

Institut de Physique Théorique

Cours de Physique Théorique

Riemann surfaces

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Fridays 12, 19, 26 January and 2, 9 February 2018 at 10 am, until 12+ noon.

Algebraic equations are widespread in mathematics and physics, and the geometry of their spaces of solutions can be complicated. In the case of an equation of two complex variables, the space of solutions is a Riemann surface.

We will provide basic tools (going back to Riemann) for studying algebraic equations and describing the geometry of compact Riemann surfaces.

We will consider a Riemann surface defined from the solution locus of a polynomial equation $P(x, y) = 0$ in $\mathbb{C} \times \mathbb{C}$. We will study its topology and geometry, and learn how to integrate differential forms along closed contours. Then we will describe the moduli space of Riemann surfaces with a given topology : its dimension, topology, etc.

We will introduce some of the many tools that have been invented since the time of Riemann for studying these objects. We will partly follow the Mumford Tata lectures, the Fay lectures, and the Farkas-Kra book.

The plan is :

1. Compact Riemann surfaces, charts, atlas, topology. Meromorphic functions and one-forms. Theorems on poles and residues. Newton's polygon.
2. Integrals, periods, Abel map, Jacobian, divisors. Theta functions, prime form, fundamental form. Basis of cycles, homology and cohomology.
3. Moduli spaces of Riemann surfaces. Deligne-Mumford compactification, Chern classes, tautological ring. Kontsevich integral and KdV hierarchy.
4. If times permits : fiber bundles, Hitchin systems, link to integrable systems.



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