Polarized parton production in DIS at small x

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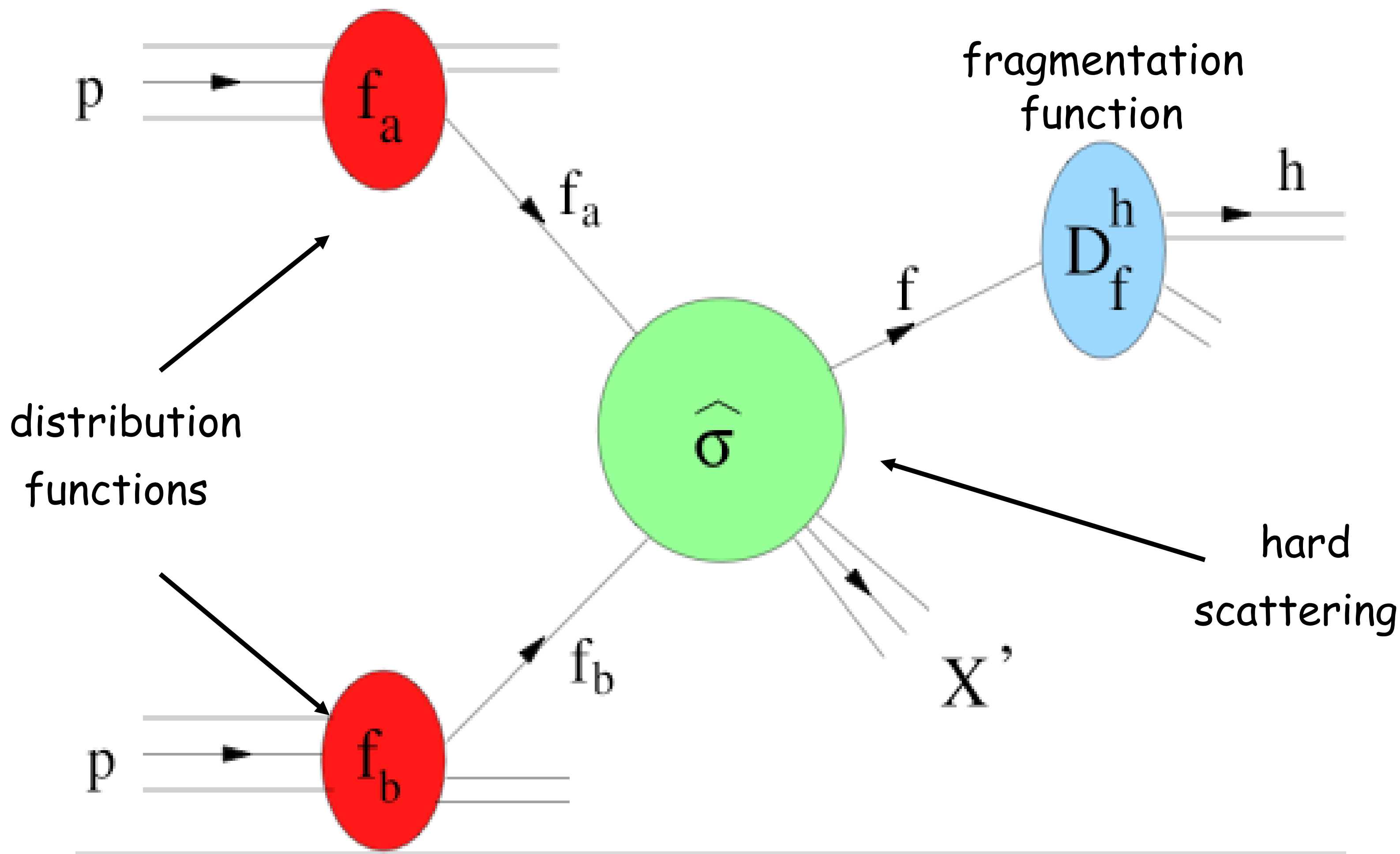
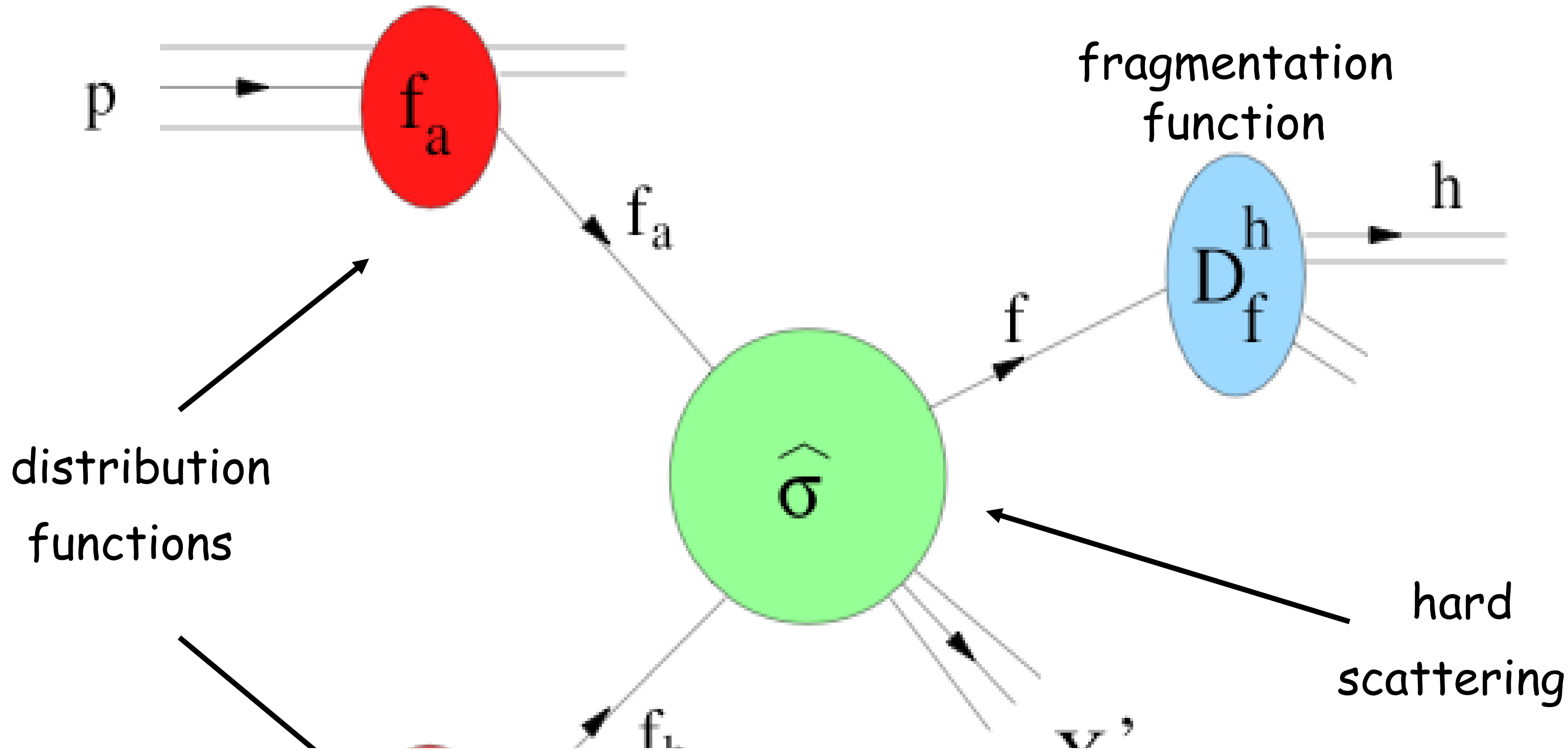
pQCD in pp Collisions

Collinear factorization :separation of long and short distances



pQCD in pp Collisions

<u>*Collinear factorization*</u> :separation of long and short distances

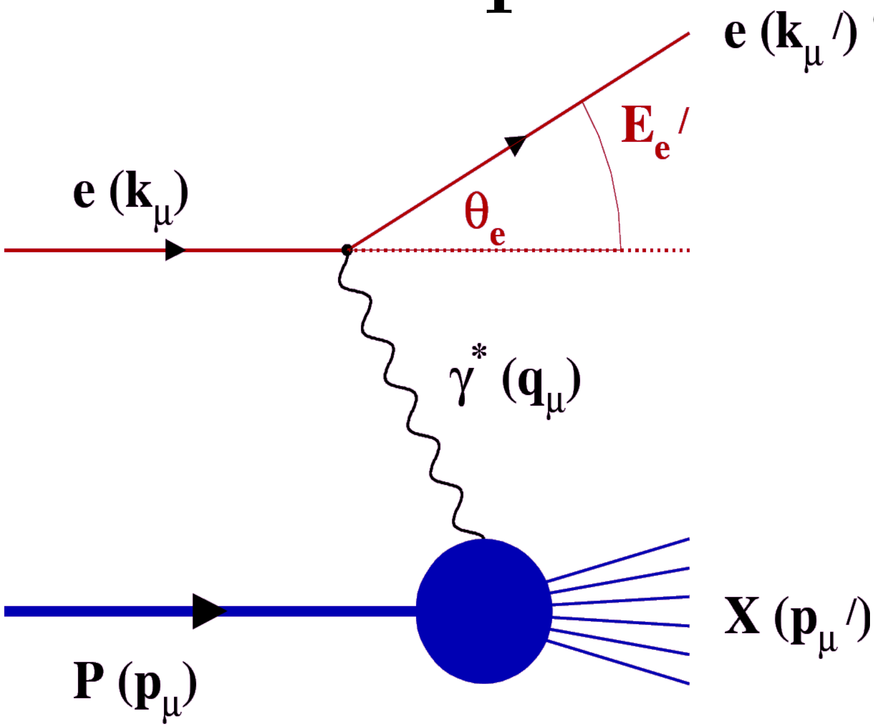


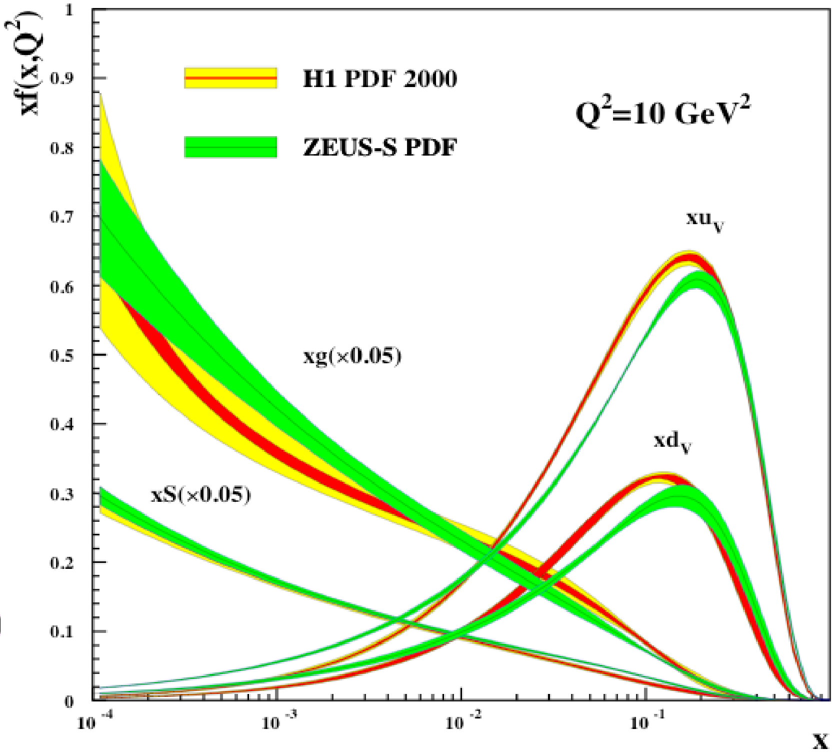
$$d\sigma = \int dx_1\, dx_2\, dz\, f_a^{H1}(x_1, M^2)\, f_b^{H2}(x_2, M^2)\, D_c^h(z, M^2)$$

$$\otimes \quad d\hat{\sigma}_{ab}^c(x_1 P_{H1}, x_2 P_{H2}, P_h/z, M^2)$$

DIS at HERA: parton distributions

$$Q^2 \equiv -q^2 \quad x = \frac{p^+}{P^+}$$





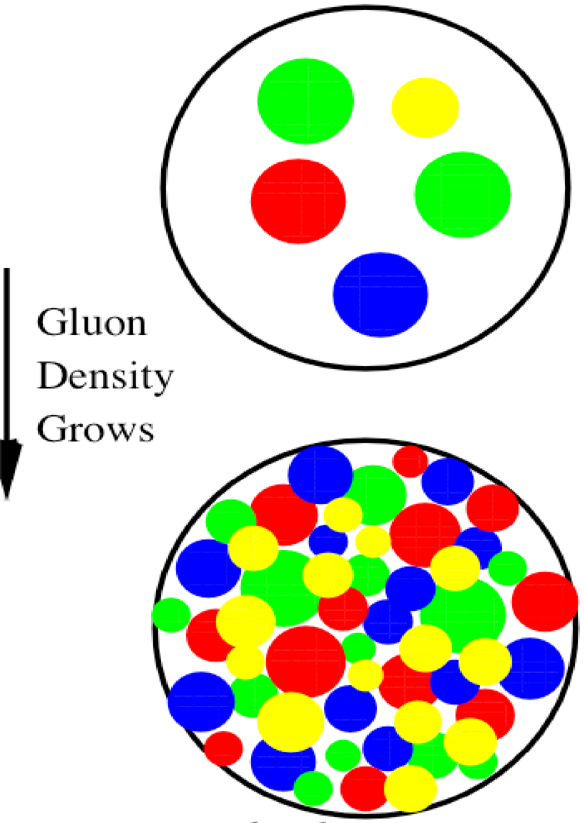
power-like growth of gluon and sea quark distributions with x

new QCD dynamics at small x?

Gluon saturation

Gribov-Levin-Ryskin
Mueller-Qiu

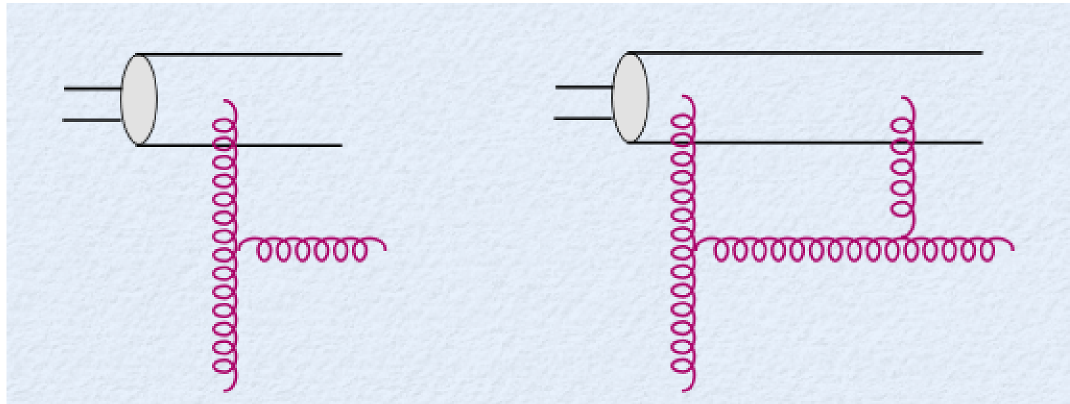
"attractive" bremsstrahlung
vs.
"repulsive" recombination



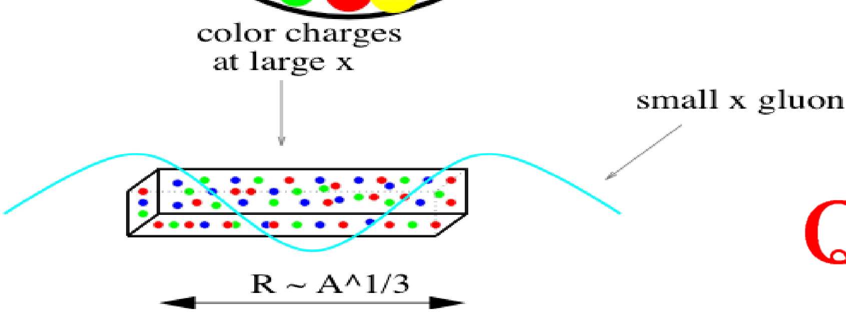
Low Energy

$$\frac{1}{x}$$ ↓

High Energy



$$\frac{\alpha_s}{Q^2}\,\frac{xG(x, b_t, Q^2)}{S_\perp} \sim 1$$



$$Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}$$

Leading twist pQCD: collinear factorization

DGLAP evolution of partons

number of partons increases with Q^2

parton "size" decreases

excellent tool for high Q^2 inclusive observables

higher twists become important at low Q^2

Does not include:

shadowing

multiple scattering

diffraction

....

Saturation effects break collinear factorization:

multiple scattering

evolution with energy (x or rapidity)

Need a new formalism

MV effective Action + Wilsonian RGE

$$\mathbf{S[A,\rho] = -\frac{1}{4}\int d^4x\, F^2_{\mu\nu} + \frac{i}{N_c}\int d^2x_t dx^-\, \delta(x^-) Tr[\rho(x_t) U(A^-)]}$$

Large x: color source ρ small x: gluon field $\mathbf{A}^\mu$

$$\mathbf{U(A^-) = \hat{P}\, Exp\left[ig \int dx^+\, A_a^-\, T_a\right]}$$

$$\mathbf{Z[j] = \int [D\rho]\, W_{\Lambda^+}[\rho] \left[\frac{\int^{\Lambda^+}[DA]\delta(A^+)e^{iS[A,\rho]-\int j\cdot A}}{\int^{\Lambda^+}[DA]\delta(A^+)e^{iS[A,\rho]}}\right]}$$

$\mathbf{W_{\Lambda^+}[\rho]}$

weight functional:
probability distribution of color source ρ
at longitudinal scale $\mathbf{\Lambda^+}$

invariance under change of $\mathbf{\Lambda^+}$ ⟶ RGE for $\mathbf{W_{\Lambda^+}[\rho]}$

static color charges ρ

classical equations of motion

$$\mathbf{D}_\mu\, \mathbf{F}_{\mathbf{a}}^{\mu\nu} = \mathbf{g}\, \mathbf{J}_{\mathbf{a}}^{\nu} \qquad \textit{with} \qquad \boxed{\mathbf{J}_{\mathbf{a}}^{\mu}(\mathbf{x}) \equiv \delta^{\mu +}\, \delta(\mathbf{x}^-)\, \rho_{\mathbf{a}}(\mathbf{x_t})}$$

solution in light cone gauge ($A^+ = 0$) :

can not be inverted

$$A_a^- = 0$$
$$A_i^a = \theta(x^-)\, \alpha_i^a(x_t)$$

and

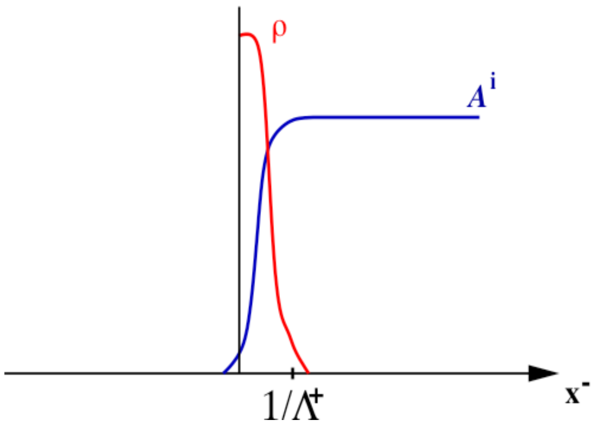
$$\partial_i \alpha_i^a(x_t) = g\, \rho^a(x_t)$$
$$\alpha_i = \frac{i}{g} U(x_t)\partial_i U^\dagger(x_t)$$

$$\mathbf{F}^{+\,\mathbf{i}} \sim \delta(\mathbf{x}^-)\alpha^{\mathbf{i}} \neq \mathbf{0}$$

solution is a 2-d pure gauge

it is (LC) time-independent

the only "physical" color fields $\mathbf{E}_\perp^{\mathbf{a}}$, $\mathbf{B}_\perp^{\mathbf{a}}$



JIMWLK evolution equation

$$\frac{d}{d\,ln 1/x}\langle O\rangle = \frac{1}{2}\left\langle \int d^2x\, d^2y\, \frac{\delta}{\delta\alpha_x^b}\,\eta_{xy}^{bd}\,\frac{\delta}{\delta\alpha_y^d}\,O\right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi}\int \frac{d^2z}{(2\pi)^2}\frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2}\left[\underbrace{1+U_x^\dagger U_y}_{\text{virtual}} - \underbrace{U_x^\dagger U_z - U_z^\dagger U_y}_{\text{real}}\right]^{bd}$$

probes

dense-dense (AA, ...) collisions

need quite a bit of modeling

↓

much less modeling

dilute-dense (pA, ...) collisions

DIS

structure functions (diffraction)
NLO di-hadron/jet correlations
3-hadron/jet angular correlations

Signatures in production spectra

multiple scattering: p_t broadening

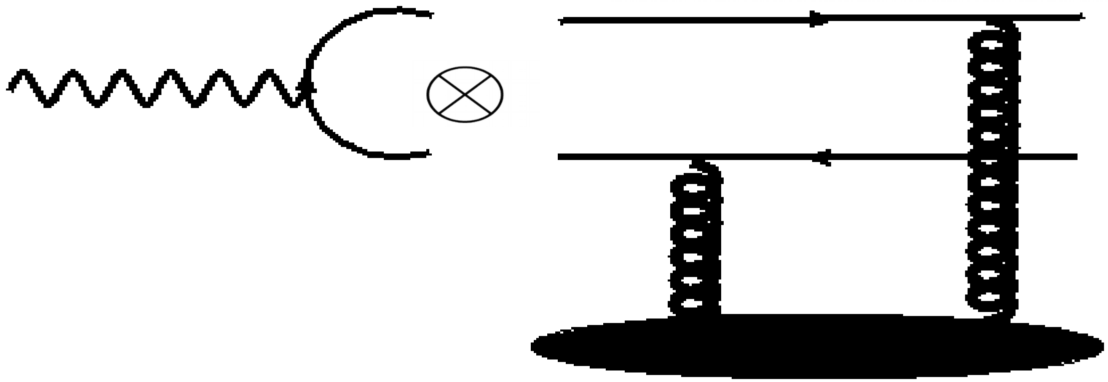
x-evolution: suppression of spectra/away side peaks

DIS total cross section (F_2, F_L)

$$\sigma^{\rm total}_{\rm DIS} = 2\int_0^1 dz \int d^2x_t\, d^2y_t \left|\Psi(k^\pm, k_t | z, x_t, y_t)\right|^2 T(x_t, y_t)$$

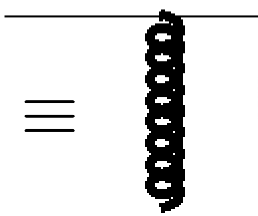
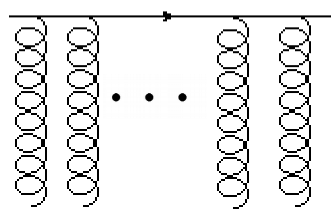
***dipole** cross section*

$$T(x_t, y_t) \equiv \frac{1}{N_c} \mathrm{Tr} \left\langle 1 - V(x_t) V^\dagger(y_t) \right\rangle$$



Golec-Biernat-Wusthoff

$$T(r_t) \sim \left[1 - e^{-r_t^2\, Q^2}\right]$$

$V(x_t) \equiv$  $\equiv$  $\sim 1 + O(g\,A) + O(g^2\, A^2)$

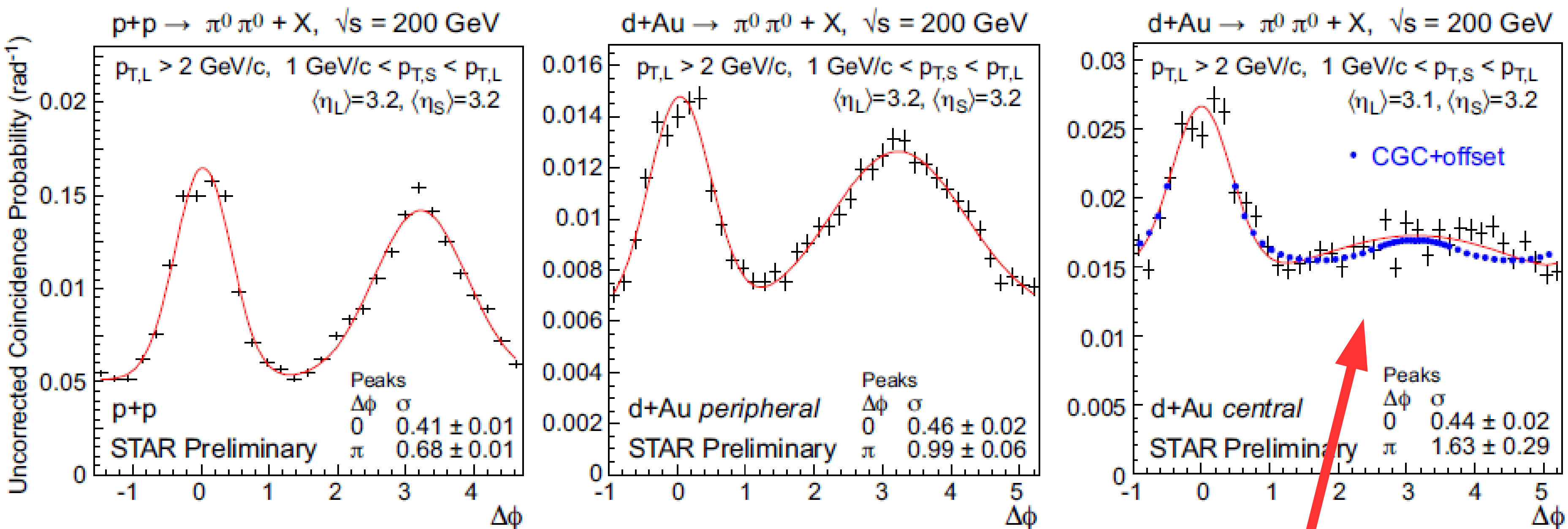
***multiple soft scatterings** encoded in Wilson line V*

***energy (rapidity or x) dependence** via JIMWLK evolution of correlators of V's*

disappearance of back to back hadrons in pA collisions

Marquet, NPA (2007)

Recent STAR measurement (arXiv:1008.3989v1):



CGC fit from Albacete + Marquet, PRL (2010)
Tuchin, NPA846 (2010)
A. Stasto, B-W. Xiao, F. Yuan, PLB716 (2012)
T. Lappi, H. Mantysaari, NPA908 (2013)

broadening + reduction

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

di-hadron (azimuthal) angular correlations in DIS

LO: $\gamma^\star(\mathbf{k})\,\mathbf{T} \rightarrow \mathbf{q}(\mathbf{p})\,\bar{\mathbf{q}}(\mathbf{q})\,\mathbf{X}$ ***momentum space***

Target (proton, nucleus) as a classical color field

building block: quark propagator in the background color field
solution of Dirac equation

$$\mathrm{S}_F(q,p) \equiv \underbrace{(2\pi)^4\delta^4(p-q)\,S_F^0(p)}_{\text{no interaction}} + S_F^0(q)\,\underbrace{\tau_f(q,p)}_{\text{interaction}}\,S_F^0(p) \quad \text{with} \quad S_F^0(p) = \frac{i}{\not{p}+i\epsilon}$$

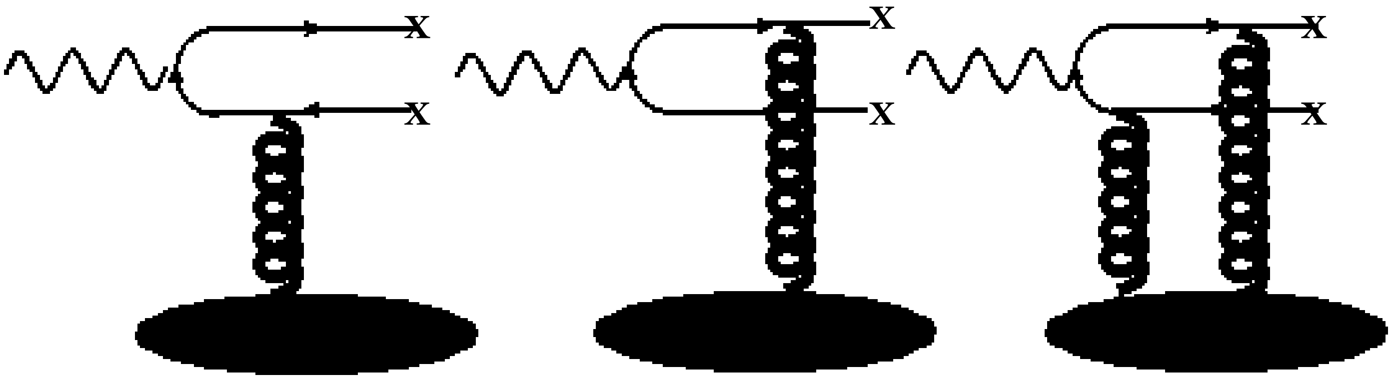
$$\tau_f(q,p) \equiv (2\pi)\delta(p^+ - q^+)\,\gamma^+ \int d^2x_t\, e^{i(q_t-p_t)\cdot x_t} \left\{\theta(p^+)[V(x_t)-1] - \theta(-p^+)[V^\dagger(x_t)-1]\right\}$$

$\gamma^\star(\mathbf{k})\,\mathbf{T} \to \mathbf{q}(\mathbf{p})\,\bar{\mathbf{q}}(\mathbf{q})\,\mathbf{X}$

$$
\begin{aligned}
i\mathcal{A} &= (ie)\bar{u}(p)\left[S_F^{(0)}(p)\right]^{-1} S_F(p, k-q)\,\not\epsilon(k)\,S_F(p-k,-q)\left[S_F^{(0)}(-q)\right]^{-1} v(q) \\
&\equiv i\mathcal{A}_1 + i\mathcal{A}_2 + i\mathcal{A}_3
\end{aligned}
$$

with

$$
\begin{aligned}
i\mathcal{A}_1 &= (ie)\bar{u}(p)\,\tau_F(p,k-q)\,S_F^{(0)}(k-q)\,\not\epsilon(k)\,v(q) \\
i\mathcal{A}_2 &= (ie)\bar{u}(p)\,\not\epsilon(k)\,S_F^{(0)}(p-k)\,\tau_F(p-k,-q)\,v(q) \\
i\mathcal{A}_3 &= (ie)\int \frac{d^4k_1}{(2\pi)^4}\bar{u}(p)\,\tau_F(p,k_1)\;S_F^{(0)}(k_1)\,\not\epsilon(k)\,S_F^{(0)}(k_1-k)\,\tau_F(k_1-k,-q)\,v(q)
\end{aligned}
$$



simplify: τ has two parts V and -1, multiply out
cancellation of A1, A2 with parts of A3

$V(x_t)\otimes(-1)$

$(-1)\otimes V^\dagger(y_t)$

di-hadron production in DIS

$$\gamma^\star(\mathbf{k})\,\mathbf{T} \to \mathbf{q}(\mathbf{p})\,\bar{\mathbf{q}}(\mathbf{q})\,\mathbf{X}$$

$$\mathcal{A}^\mu(k,q,p) = \frac{i}{2}\int \frac{d^2 l_t}{(2\pi)^2} d^2x_t d^2y_t\, e^{i(p_t+q_t-k_t-l_t)\cdot y_t}$$
$$e^{il_t\cdot x_t}\,\overline{u}(p)\,\Gamma^\mu\, v(q)\;\left[V(x_t)V^\dagger(y_t) - 1\right]$$

with

quadrupoles

$$\Gamma^\mu \equiv$$
$$\frac{\gamma^+(\not{p}-\not{l}+m)\gamma^\mu(\not{p}-\not{k}-\not{l}+m)\gamma^+}{p^+[(p_t-l_t)^2+m^2-2q^+k^-]+q^+[(p_t-k_t-l_t)^2+m^2]}$$

F. Gelis and J. Jalilian-Marian, PRD67 (2003) 074019
Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037

spinor helicity methods

Review:
L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_\pm(k) \equiv \frac{1}{2}(1\pm\gamma_5)\,u(k) \qquad \overline{u_\pm(k)} \equiv \overline{u(k)}\,\frac{1}{2}(1\mp\gamma_5)$$

$$v_\mp(k) \equiv \frac{1}{2}(1\pm\gamma_5)\,v(k) \qquad \overline{v_\mp(k)} \equiv \overline{v(k)}\,\frac{1}{2}(1\mp\gamma_5)$$

helicity operator

$$\mathrm{h} \equiv \vec{\Sigma}\cdot\hat{p} = \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix}$$

$$\begin{aligned} \vec{\Sigma}\cdot\hat{p}\,u_\pm(p) &= \pm u_\pm(p) \\ -\vec{\Sigma}\cdot\hat{p}\,v_\pm(p) &= \pm v_\pm(p) \end{aligned}$$

$$u_+(k) = v_-(k) = \frac{1}{2^{1/4}}\begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-}e^{i\phi_k} \\ \sqrt{k^+} \\ \sqrt{k^-}e^{i\phi_k} \end{bmatrix} \qquad u_-(k) = v_+(k) = \frac{1}{2^{1/4}}\begin{bmatrix} \sqrt{k^-}e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-}e^{-i\phi_k} \\ \sqrt{k^+} \end{bmatrix}$$

with $e^{\pm i\phi_k} \equiv \dfrac{k_x \pm i k_y}{\sqrt{2k^+\,k^-}} = \sqrt{2}\,\dfrac{k_t\cdot\epsilon_\pm}{k_t}$ and

$$\begin{aligned} k^\pm &= \frac{E\pm k_z}{\sqrt{2}} \\ \epsilon_\pm &= \frac{1}{\sqrt{2}}(1,\pm i) \end{aligned}$$

$$\begin{aligned} n^\mu &= (n^+ = 0, n^- = 1, n_\perp = 0) \\ \bar{n}^\mu &= (\bar{n}^+ = 1, \bar{n}^- = 0, \bar{n}_\perp = 0) \end{aligned}$$

spinor helicity methods

notation:

$$|i^{\pm}>\equiv|k_i^{\pm}>\equiv u_{\pm}(k_i)=v_{\mp}(k_i) \qquad <i^{\pm}|\equiv<k_i^{\pm}|\equiv\overline{u}_{\pm}(k_i)=\overline{v}_{\mp}(k_i)$$

basic spinor products:

$$<i\,j> \;\equiv\; <i^-|j^+>=\overline{u}_-(k_i)\,u_+(k_j) \;=\; \sqrt{|s_{ij}|}e^{i\phi_{ij}}$$

$$[i\,j] \;\equiv\; <i^+|j^->=\overline{u}_+(k_i)\,u_-(k_j) \;=\; -\sqrt{|s_{ij}|}e^{-i\phi_{ij}}$$

$$\cos\phi_{ij} = \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

$$\sin\phi_{ij} = \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

with

$$s_{ij} = (k_i+k_j)^2 = 2k_i\cdot k_j$$
$$= -<ij>[ij]$$

and

$$<ii> = [ii]=0$$
$$<ij] = [ij>=0$$

charge conjugation $<i^+|\gamma^\mu|j^+>=<j^-|\gamma^\mu|i^->$

Fierz identity $<i^+|\gamma^\mu|j^+><k^+|\gamma^\mu|l^+>=2[ik]\;<lj>$

any off-shell momentum $k^\mu\equiv\bar{k}^\mu+\dfrac{k^2}{2k^+}n^\mu$ where $\bar{k}^\mu$ is on-shell $\bar{k}^2=0$

any on-shell momentum $\not{p}=|p^+><p^+|+|p^-><p^-|$

spinor helicity methods

<u>*efficient way to handle the Dirac Algebra*</u>

$$\gamma^\star(\mathbf{k})\,\mathbf{T} \to \mathbf{q}(\mathbf{p})\,\bar{\mathbf{q}}(\mathbf{q})\,\mathbf{X}$$

$$P^\mu = P^- \, n^\mu$$

$$N \equiv \bar{u}(p) \not{n} \not{k}_1 \not{\epsilon}(k) \, (\not{k}_1 - \not{k}) \not{n} v(q)$$

$$k^\mu = k^+ \bar{n}^\mu - \frac{Q^2}{2k^+} n^\mu$$

work with a given helicity: <u>longitudinal photon, quark +, anti-quark -</u>

$$\begin{aligned}
N^{L;+-} &= \frac{Q}{k^+} < p^+|n^- >< n^-|\bar{k}_1^+ >< \bar{k}_1^+|n^- >< n^-| \left(|\bar{k}_1^+ >< \bar{k}_1^+| - k^+|\bar{n}^+ >< \bar{n}^+|\right) \\
&\quad |n^- >< n^-|q^+ > \\
&= \frac{Q}{k^+} [pn] < n\bar{k}_1 > [\bar{k}_1 n] < nq > \left(< n\bar{k}_1 > [\bar{k}_1 n] - k^+ < n\bar{n} > [\bar{n}n] \right) \\
&= -2^3 \frac{Q}{k^+} p^+ q^+ \sqrt{p^+ q^+} = -2^3 \, Q k^+ k^+ \, z(1-z)\sqrt{z(1-z)}
\end{aligned}$$

with

$$\begin{aligned}
z &\equiv p^+/k^+ \\
(1-z) &\equiv q^+/k^+
\end{aligned}$$

and

$$N^{L;-+} = N^{L;+-}$$

$\gamma^\star(\mathbf{k})\,\mathbf{T} \to \mathbf{q}(\mathbf{p})\,\bar{\mathbf{q}}(\mathbf{q})\,\mathbf{X}$

the rest is standard integration $\int \frac{d^2 l_t}{(2\pi)^2} \frac{e^{i l_t \cdot (x_t - y_t)}}{[l_t^2 + z(1-z)Q^2]} = \frac{1}{2\pi} K_0[\sqrt{z(1-z)|x_t - y_t|Q^2}\,]$

the amplitude is

$$i\mathcal{A}^{L;+-} = -4i\delta(k^+ - p^+ - q^+)\,Qk^+\,z(1-z)\sqrt{z(1-z)}\int d^2x_t d^2y_t e^{-i(p_t\cdot x_t + q_t \cdot y_t)}$$
$$K_0[\sqrt{z(1-z)|x_t - y_t|Q^2}]\,\left[V(x_t)V^\dagger(y_t) - 1\right]$$

repeat for transverse polarization

$$N^{\perp=+;+-} = -2^3 k^+ k^+ z\sqrt{z(1-z}\,\bar{k}_{1t}\cdot\epsilon_\perp$$

$$i\mathcal{A}^{\perp=+;+-} = 4ie\,\delta(l^+ - p^+ - q^+)\,zk^+\,z(1-z)\int d^2x_t d^2y_t e^{-i(p_t\cdot x_t + q_t\cdot y_t)}$$
$$\otimes \quad \frac{(\vec{x}_t - \vec{y}_t)\cdot\epsilon_\perp}{|\vec{x}_t - \vec{y}_t|}\,QK_1\left[\sqrt{z(1-z)(x_t^2 - y_t)^2Q^2}\right]\left[V(x_t)V^\dagger(y_t) - 1\right]$$

square,.... to get the inclusive di-jet production cross section

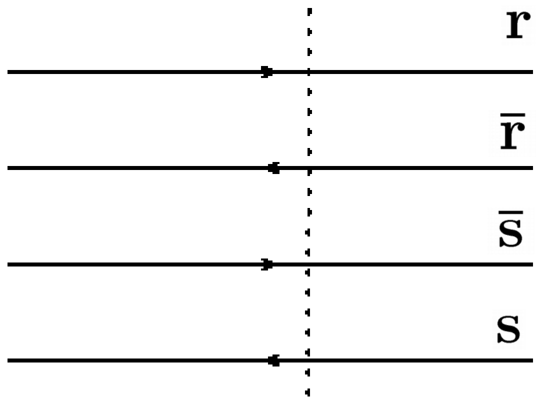
integrating over the final state momenta gives the total cross section (structure functions)

Evolution of quadrupole from JIMWLK

$$\mathbf{Q}(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \equiv \frac{1}{\mathbf{N_c}} < \mathbf{Tr}\, \mathbf{V}(\mathbf{r})\, \mathbf{V}^\dagger(\bar{\mathbf{r}})\, \mathbf{V}(\bar{\mathbf{s}})\, \mathbf{V}^\dagger(\mathbf{s}) >$$

radiation kernels as in dipole

$$\frac{\mathbf{d}}{\mathbf{dy}}$$



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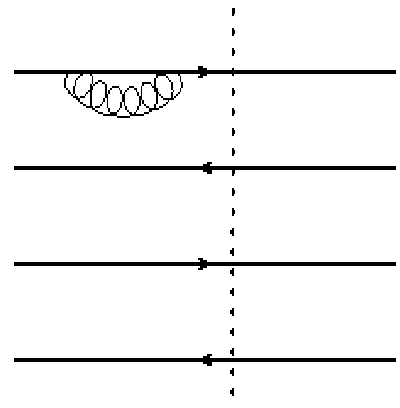

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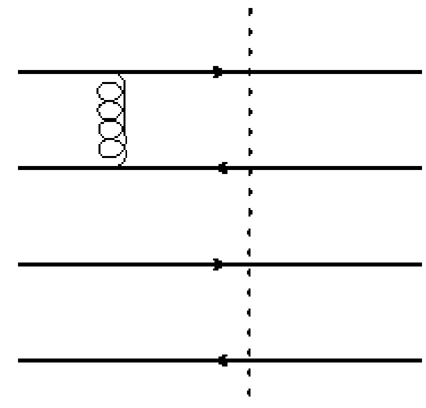
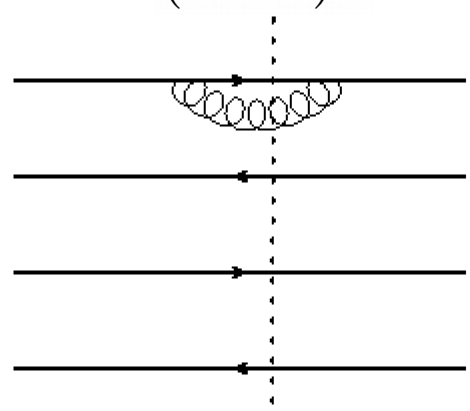
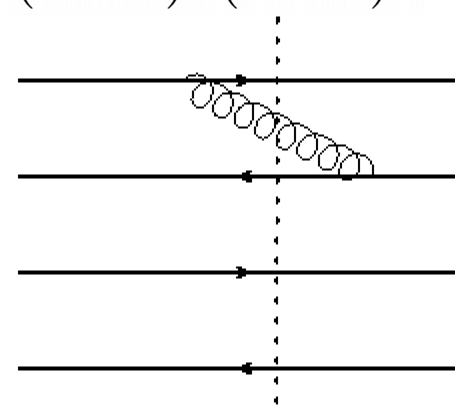
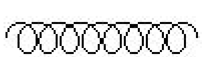


$$\frac{\mathbf{1}}{(\mathbf{r}-\mathbf{z})^2} \qquad \frac{(\mathbf{r}-\mathbf{z})\cdot(\bar{\mathbf{r}}-\mathbf{z})}{(\mathbf{r}-\mathbf{z})^2(\bar{\mathbf{r}}-\mathbf{z})^2}$$





large N_c



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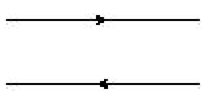


$+ \cdots$

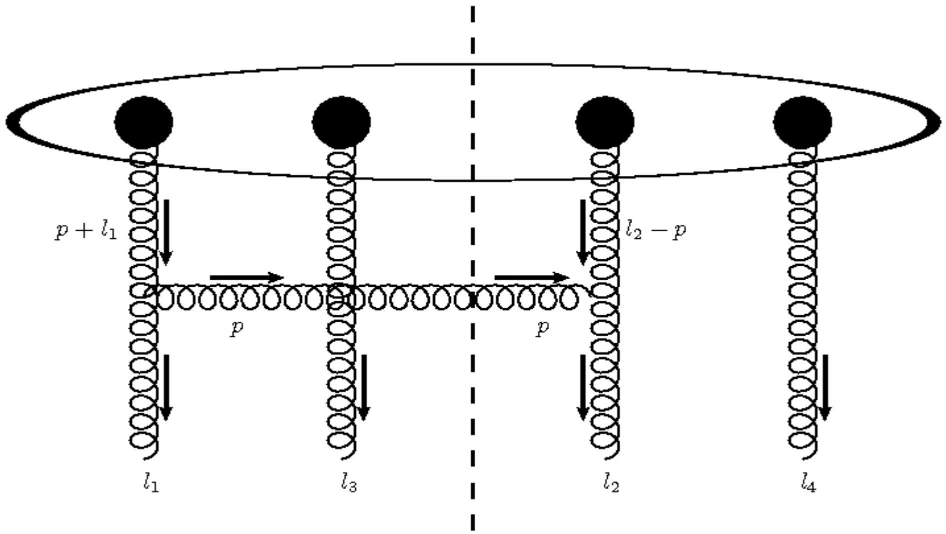
Evolution of quadrupole from JIMWLK

$$
\begin{aligned}
&\frac{d}{dy}\,\langle Q(r,\bar r,\bar s,s)\rangle \\
= \quad & \frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z\Bigg\{\Bigg\langle\Bigg[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\Bigg]\,Q(z,\bar r,\bar s,s)\,S(r,z) \\
+ \quad & \Bigg[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\Bigg]\,Q(r,z,\bar s,s)\,S(z,\bar r) \\
+ \quad & \Bigg[\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(s-z)^2(\bar r-z)^2}\Bigg]\,Q(r,\bar r,z,s)\,S(\bar s,z) \\
+ \quad & \Bigg[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\Bigg]\,Q(r,\bar r,\bar s,z)\,S(z,s) \\
- \quad & \Bigg[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}\Bigg]\,Q(r,\bar r,\bar s,s) \\
- \quad & \Bigg[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\Bigg]\,S(r,s)\,S(\bar r,\bar s) \\
- \quad & \Bigg[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\Bigg]\,S(r,\bar r)\,S(\bar s,s)\Bigg\rangle\Bigg\}
\end{aligned}
$$

$$\frac{d}{d\,y}Q = \int P_1\,[Q\,S] - P_2\,[Q] + P_3[S\,S]$$ with $$P_1 - P_2 + P_3 = 0$$

J. Jalilian-Marian, Y. Kovchegov: PRD70 (2004) 114017
Dominguez, Mueller, Munier, Xiao: PLB705 (2011) 106
J. Jalilian-Marian: Phys.Rev. D85 (2012) 014037

quadrupole evolution: linear regime



$\mathbf{O}(\mathbf{A}^4)$: 4-gluon exchange

BJKP equation

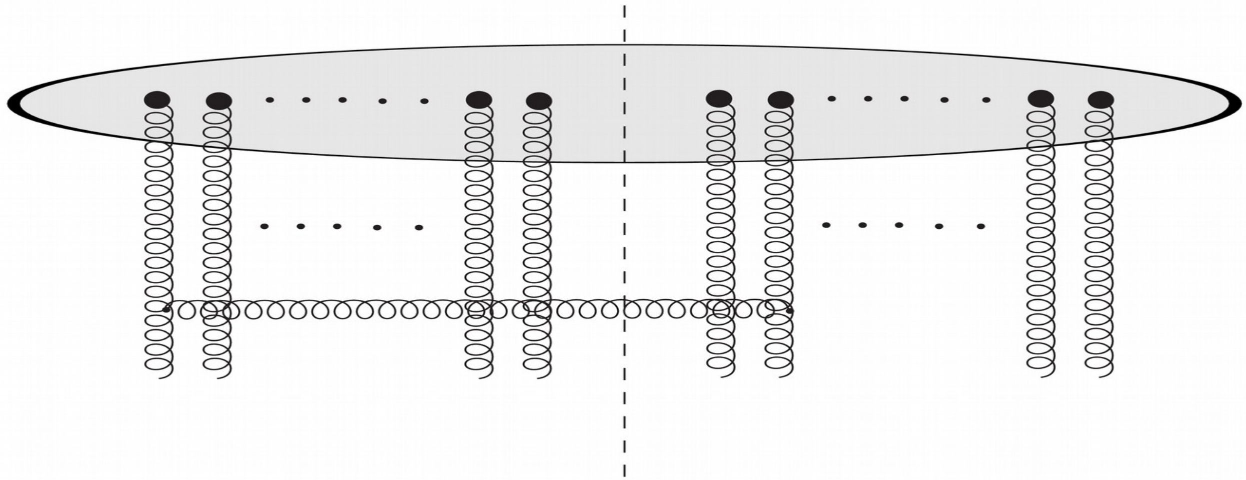
$$\frac{d}{dy}\hat{T}_4(l_1,l_2,l_3,l_4) = \frac{N_c\,\alpha_s}{\pi^2}\int d^2p_t\left[\frac{p^i}{p_t^2}-\frac{(p^i-l_1^i)}{(p_t+l_1)^2}\right]\cdot\left[\frac{p^i}{p_t^2}-\frac{(p^i-l_2^i)}{(p_t+l_2)^2}\right]$$
$$\hat{T}_4(p_t+l_1,l_2-p_t,l_3,l_4)+\cdots$$
$$-\frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2p_t\left[\frac{l_1^2}{p_t^2(l_1-p_t)^2}+\{l_1\to l_2,l_3,l_4\}\right]\hat{T}_4(l_1,l_2,l_3,l_4)$$

BJKP is recovered from JIMWLK in the linear regime

2n-Wilson line evolution: linear regime

$\mathbf{O}(\alpha^{\mathbf{2n}})$: 2n-gluon exchange

Ayala, Cazaroto, Hernandez, Jalilian-Marian, Tejeda-Yeomans, PRD90 (2014) no.7, 074037



$$\frac{d}{dY}T^{(2n)}_{\left(\prod_{k=1}^{2n} l_k\right)} = -\frac{\bar{\alpha}}{2\pi}\sum_{j=1}^{2n}\int d^2p_t\left[\frac{l_j^2}{p_t^2\left[p_t^2+(p_t-l_j)^2\right]}\right]T^{(2n)}_{\left(\prod_{k=1}^{2n} l_k\right)}$$

$$+\quad \frac{\bar{\alpha}}{4\pi}\int d^2p_t\left[\frac{l_1^2}{p_t^2\,(p_t+l_1)^2}+\frac{l_{2n}^2}{p_t^2\,(p_t-l_{2n})^2}-\frac{(l_1+l_{2n})^2}{(p_t+l_1)^2\,(p_t-l_{2n})^2}\right]T^{(2n)}_{(l_1+p_t)\left(\prod_{k=2}^{2n-1} l_k\right)(l_{2n}-p_t)}$$

$$+\quad \frac{\bar{\alpha}}{4\pi}\sum_{j=2}^{2n}\int d^2p_t\left[\frac{l_{j-1}^2}{p_t^2\,(p_t+l_{j-1})^2}+\frac{l_j^2}{p_t^2\,(p_t-l_j)^2}-\frac{(l_{j-1}+l_j)^2}{(p_t+l_{j-1})^2\,(p_t-l_j)^2}\right]T^{(2n)}_{\left(\prod_{k=1}^{j-2} l_k\right)(l_{j-1}+p_t)(l_j-p_t)\left(\prod_{k=j+1}^{2n} l_k\right)}.$$

A Mathematica program that gives the equation can be downloaded from

faculty.baruch.cuny.edu/naturalscience/physics/Jalilian-Marian/

paginas.fisica.uson.mx/elena.tejeda/code.nb

quadrupole: limits

$$< Q(r,\bar{r},\bar{s},s) > \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$$

line config.: $r = \bar{s},\ \bar{r} = s,\ z \equiv r - \bar{r}$

square config.: $r - \bar{s} = \bar{r} - s = r - \bar{r} = \cdots \equiv z$

"naive" Gaussian: $Q = S^2$

Gaussian $Q_|(z) \approx \frac{N_c+1}{2} [S(z)]^{2\frac{N_c+2}{N_c+1}} - \frac{N_c-1}{2} [S(z)]^{2\frac{N_c-2}{N_c-1}}$

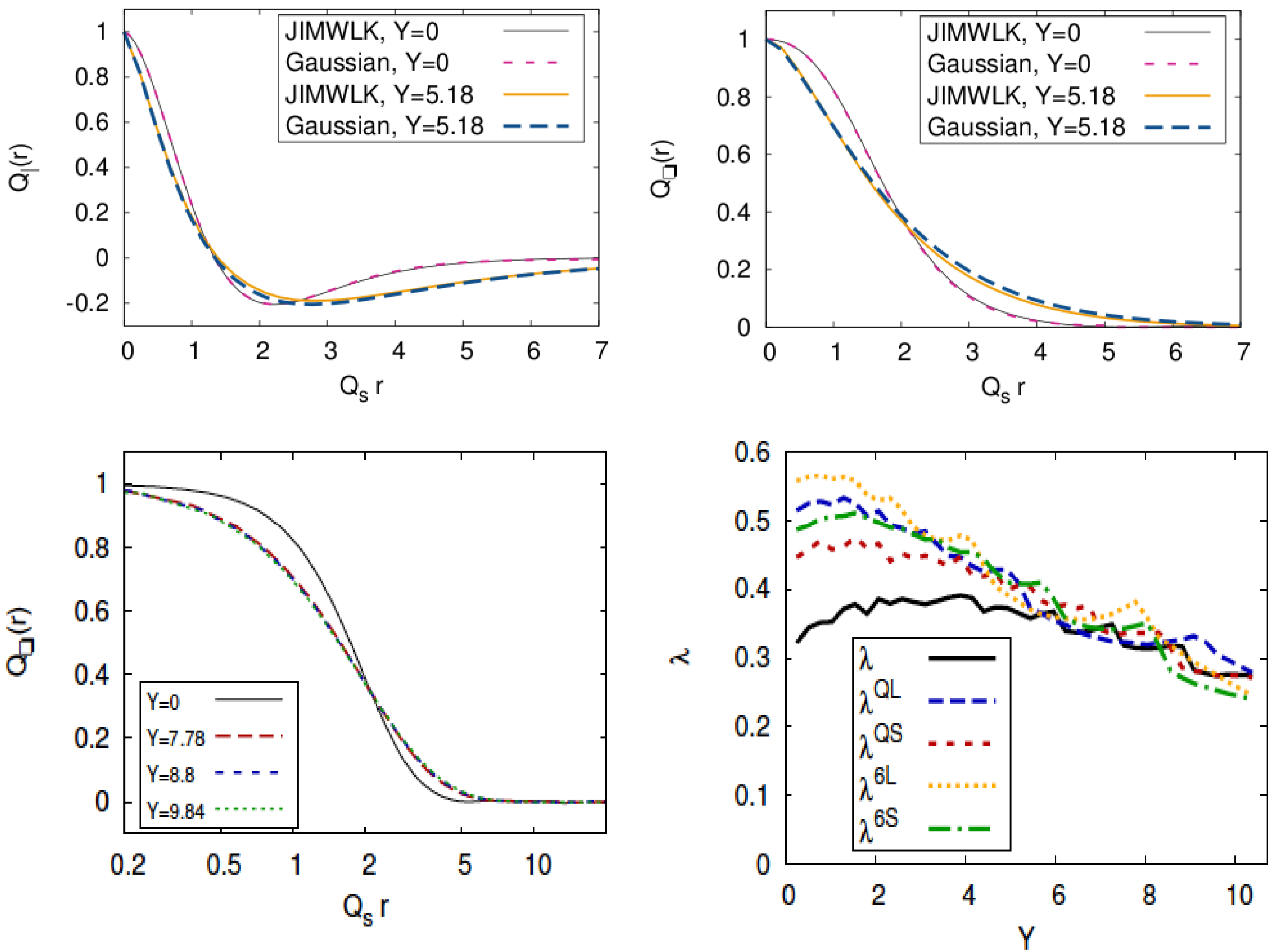
$$Q_{sq}(z) = [S(z)]^2 \left[\frac{N_c+1}{2} \left(\frac{S(z)}{S(\sqrt{2}z)} \right)^{\frac{2}{N_c+1}} - \frac{N_c-1}{2} \left(\frac{S(\sqrt{2}z)}{S(z)} \right)^{\frac{2}{N_c-1}} \right]$$

Gaussian + large N_c $Q_|(z) \to S^2(z)[1 + 2\, log[S(z)]]$

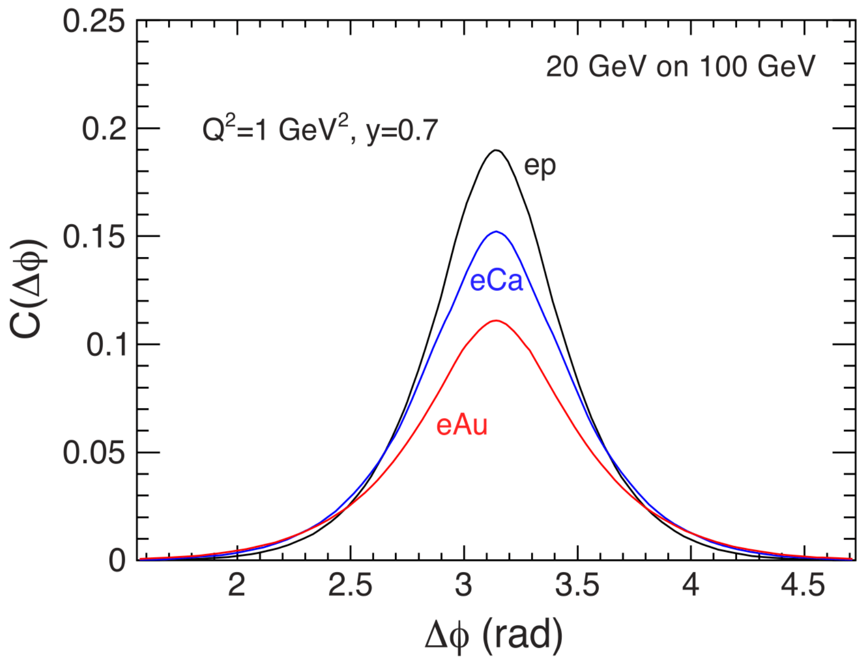
$$Q_{sq}(z) = S^2(z) \left[1 + 2 \ln \left(\frac{S(z)}{S(\sqrt{2}z)} \right) \right]$$

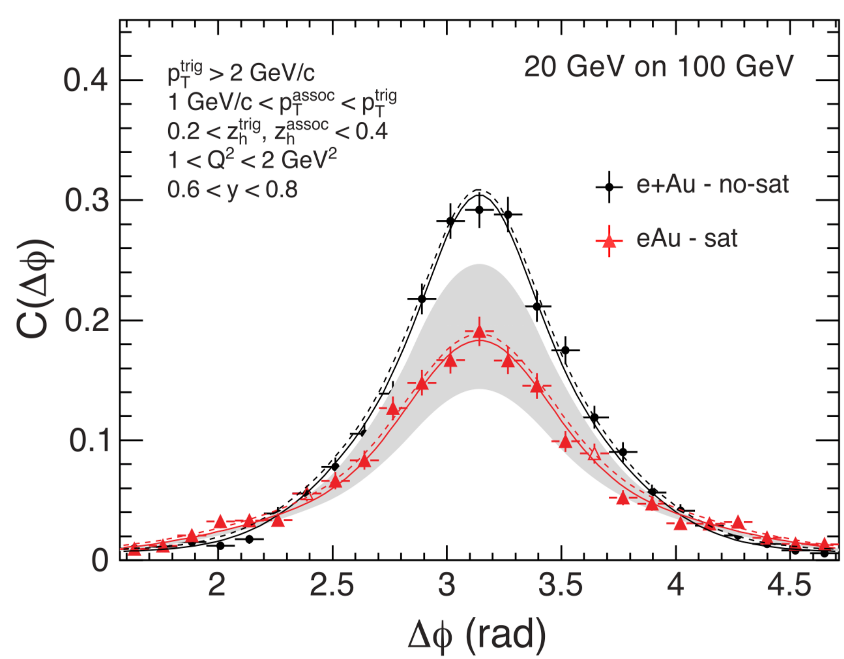
Quadrupole: $< Q(r, \bar{r}, \bar{s}, s) > \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$

Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:PLB706 (2011) 219



di-hadron azimuthal correlations in DIS

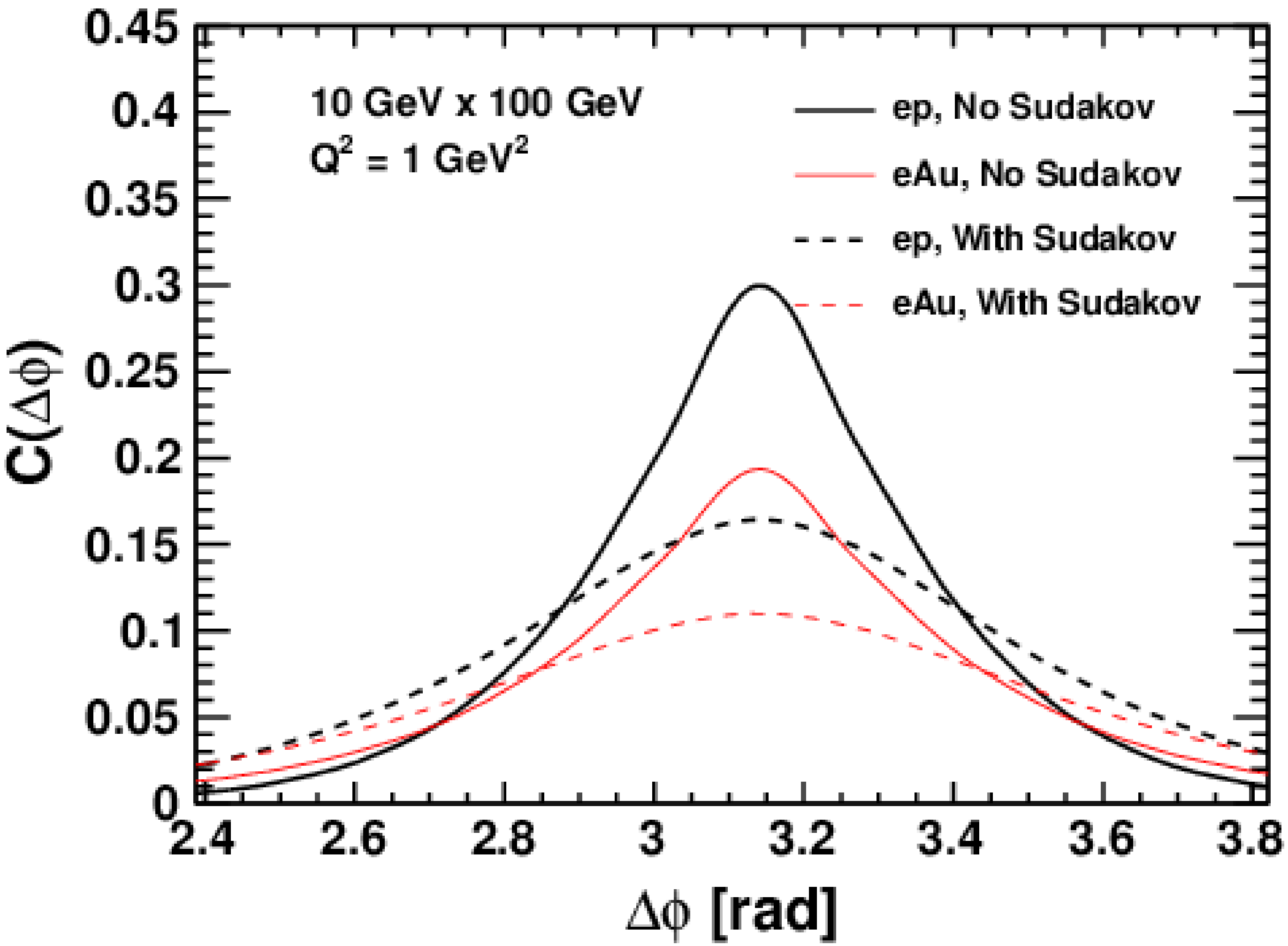




Electron Ion Collider…., A. Accardi et al., arXiv:1212.1701

Zheng-Aschenauer-Lee-Xiao, PRD89 (2014)7, 074037

di-hadron azimuthal correlations in DIS

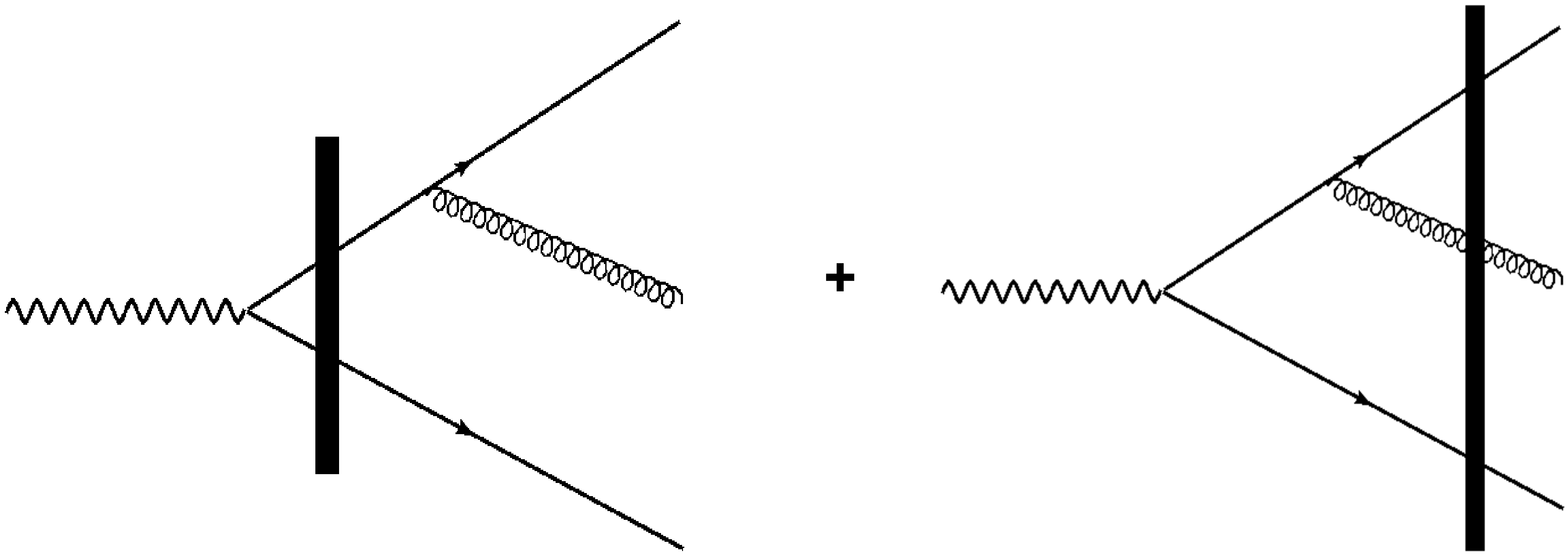


Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037

something with more discriminating power

angular correlations in 3-parton production in DIS

$$\gamma^\star \mathbf{T} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$$



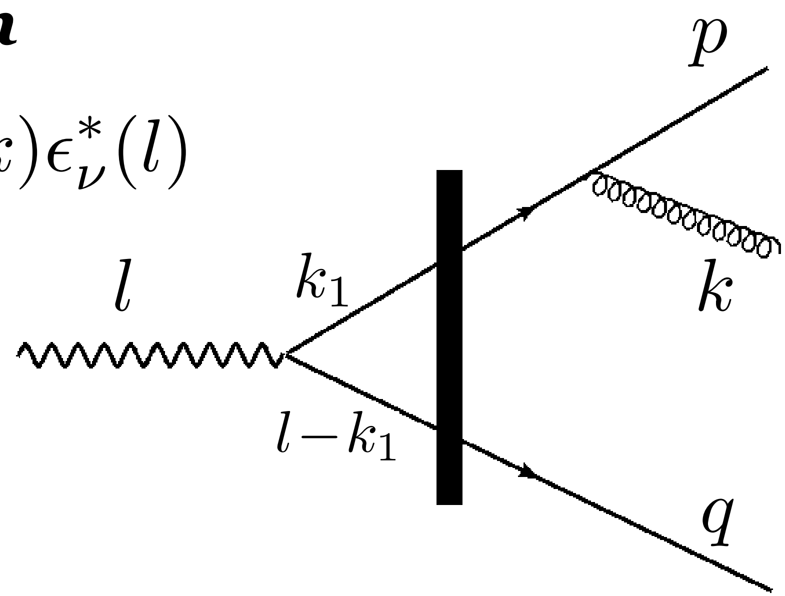
+ radiation from anti-quark

Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans; PLB761 (2016) 229

[NLO diffractive di-jets: Boussarie, Grabovsky, Szymanowski, Wallon, JHEP 1611 (2016) 149]

1^{st} diagram

$$\mathcal{A} \equiv -eg\,\bar{u}(p)\,[A]^{\mu\nu}\,v(q)\,\epsilon_\mu\,(k)\epsilon^*_\nu(l)$$



$$A_1^{\mu\nu} = \gamma^\mu\,t^a\,S_F^0(p+k)\,\tau_F(p+k,k_1)\,S_F^0(k_1)\,\gamma^\nu\,S_F^0(l-k_1)\,\tau_F(l-k_1,q)\,\frac{d^4k_1}{(2\pi)^4}$$

$$= \frac{i}{2\,l^-}\frac{\delta(l^- - p^- - q^- - k^-)}{(p+k)^2}\int d^2x_t\,d^2y_t\,e^{-i(p_t+k_t)\cdot x_t}\,e^{-iq_t\cdot y_t}$$

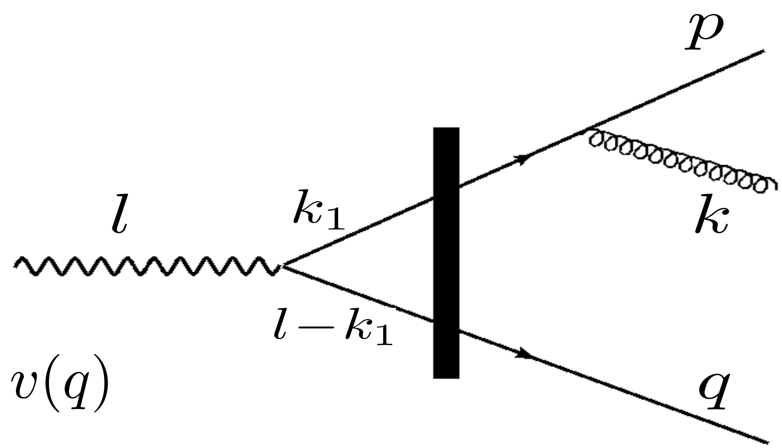
$$\gamma^\mu\,t^a\,i(\not{p}+\not{k})\gamma^-\,i\not{k}_1\,\gamma^\nu\,i(\not{l}-\not{k}_1)\,\gamma^-\,K_0\,[L\,(x_t-y_t)]$$

$$V(x_t)\,V^\dagger(y_t)$$

with

$$L^2 = \frac{q^-(p^-+k^-)}{l^-\,l^-}Q^2 \qquad k_1^- = p^- - k^- \qquad k_1^+ = \frac{k_{1t}^2 - i\epsilon}{2(p^-+k^-)} \qquad k_{1t} = -i\,\partial_{x_t-y_t}$$

Diagram A1



Numerator: Dirac Algebra

$$a_1 \equiv \overline{u}(p)\, \not\epsilon^{\star}(k)\, (\not p + \not k)\, \not n\, \not k_1\, \not\epsilon(l)\, (\not k_1 - \not l)\, \not n\, v(q)$$

longitudinal photons

quark anti-quark gluon helicity: + - +

$$\not l = l^+ \not\overline{n} - \frac{Q^2}{2l^+} \not n$$

$$a_1^{L;+-+} \;=\; -\frac{\sqrt{2}}{[n\,k]}\frac{Q}{l^+}\,[n\,p] < k\,p > [n\,p] < n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] < n\,q >$$

$$(< n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] - l^+ < n\,\overline{n} > [n\,\overline{n}])$$

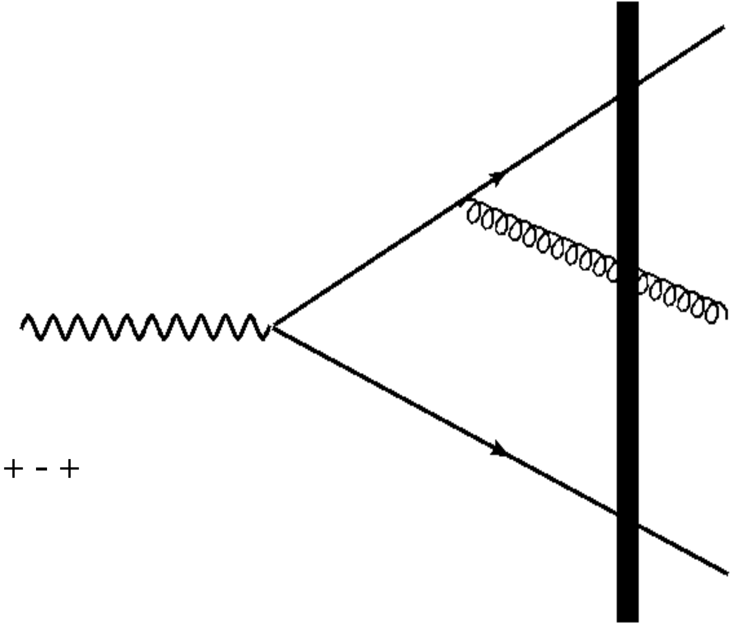
with

$$< np >= -[np] = \sqrt{2p^+}$$

transverse photons: +

$$a_1^{\perp=+;+-+} = -\frac{\sqrt{2}}{[nk]}[pn] < kp > [pn] < nk_1 > [k_1 n] < \bar{n}k_1 > [k_1 n] < nq >$$

Diagram A3



Numerator: Dirac Algebra

longitudinal photons

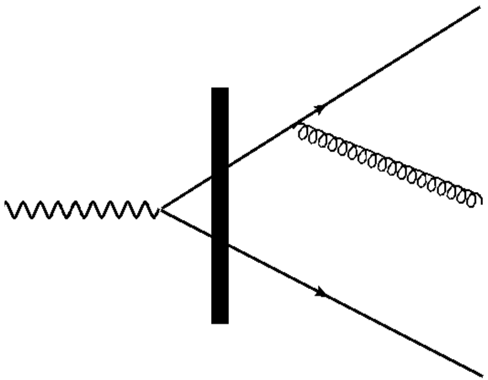
quark anti-quark gluon helicity: + - +

$$a_3^{L;+-+} = \frac{\sqrt{2}Q}{l^+ [n\bar{k}_2]}[pn]\left(< n\bar{k}_1 > [\bar{k}_1 n] - < n\bar{k}_2 > [\bar{k}_2 n]\right) < \bar{k}_2\bar{k}_1 > [\bar{k}_1 n]$$
$$\left(< n\bar{k}_1 > [\bar{k}_1 n] - l^+ < n\bar{n} > [\bar{n}n]\right) < nq >$$
$$= -2^4 Q (l^+)^2 \frac{(z_1 z_2)^{3/2}}{z_3} \left[z_3\, k_{1\,t}\cdot\epsilon - (z_1 + z_3) k_{2\,t}\cdot\epsilon\right]$$

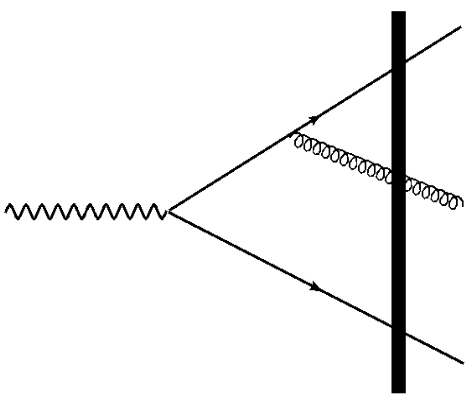
the rest is some standard integrals

add up the amplitudes, square.., still need to deal with products of Wilson lines: Quadrupoles

structure of Wilson lines: amplitude

$\Bigg($  $\Bigg)_{ij}$

$$= \left[V^\dagger(y_t)\, V(x_t)\, t^a\right]_{ij}$$

$\Bigg($  $\Bigg)_{ij}$

$$= \left[V^\dagger(y_t)\, t^b\, V(x_t)\right]_{ij}\; U^{ba}(z_t)$$

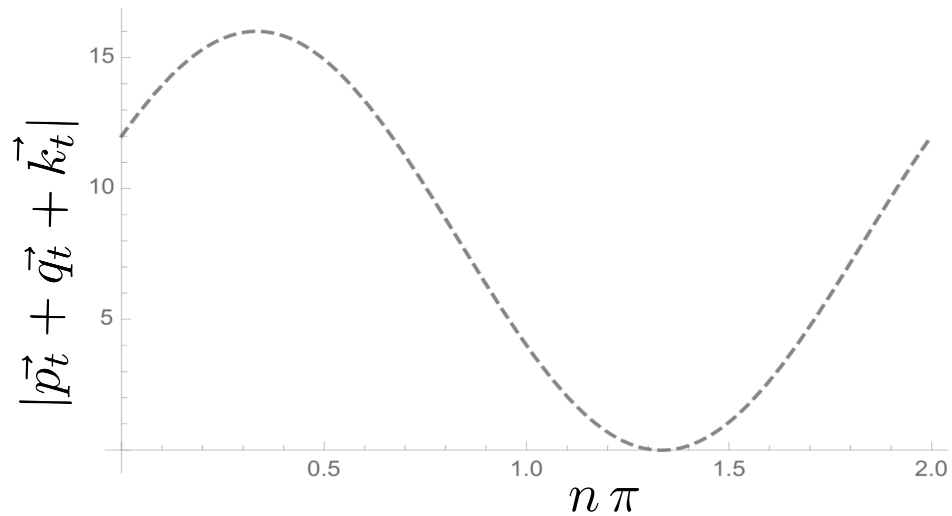
3-parton kinematics

<u>linear regime</u>: <u>*use ugd's*</u>

$$z_1 = z_2 = 0.2, z_3 = 0.6$$

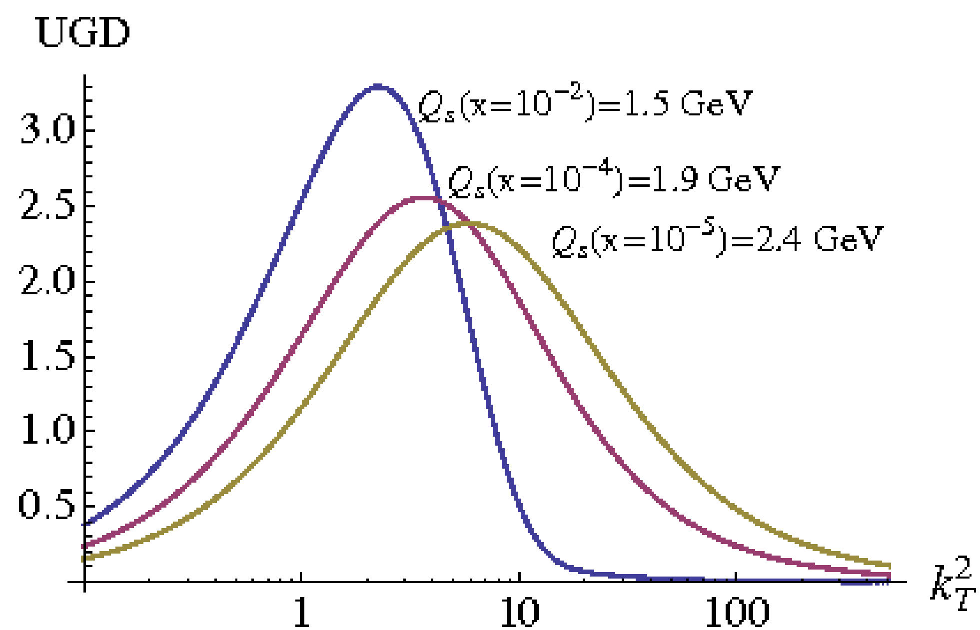
$$p_t = q_t = k_t = 4\, GeV$$

$$Q^2 = 16\, GeV^2$$



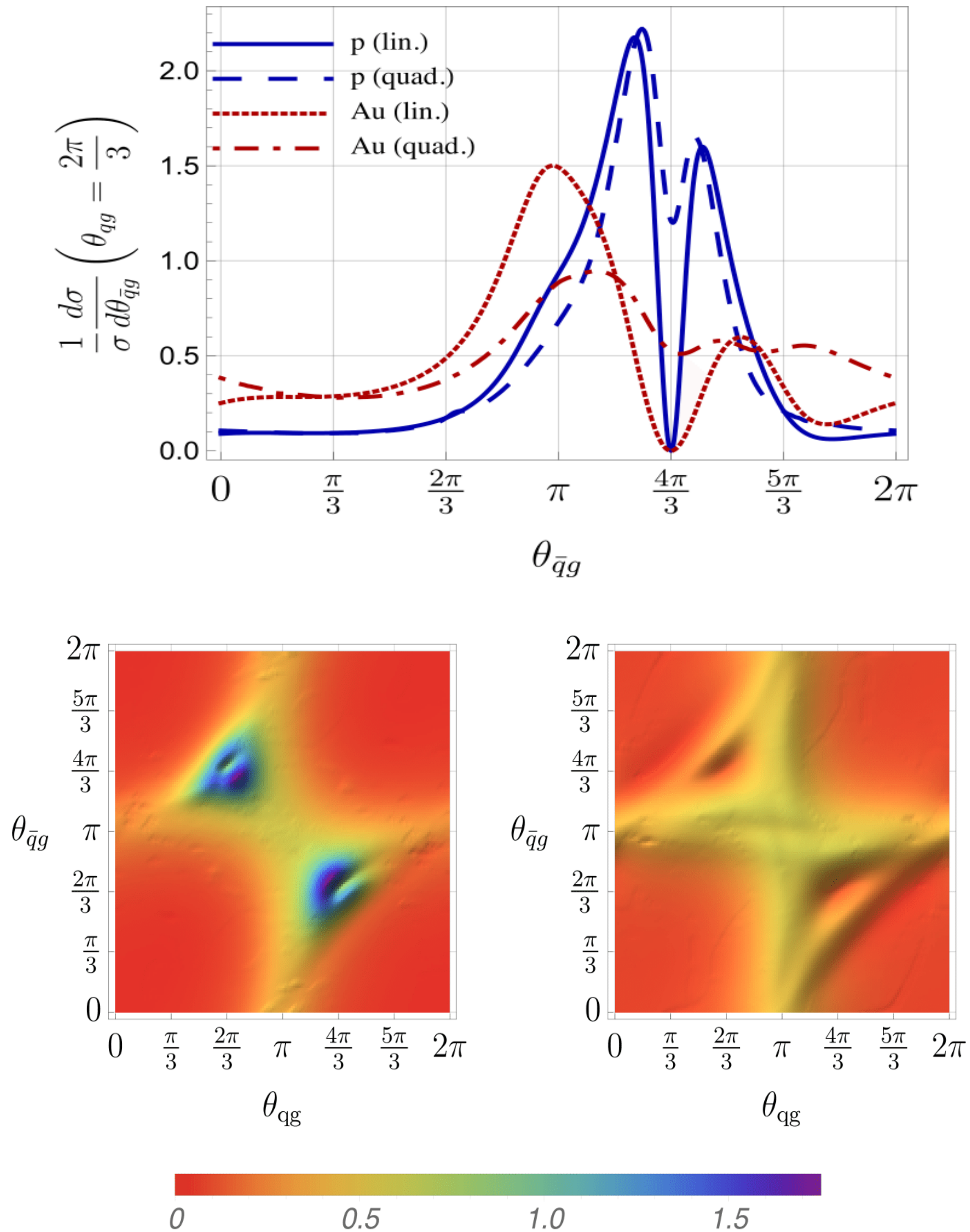
$$\Delta\phi_{12} = \frac{2\pi}{3} \qquad \text{vary} \qquad \Delta\phi_{13}$$

$$k_t^2\, \tilde{T}(k_t)$$



multiple scattering: <u>broadening</u> of the peak
x-evolution: <u>reduction</u> of magnitude

3-parton azimuthal angular correlations



Possible extensions to other processes?

real photons: $Q^2 \to 0$

ultra-peripheral nucleus-nucleus collisions

inclusive 3-jet production

NLO inclusive di-jet production

crossing symmetry:

$$\gamma^{(\star)}\, T \longrightarrow q\,\bar{q}\,g\,X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\,T \longrightarrow q\,g\,\gamma^{(\star)}\,X \\ \bar{q}\,T \longrightarrow \bar{q}\,g\,\gamma^{(\star)}\,X \\ g\,T \longrightarrow q\,\bar{q}\,\gamma^{(\star)}\,X \end{array} \right\}$$

proton-nucleus collisions (collinear factorization in proton?)

di-jet + photon production in pA $\qquad pA \longrightarrow h_1\,h_2\,\gamma^{(\star)}\,X$

Possible extensions to other processes?

MPI (double/triple parton scattering)

$$\gamma^{(\star)}\, T \longrightarrow q\,\bar{q}\,g\,X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\,\bar{q}\,T \longrightarrow g\,\gamma^{(\star)}\,X \\ g\,\bar{q}\,T \longrightarrow \bar{q}\,\gamma^{(\star)}\,X \\ g\,q\,T \longrightarrow q\,\gamma^{(\star)}\,X \end{array} \right\}$$

$$pA \longrightarrow h\,\gamma^{(\star)}\,X$$

if one *assumes* target is accurately described by CGC at small x
this will tell us about DPS (proton GPD at large x)

some thoughts/ideas/dreams/......

cold matter energy loss

how important is cold matter Eloss in single inclusive production in the forward rapidity region?

cold matter energy loss?
Kopeliovich, Frankfurt and Strikman
Neufeld,Vitev,Zhang, PLB704 (2011) 590

Munier, Peigne, Petreska, arXiv:1603.01028

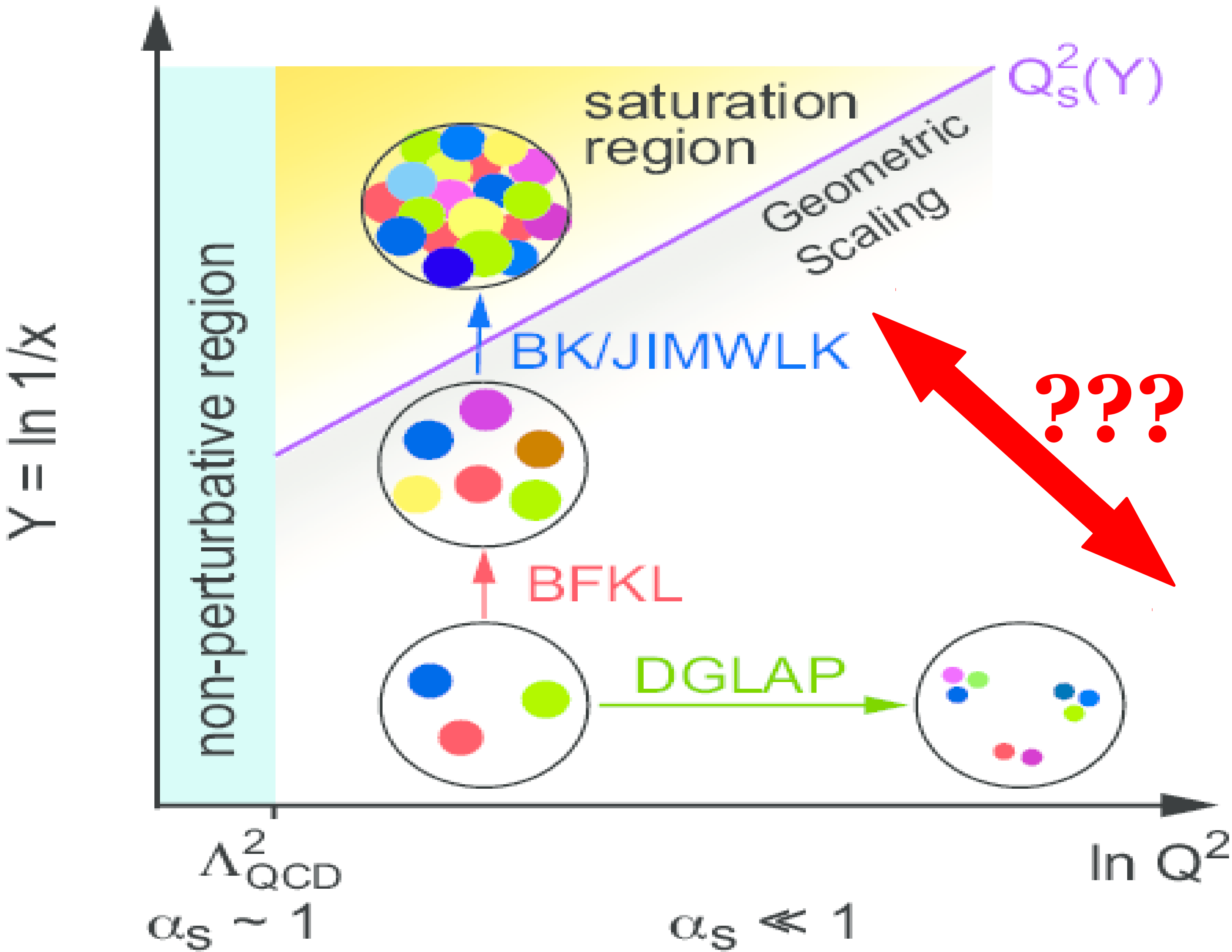
$$z\frac{dI}{dz} \equiv \frac{\frac{d\sigma a+A\to a+g+X}{dydy'd^2p_t}}{\frac{d\sigma a+A\to a+X}{dyd^2p_t}}$$

the difference between a nuclear target and a proton target is the medium induced energy loss

one can use this to estimate the energy loss in single inclusive processes in the forward kinematics at RHIC and the LHC

can also do this for di-jets in DIS (3-parton production/2-parton production)

QCD kinematics phases at high energy



SUMMARY

CGC is a systematic approach to high energy collisions

it has been used to fit a wealth of data; ep, eA, pp, pA, AA

Leading Log CGC works (too) well for a qualitative/semi-quantitative description of data, NLO is needed

Azimuthal angular correlations offer a unique probe of CGC

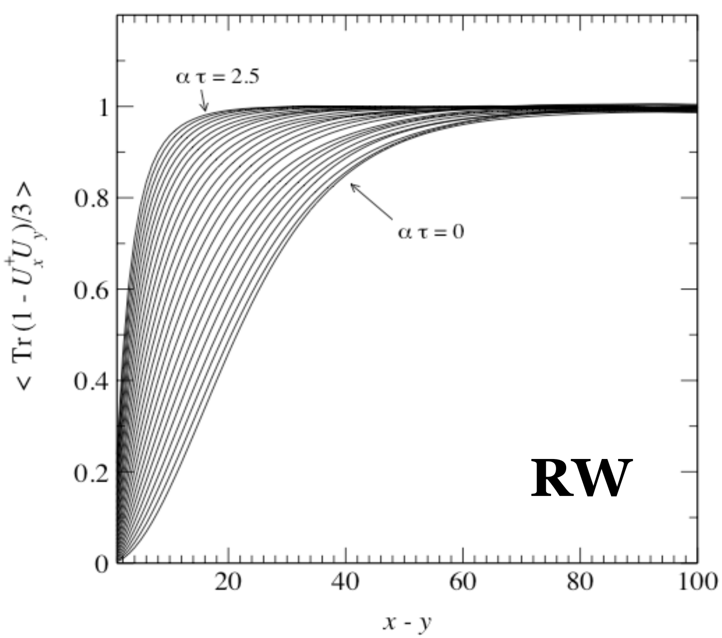
3-hadron/jet correlations should be even more discriminatory

generalize CGC to include high p_t (DGLAP) physics?

Dipoles at large N_c : BK eq.

$$\frac{d}{dy}T(x_t - y_t) = \frac{\bar{\alpha}_s}{2\pi}\int d^2z_t \, \frac{(x_t - y_t)^2}{(x_t - z_t)^2(y_t - z_t)^2} \times$$

$$\left[T(x_t - z_t) + T(z_t - y_t) - T(x_t - y_t) - T(x_t - z_t)T(z_t - y_t)\right]$$



$\tilde{T}(p_t) \to \log\left[\frac{Q_s^2}{p_t^2}\right]$ **saturation region**

$\tilde{T}(p_t) \to \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^\gamma$ **extended scaling region**

$\tilde{T}(p_t) \to \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]$ **pQCD region**

Rummukainen-Weigert, NPA739 (2004) 183
NLO: Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)