Quarkonium suppression in heavy-ion collisions: an open quantum system approach

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Motivation

We will explore concepts like

- environment,
- open quantum system,
- multiscale problem,
- perturbative and non-perturbative (numerical, lattice field theory) physics,

on the example of quarkonium formation in heavy-ion collisions at the LHC.

Open Quantum Systems

Density matrix

An arbitrary statistical ensemble of quantum states can be represented by a density matrix ρ, which is

- Hermitian: $\rho^\dagger = \rho$;
- positive: $\langle\psi|\rho|\psi\rangle \geq 0$ for all nonzero states $|\psi\rangle$;
- and can be normalized to have unit trace: $\mathrm{Tr}\{\rho\} = 1$.

The time evolution of the density matrix is described by the von Neumann equation:

$$i\frac{d\rho}{dt} = [H, \rho]$$

which follows from the Schrödinger equation for $|\psi\rangle$. The evolution equation

- is linear in ρ;
- preserves the trace of ρ;
- is Markovian.

Open quantum system

In quantum information theory, one separates the full system into a subsystem of interest and its environment. A density matrix ρ for the subsystem can be obtained from the density matrix ρ_{full} for the full system by the partial trace over the environment states:

$$\rho = \mathrm{Tr}_{\text{environment}} \{\rho_{\text{full}}\}$$

In general the evolution of ρ is non-Markovian.

The evolution is Markovian if the time during which the subsystem is observed is much larger than the time scale for correlations between the subsystem and the environment. We must also restrict to the low-frequency behavior of the subsystem, which can be accomplished by smoothing out over times larger than the correlation time scale.

Lindblad equation

The density matrix ρ for the subsystem necessarily satisfies the three basic properties: it is Hermitian, positive, and it can be normalized.

If further the time evolution is linear in ρ, preserves the trace of ρ, is Markovian and the linear operator that determines the time evolution of ρ is completely positive
$\Rightarrow$ then this require the time evolution equation to have the Lindblad form

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\})$$

where H is a Hermitian operator and the C_n's are an additional set of operators called collapse operators.

- Lindblad CMP 48 (1976) 119
  Gorini Kossakowski Sudarshan JMP 17 (1976) 821

Numerical solutions: QuTiP

There exist numerical toolboxes for open quantum systems. An example is QuTiP.

QuTiP

Quantum Toolbox in Python

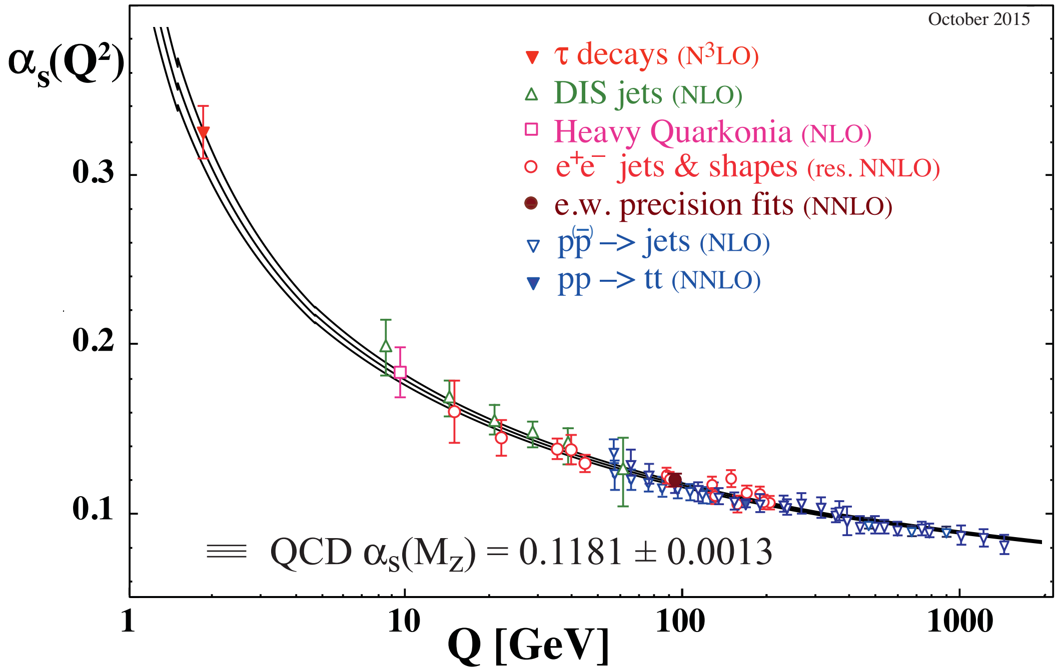
- Johansson, Nation, Nori, CPC 183 (2012) 1760, 184 (2013) 1234

The Environment:

Quark Gluon Plasma

Quantum Chromodynamics (QCD)

The fundamental theory of the strong interaction is Quantum Chromodynamics (QCD), which is part of the Standard Model. Its degrees of freedom are quarks and gluons, labeled by quantum numbers like flavor for quarks (u,d,s,c,b,t) and color (quarks transform as fundamental, gluons as adjoint representations of SU(3)).
It depends on one coupling: $\alpha_s = g^2/(4\pi)$, or on one fundamental scale: Λ_{QCD}.



- PDG (2015)

Perturbative and non-perturbative QCD

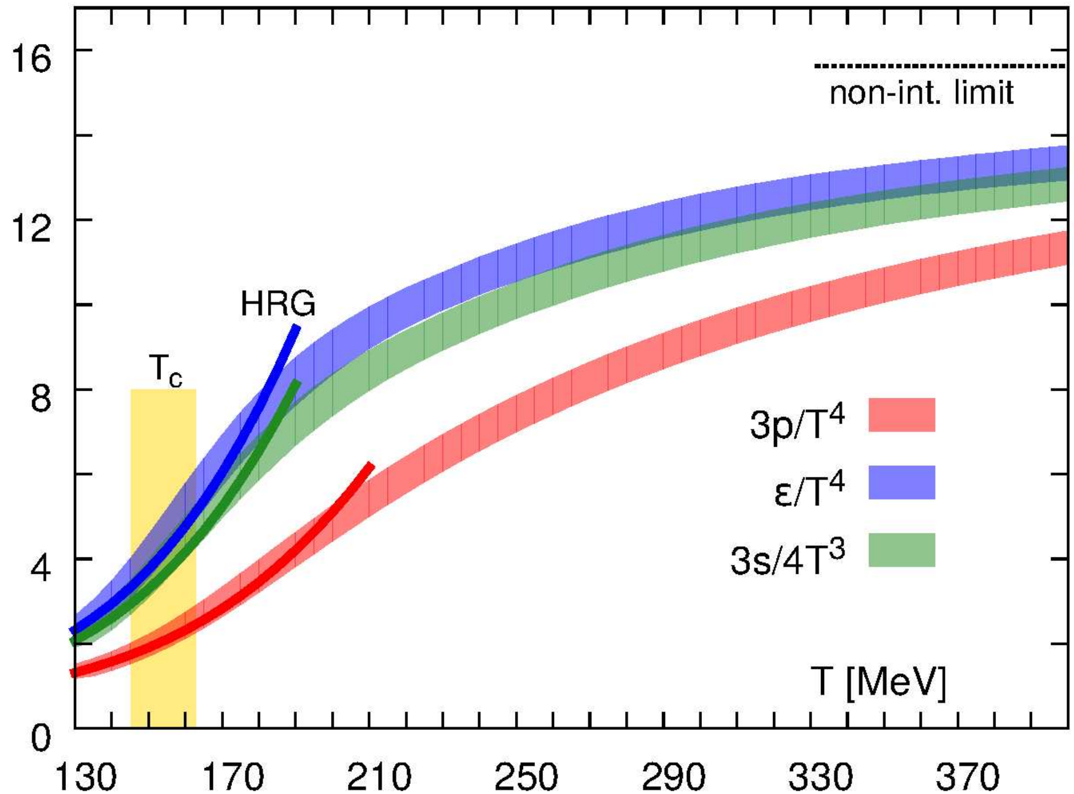
- At high energies: asymptotic freedom.
- At low energies: non-perturbative QCD.

  It is eventually responsible for the confinement of color, i.e., final states, those detected in experiments, are color neutral states.
  These are hadrons but neither quarks nor gluons.

- In a thermal environment like the one created by heavy-ion collisions (at the LHC, lead-lead collisions, the number of nucleons in lead is 207) more scales appear.

  In thermal equilibrium these are $T,\ gT,\ g^2T$.
  May need some non-perturbative treatment at any thermal scale since $g = \sqrt{4\pi\alpha_{\rm s}} \gtrsim 1$.
  $g \sim 1$ qualifies the plasma as strongly coupled.

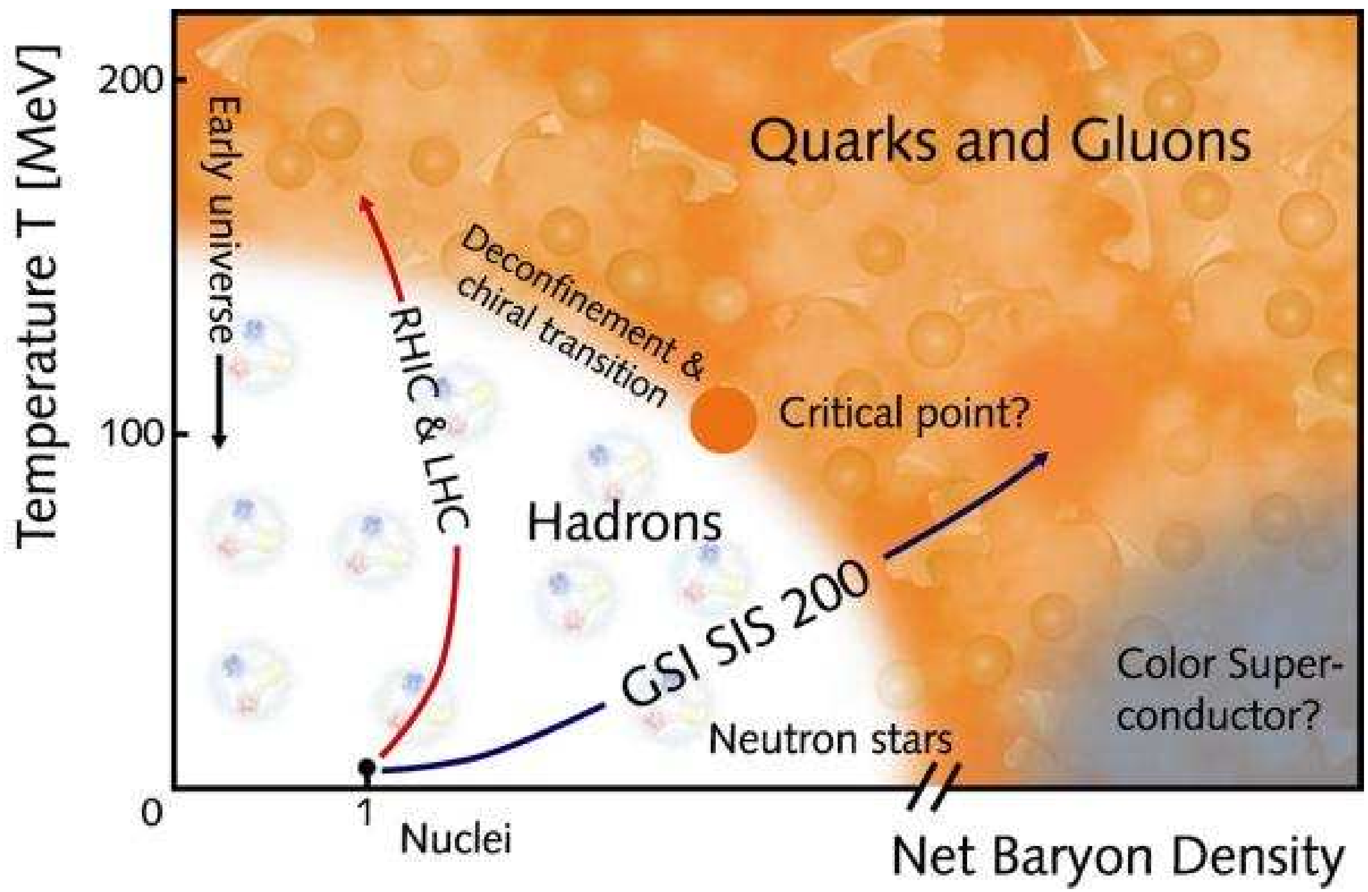
Color deconfinement



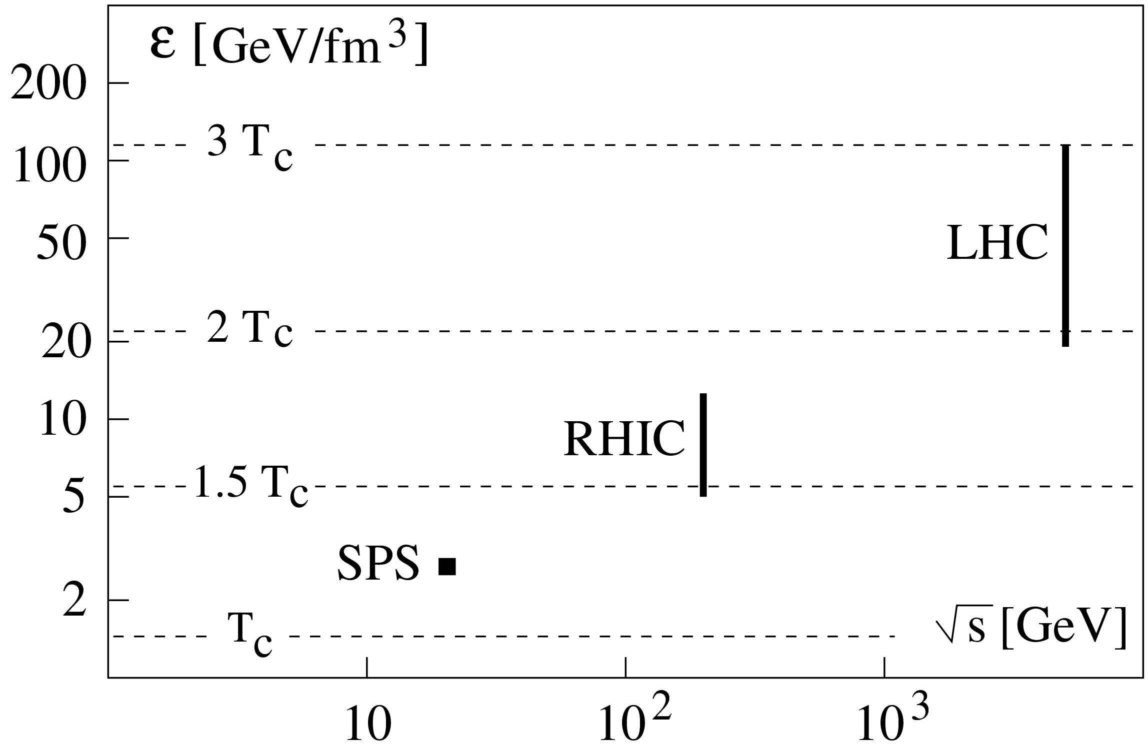
Transition from hadronic matter to a plasma of deconfined quarks and gluons as studied in finite temperature lattice QCD.

- Wuppertal-Budapest JHEP 1009 (2010) 073
  HotQCD PRD 90 (2014) 094503, TUMQCD PRD 93 (2016) 114502

QCD phase diagram



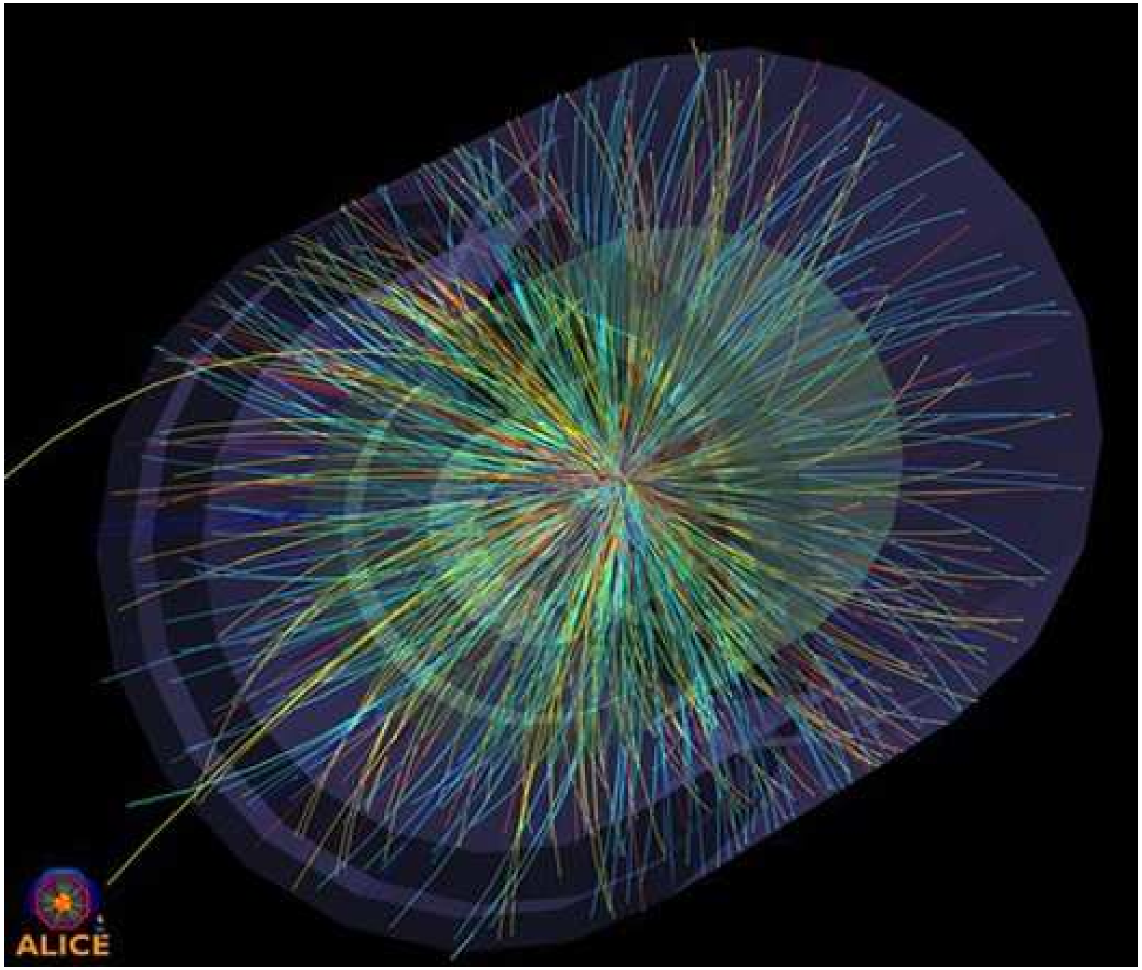
Heavy-ion experiments



High energy densities and temperatures $> T_c$ as explored by the heavy-ion experiments at RHIC and LHC.

- Satz JPG 32 (2006) R25

Heavy-ion experiments



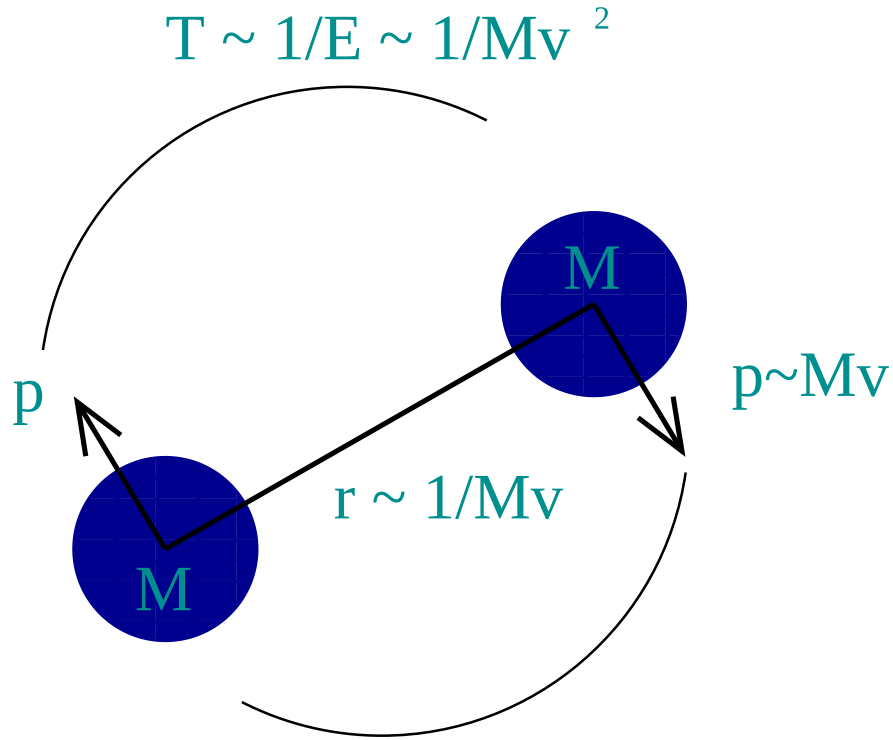
We need probes to identify the state of matter that is formed.

The Subsystem:

Quarkonium

Quarkonium

Quarkonium is a hadron made of a heavy quark (c, b) and a heavy anti-quark.



It is a non-relativistic bound state: relative velocity $= v \ll 1$.
Hence characterized by the typical hierarchy of scales of a non-relativistic bound state.

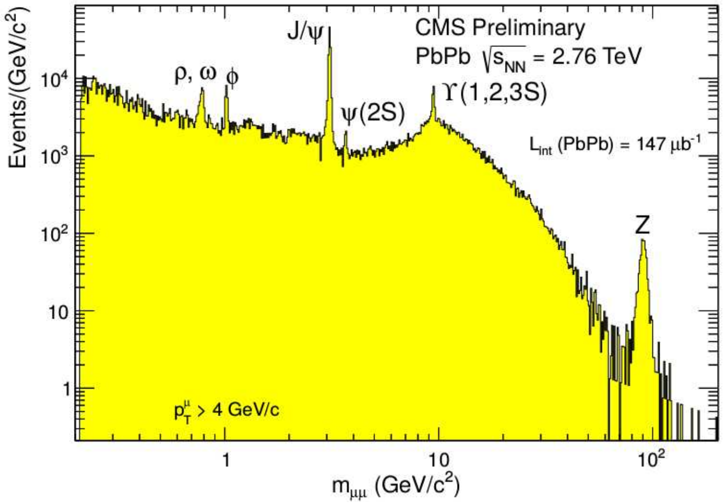
Quarkonium as a quark-gluon plasma probe

In 1986, Matsui and Satz suggested quarkonium as an ideal quark-gluon plasma probe.

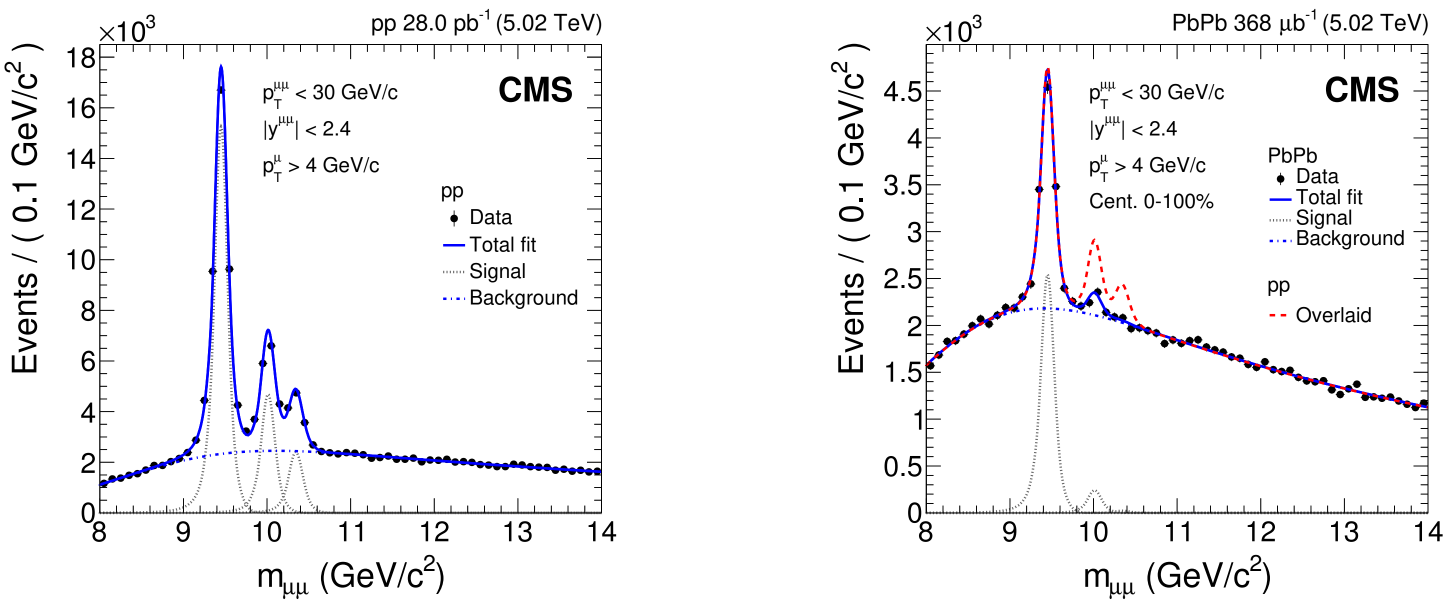
◦ `Matsui Satz PLB 178 (1986) 416`

Experimentally

- Heavy quarks are formed early in heavy-ion collisions: $1/M \sim 0.1$ fm < 0.6 fm.
- Heavy quarkonium formation will be sensitive to the medium.
- The dilepton signal (happening after the quark-gluon plasma has faded away) makes the quarkonium a clean experimental probe.

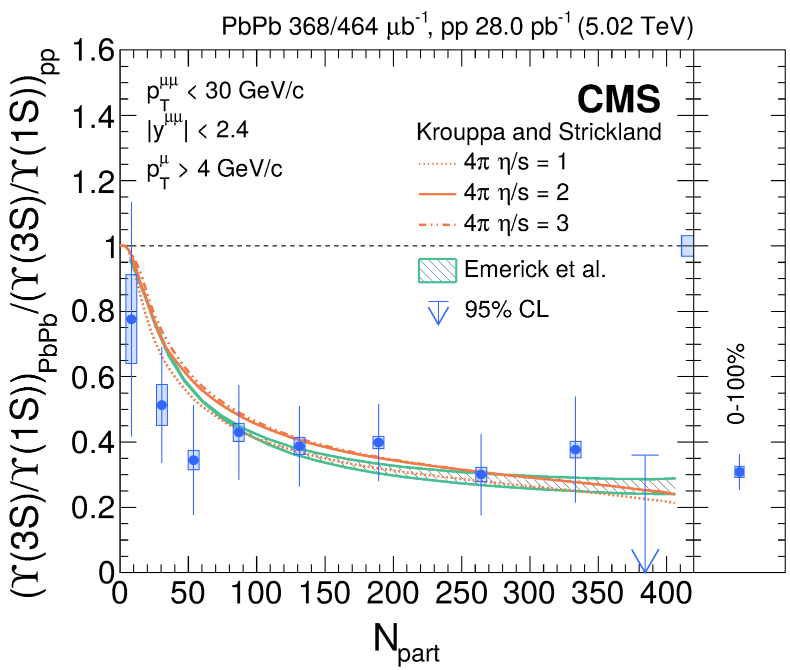


Υ suppression at CMS



- CMS arXiv:1706.05984

Υ suppression at CMS



- CMS arXiv:1706.05984

Quarkonium as a quark-gluon plasma probe

In 1986, Matsui and Satz suggested quarkonium as an ideal quark-gluon plasma probe.

Theoretically

- The heavy-quark mass introduces one or more large scales, whose contributions may be factorized and computed in perturbation theory ($\alpha_{\rm s}(M) \ll 1$).
- Low-energy scales are sensitive to the medium.
  Low-energy contributions may be accessible via lattice calculations.

Quarkonium as a multiscale system

Quarkonium being a composite system is characterized by several energy scales:

- the scales of a non-relativistic bound state
  (v is the relative heavy-quark velocity; $v \sim \alpha_s$ for a Coulombic bound state):
  M (mass),
  Mv (momentum transfer, inverse distance),
  Mv^2 (kinetic energy, binding energy, potential V), ...

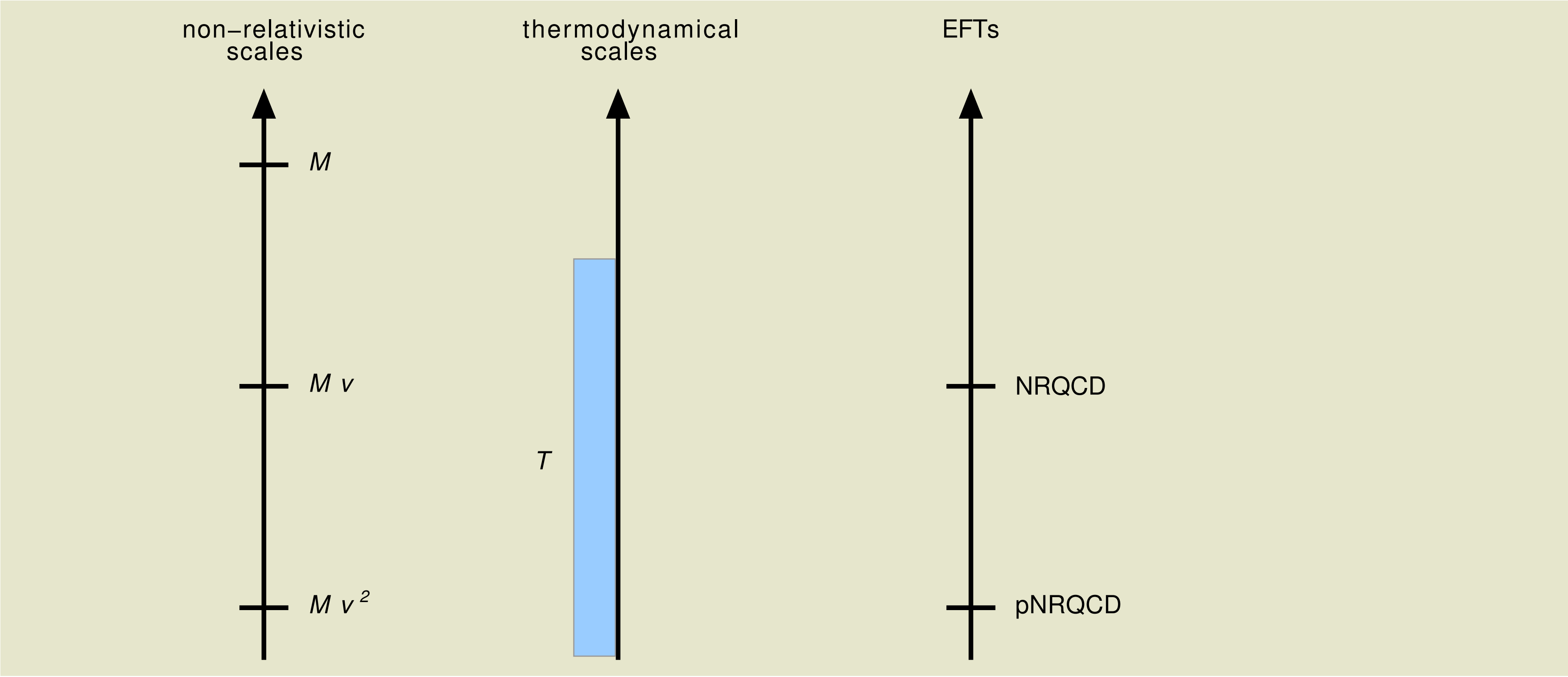
- the thermodynamical scales:
  T (temperature), ...

  T may stand for the inverse correlation length between system and environment. For definiteness we will assume that the system is locally in thermal equilibrium so that a slowly varying time-dependent temperature can be defined.

The non-relativistic scales are hierarchically ordered: $M \gg Mv \gg Mv^2$

Non-relativistic EFTs of QCD

The existence of a hierarchy of energy scales calls for a description of the system (quarkonium at rest in a thermal bath) in terms of a hierarchy of EFTs.



- Brambilla Pineda Soto Vairo RMP 77 (2005) 1423
  Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017

Effective Field Theories (Weinberg)

Whenever a system H, described by a Lagrangian $\mathcal{L}$, is characterized by 2 scales $\Lambda \gg \lambda$, observables may be calculated by expanding one scale with respect to the other. An effective field theory makes the expansion in λ/Λ explicit at the Lagrangian level.

The EFT Lagrangian, $\mathcal{L}_{\rm EFT}$, suitable to describe H at scales lower than Λ is defined by
(1) a cut off $\Lambda \gg \mu \gg \lambda$;
(2) by some degrees of freedom that exist at scales lower than μ

$\Rightarrow \mathcal{L}_{\rm EFT}$ is made of all operators O_n that may be built from the effective degrees of freedom and are consistent with the symmetries of $\mathcal{L}$.

Effective Field Theories (Weinberg)

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(\Lambda/\mu) \frac{O_n(\mu,\lambda)}{\Lambda^n}$$

- Since at $\mu \sim \lambda$, $\langle O_n \rangle \sim \lambda^n$, the EFT is organized as an expansion in λ/Λ.
- The EFT is renormalizable order by order in λ/Λ.
- The matching coefficients $c_n(\Lambda/\mu)$ encode the non-analytic behaviour in Λ. They are calculated by imposing that $\mathcal{L}_{\mathrm{EFT}}$ and $\mathcal{L}$ describe the same physics at any finite order in the expansion: matching procedure.
- In QCD, if $\Lambda \gg \Lambda_{\mathrm{QCD}}$ then $c_n(\Lambda/\mu)$ may be calculated in perturbation theory.

NRQCD

NRQCD is obtained by integrating out modes associated with the scale M.

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i + \dots \\ &+\psi^\dagger\left(iD_0 + \frac{\mathbf{D}^2}{2M} + \dots\right)\psi + \chi^\dagger\left(iD_0 - \frac{\mathbf{D}^2}{2M} + \dots\right)\chi \\ &+\frac{d_{ss}}{M^2}\psi^\dagger\psi\chi^\dagger\chi + \dots\end{aligned}$$

ψ (χ) is the field that annihilates (creates) the (anti)quark.

- The Lagrangian is systematically organized as a double expansion in $1/M$ and $\alpha_s(M)$: $\mathcal{L} = \sum_n c(\alpha_s(M/\mu)) \times O_n(\mu,\lambda)/M^n$.
- The relevant dynamical scales are: Mv, Mv^2, ... thermodynamical scales.

pNRQCD

pNRQCD is obtained by integrating out modes associated with the scale Mv.

- The Lagrangian is organized as an expansion in r:

$$\mathcal{L} = \sum_n V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

- Degrees of freedom:

  - Q-$\bar{Q}$ states, with energy $\sim$ scales lower than Mv and momentum $\lesssim Mv$
  $\Rightarrow$ (i) singlet S (ii) octet O

  - Gluons with energy and momentum $\sim$ scales lower than Mv.
  All gauge fields are multipole expanded: $A(R, r, t) = A(R, t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R, t) + \ldots$

pNRQCD: the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\,a} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i + \int d^3 r \mathrm{Tr} \left\{ \mathrm{S}^\dagger \left(i\partial_0 - h_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_o \right) \mathrm{O} \right\}$$

- LO in r

$\theta(T)\, e^{-iTh_s}$ $\quad$ $\theta(T)\, e^{-iTh_o} \left(e^{-i\int dt\, A^{\mathrm{adj}}} \right)$

pNRQCD: the Lagrangian

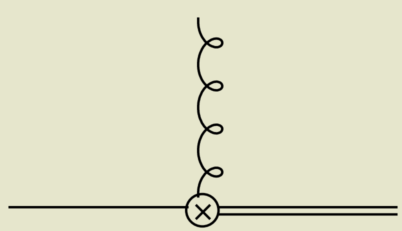
$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i + \int d^3 r \mathrm{Tr} \left\{ \mathrm{S}^\dagger \left(i\partial_0 - h_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_o \right) \mathrm{O} \right\}$$

- LO in r

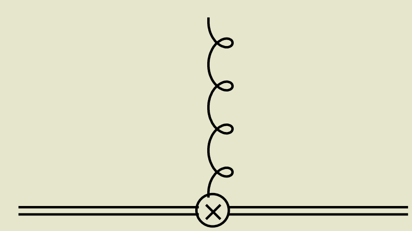
$\theta(T)\, e^{-iTh_s}$ $\qquad$ $\theta(T)\, e^{-iTh_o} \left(e^{-i \int dt\, A^{\mathrm{adj}}} \right)$

$$+ V_A \mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} \right\} + \frac{V_B}{2} \mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E} \right\}$$

- NLO in r



$O^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S}$



$O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, \, \mathrm{O} \}$

$$+ \cdots$$

Quarkonium in medium
as an open quantum system

Quarkonium in a fireball

- After the heavy-ion collisions, heavy quark-antiquarks propagate freely up to 0.6 fm.
- From 0.6 fm to the freeze-out time t_F they propagate in the medium.
- We assume the medium infinite, homogeneous and isotropic.
- We assume the medium to be locally in thermal equilibrium,
  i.e., the temperature T of the medium changes (slowly) with time:

$$T = T_0 \left(\frac{t_0}{t}\right)^{v_s^2}, \quad t_0 = 0.6 \text{ fm}, \quad v_s^2 = \frac{1}{3} \text{ (sound velocity)}$$

  ○ Bjorken PRD 27 (1983) 140

  The initial temperature T_0 may account for different centralities

centrality (%)	$\langle b \rangle$ (fm)	T_0 (MeV) @ LHC
$0 - 10$	3.4	471
$10 - 20$	6.0	461
$20 - 30$	7.8	449
$30 - 50$	9.9	425
$50 - 100$	13.6	304

- We assume the heavy quarks comoving with the medium.

Quarkonium as a Coulombic bound state

The lowest quarkonium states (1S bottomonium and charmonium, 2S bottomonium) are the most tightly bound. For these we can assume the hierarchy of energy scales

$$M \gg \frac{1}{r} \sim M\alpha_s \gg T \sim gT \gg \text{any other scale}, \qquad v \sim \alpha_s$$

This qualifies the bound state as Coulombic:

- quark-antiquark color singlet Hamiltonian $= h_s = \dfrac{\mathbf{p}^2}{M} - \dfrac{4}{3}\dfrac{\alpha_s}{r}$
- quark-antiquark color octet Hamiltonian $= h_o = \dfrac{\mathbf{p}^2}{M} + \dfrac{\alpha_s}{6r}$

The attractive factor $-4/3$ and repulsive factor $1/6$ come from the color quantum numbers. The octet potential describes an unbound quark-antiquark pair.

Density matrices

- Subsystem: heavy quarks/quarkonium
- Environment: quark gluon plasma

We may define a density matrix in pNRQCD for the heavy quark-antiquark pair in a singlet and octet configuration:

$$\begin{aligned} \langle \mathbf{r}', \mathbf{R}' | \rho_s(t';t) | \mathbf{r}, \mathbf{R} \rangle &\equiv \mathrm{Tr}\{\rho_{\mathrm{full}}(t_0)\, S^\dagger(t,\mathbf{r},\mathbf{R}) S(t',\mathbf{r}',\mathbf{R}')\} \\ \langle \mathbf{r}', \mathbf{R}' | \rho_o(t';t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{N_c^2-1} &\equiv \mathrm{Tr}\{\rho_{\mathrm{full}}(t_0)\, O^{a\dagger}(t,\mathbf{r},\mathbf{R}) O^b(t',\mathbf{r}',\mathbf{R}')\} \end{aligned}$$

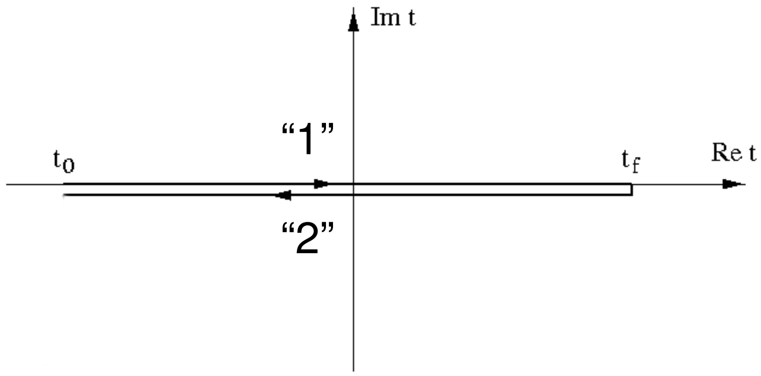
$t_0 \approx 0.6$ fm is the time formation of the plasma.

The system is in non-equilibrium because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although the number of heavy quarks is conserved: $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\} = 1$.

Closed-time path formalism

In the closed-time path formalism we can represent the density matrices as 12 propagators on a closed time path:

$$\begin{aligned}\langle \mathbf{r}', \mathbf{R}' | \rho_s(t';t) | \mathbf{r}, \mathbf{R} \rangle &= \langle S_1(t', \mathbf{r}', \mathbf{R}') S_2^\dagger(t, \mathbf{r}, \mathbf{R}) \rangle \\ \langle \mathbf{r}', \mathbf{R}' | \rho_o(t';t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{N_c^2 - 1}, &= \langle O_1^b(t', \mathbf{r}', \mathbf{R}') O_2^{a\dagger}(t, \mathbf{r}, \mathbf{R}) \rangle\end{aligned}$$

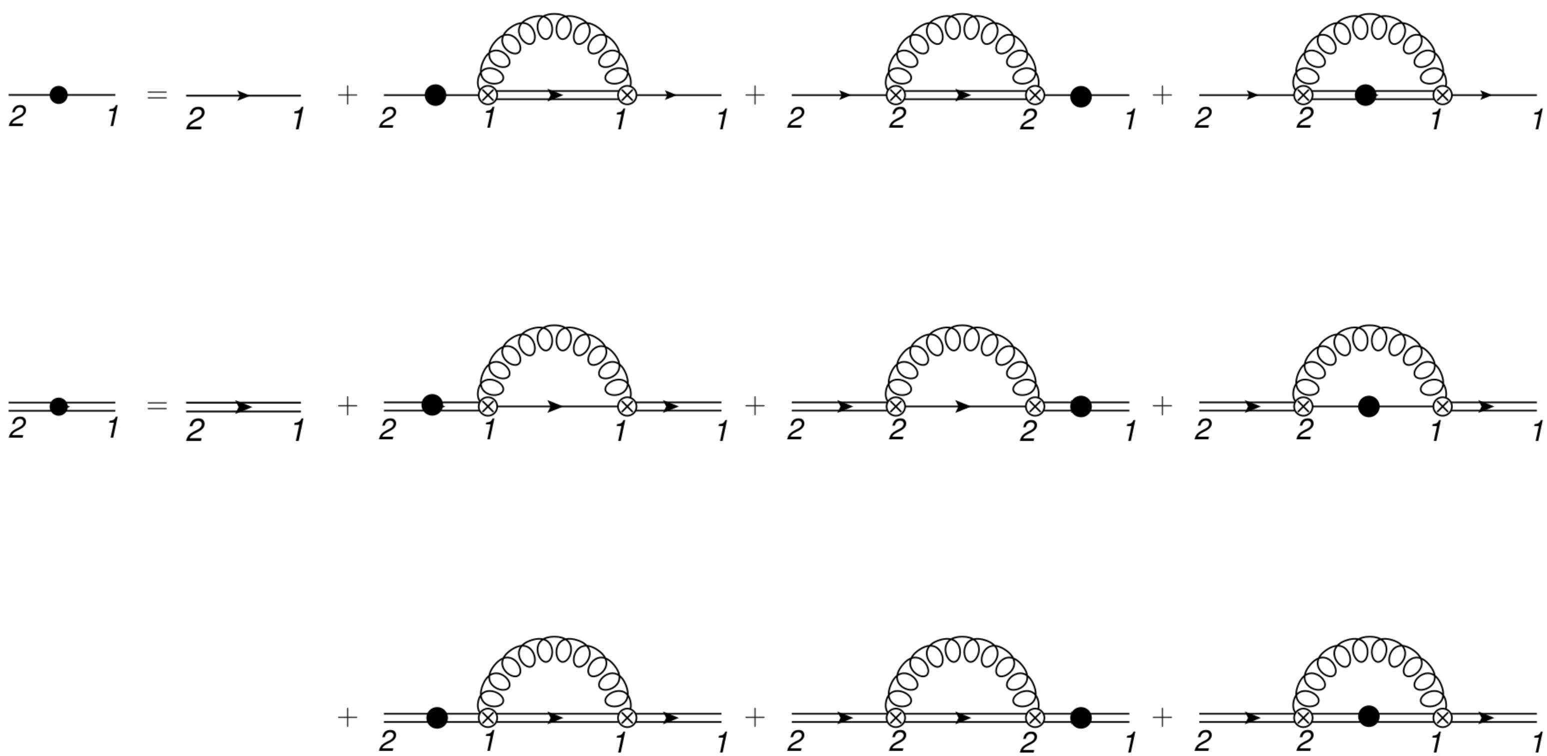


Differently from the thermal equilibrium case 12 propagators are relevant (in thermal equilibrium they are exponentially suppressed).

12 propagators are not time ordered, while 11 and 22 operators select the forward time direction $\propto \theta(t - t')$, $\theta(t' - t)$.

Resummation

Resumming $(t - t_0)\times$ self-energy contributions à la Schwinger–Dyson ...



Evolution equations

... and differentiating over time we obtain the coupled evolution equations:

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t), t)$$

$$\frac{d\rho_o(t;t)}{dt} = -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t), t) + \Xi_{oo}(\rho_o(t;t), t)$$

Interpretation

- The self energies Σ_s and Σ_o provide the in-medium induced mass shifts, $\delta m_{s,o}$, and widths, $\Gamma_{s,o}$, for the color-singlet and color-octet heavy quark-antiquark systems respectively:

$$
\begin{aligned}
-i\Sigma_{s,o}(t) + i\Sigma^\dagger_{s,o}(t) &= 2\,\mathrm{Re}\left(-i\Sigma_{s,o}(t)\right) = 2\delta m_{s,o}(t) \\
\Sigma_{s,o}(t) + \Sigma^\dagger_{s,o}(t) &= -2\,\mathrm{Im}\left(-i\Sigma_{s,o}(t)\right) = \Gamma_{s,o}(t)
\end{aligned}
$$

- Ξ_{so} accounts for the production of singlets through the decay of octets, and Ξ_{os} and Ξ_{oo} account for the production of octets through the decays of singlets and octets respectively. There are two octet production mechanisms/octet chromoelectric dipole vertices in the pNRQCD Lagrangian.
- The conservation of the trace of the sum of the densities, i.e., the conservation of the number of heavy quarks, provides the constraints

$$
\begin{aligned}
\mathrm{Tr}\left\{\rho_s(t;t)\left(\Sigma_s(t) + \Sigma^\dagger_s(t)\right)\right\} &= \mathrm{Tr}\left\{\Xi_{os}(\rho_s(t;t),t)\right\} \\
\mathrm{Tr}\left\{\rho_o(t;t)\left(\Sigma_o(t) + \Sigma^\dagger_o(t)\right)\right\} &= \mathrm{Tr}\left\{\Xi_{so}(\rho_o(t;t),t) + \Xi_{oo}(\rho_o(t;t),t)\right\}
\end{aligned}
$$

Lindblad equations for the case $T \gg E$

If $(1/T)\, dT/dt \ll E$, $E \ll T$ the evolution equations can be written in the Lindblad form.

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\})$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\,\gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix},$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \qquad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

○ Brambilla Escobedo Soto Vairo PRD 96 (2017) 034021

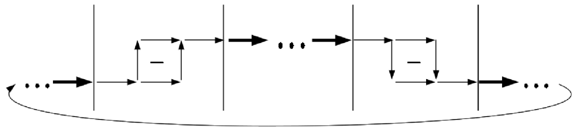
Results:

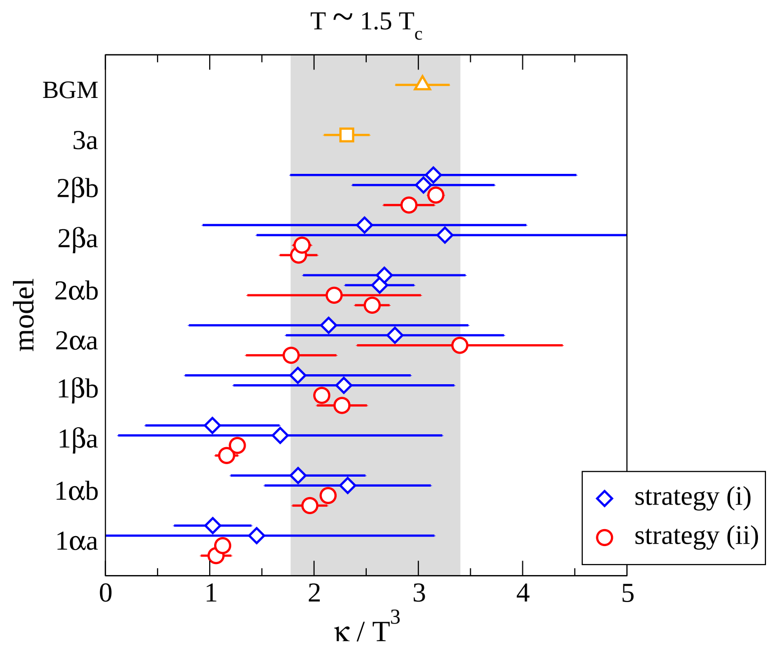
The Out of Equilibrium Evolution

Lattice QCD and low-energy coefficients

Low energy parameters may be determined by numerical calculations in lattice QCD.
κ is the heavy-quark momentum diffusion coefficient:

$$\kappa = \frac{g^2}{6\,N_c}\,\mathrm{Re}\int_{-\infty}^{+\infty} ds\,\langle\, \mathrm{T}\, E^{a,i}(s,\mathbf{0})\,\phi^{ab}(s,0)\,E^{b,i}(0,\mathbf{0})\rangle =$$





○ Francis Kaczmarek Laine Neuhaus Ohno PRD 92 (2015) 116003

Lattice QCD and low-energy coefficients

Low energy parameters may be determined by numerical calculations in lattice QCD.

$$\gamma = \frac{g^2}{6\,N_c}\,\mathrm{Im}\int_{-\infty}^{+\infty} ds\,\langle\,\mathrm{T}\,E^{a,i}(s,\mathbf{0})\,\phi^{ab}(s,0)\,E^{b,i}(0,\mathbf{0})\rangle$$

γ is known only in perturbation theory.

Initial conditions

- The production of singlets is α_s suppressed compared to that of octets.

○ `Cho Leibovich PRD 53 (1996) 6203`

- Our choice at $t = 0$ is

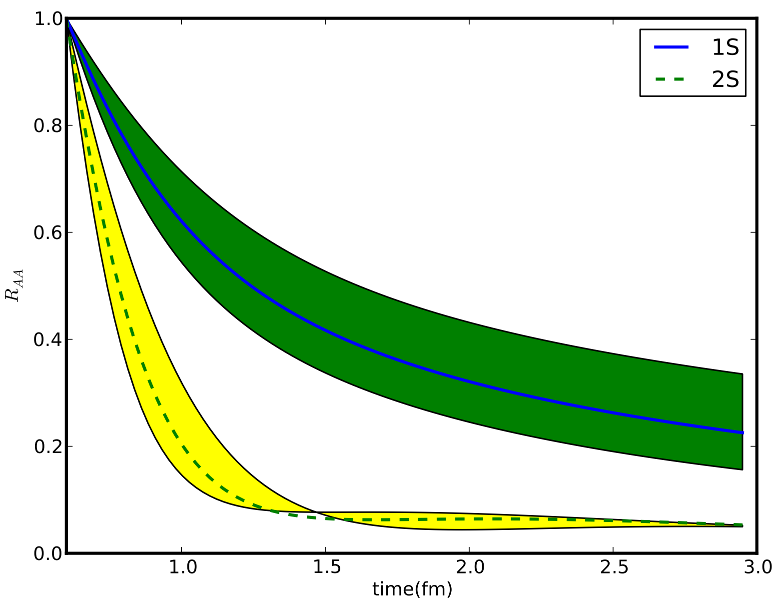
$$\rho_s = N|\mathbf{0}\rangle\langle\mathbf{0}|, \qquad \rho_o = \frac{\delta}{\alpha_s(M)}\rho_s$$

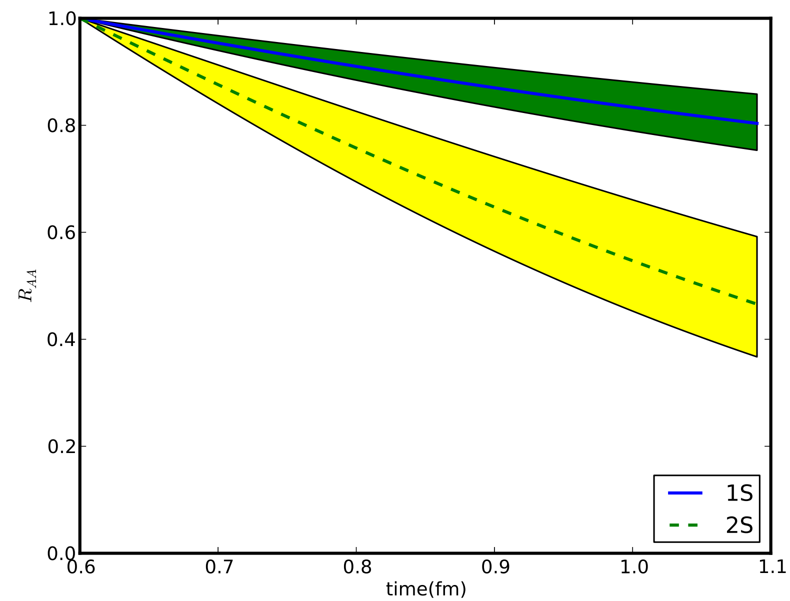
N is fixed by $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\} = 1$
δ fixes the octet fraction with respect to the singlet.

- We compute the nuclear modification factor R_{AA}:

$$R_{AA} \sim \frac{\rho_S|^{AA}_{1S\,1S}}{\rho_S|^{pp}_{1S\,1S}}$$

Time evolution of R_{AA} for $1S$ and $2S$ bottomonium

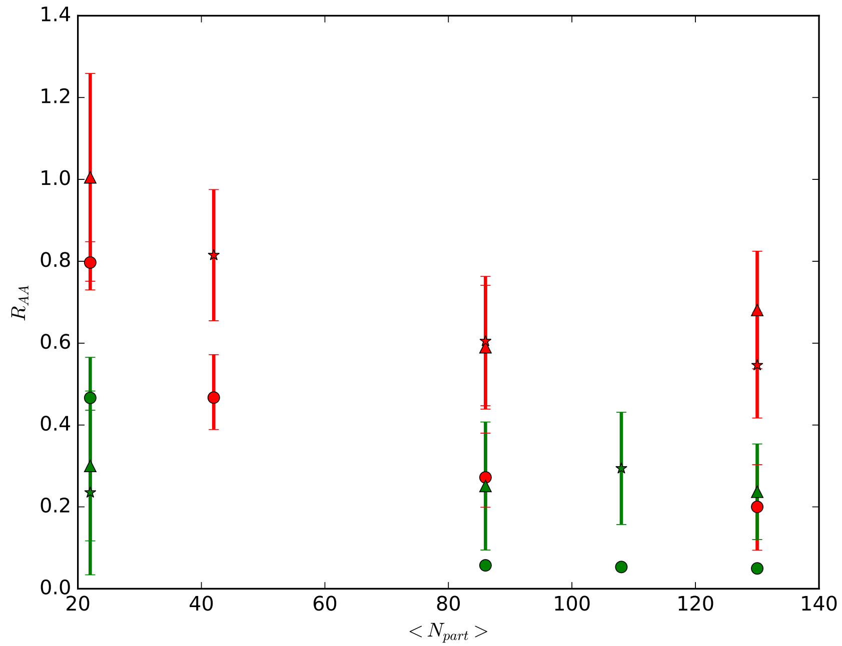




$30 - 50\%$ centrality (left) and $50 - 100\%$ centrality (right)

$\gamma = 0$ and $\delta = 1$

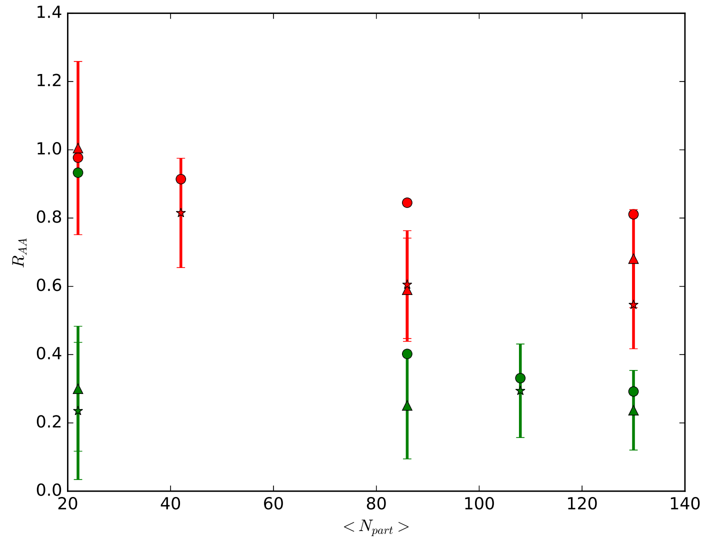
Bottomonium nuclear modification factor vs CMS data

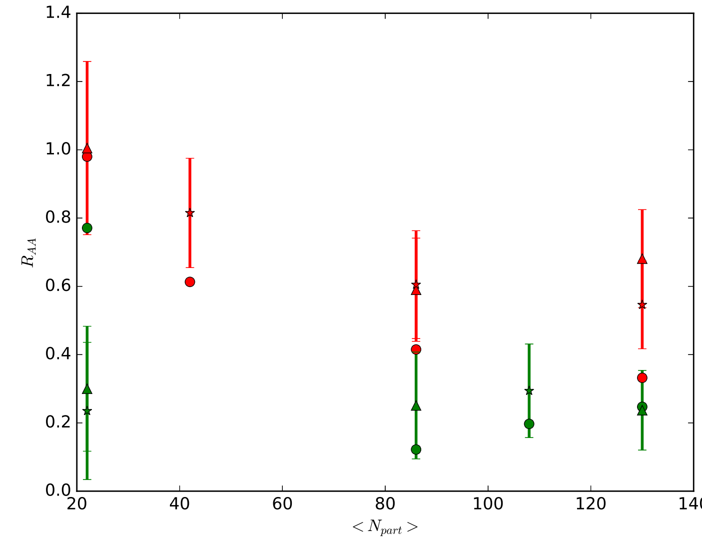


○: theory results; △: CMS data; red: $\Upsilon(1S)$; green: $\Upsilon(2S)$.

- CMS coll. PRL 109 (2012) 222301 and PLB 770 (2017) 357
  Brambilla Escobedo Soto Vairo PRD 96 (2017) 034021

Bottomonium nuclear modification factor vs CMS data





$\kappa/T^3 = 0.25$ and $\gamma = 0$;

$\kappa/T^3 = 2.6$ and $\gamma/T^3 = 6$.

- Brambilla Escobedo Soto Vairo (2017) in preparation

Conclusions

We have shown a realistic particle physics example where a complex full system made out of a multiscale subsystem (quarkonium) interacting with a rich and inherently non-perturbative environment (the quark-gluon plasma) could be studied in its out-of-equilibrium evolution with the methods of

- effective field theories, to factorize contributions coming from different energy scales. Contributions coming from high-energy scales (mass, ...) can be computed in perturbation theory.
- lattice QCD, to compute numerically on a space-time lattice low-energy non-perturbative contributions.
- open quantum systems, to compute the out-of-equilibrium evolution of the subsystem and its non-trivial interaction with the environment (production, dissociation and recombination of quarkonium).

As a result we could describe the evolution of the density of heavy quark and antiquarks taking into account the conservation of the total number of heavy quarks, the non-abelian nature of QCD, without any classical approximation.