k^+_0 k^+_1}}\right) + \frac{3}{4}\right]\left[\frac{\Gamma\left(2-\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}-2}} + \log\left(\frac{\mathbf{x}_{01}{}^2\,\mu^2}{4}\right) - 2\Psi(1)\right] \\
&\quad + \frac{1}{2}\left[\log\left(\frac{k^+_0}{k^+_1}\right)\right]^2 - \frac{\pi^2}{6} + 3 + O(D-4)
\end{aligned}$$

- In mixed space: NLO corrections $\Rightarrow$ rescaling of the LO $\gamma_{T,L} \to q\bar{q}$ LFWFs by a factor independent of the photon polarization and virtuality !
- Leftover logarithmic *UV* and soft divergences to be dealt with at cross-section level.

From LFWFs to DIS cross-section



$\widetilde{\psi}^{\gamma^*_{T,L}}_{\gamma^*_{T,L}\rightarrow \bar{q}}$ now known at NLO accuracy in Dim Reg.

$\Rightarrow$ Need to be combined with the $q\bar{q}g$ contribution in the dipole factorization formula at NLO

$\Rightarrow$ $\widetilde{\psi}_{\gamma^*_{T,L}q\bar{q}g}$ is required also in Dim Reg, in order to cancel UV divergences as well as scheme dependent artifacts.

Only the case of σ^{γ}_{L} will be discussed in the following for simplicity. The case of σ^{γ}_{T} can be dealt with in the same way, but gives much longer expressions.

$q\bar{q}$ contribution to σ_L^{γ} at NLO in dim. reg.

$$\widetilde{\psi}^{\text{tree}}_{\gamma_T^* \to q_0 \bar{q}_1} = -e\, e_f\, \mu^{2-\frac{D}{2}}\, (2\pi)^{1-\frac{D}{2}}\, 2Q\, \frac{k_0^+ k_1^+}{(q^+)^2} \left(\frac{\overline{Q}}{|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2} \mathrm{K}_{\frac{D}{2}-2}\Big(|\mathbf{x}_{01}|\,\overline{Q}\Big)\, \overline{u_G}(0)\, \gamma^+ v_G(1)$$

$$\sigma_L^{\gamma}\Big|_{q\bar{q}} = 2N_c \widetilde{\sum_{q_0 \bar{q}_1 \text{ F. states}}} \frac{2\pi\delta(k_0^+ + k_1^+ - q^+)}{2q^+} \left|\widetilde{\psi}^{\text{tree}}_{\gamma_L \to q_0 \bar{q}_1}\right|^2 \mathrm{Re}\left[1 - \mathcal{S}_{01}\right] \times \left[1 + \left(\frac{\alpha_s C_F}{2\pi}\right) \widetilde{\mathcal{V}}^{T,L}\right]^2 + O(\alpha_{em}\, \alpha_s^2)$$

$$\sigma_L^{\gamma}\Big|_{q\bar{q}} = 4N_c\, \alpha_{em} \sum_f e_f^2 \int \frac{\mathrm{d}^{D-2}\mathbf{x}_0}{2\pi} \int \frac{\mathrm{d}^{D-2}\mathbf{x}_1}{2\pi} \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+\, \delta(k_0^+ + k_1^+ - q^+) \times \frac{4Q^2}{(q^+)^5} (k_0^+ k_1^+)^2 \left[\frac{\overline{Q}^2}{(2\pi)^2 \mu^2 x_{01}^2}\right]^{\frac{D}{2}-2} \left[\mathrm{K}_{\frac{D}{2}-2}\Big(|\mathbf{x}_{01}|\,\overline{Q}\Big)\right]^2 \times \left[1 + \left(\frac{\alpha_s C_F}{\pi}\right) \widetilde{\mathcal{V}}^{L}\right] \mathrm{Re}\left[1 - \mathcal{S}_{01}\right] + O(\alpha_{em}\, \alpha_s^2)$$

Tree-level diagrams for $\gamma_L \to q\bar{q}g$ LFWFs

2 diagrams contribute to $\gamma_L \to q\bar{q}g$ (and 4 to $\gamma_T \to q\bar{q}g$):

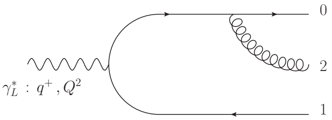


Diagram (a)

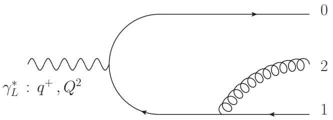


Diagram (b)

$\rightarrow$ Standard calculation in momentum space using LFPT rules, but to be done in dimensional regularization

Then: Fourier transform to mixed space

$\gamma_L \to q\bar{q}g$ LFWF in mixed space

Result:

$$\widetilde{\psi}^{\mathrm{Tree}}_{\gamma_L^* \to q_0 \bar{q}_1 g_2} = e\, e_f\, g\, \varepsilon^{j*}_{\lambda_2} \frac{2Q}{(q^+)^2}$$

$$\times \Bigg\{ k_1^+ \;\; \overline{u_G}(0)\gamma^+ \Big[(2k_0^+ + k_2^+)\delta^{jm} + \frac{k_2^+}{2}[\gamma^j, \gamma^m]\Big] v_G(1) \;\; \mathcal{I}^m\Big(\mathbf{x}_{0+2;1}, \mathbf{x}_{20}; \overline{Q}^2_{(a)}, \mathcal{C}_{(a)}\Big)$$

$$- k_0^+ \;\; \overline{u_G}(0)\gamma^+ \Big[(2k_1^+ + k_2^+)\delta^{jm} - \frac{k_2^+}{2}[\gamma^j, \gamma^m]\Big] v_G(1) \;\; \mathcal{I}^m\Big(\mathbf{x}_{0;1+2}, \mathbf{x}_{21}; \overline{Q}^2_{(b)}, \mathcal{C}_{(b)}\Big) \Bigg\}$$

with the notations:

$$\overline{Q}^2_{(a)} = \frac{k_1^+(q^+ - k_1^+)}{(q^+)^2}\, Q^2 \quad \text{and} \quad \overline{Q}^2_{(b)} = \frac{k_0^+(q^+ - k_0^+)}{(q^+)^2}\, Q^2$$

$$\mathcal{C}_{(a)} = \frac{q^+\, k_0^+\, k_2^+}{k_1^+(k_0^+ + k_2^+)^2} \quad \text{and} \quad \mathcal{C}_{(b)} = \frac{q^+\, k_1^+\, k_2^+}{k_0^+(k_1^+ + k_2^+)^2}$$

And parent dipole vectors defined as:

$$\mathbf{x}_{n+m;p} \;=\; -\mathbf{x}_{p;n+m} \equiv \left(\frac{k_n^+ \mathbf{x}_n + k_m^+ \mathbf{x}_m}{k_n^+ + k_m^+}\right) - \mathbf{x}_p$$

First look at the Fourier integral

$$\mathcal{I}^m\left(\mathbf{r},\mathbf{r}';\overline{Q}^2,\mathcal{C}\right) \equiv (\mu^2)^{2-\frac{D}{2}} \int \frac{d^{D-2}\mathbf{P}}{(2\pi)^{D-2}} \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}} \frac{\mathbf{K}^m\, e^{i\mathbf{K}\cdot\mathbf{r}'}\, e^{i\mathbf{P}\cdot\mathbf{r}}}{\left[\mathbf{P}^2+\overline{Q}^2\right]\left\{\mathbf{K}^2+\mathcal{C}\left[\mathbf{P}^2+\overline{Q}^2\right]\right\}}$$

Introducing Schwinger variables:

$$\begin{aligned}\mathcal{I}^m\left(\mathbf{r},\mathbf{r}';\overline{Q}^2,\mathcal{C}\right) &= \mathbf{r}'^m \left(\mathbf{r}'^2\right)^{1-\frac{D}{2}} \frac{i}{2}\,(2\pi)^{2-D} \left(\mu^2\right)^{2-\frac{D}{2}} \\ &\times \int_0^{+\infty} d\sigma\; \sigma^{1-\frac{D}{2}}\; e^{-\sigma\overline{Q}^2}\; e^{-\frac{\mathbf{r}^2}{4\sigma}}\; \Gamma\left(\frac{D}{2}-1, \frac{\mathbf{r}'^2\mathcal{C}}{4\sigma}\right)\end{aligned}$$

For $D=4$:

$$\mathcal{I}^m\left(\mathbf{r},\mathbf{r}';\overline{Q}^2,\mathcal{C}\right) = \frac{i}{(2\pi)^2} \left(\frac{\mathbf{r}'^m}{\mathbf{r}'^2}\right) \mathrm{K}_0\left(\overline{Q}\sqrt{\mathbf{r}^2+\mathcal{C}\,\mathbf{r}'^2}\right)$$

$q\bar{q}g$ contribution to σ_L^γ at NLO in dim. reg.

$$\sigma_L^\gamma\big|_{q\bar{q}g} = 2N_c C_F \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. states}}} \frac{2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+)}{2q^+} \left|\widetilde{\psi}_{\gamma_L\to q_0\bar{q}_1 g_2}\right|^2 \text{Re}\left[1-\mathcal{S}_{012}^{(3)}\right]$$

$$= 4N_c\,\alpha_{em}\sum_f e_f^2 \int \mathrm{d}^{D-2}\mathbf{x}_0 \int \mathrm{d}^{D-2}\mathbf{x}_1 \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+ \,\frac{4Q^2}{(q^+)^5}$$

$$\times\, 2\alpha_s C_F \int_{k^+_{\min}}^{+\infty} \frac{dk_2^+}{k_2^+}\,\delta(k_0^+ + k_1^+ + k_2^+ - q^+) \int \mathrm{d}^{D-2}\mathbf{x}_2\, \text{Re}\left[1-\mathcal{S}_{012}^{(3)}\right]$$

$$\times\Bigg\{(k_1^+)^2(q^+ - k_1^+)^2\left[2-2\frac{k_2^+}{k_0^+ + k_2^+} + \frac{D-2}{2}\left(\frac{k_2^+}{k_0^+ + k_2^+}\right)^2\right]\left|\mathcal{I}^m\left((a)\right)\right|^2$$

$$+(k_0^+)^2(q^+ - k_0^+)^2\left[2-2\frac{k_2^+}{k_1^+ + k_2^+} + \frac{D-2}{2}\left(\frac{k_2^+}{k_1^+ + k_2^+}\right)^2\right]\left|\mathcal{I}^m\left((b)\right)\right|^2$$

$$-k_0^+ k_1^+\left[2(k_0^+ + k_2^+)k_1^+ + 2k_0^+(k_1^+ + k_2^+) - (D-2)(k_2^+)^2\right]$$

$$\times\text{Re}\left(\mathcal{I}^m\left((a)\right)^* \mathcal{I}^m\left((b)\right)\right)\Bigg\} + O(\alpha_{em}\,\alpha_s^2)$$

UV divergences of the $q\bar{q}g$ contribution to σ_L^γ

UV divergences :

- At $\mathbf{x}_2 \to \mathbf{x}_0$ for $|(a)|^2$ contribution
- At $\mathbf{x}_2 \to \mathbf{x}_1$ for $|(b)|^2$ contribution

For example, for $\mathbf{x}_2 \to \mathbf{x}_0$:

$$\mathrm{d}^{D-2}\mathbf{x}_2 \left|\mathcal{I}^m\left((a)\right)\right|^2 \mathrm{Re}\left[1-\mathcal{S}_{012}^{(3)}\right] \propto \mathrm{d}^{D-2}\mathbf{x}_2 \left({\mathbf{x}_{20}}^2\right)^{3-D} \mathrm{Re}\left[1-\mathcal{S}_{01}\right]$$

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Traditional method to deal with these UV divergences:

1. Make the subtraction $\left[1 - \mathcal{S}_{012}^{(3)}\right] \to \left[1 - \mathcal{S}_{012}^{(3)}\right] - \left[1 - \mathcal{S}_{01}\right]$ in $\sigma_L^\gamma|_{q\bar{q}g}$
2. Add the corresponding term to $\sigma_L^\gamma|_{q\bar{q}}$

It works, but it is far from optimal in the present case!
$\Rightarrow$ Let us present an improvement of that method.

Building the UV subtraction terms

→ Let us define a UV approximation of the Fourier integral:

$$\text{For} \quad |\mathbf{r}'| \to 0 : \qquad \mathcal{I}^m\left(\mathbf{r}, \mathbf{r}'; \overline{Q}^2, \mathcal{C}\right) \;\sim\; \mathcal{I}^m_{UV}\left(\mathbf{r}, \mathbf{r}'; \overline{Q}^2\right)$$

where

$$\mathcal{I}^m_{UV}\left(\mathbf{r}, \mathbf{r}'; \overline{Q}^2\right) \;\equiv\; \mathbf{r}'^m \left(\mathbf{r}'^2\right)^{1-\frac{D}{2}} \frac{i}{(2\pi)^2} \Gamma\left(\frac{D}{2}-1\right) \left(\frac{2\overline{Q}}{(2\pi)^2 \mu^2 |\mathbf{r}|}\right)^{\frac{D}{2}-2} \mathrm{K}_{\frac{D}{2}-2}\left(\overline{Q}\,|\mathbf{r}|\right)$$

⇒ Factorized power-like dependence on the daughter dipole vector $\mathbf{r}'$

Building the UV subtraction terms

$\rightarrow$ Let us define a UV approximation of the Fourier integral:

$$\text{For} \quad |\mathbf{r}'| \to 0 : \qquad \mathcal{I}^m\left(\mathbf{r},\mathbf{r}';\overline{Q}^2,\mathcal{C}\right) \;\sim\; \mathcal{I}^m_{UV}\left(\mathbf{r},\mathbf{r}';\overline{Q}^2\right)$$

where

$$\mathcal{I}^m_{UV}\left(\mathbf{r},\mathbf{r}';\overline{Q}^2\right) \;\equiv\; \mathbf{r}'^m\left(\mathbf{r}'^2\right)^{1-\frac{D}{2}} \frac{i}{(2\pi)^2}\Gamma\left(\frac{D}{2}-1\right)\left(\frac{2\overline{Q}}{(2\pi)^2\mu^2|\mathbf{r}|}\right)^{\frac{D}{2}-2} \mathrm{K}_{\frac{D}{2}-2}\left(\overline{Q}\,|\mathbf{r}|\right)$$

$\Rightarrow$ Factorized power-like dependence on the daughter dipole vector $\mathbf{r}'$

Next idea to deal with the UV divergences : make the subtraction

$$\left\{\left|\mathcal{I}^m((a))\right|^2 \mathrm{Re}\left[1-\mathcal{S}^{(3)}_{012}\right] - \left|\mathcal{I}^m_{UV}\left(\mathbf{x}_{01},\mathbf{x}_{20};\overline{Q}^2_{(a)}\right)\right|^2 \mathrm{Re}\left[1-\mathcal{S}_{01}\right]\right\}$$

Cancels indeed the UV divergence at $\mathbf{x}_2 \to \mathbf{x}_0$, but produces an IR divergence at $|\mathbf{x}_{20}| \to +\infty$, absent in the original term!

Building the UV subtraction terms

Final idea: subtract the IR divergence from the UV subtraction term, as

$$\left\{ \left|\mathcal{I}^m\left((a)\right)\right|^2 \mathrm{Re}\left[1-\mathcal{S}^{(3)}_{012}\right] - \left[\left|\mathcal{I}^m_{UV}\left(\mathbf{x}_{01},\mathbf{x}_{20};\overline{Q}^2_{(a)}\right)\right|^2 \right.\right.$$
$$\left.\left. -\mathrm{Re}\left(\mathcal{I}^{m*}_{UV}\left(\mathbf{x}_{01},\mathbf{x}_{20};\overline{Q}^2_{(a)}\right)\mathcal{I}^m_{UV}\left(\mathbf{x}_{01},\mathbf{x}_{21};\overline{Q}^2_{(a)}\right)\right)\right]\mathrm{Re}\left[1-\mathcal{S}_{01}\right]\right\}$$

This difference leads to a UV and IR finite integral in $\mathbf{x}_2$.

Building the UV subtraction terms

Final idea: subtract the IR divergence from the UV subtraction term, as

$$\left\{ \left|\mathcal{I}^m\left((a)\right)\right|^2 \mathrm{Re}\left[1-\mathcal{S}^{(3)}_{012}\right] - \left[\left|\mathcal{I}^m_{UV}\left(\mathbf{x}_{01},\mathbf{x}_{20};\overline{Q}^2_{(a)}\right)\right|^2 \right.\right.$$
$$\left.\left. -\mathrm{Re}\left(\mathcal{I}^{m*}_{UV}\left(\mathbf{x}_{01},\mathbf{x}_{20};\overline{Q}^2_{(a)}\right)\mathcal{I}^m_{UV}\left(\mathbf{x}_{01},\mathbf{x}_{21};\overline{Q}^2_{(a)}\right)\right)\right] \mathrm{Re}\left[1-\mathcal{S}_{01}\right] \right\}$$

This difference leads to a UV and IR finite integral in $\mathbf{x}_2$.
$\Rightarrow$ The $D \to 4$ limit is now safe to take:

$$\to \frac{1}{(2\pi)^4}\left\{ \frac{1}{\mathbf{x}_{20}^2}\left[\mathrm{K}_0\left(Q\,X_{012}\right)\right]^2 \mathrm{Re}\left[1-\mathcal{S}^{(3)}_{012}\right]\right.$$
$$\left. -\left[\frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2}\cdot\left(\frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2}-\frac{\mathbf{x}_{21}}{\mathbf{x}_{21}^2}\right)\right]\left[\mathrm{K}_0\left(\overline{Q}^2_{(a)}\,|\mathbf{x}_{01}|\right)\right]^2 \mathrm{Re}\left[1-\mathcal{S}_{01}\right]\right\}$$

$$Q^2 X_{012}^2 \equiv \frac{Q^2}{(q^+)^2}\left[k_0^+k_1^+x_{01}^2 + k_0^+k_2^+x_{02}^2 + k_1^+k_2^+x_{12}^2\right] = \frac{q\bar{q}g \text{ form. time}}{\gamma^* \text{ life time}}$$

UV-subtracted $q\bar{q}g$ contribution to σ_L^γ

Subtracting both UV divergences this way:

$$\sigma_L^\gamma|_{q\bar{q}g} - \sigma_L^\gamma|_{UV,|(a)|^2} - \sigma_L^\gamma|_{UV,|(b)|^2}$$
$$= 4N_c\,\alpha_{em}\sum_f e_f^2 \int \frac{\mathrm{d}^2\mathbf{x}_0}{2\pi}\int \frac{\mathrm{d}^2\mathbf{x}_1}{2\pi}\int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+\, \frac{4Q^2}{(q^+)^5}\,\frac{\alpha_s C_F}{\pi}$$
$$\times \int_{k^+_{\min}}^{+\infty}\frac{dk_2^+}{k_2^+}\,\delta(k_0^+ + k_1^+ + k_2^+ - q^+)\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\Big[q \text{ term} + \bar{q} \text{ term} + \text{leftover}\Big]$$

UV-subtracted $q\bar{q}g$ contribution to σ_L^γ

Subtracting both UV divergences this way:

$$\sigma_L^\gamma\big|_{q\bar{q}g} - \sigma_L^\gamma\big|_{UV,|(a)|^2} - \sigma_L^\gamma\big|_{UV,|(b)|^2}$$
$$= 4N_c\,\alpha_{em} \sum_f e_f^2 \int \frac{\mathrm{d}^2\mathbf{x}_0}{2\pi} \int \frac{\mathrm{d}^2\mathbf{x}_1}{2\pi} \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+ \, \frac{4Q^2}{(q^+)^5}\, \frac{\alpha_s C_F}{\pi}$$
$$\times \int_{k^+_{\min}}^{+\infty} \frac{dk_2^+}{k_2^+}\, \delta(k_0^+ + k_1^+ + k_2^+ - q^+) \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \Big[q \text{ term} + \bar{q} \text{ term} + \text{leftover} \Big]$$

With:

$$q \text{ term} = (k_1^+)^2 (q^+ - k_1^+)^2 \left[2 - 2\frac{k_2^+}{k_0^+ + k_2^+} + \frac{D-2}{2}\left(\frac{k_2^+}{k_0^+ + k_2^+}\right)^2 \right] \left[\frac{\mathbf{x}_{20}}{x_{20}^2} \cdot \left(\frac{\mathbf{x}_{20}}{x_{20}^2} - \frac{\mathbf{x}_{21}}{x_{21}^2} \right) \right]$$
$$\times \left\{ \Big[\mathrm{K}_0(QX_{012}) \Big]^2 \, \mathrm{Re}\Big[1 - \mathcal{S}_{012} \Big] - \Big(\mathbf{x}_2 \to \mathbf{x}_0 \Big) \right\}$$

UV-subtracted $q\bar{q}g$ contribution to σ_L^γ

Subtracting both UV divergences this way:

$$\sigma_L^\gamma\big|_{q\bar{q}g} - \sigma_L^\gamma\big|_{UV,|(a)|^2} - \sigma_L^\gamma\big|_{UV,|(b)|^2}$$
$$= 4N_c\,\alpha_{em} \sum_f e_f^2 \int \frac{\mathrm{d}^2\mathbf{x}_0}{2\pi} \int \frac{\mathrm{d}^2\mathbf{x}_1}{2\pi} \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+ \,\frac{4Q^2}{(q^+)^5}\, \frac{\alpha_s C_F}{\pi}$$
$$\times \int_{k^+_{\min}}^{+\infty} \frac{dk_2^+}{k_2^+}\, \delta(k_0^+ + k_1^+ + k_2^+ - q^+) \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \Big[q \text{ term} + \bar{q} \text{ term} + \text{leftover} \Big]$$

With:

$$\bar{q}\text{ term} = (k_0^+)^2 (q^+ - k_0^+)^2 \left[2 - 2\frac{k_2^+}{k_1^+ + k_2^+} + \frac{D-2}{2}\left(\frac{k_2^+}{k_1^+ + k_2^+}\right)^2 \right] \left[\frac{\mathbf{x}_{21}}{x_{21}^2} \cdot \left(\frac{\mathbf{x}_{21}}{x_{21}^2} - \frac{\mathbf{x}_{20}}{x_{20}^2} \right) \right]$$
$$\times \left\{ \Big[\mathrm{K}_0(QX_{012}) \Big]^2 \mathrm{Re}\Big[1 - \mathcal{S}_{012} \Big] - \Big(\mathbf{x}_2 \to \mathbf{x}_1 \Big) \right\}$$

UV-subtracted $q\bar{q}g$ contribution to σ_L^γ

Subtracting both UV divergences this way:

$$\sigma_L^\gamma|_{q\bar{q}g} - \sigma_L^\gamma|_{UV,|(a)|^2} - \sigma_L^\gamma|_{UV,|(b)|^2}$$

$$= 4N_c\,\alpha_{em}\sum_f e_f^2 \int\frac{\mathrm{d}^2\mathbf{x}_0}{2\pi}\int\frac{\mathrm{d}^2\mathbf{x}_1}{2\pi}\int_0^{+\infty}dk_0^+\int_0^{+\infty}dk_1^+\,\frac{4Q^2}{(q^+)^5}\,\frac{\alpha_s C_F}{\pi}$$

$$\times\int_{k^+_{\min}}^{+\infty}\frac{dk_2^+}{k_2^+}\,\delta(k_0^+ + k_1^+ + k_2^+ - q^+)\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\Big[q\ \mathrm{term} + \bar{q}\ \mathrm{term} + \mathrm{leftover}\Big]$$

With:

$$\mathrm{leftover} \;=\; (k_2^+)^2(q^+ - k_2^+)^2\left(\frac{\mathbf{x}_{20}}{x_{20}^2}\cdot\frac{\mathbf{x}_{21}}{x_{21}^2}\right)\Big[\mathrm{K}_0(QX_{012})\Big]^2\,\mathrm{Re}\Big[1-\mathcal{S}_{012}\Big]$$

Combining the UV terms with the $q\bar{q}$ contribution to σ_L^γ

In dim. reg., the UV subtraction terms can be written as

$$\sigma_L^\gamma\big|_{UV,|(a)|^2} + \sigma_L^\gamma\big|_{UV,|(b)|^2}$$
$$= 4N_c\,\alpha_{em} \sum_f e_f^2 \int \frac{\mathrm{d}^{D-2}\mathbf{x}_0}{2\pi} \int \frac{\mathrm{d}^{D-2}\mathbf{x}_1}{2\pi} \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+\, \delta(k_0^+ + k_1^+ - q^+)$$
$$\times \frac{4Q^2}{(q^+)^5}\,(k_0^+ k_1^+)^2 \left[\frac{\overline{Q}^2}{(2\pi)^2 \mu^2 x_{01}^2}\right]^{\frac{D}{2}-2} \left[\mathrm{K}_{\frac{D}{2}-2}\Big(|\mathbf{x}_{01}|\,\overline{Q}\Big)\right]^2$$
$$\times \left(\frac{\alpha_s\, C_F}{\pi}\right) \left[\widetilde{\mathcal{V}}^L_{UV,|(a)|^2} + \widetilde{\mathcal{V}}^L_{UV,|(b)|^2}\right] \;\mathrm{Re}\left[1 - \mathcal{S}_{01}\right]$$

Combining the UV terms with the $q\bar{q}$ contribution to σ_L^γ

In dim. reg., the UV subtraction terms can be written as

$$\sigma_L^\gamma\big|_{UV,|(a)|^2} + \sigma_L^\gamma\big|_{UV,|(b)|^2}$$
$$= 4N_c\,\alpha_{em}\sum_f e_f^2 \int \frac{\mathrm{d}^{D-2}\mathbf{x}_0}{2\pi}\int \frac{\mathrm{d}^{D-2}\mathbf{x}_1}{2\pi}\int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+\,\delta(k_0^+ + k_1^+ - q^+)$$
$$\times \frac{4Q^2}{(q^+)^5}\,(k_0^+ k_1^+)^2 \left[\frac{\overline{Q}^2}{(2\pi)^2\mu^2 x_{01}^2}\right]^{\frac{D}{2}-2} \left[\mathrm{K}_{\frac{D}{2}-2}\Big(|\mathbf{x}_{01}|\,\overline{Q}\Big)\right]^2$$
$$\times \left(\frac{\alpha_s\, C_F}{\pi}\right) \left[\widetilde{\mathcal{V}}^L_{UV,|(a)|^2} + \widetilde{\mathcal{V}}^L_{UV,|(b)|^2}\right]\ \mathrm{Re}\left[1 - \mathcal{S}_{01}\right]$$

With:

$$\widetilde{\mathcal{V}}^L_{UV,|(a)|^2} = \Gamma\left(\frac{D}{2}-2\right)\,\left(\pi\mu^2\mathbf{x}_{01}^2\right)^{2-\frac{D}{2}} \left[\log\left(\frac{k_{\min}^+}{k_0^+}\right) + \frac{3}{4} - \frac{(D-4)}{8}\right]$$

Combining the UV terms with the $q\bar{q}$ contribution to σ_L^γ

In dim. reg., the UV subtraction terms can be written as

$$\sigma_L^\gamma\Big|_{UV,|(a)|^2} + \sigma_L^\gamma\Big|_{UV,|(b)|^2}$$
$$= 4N_c\,\alpha_{em} \sum_f e_f^2 \int \frac{\mathrm{d}^{D-2}\mathbf{x}_0}{2\pi} \int \frac{\mathrm{d}^{D-2}\mathbf{x}_1}{2\pi} \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+\, \delta(k_0^+ + k_1^+ - q^+)$$
$$\times \frac{4Q^2}{(q^+)^5}\,(k_0^+ k_1^+)^2 \left[\frac{\overline{Q}^2}{(2\pi)^2\mu^2 x_{01}^2}\right]^{\frac{D}{2}-2} \left[\mathrm{K}_{\frac{D}{2}-2}\Big(|\mathbf{x}_{01}|\,\overline{Q}\Big)\right]^2$$
$$\times \left(\frac{\alpha_s\, C_F}{\pi}\right) \left[\widetilde{\mathcal{V}}^L_{UV,|(a)|^2} + \widetilde{\mathcal{V}}^L_{UV,|(b)|^2}\right]\ \mathrm{Re}\left[1 - \mathcal{S}_{01}\right]$$

With:

$$\widetilde{\mathcal{V}}^L_{UV,|(b)|^2} = \Gamma\left(\frac{D}{2}-2\right)\,\left(\pi\mu^2\mathbf{x}_{01}^2\right)^{2-\frac{D}{2}} \left[\log\left(\frac{k_{\min}^+}{k_1^+}\right) + \frac{3}{4} - \frac{(D-4)}{8}\right]$$

Combining the UV terms with the $q\bar{q}$ contribution to σ_L^γ

Expanding around $D = 4$:

$$\widetilde{\mathcal{V}}^L_{UV,|(a)|^2} + \widetilde{\mathcal{V}}^L_{UV,|(b)|^2} = -2 \left[\frac{1}{\left(2-\frac{D}{2}\right)} - \Psi(1) + \log\left(\pi\, \mathbf{x}_{01}{}^2\, \mu^2\right) \right] \times \left[\log\left(\frac{k^+_{\text{min}}}{\sqrt{k^+_0\, k^+_1}} \right) + \frac{3}{4} \right] - \frac{1}{2} + O(D-4)$$

Combining the UV terms with the $q\bar{q}$ contribution to σ_L^γ

Expanding around $D = 4$:

$$\widetilde{\mathcal{V}}^L_{UV,|(a)|^2} + \widetilde{\mathcal{V}}^L_{UV,|(b)|^2} = -2\left[\frac{1}{(2-\frac{D}{2})} - \Psi(1) + \log\left(\pi\, \mathbf{x}_{01}{}^2\, \mu^2\right)\right] \times \left[\log\left(\frac{k^+_{\min}}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right] - \frac{1}{2} + O(D-4)$$

But in the $q\bar{q}$ contribution to σ_L^γ:

$$\widetilde{\mathcal{V}}^L = 2\left[\frac{1}{(2-\frac{D}{2})} - \Psi(1) + \log\left(\pi\, \mathbf{x}_{01}{}^2\, \mu^2\right)\right]\left[\log\left(\frac{k^+_{\min}}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right] + \frac{1}{2}\left[\log\left(\frac{k_0^+}{k_1^+}\right)\right]^2 - \frac{\pi^2}{6} + \frac{5}{2} + \frac{1}{2} + O(D-4)$$

$\Rightarrow$ Cancelation of:

- the UV divergence
- the $k^+_{\min}$ dependence
- the $\pm 1/2$ rational term : strong hint of UV regularization scheme independence

Combining the UV terms with the $q\bar{q}$ contribution to σ_L^γ

Final result for the dipole-like terms:

$$\sigma_L^\gamma|_{q\bar{q}} + \sigma_L^\gamma|_{UV,|(a)|^2} + \sigma_L^\gamma|_{UV,|(b)|^2}$$
$$= 4N_c\, \alpha_{em} \sum_f e_f^2 \int \frac{\mathrm{d}^2\mathbf{x}_0}{2\pi} \int \frac{\mathrm{d}^2\mathbf{x}_1}{2\pi} \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+\, \delta(k_0^+ + k_1^+ - q^+)\, \frac{4Q^2}{(q^+)^5}$$
$$\times (k_0^+ k_1^+)^2 \left[\mathrm{K}_0\left(|\mathbf{x}_{01}|\, \overline{Q}\right)\right]^2 \left(\frac{\alpha_s\, C_F}{\pi}\right) \left[1 + \left(\frac{\alpha_s\, C_F}{\pi}\right)\, \widetilde{\mathcal{V}}^L_{\mathrm{reg.}}\right]\ \mathrm{Re}\left[1 - \mathcal{S}_{01}\right]$$

With:

$$\widetilde{\mathcal{V}}^L_{\mathrm{reg.}} \equiv \widetilde{\mathcal{V}}^L + \widetilde{\mathcal{V}}^L_{UV,|(a)|^2} + \widetilde{\mathcal{V}}^L_{UV,|(b)|^2}$$
$$= \frac{1}{2}\left[\log\left(\frac{k_0^+}{k_1^+}\right)\right]^2 - \frac{\pi^2}{6} + \frac{5}{2}$$

To do next: BK/JIMWLK resummation

1. Assign $k^+_{\min}$ to the scale set by the target: $k^+_{\min} = \frac{Q_0^2}{2x_0\, P^-} = \frac{x_{Bj}\, Q_0^2}{x_0\, Q^2}\, q^+$
2. Choose a factorization scale $k_f^+ \lesssim k_0^+, k_1^+$, corresponding to a range for the high-energy evolution $Y_f^+ \equiv \log\left(\frac{k_f^+}{k^+_{\min}}\right) = \log\left(\frac{x_0\, Q^2\, k_f^+}{x_{Bj}\, Q_0^2\, q^+}\right)$
3. In the LO term in the observable, make the replacement

$$\langle \mathcal{S}_{01}\rangle_0 \;=\; \langle \mathcal{S}_{01}\rangle_{Y_f^+} - \int_0^{Y_f^+} dY^+ \left(\partial_{Y^+}\langle \mathcal{S}_{01}\rangle_{Y^+}\right)$$

   with both terms calculated with the same evolution equation
4. Combine the second term with the NLO correction to cancel its $k^+_{\min}$ dependence and the associated large logs.

$\Rightarrow$ Works straightforwardly in the case of

- the naive LL BK equation
- the kinematically improved BK equation as implemented in G.B., PRD89 (2014)

Should also work with the other implementation (Iancu *et al.*, PLB744 (2015)), but might require a bit more work.

Conclusions

- Direct calculation of $\gamma_{T,L} \to q\bar{q}$ LFWFs at one-gluon-loop order, both in momentum and in mixed space
- Full NLO correction to F_L and F_T obtained from the combination of the $q\bar{q}$ and $q\bar{q}g$ contributions:
  UV Dim. Reg. used in both cases, in order to have the finite terms under control.
- More refined method proposed to cancel the UV divergences between the $q\bar{q}$ and $q\bar{q}g$ contributions:
  1. Minimally transfers terms from one to the other
  2. Each of the 4 terms in the final result seems well behaved
     $\Rightarrow$ good numerical stability is expected
  3. Only the q-term and the $\bar{q}$-term relevant for High-Energy LL resummation

Ambiguity in the way of cancelling UV divergences is reminiscent of the similar issue for the cancellation of soft and collinear divergence in higher order pQCD calculations (antenna subtraction, etc...)

Outlook

Phenomenology outlook : All ingredients soon available for fits to HERA data at NLO+LL accuracy, and hopefully NLO+NLL accuracy, in the dipole factorization, including gluon saturation.

Theory outlook :

- Application of the NLO $\gamma_{T,L} \to q\bar{q}(g)$ LFWFs to calculate other DIS observables at NLO?
- Extension to the case of massive quarks?
- The techniques developed here should be useful for future NLO calculations in the CGC