Generalized Heisenberg-Euler Formula and its Application to Vacuum Magnetic Birefringence Experiment

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arXiv:1707.03308, 1707.03609

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Contents

1. Introduction

In QED the effective action is known since 1936

W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936)

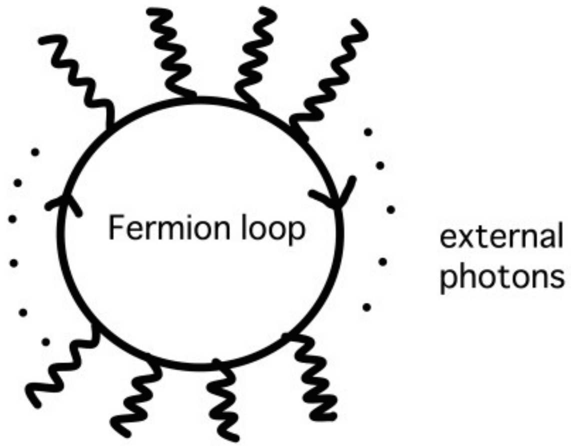
where the electron ψ couples to the photon A

$$S_\psi(m) = \int d^4x \; \bar{\psi} \left[\gamma^\mu \left(i\partial_\mu + eA_\mu \right) - m \right] \psi(x)$$

which gives the effective Lagrangian for photon:

$$e^{iS_{\text{eff}}[A_\mu]} = e^{i\int d^4x \, \mathcal{L}_{\text{eff}}[A_\mu]} = \int \mathcal{D}\psi(x)\mathcal{D}\bar{\psi}(x) e^{iS_\psi(m)}$$

$$\mathcal{L}_{\text{eff}}[A_\mu] = \sum_{\#\text{ of }\gamma}$$

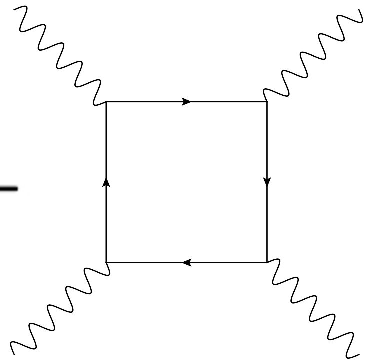


The effective Lagrangian Is known at any order
in the expansion of constant electromagnetic fields
In case of Parity conserving theory (QED).

$$\mathfrak{L} = -\mathfrak{F} - \frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3}\exp(-m^2 s)$$
$$\times\left[(es)^2\mathfrak{G}\frac{\mathrm{Re}\cosh esX}{\mathrm{Im}\cosh esX} - 1 - \tfrac{2}{3}(es)^2\mathfrak{F}\right]$$
$$= \tfrac{1}{2}(\mathbf{E}^2-\mathbf{H}^2)+\frac{2\alpha^2}{45}\frac{(\hbar/mc)^3}{mc^2}\times[(\mathbf{E}^2-\mathbf{H}^2)^2+7(\mathbf{E}\cdot\mathbf{H})^2]+\cdots$$

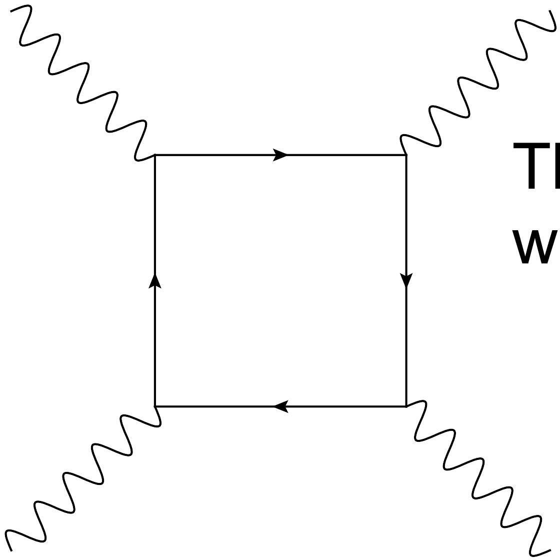
$$X = \sqrt{2(\mathcal{F}+i\mathcal{G})},$$
$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{H}^2 - \vec{E}^2\right)$$
$$\mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{H}$$

see J. Schwinger, Phys. Rev. **82**, 664 (1951)

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}^2 +$$  $$+ \quad \cdots$$

We have generalized the Heisenberg-Euler formula to the case with Parity violation.

arXiv:1707.03308



Then, the effective action is formed with $\mathcal{F}^2$, $\mathcal{G}^2$ plus $\mathcal{F}\mathcal{G}$ (P, CP violating)

At the 4th order, the effective action becomes

$$\mathcal{L}_{\text{eff}} = -\mathcal{F} + a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}$$

(a, b, c) are given by the coupling constant & mass.

2-1. Effective Action in Proper-time Method

$\not{P}$ Action:

$$S_\psi(m) = \int d^4x\ \bar{\psi}\left[\gamma^\mu\left(i\partial_\mu - (g_V + g_A\gamma_5)A_\mu\right) - m\right]\psi(x)$$

Effective Action:

$$S_{\rm eff}[A_\mu] \quad = \quad \int d^4x\ \mathcal{L}_{\rm eff}[A_\mu] = -i\ln\left[\int \mathcal{D}\psi(x)\mathcal{D}\bar{\psi}(x)e^{iS_\psi(m)}\right]$$

Integrated out with

$$A_\mu(x) = \tfrac{1}{2}x^\lambda F_{\lambda\mu}$$

$$(F_{\mu\nu} = const)$$

$$= (-i)\frac{1}{2}{\rm Tr}\ln(\hat{H} + m^2)$$

Pauli form

$$\hat{H} = -\left(i\partial_\mu - g_V\frac{1}{2}x^\nu F_{\nu\mu}(x)\right)^2 - \frac{1}{4}x^\mu\left(g_A^2 F_{\mu\lambda}F^{\lambda\nu}\right)x_\nu$$
$$+ \frac{1}{2}(g_V + g_A\gamma_5)\sigma^{\mu\nu}F_{\mu\nu} + i\frac{1}{2}\sigma^{\mu\nu}g_A\gamma_5(x^\lambda F_{\lambda\mu}\, i\partial_\nu - x^\lambda F_{\lambda\nu}\, i\partial_\mu)$$

$$S_{\text{eff}}(A) = (-i)\frac{1}{2}\mathrm{Tr}\ln(\hat{H} + m^2)$$

traces of x^μ and spin

Proper time description:

$$= \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s}\, \mathrm{Tr}(e^{-i\hat{H}s})$$

V. Fock, Physik. Z. Sowjetunion, **12**, 404 (1937),
Y. Nambu, Prog. Theor. Phys. **5**, 82 (1950)

Then, the quantum field theory is described by

A quantum mechanics of a point particle, located at position $x^\mu(s)$ at a proper time s, and the position $x^\mu(s)$ and the spin couples.

2-2. Path Integral Representation

$$S_{\rm eff}(A) = \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s}\, {\rm Tr}(e^{-i\hat{H}s})$$

$$= \int d^4x \, \underset{\text{spin}}{\rm tr} \int_{x^\mu(0)=x^\mu}^{x^\mu(s)=x^\mu} \mathcal{D}x^\mu(s')\, e^{i\int_0^s ds'\, \tilde{L}(x(s'),\, \dot{x}(s'))}$$

$$\tilde{L}(x^\mu(s), \dot{x}^\mu(s))$$

$$= -\frac{1}{4}(\dot{x}^\mu)^2 + \frac{1}{2}x^\mu F_{\mu\lambda}\left(g_V\, g^{\lambda\nu} + ig_A\, \gamma_5\sigma^{\lambda\nu}\right)\dot{x}_\nu - \frac{1}{2}g_A^2\, x^\mu F_{\mu\lambda}F^{\lambda\nu}x_\nu$$

$$- \frac{1}{2}\sigma^{\mu\nu}(g_V + g_A\gamma_5)F_{\mu\nu}$$

This is the difficult part

If $g_A = 0$, x^μ and spin $\sigma^{\mu\nu}$ decouple (this is Heisenberg-Euler case), but <u>they do couple</u> when Parity is violated (our case).

Take "tr" for the spin easily, then

$$S_{\text{eff}}(A) = \frac{i}{2}\int_0^\infty \frac{ds}{s} e^{-im^2 s} \times$$

$$\int \mathcal{D}x^\mu(s')\, e^{i\bar{A}(s)} \times 2\left(\cos\sqrt{2(\bar{\mathcal{F}}'(s)+i\bar{\mathcal{G}}'(s))} + \cos\sqrt{2(\bar{\mathcal{F}}'(s)-i\bar{\mathcal{G}}'(s))}\right)$$

$= \bar{X}'_+(s)$ $= \bar{X}'_-(s)$

Lagrangian is separated into
A(s) (free part) and B(s) (interaction part):

$$\bar{A}(s) = \int_0^s ds'\left[-\frac{1}{4}(\dot{x}^\mu)^2 + \frac{1}{2}g_V x^\mu(F_{\mu\nu})\dot{x}^\nu - \frac{1}{2}g_A^2 x^\mu(F_{\mu\lambda}F^{\lambda\nu})x_\nu\right]$$

$$\bar{\mathcal{F}}'(s) = \frac{1}{4}\bar{B}_{\mu\nu}(s)\bar{B}^{\mu\nu}(s),\quad \bar{\mathcal{G}}'(s) = \frac{1}{4}\bar{B}_{\mu\nu}(s)\tilde{\bar{B}}^{\mu\nu}(s).$$

$$\bar{B}_{\mu\nu}(s) = \int_0^s ds'\left[g_A\frac{1}{2}\epsilon_{\mu\nu\beta\gamma}x_\alpha F^{\alpha\beta}\dot{x}^\gamma - (g_V F_{\mu\nu} - i g_A \tilde{F}_{\mu\nu})\right]$$

The path integration can't be performed exactly.

A general expression of the effective action can be obtained, even in case of parity violation. However, the contractions by the propagator $\langle \cdots \rangle'$ remain.

$$\mathcal{L}_{\rm eff} = -\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s} \frac{(g_+ s)^2 \mathcal{G}}{Im \cosh(g_+ X s)} \times \frac{(g_- s)^2 \mathcal{G}}{Im \cosh(g_- X s)}$$

$$\times \frac{1}{2}\left\langle \left(\cos \bar{\boldsymbol{X}}'_+(s \to -is) + \cos \bar{\boldsymbol{X}}'_-(s \to -is) \right) \right\rangle'$$

$$g_\pm = \frac{1}{2}(g_V \pm \sqrt{g_V^2 + 2g_A^2})$$

g_V=-e, g_A=0 (g_+=0, g_-=-e)

$$\bar{\boldsymbol{X}}'_+ = s g_V X, \text{ and } \bar{\boldsymbol{X}}'_- = s g_V X^\dagger$$

reproduce QED result.

$$\mathfrak{L} = -\mathfrak{F} - \frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3} \exp(-m^2 s) \times \left[(es)^2 \mathfrak{G} \frac{\mathrm{Re}\cosh esX}{\mathrm{Im}\cosh esX} - 1 - \tfrac{2}{3}(es)^2 \mathfrak{F} \right]$$

see J. Schwinger, Phys. Rev. **82**, 664 (1951)

3. Effective Lagrangian at Fourth Order

$$\mathcal{L}_{\text{eff}} = -\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s} \frac{(g_+ s)^2 \mathcal{G}}{Im\cosh(g_+ X s)} \times \frac{(g_- s)^2 \mathcal{G}}{Im\cosh(g_- X s)}$$

$$\times \frac{1}{2}\left\langle\left(\cos \bar{X}'_+(s \to -is) + \cos \bar{X}'_-(s \to -is)\right)\right\rangle'$$

$$\parallel$$

$$1 - \langle \bar{\mathcal{F}}'(s)\rangle' + \frac{1}{6}\langle(\bar{\mathcal{F}}'(s))^2 - (\bar{\mathcal{G}}'(s))^2\rangle' + \cdots$$

dimension 4

extract s^4 terms

$O(s^4)$ corresponds to $O(F^4)$

$sF_{\mu\nu}$ is dimensionless combination, and it becomes $F_{\mu\nu}/m^2$, after s-integration.

$$a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}$$

Now, the original fermion action

$$S_\psi(m) = \int d^4x\ \bar{\psi}_{\rm DM} \left[\gamma^\mu \left(i\partial_\mu - (g_V + g_A\gamma_5)A'_\mu\right) - m\right] \psi_{\rm DM}$$

gives the following effective action at the forth order

$$\mathcal{L}_{\rm eff} = -\mathcal{F} + a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}$$

P̸

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \quad \mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

$$a = \frac{1}{(4\pi)^2 m^4}\left(\frac{8}{45}\ g_V^4 - \frac{4}{5}\ g_V^2 g_A^2 - \frac{1}{45}\ g_A^4\right)$$

H-E

$$b = \frac{1}{(4\pi)^2 m^4}\left(\frac{14}{45}\ g_V^4 + \frac{34}{15}\ g_V^2 g_A^2 + \frac{97}{90}\ g_A^4\right)$$

$$c = \frac{1}{(4\pi)^2 m^4}\left(\frac{4}{3}\ g_V^3 g_A + \frac{28}{9}\ g_V g_A^3\right)$$

c=0 when g_A or g_V is 0

4. Dark Matter Model

arXiv:1707.03609

- Couple Fermionic DM to U(1)'Y' gauge field B' in the DS,

$$\mathcal{L}_{\psi'_{DM}} = \bar{\psi}'_{DM}\left[\gamma^{\mu}\left(i\partial_{\mu} - (g'_V + g'_A\gamma_5)B'_{\mu}\right) - m'\right]\psi'(x)_{DM}$$

- Couple a messenger scalar S to

$SM\ B_{\mu}(U(1)\text{gauge field})$ and $DS\ B'_{\mu}$ gauge field)

$$\mathcal{L}_S = \left|\left(i\partial_{\mu} - g_1 Y_s B_{\mu} - g'_1 Y'_s B'_{\mu}\right) S(x)\right|^2$$

Spontaneous breaking $\langle S \rangle = v_s/\sqrt{2}$,

B_{μ} and B'_{μ} are mixed

$$\mathcal{L}_{\text{mixing}} = \frac{1}{2} m_{B'}^2 \left(\varepsilon^2 B_{\mu} B^{\mu} + 2\varepsilon B_{\mu} B'^{\mu} + B'_{\mu} B'^{\mu}\right)$$

$$m_{B'} = g'_1 Y'_s v_s \qquad \varepsilon \equiv \frac{g_1 Y_s}{g'_1 Y'_s}$$

Including $A^3_\mu(x)$ the 3rd component of SU(2)L gauge boson

$$\mathcal{L}_{mass} = \frac{v^2}{8}\left(A^{3\mu}(x),\ B^\mu(x),\ B'^\mu(x) \right) \begin{pmatrix} g_2^2 & -g_1 g_2 & 0 \\ -g_1 g_2 & g_1^2 + \alpha' \varepsilon^2 & \alpha' \varepsilon \\ 0 & \alpha' \varepsilon & \alpha' \end{pmatrix} \begin{pmatrix} A^3_\mu(x) \\ B_\mu(x) \\ B'_\mu(x) \end{pmatrix}$$

mass diagonalization

$$\alpha' = 4(m_{B'}/v)^2$$

$$(m_{\tilde{A}})^2 = 0,\ (m_{\tilde{Z}})^2 = \frac{1}{4}v^2(g_1^2 + g_2^2) + \varepsilon^2 \frac{g_1^2}{g_1^2 + g_2^2 - \alpha'}(m_{B'})^2,\ \text{and}$$

$$(m_{\tilde{B}'})^2 = (m_{B'})^2 \left(1 + \varepsilon^2 \frac{g_2^2 - \alpha'}{g_1^2 + g_2^2 - \alpha'}\right).$$

$$\tilde{A}_\mu = \frac{g_1 A^3_\mu + g_2 B_\mu}{\sqrt{g_1^2 + g_2^2}} - \varepsilon \frac{g_2}{\sqrt{g_1^2 + g_2^2}} B'_\mu$$

We assume the mixing parameter $\varepsilon \ll 1$

$$\tilde{A}_\mu = \frac{g_1 A^3_\mu + g_2 B_\mu}{\sqrt{g_1^2 + g_2^2}} - \varepsilon \frac{g_2}{\sqrt{g_1^2 + g_2^2}} B'_\mu$$

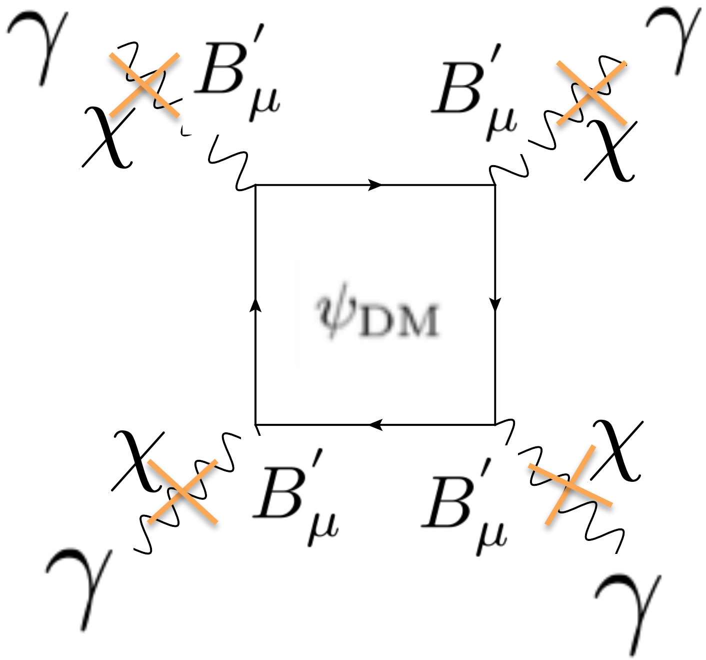
Real photon

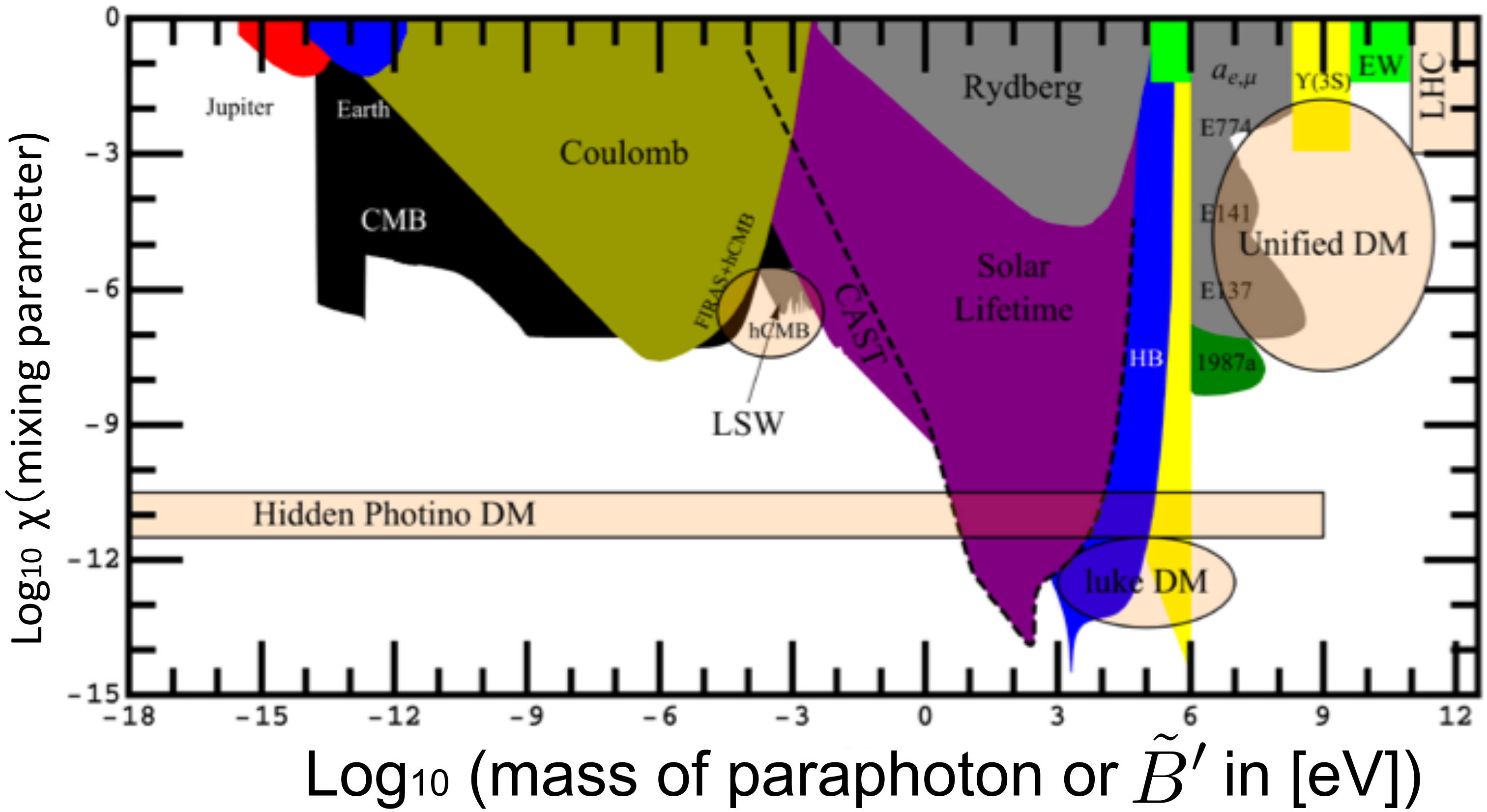
Paraphoton

$$\varepsilon \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \chi$$

The mixing parameter between SM and DS

$$\mathcal{L}'_{\rm eff} = \chi^4 \left\{ a\, \mathcal{F}^2 + b\, \mathcal{G}^2 + ic\, \mathcal{F}\mathcal{G} \right\}$$





The allowed region is

$$\chi \leq 10^{-6} \text{ for } m_{\tilde{B}'} \geq 1 \text{ MeV}, \text{ or}$$

$$10^{-6} \leq \chi \leq 10^{-3} \text{ for } m_{\tilde{B}'} \geq 1 \text{ GeV}.$$

5. Vacuum Magnetic Birefringence Experiment

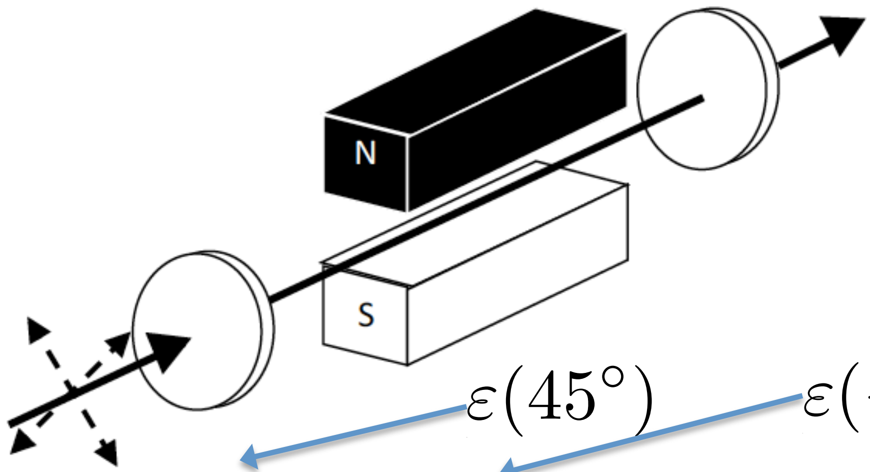
arXiv: 1707.03609

Observe the polarization change under the strong magnetic field.

Conventional

Fabry-Perot resonator

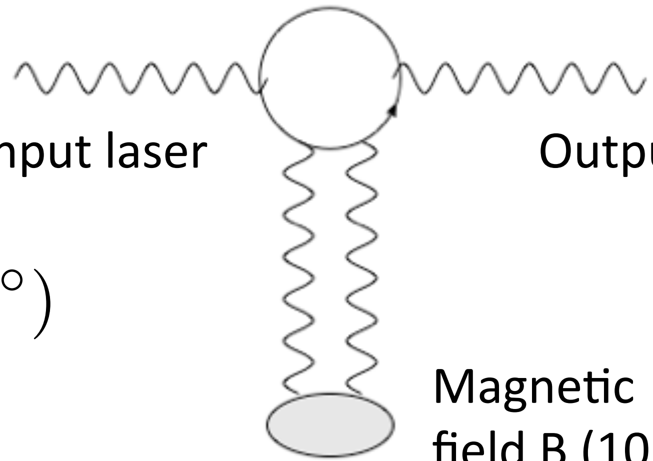
$\boldsymbol{\varepsilon}_f = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$ or $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$



$\varepsilon(45^\circ)$ $\varepsilon(-45^\circ)$

$\boldsymbol{\varepsilon}_i = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$ or $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$

- OVAL (Observing Vacuum with Laser) arXiv:1705.00495
- BMV Eur. Phys. J. D (2014) **68**: 16
- PVLAS Eur. Phys. J. C (2016) **76**: 24



Initial linear polarization becomes elliptic polarization, after propagating under the magnetic field.

$\varepsilon(45^\circ)$ Input this linear polarization, bended 45 degree against the magnetic field.

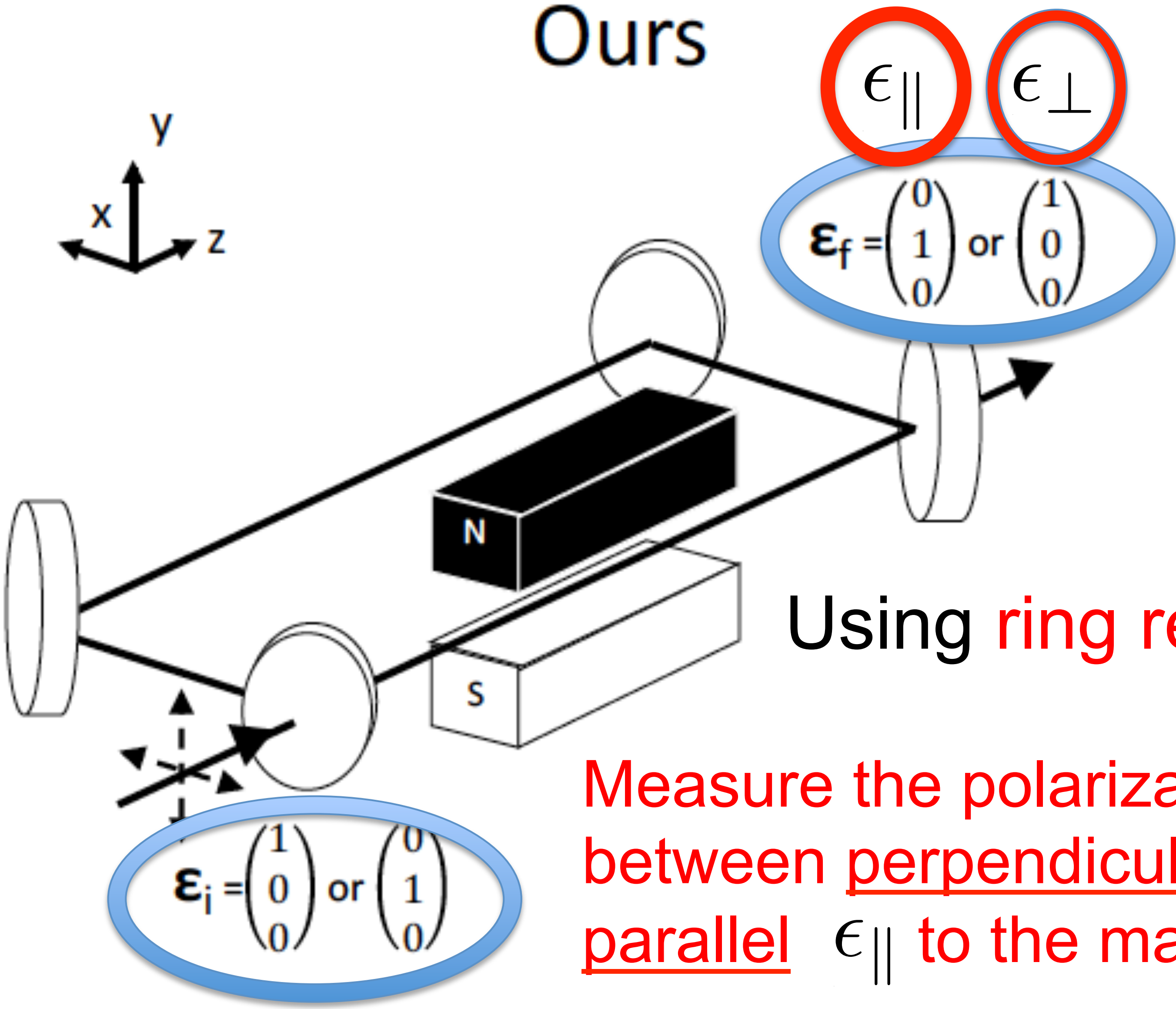
If it becomes $\varepsilon(45^\circ) \pm i\varepsilon(-45^\circ)$
then it is called the circular polarization appears.

Similarly, if it becomes $a \times \varepsilon(45^\circ) \pm ib \times \varepsilon(-45^\circ)$
then it is called the elliptic polarization appears.
This occurs in QED.

Then, the ellipticity Ψ is naturally defined by b/a.

To detect the Parity violation, we propose a new method:

arXiv: 1707.03609



Using ring resonator

Measure the polarization change between perpendicular $\epsilon_{\perp}$ and parallel $\epsilon_{||}$ to the magnetic field.

Why is the ring resonator efficient ?

In the conventional Fabry-Perot resonator, each reflection by mirror reverses Party. So, even if the high finness (# of times of forward and backward trips) is realized, the effect is not better than a one way trip. On the other hand, the ring resonator can keep the definite party state during clockwise and couter-clockwise round trips by light.

More explicitly,

$$\mathcal{L}_{\text{eff}} = -\mathcal{F} + a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}$$

Equation of Motion for laser gives the two eigen-functions of the polarization vector $\epsilon_{\pm}$, and their refraction indices $n_{\pm}$ (= the inverse of the phase velocity)

$$\boldsymbol{\epsilon}_{\pm} \propto \begin{cases} -ic\boldsymbol{\epsilon}_{\parallel} + \left(a - b \pm \sqrt{(a-b)^2 - c^2}\right)\boldsymbol{\epsilon}_{\perp} & (D > 0) \\ -ic\boldsymbol{\epsilon}_{\parallel} + \left(a - b \pm i\sqrt{c^2 - (a-b)^2}\right)\boldsymbol{\epsilon}_{\perp} & (D < 0) \end{cases}$$

$$n_{\pm} \;=\; 1 + \frac{1}{2}\boldsymbol{B}^2\left\{(a+b) \pm \sqrt{(a-b)^2 - c^2}\right\}$$

$(a-b)^2 - c^2$ =D

Define

$$\sin\phi = \frac{c}{\left\{\left((a-b)+\sqrt{(a-b)^2-c^2}\right)^2+c^2\right\}^{\frac{1}{2}}}$$

$= 0$ when Parity is conserved

$$\cos\phi = \frac{(a-b)+\sqrt{(a-b)^2-c^2}}{\left\{\left((a-b)+\sqrt{(a-b)^2-c^2}\right)^2+c^2\right\}^{\frac{1}{2}}}$$

$$\sinh\theta = \frac{a-b}{(c^2-(a-b)^2)^{\frac{1}{2}}} \times sign\,(c)$$

$$\cosh\theta = \frac{|c|}{(c^2-(a-b)^2)^{\frac{1}{2}}}$$

$$\Psi = \pi|B|^2\frac{L}{\lambda}\sqrt{|(a-b)^2-c^2|},$$

$(= 0$ when D=0)

where, B(magnetic field)=10 [T],
λ(laser beam wave length)=200-50 [nm] = 1-4 [eV] ,
L(beam propagating distance)=0.2 [m] x 100,000 (finness).

After passing distance L under the magnetic field

Conventional: $\epsilon(45^\circ)$

$$\epsilon(45^\circ) \rightarrow \begin{cases} (\cos(\Psi - 2\phi)\epsilon(45^\circ) - i\sin\Psi\epsilon(-45^\circ))\,/\cos 2\phi & (D>0) \\ ((\cosh\theta\sinh\Psi - \cosh\Psi)\epsilon(45^\circ) - i\sinh\theta\sinh\Psi\epsilon(-45^\circ))\,/\cosh\theta & (D<0) \end{cases}$$

ellipticity * appears

$$\sin\Psi / \cos(\Psi - 2\phi) \quad \text{for} \quad \epsilon_i = \epsilon(45^\circ)\ (D > 0)$$

$$\sinh\theta\sinh\Psi/(\cosh\Psi - \cosh\theta\sinh\Psi) \quad \text{for} \quad \epsilon_i = \epsilon(45^\circ)\ (D < 0)$$

(*) D > 0 in QED (pure vector),
but D < 0 for Dark sector neutrino (pure V-A)

To Detect $\not{P}$ interaction:

After passing distance L under the magnetic field

$$\epsilon_{\parallel} \rightarrow \begin{cases} \left((-i\sin\Psi + \cos 2\phi\cos\Psi)\epsilon_{\parallel} + \sin\Psi\sin 2\phi\epsilon_{\perp}\right)/\cos 2\phi & (D>0) \\ (\cosh\Psi + i\sinh\theta\sinh\Psi)\epsilon_{\parallel} - \cosh\theta\sinh\Psi\epsilon_{\perp} & (D<0) \end{cases}$$

$$\epsilon_{\perp} \rightarrow \begin{cases} \left(\sin 2\phi\sin\Psi\epsilon_{\parallel} + (i\sin\Psi + \cos 2\phi\cos\Psi)\epsilon_{\perp}\right)/\cos 2\phi & (D>0) \\ -\cosh\theta\sinh\Psi\epsilon_{\parallel} + (\cosh\Psi - i\sinh\theta\sinh\Psi)\epsilon_{\perp} & (D<0) \end{cases}$$

These polarization changes between $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are zero for QED, but non-zero for DS neutrino.

Since,

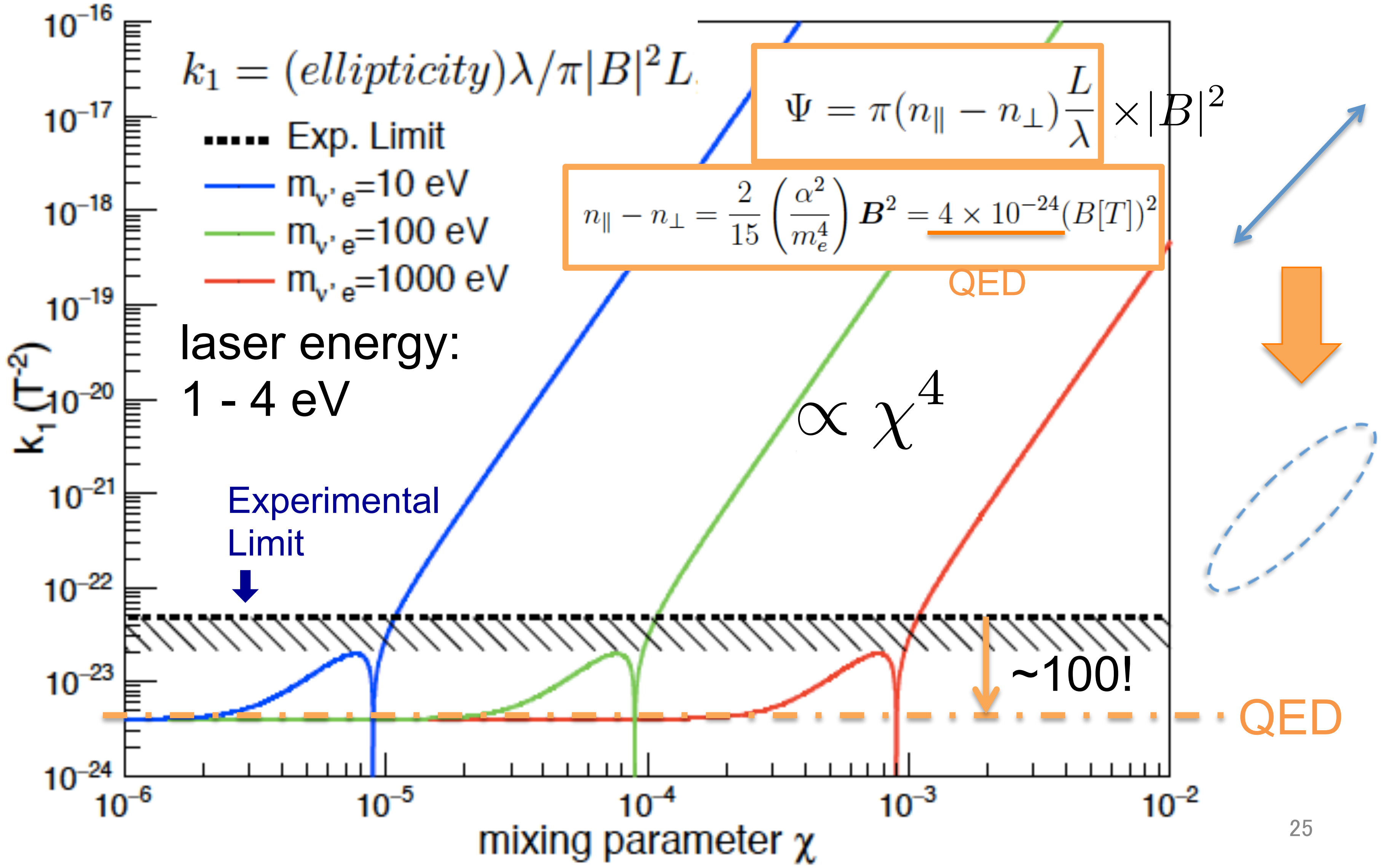
$$\sin 2\phi = 0 \text{ (QED)}, \quad \cosh\theta \neq 0 \text{ (DS neutrino)}$$

As an example, we examine the case, having both the electron and the lightest neutrino in the DS. If the latter is the target, it gives the signal, while the former gives the QED background.

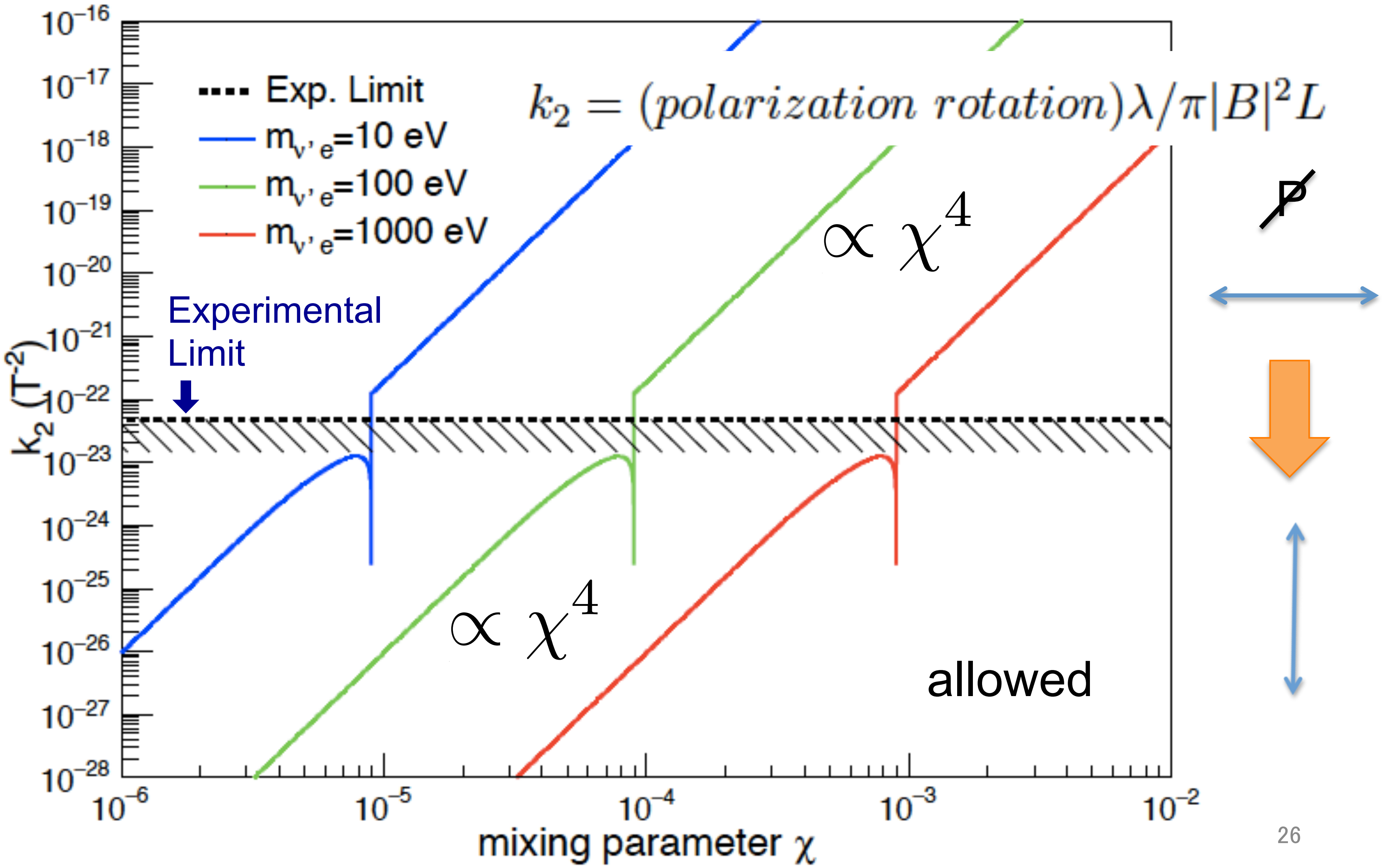
$$a = a_{\mathrm{QED}} + \chi^4 a_{\mathrm{DS}\nu'}, \;\; b = b_{\mathrm{QED}} + \chi^4 b_{\mathrm{DS}\nu'}, \;\; \text{and } c = \chi^4 c_{\mathrm{DS}\nu'}$$

For the small χ, D >0 holds, while for the large χ, D<0 holds. Therefore, in between D=0 is attained, where the ellipticity or the polarization change disappears due to ψ＝0, that forms a dip in the next Figures.

Conventional signal for QED plus DM



P̸ signal for QED plus DM



6. Summary

1. The generalized Heisenberg-Euler formula is derived, in a general case with parity violation.
2. Effective Lagrangian at the fourth order is explicitly calculated:

$$\mathcal{L}_{\rm eff} = -\mathcal{F} + a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}$$

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \quad \mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

$$a = \frac{1}{(4\pi)^2 m^4}\left(\frac{8}{45}\, g_V^4 - \frac{4}{5}\, g_V^2 g_A^2 - \frac{1}{45}\, g_A^4\right)$$

$$b = \frac{1}{(4\pi)^2 m^4}\left(\frac{14}{45}\, g_V^4 + \frac{34}{15}\, g_V^2 g_A^2 + \frac{97}{90}\, g_A^4\right)$$

$$c = \frac{1}{(4\pi)^2 m^4}\left(\frac{4}{3}\, g_V^3 g_A + \frac{28}{9}\, g_V g_A^3\right)$$

3. Apply the formula to the Vacuum Magnetic Birefringence Experiment, intending to probe the dark sector (neutrino).

   A sensitive way to observe the parity violation is proposed:

   The signal is free from the QED background, and is more efficient, if the ring resonator is used instead of the conventional Fabry-Perot resonator.

Backup

●Heseiberg-Euler assumes the fields be constant.
This is guaranteed, when the followings hold:

$$\lambda_{\rm laser} \gg \lambda_{\rm vac} = \frac{\hbar c}{mc^2} \ , \quad \text{or}$$

$$i\partial_\mu/m = k_\mu(\text{wave vector of the laser})/m \ll 1$$

$\lambda_{\rm laser} = 200 - 50$ [nm] implies $mc^2 \gg 1 - 4$ [eV]

●To guarantee the expansion in terms of fields:

$$\left(\sqrt{eE} = \sqrt{e|\boldsymbol{E}|}, \sqrt{eB} = \sqrt{e|\boldsymbol{B}|}\right) \ll m \quad \text{or} \quad m'/\chi$$

10 [T]=24 [eV]

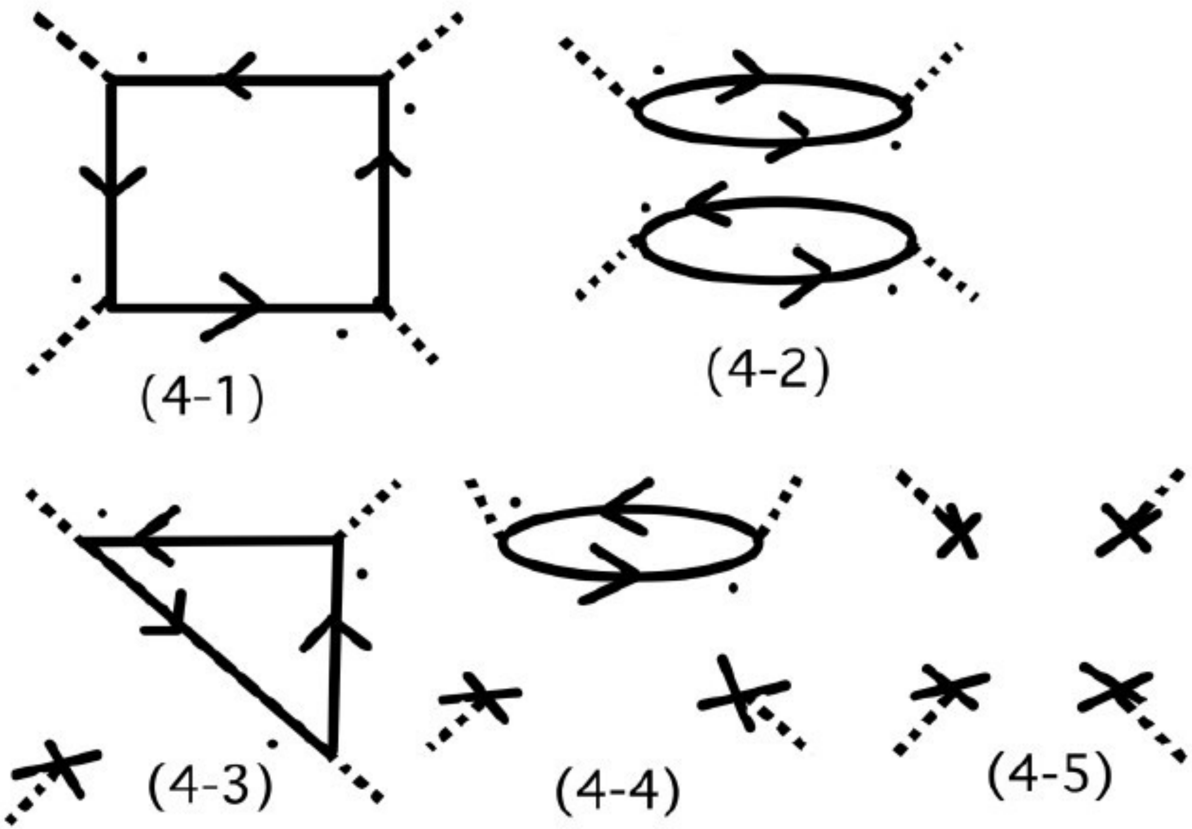


Figure 4: Feynman diagrams at the forth order

The propagator in the diagonal frame of the field reads

$$\Delta(s'-s'')_{(\lambda)} = \frac{-1}{4(g_+ - g_-)F'_{(\lambda)}} \times \left\{ \frac{e^{-2g_+F'_{(\lambda)}(s'-s''-\epsilon(s'-s'')\frac{s}{2})}}{\sinh(g_+F'_{(\lambda)}s)} - \frac{e^{-2g_-F'_{(\lambda)}(s'-s''-\epsilon(s'-s'')\frac{s}{2})}}{\sinh(g_-F'_{(\lambda)}s)} \right\}$$