

ANEC and String Theory

Sunny Itzhaki

Based on 1808.02259

and work in progress

ANEC and String Theory

Outline:

- Why we like the Averaged Null Energy Condition (ANEC) so much.

ANEC and String Theory

Outline:

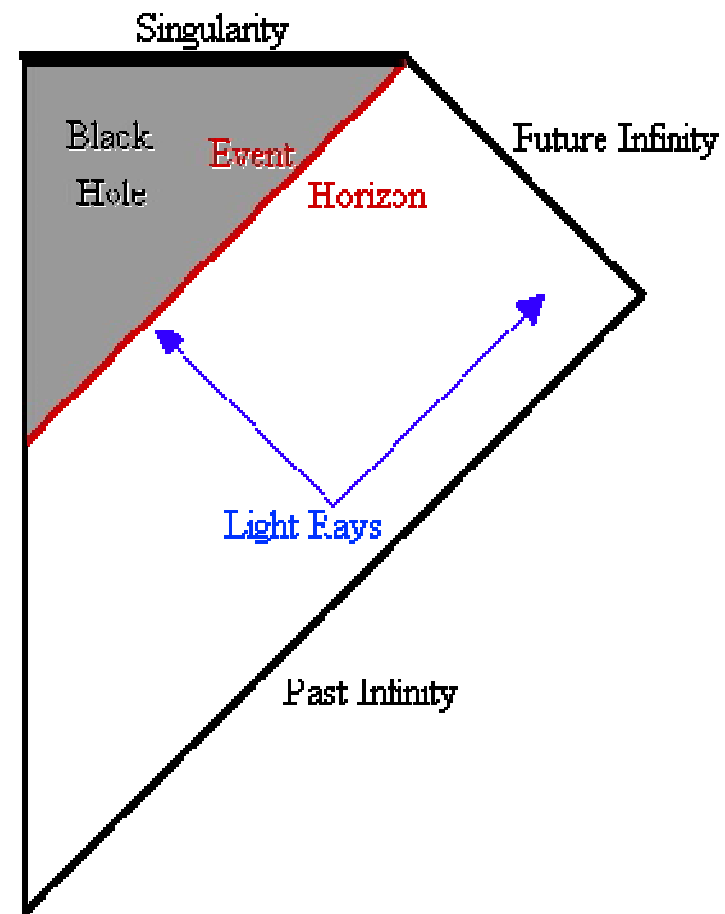
- Why we like the Averaged Null Energy Condition (ANEC) so much.
- Why we are certain it is correct.

ANEC and String Theory

Outline:

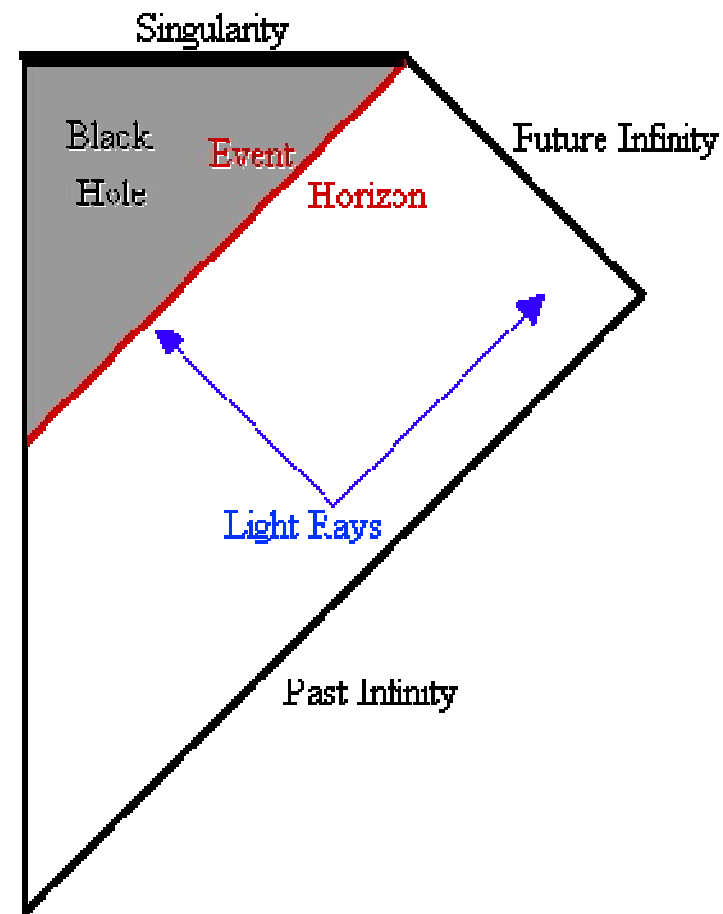
- Why we like the Averaged Null Energy Condition (ANEC) so much.
- Why we are certain it is correct.
- What string theory has to say about that.

In GR the BH horizon is smooth



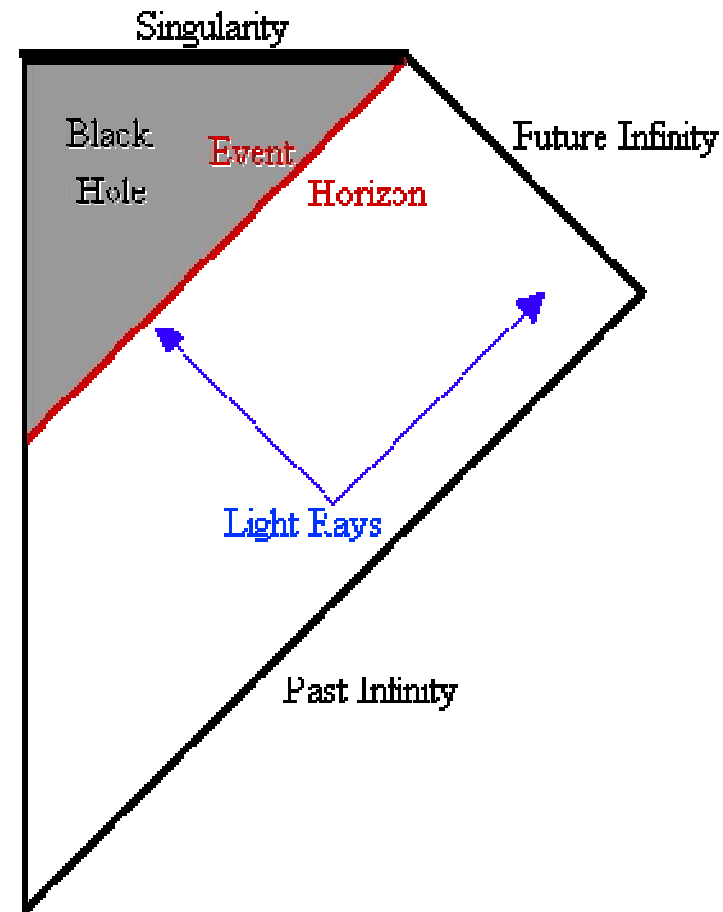
In GR the BH horizon is smooth:

- No local way to know we are crossing the horizon.



In GR the BH horizon is smooth:

- No local way to know we are crossing the horizon.
- Once crossing the horizon we must reach the singularity.

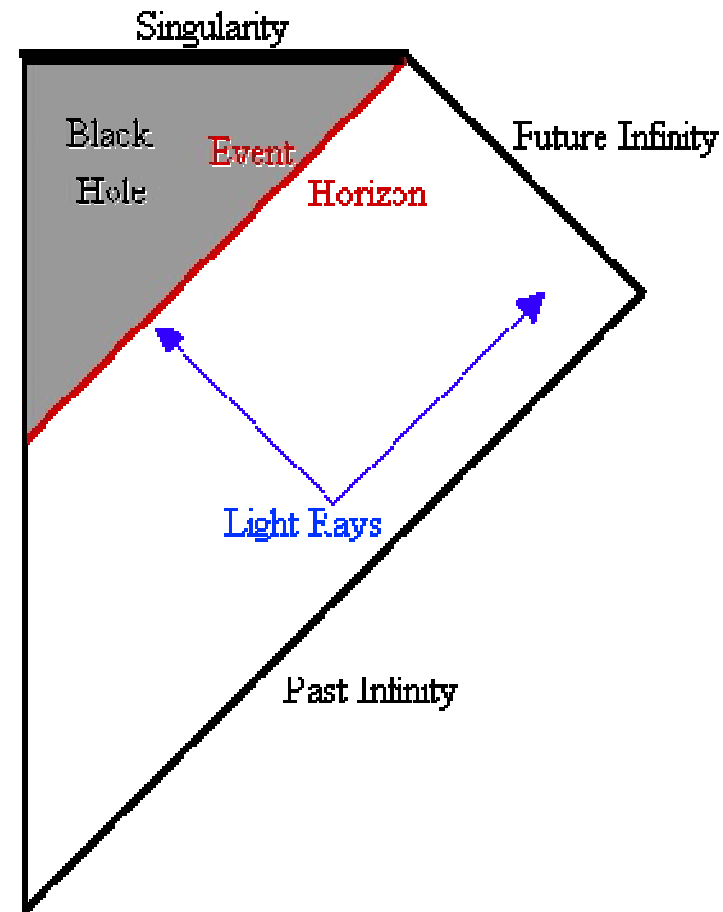


In GR the BH horizon is smooth:

- No local way to know we are crossing the horizon.
- Once crossing the horizon we **must** reach the singularity.

Energy conditions:

1- Weak



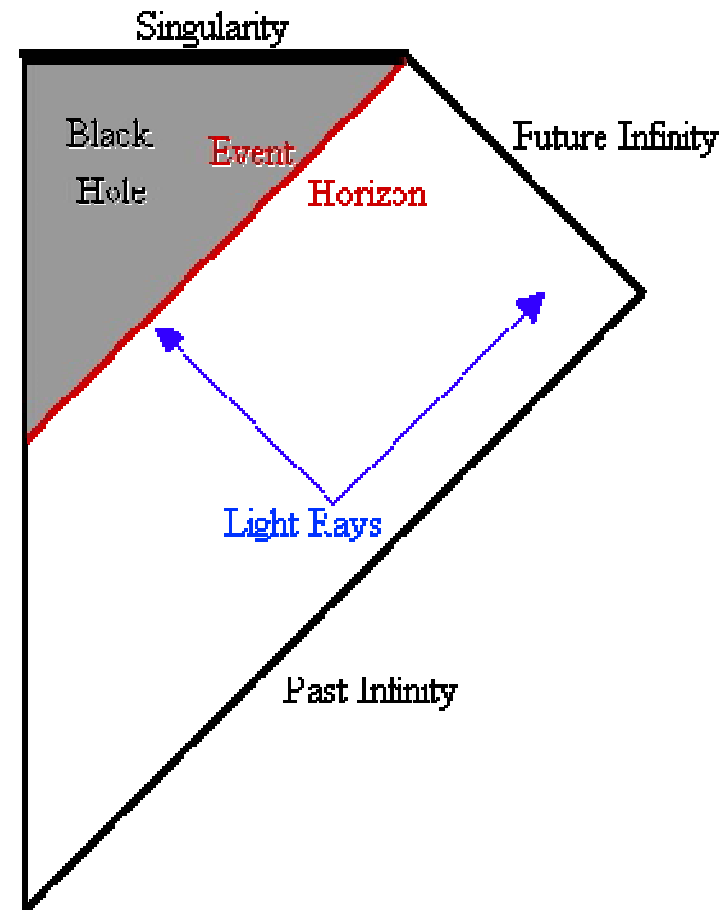
In GR the BH horizon is smooth:

- No local way to know we are crossing the horizon.
- Once crossing the horizon we **must** reach the singularity.

Energy conditions:

1- Weak X

2- Null

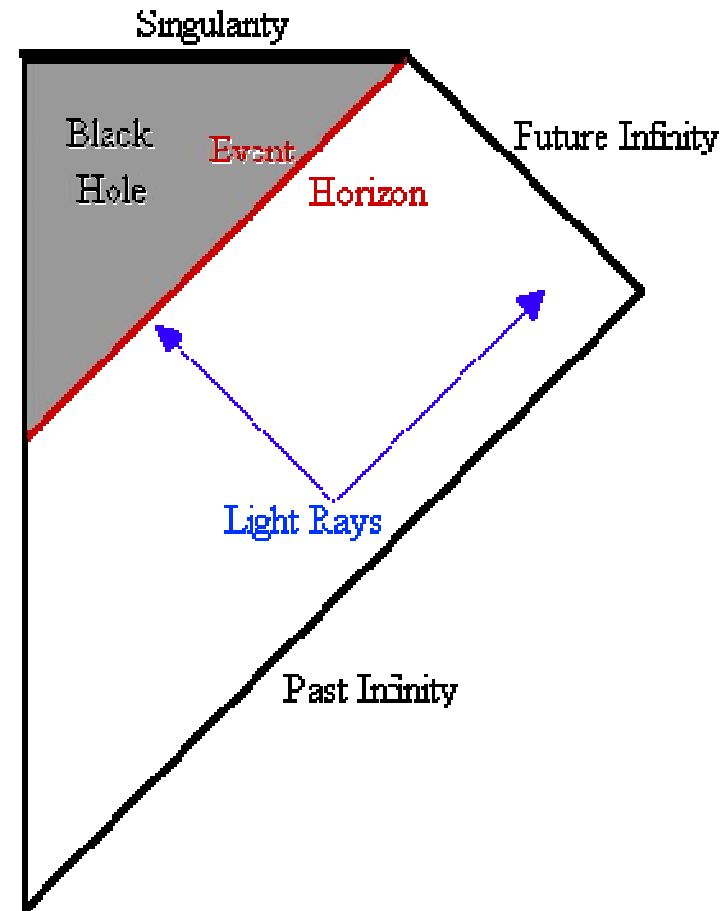


In GR the BH horizon is smooth:

- No local way to know we are crossing the horizon.
- Once crossing the horizon we **must** reach the singularity.

Energy conditions:

- 1- Weak \times
- 2- Null \times (at the quantum level)
- 3- ANEC (Averaged null energy condition)



A bit on ANEC

1. Once it was found that in QFT it is easy to violate

the null energy condition: $\langle \Psi | T_{uu} | \Psi \rangle < 0$

It was quickly realized that in fact the ANEC is sufficient for most of the GR needs [Borde, 87].

A bit on ANEC

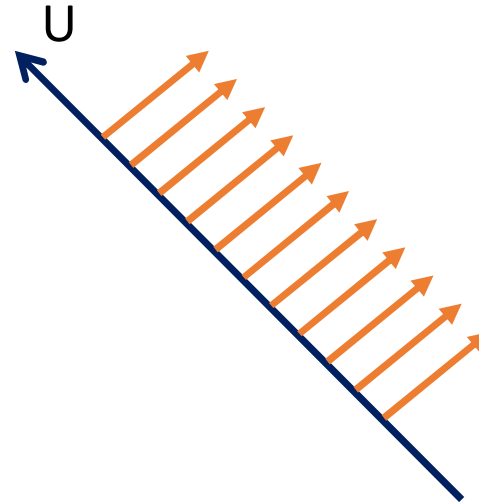
1. Once it was found that in QFT it is easy to violate

the null energy condition: $\langle \Psi | T_{uu} | \Psi \rangle < 0$

It was quickly realized that in fact the ANEC is sufficient for most of the GR needs [\[Borde, 87\]](#).

2. Definition:

$$\int_{\gamma} du T_{uu} \geq 0$$



A bit on ANEC

3. Classical examples (that we'll not discuss too much):

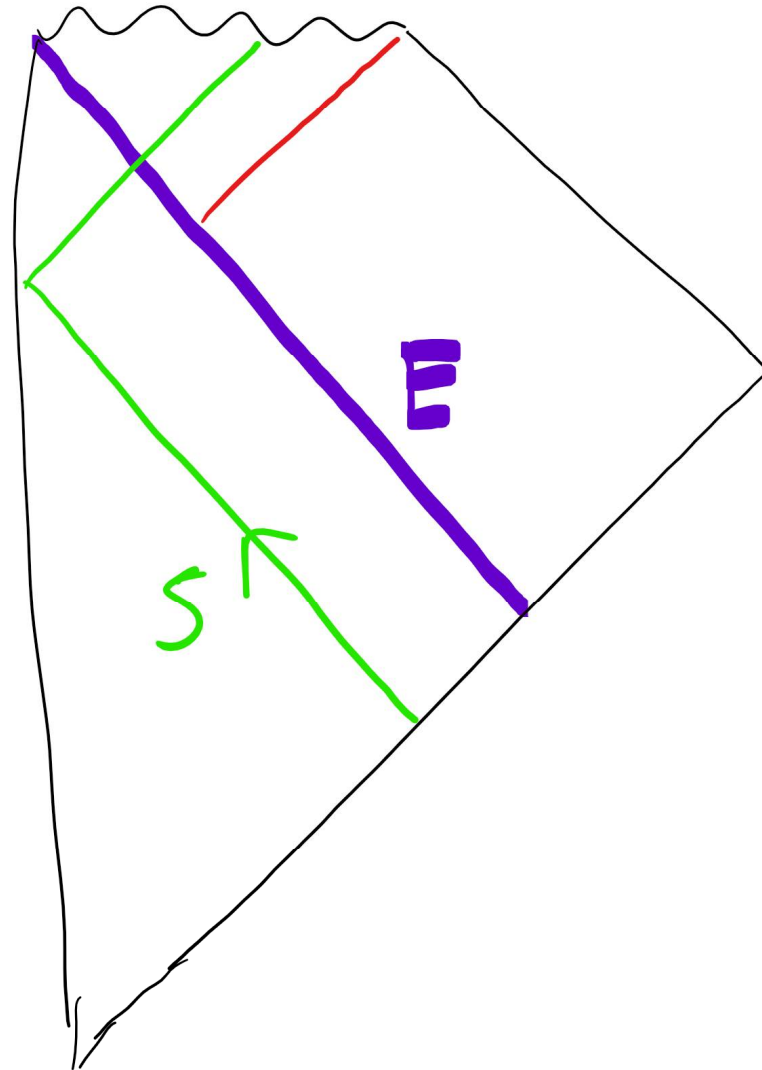
Focusing theorems, no traversable wormhole, topo. censorship etc.

A bit on ANEC

3. An example:

Firewall is not enough

A firewall (red line) cannot prevent
from all the info. to reach the singularity.



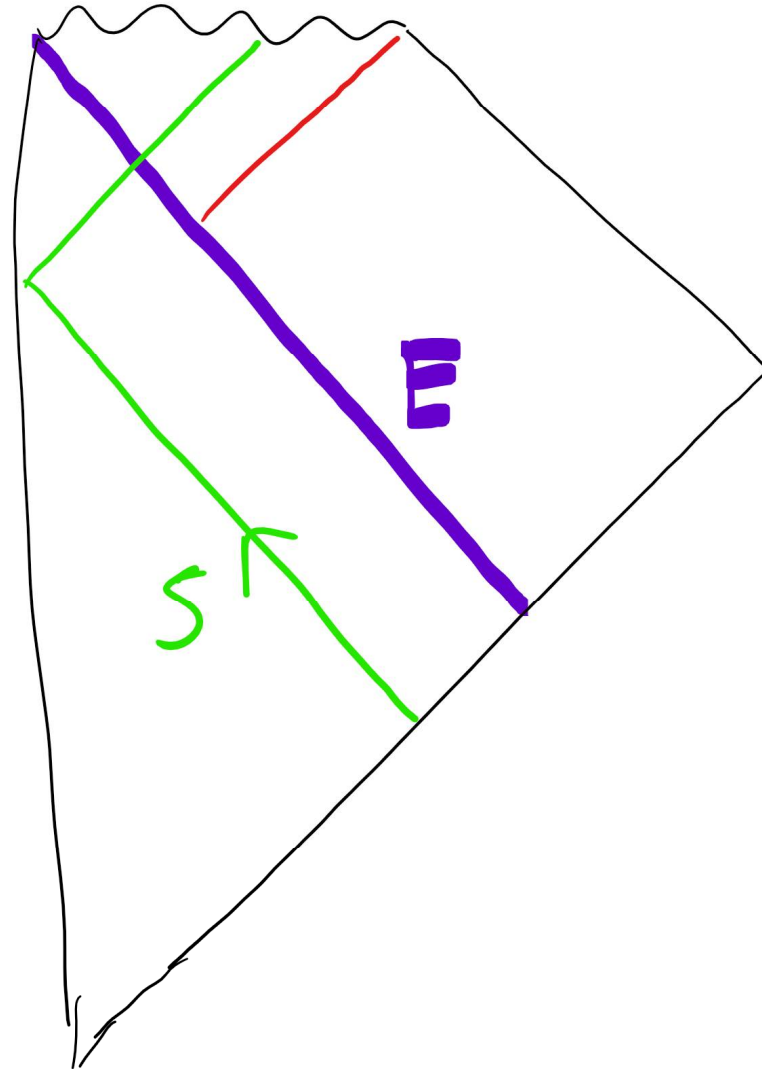
A bit on ANEC

3. An example:

Firewall is not enough

A firewall (red line) cannot prevent from all the info. to reach the singularity.

- ANEC implies that this must be the case.



A bit on ANEC

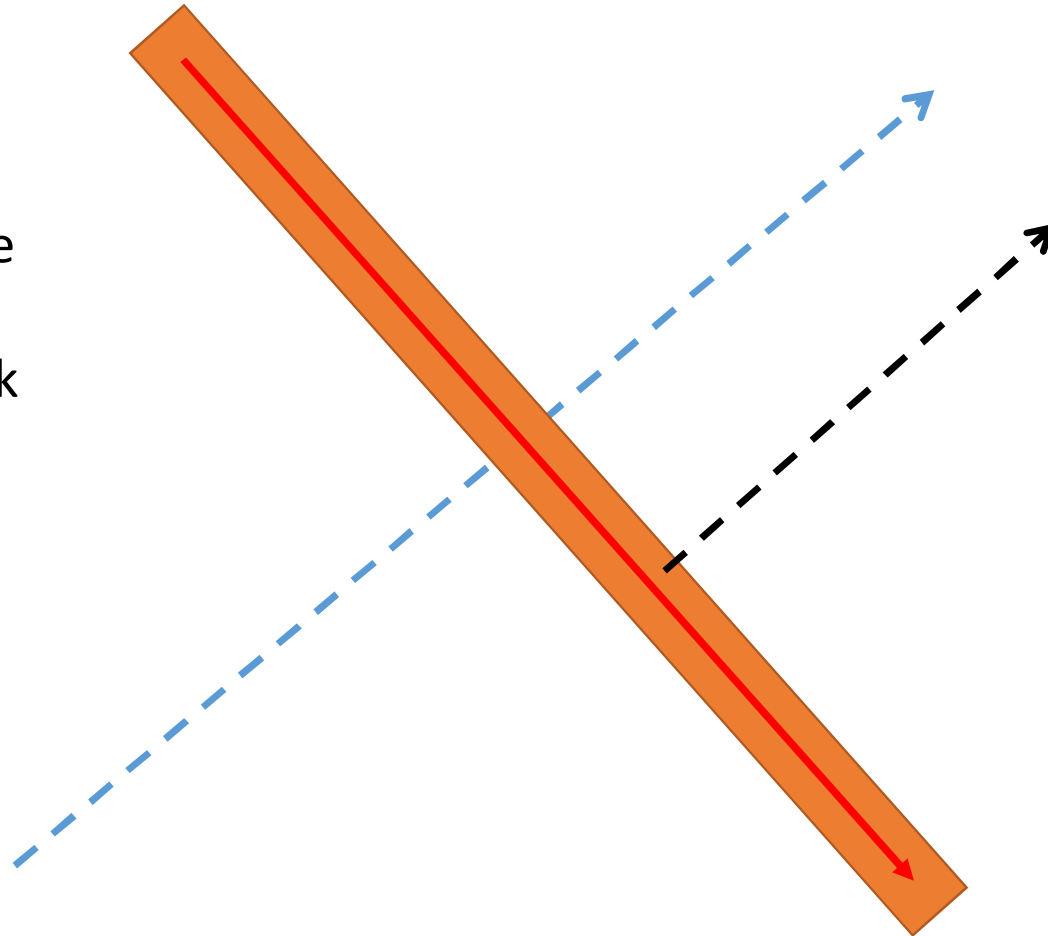
4. In QFT ANEC was proved in various methods:

- Quantum Information Theory [[Faulkner, Leigh, Parrikar and Wang, 1605.08072](#)]
- Causality [[Hartman, Kundu, and Tajdini, 1610.05308](#)]
- Holography [[Kelly, Wall 1408.3566](#)]

A bit on ANEC

- Causality [Hartman, Kundu, and Tajdini, 1610.05308]

If the flux is negative
then the shift is back
in time.



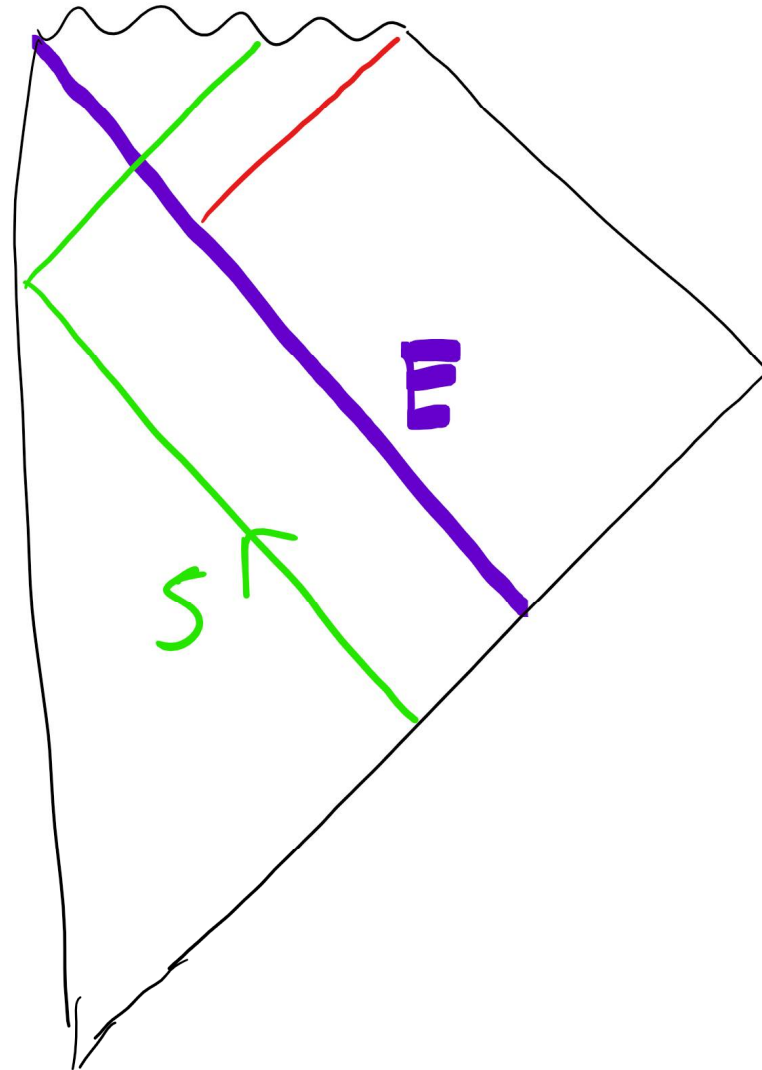
A bit on ANEC

3. An example:

Firewall is not enough

A firewall (red line) cannot prevent
from all the info. to reach the singularity.

ANEC implies that this **must** be the case.



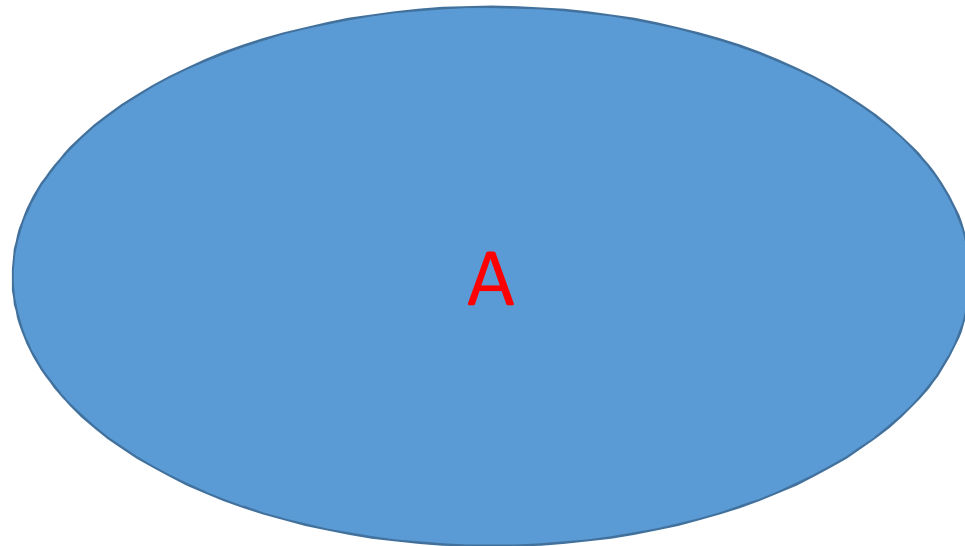
A bit on ANEC

- Quantum Information Theory [[Faulkner, Leigh, Parrikar and Wang, 1605.08072](#)]

Relies on basic properties of QIF:

the larger the region is the easier is to distinguish between two states

(monotonicity of relative entropy)



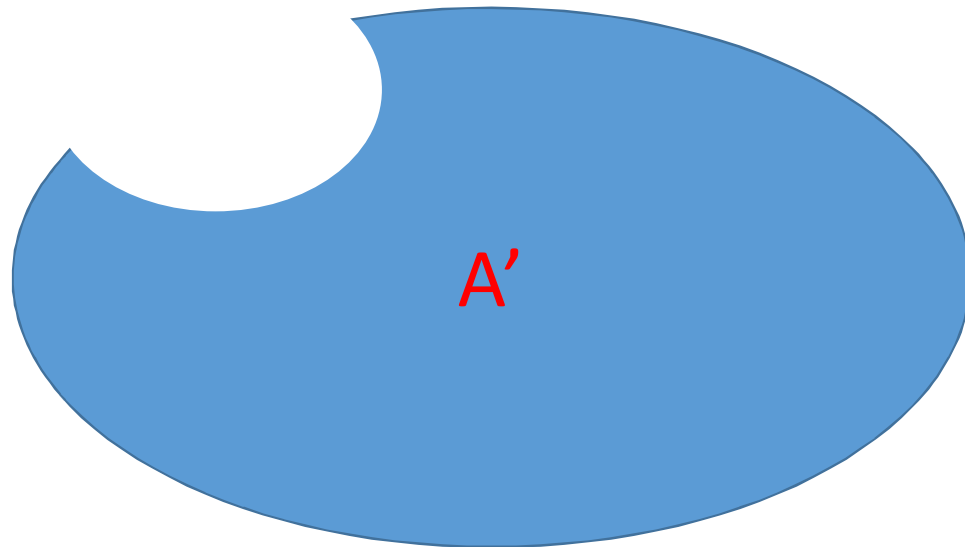
A bit on ANEC

- Quantum Information Theory [[Faulkner, Leigh, Parrikar and Wang, 1605.08072](#)]

Relies on basic properties of QIF:

the larger the region is the easier is to distinguish between two states

(monotonicity of relative entropy)



A bit on ANEC

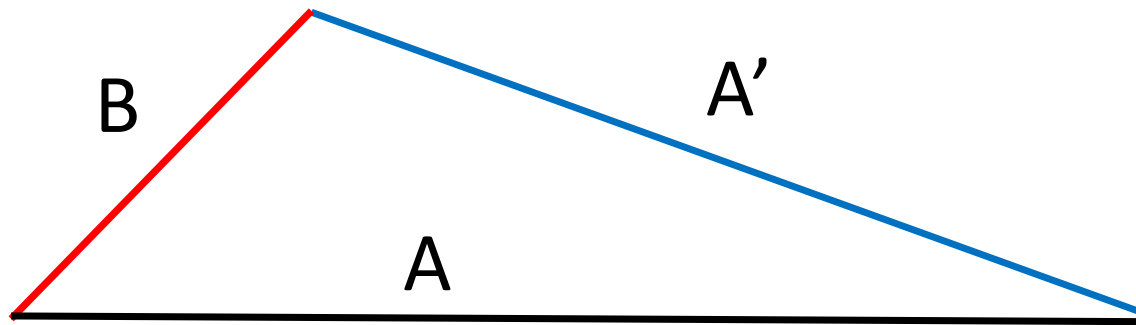
- Quantum Information Theory [[Faulkner, Leigh, Parrikar and Wang, 1605.08072](#)]

Relies on basic properties of QIF:

the larger the region is the easier is to distinguish between two states

(monotonicity of relative entropy)

In relativistic theories we can subtract a null surface:



A bit on ANEC

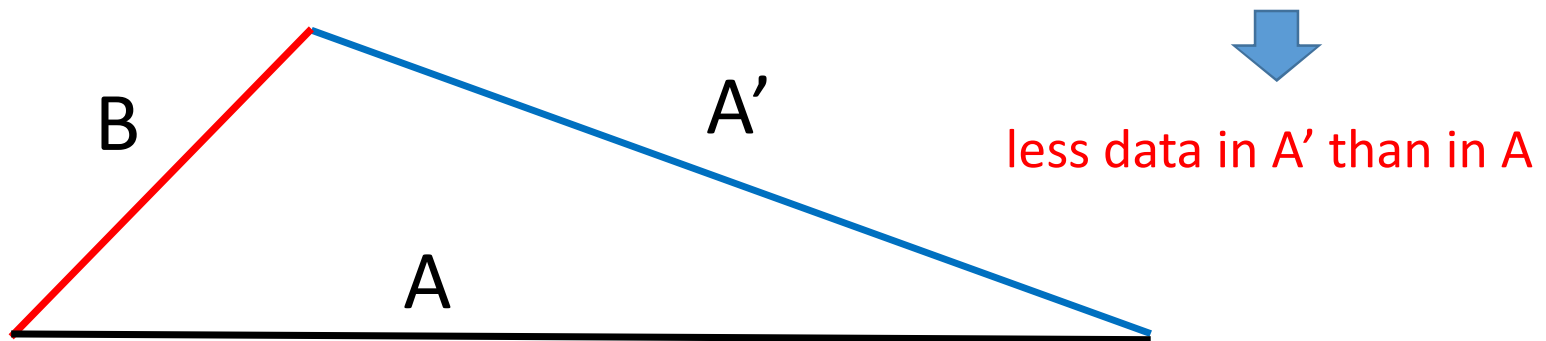
- Quantum Information Theory [Faulkner, Leigh, Parrikar and Wang, 1605.08072]

Relies on basic properties of QIF:

the larger the region is the easier is to distinguish between two states

(monotonicity of relative entropy)

In relativistic theories we can subtract a null surface: **The data in A is the same as in A' + B**

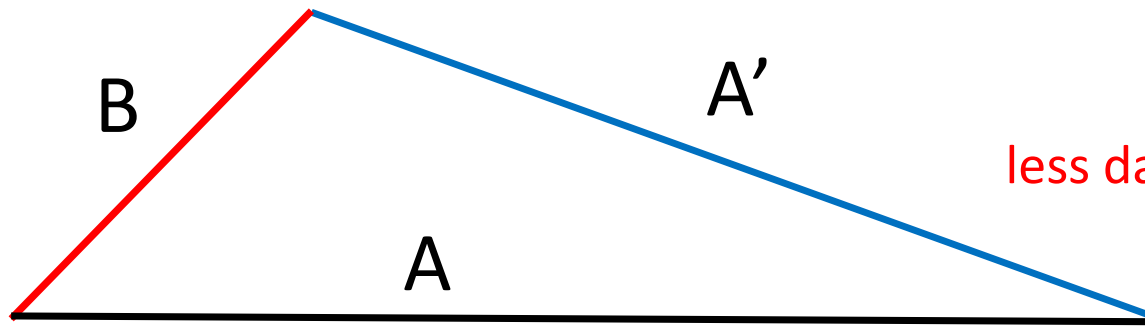


A bit on ANEC

- Quantum Information Theory [Faulkner, Leigh, Parrikar and Wang, 1605.08072]

Tracing B gives the ANEC

In relativistic theories we can subtract a null surface: The data in A is the same as in $A' + B$



less data in A' than in A

A bit on ANEC

4. In QFT ANEC was proved in various methods:

- Quantum Information Theory [[Faulkner, Leigh, Parrikar and Wang, 1605.08072](#)]
- Causality [[Hartman, Kundu, and Tajdini, 1610.05308](#)]
- Holography [[Kelly, Wall 1408.3566](#)]

All assume locality, but string theory is a non local theory.

In the rest of the talk:

- I'll argue that in string theory there are, in certain situations, objects that violate the ANEC at the **classical level** and **at macroscopic scales**.

In the rest of the talk:

- I'll argue that in string theory there are, in certain situations, objects that violate the ANEC at the classical level and at macroscopic scales.
- The region behind the horizon of $SL(2)/U(1)$ BH is filled with these objects.

Let's start by considering a 2D linear dilaton background:

$$ds^2 = -dt^2 + dr^2, \quad \Phi = -Qr,$$

Let's start by considering a 2D linear dilaton background:

$$ds^2 = -dt^2 + dr^2, \quad \Phi = -Qr,$$

1. The string coupling blows up when $r \rightarrow -\infty$.

Let's start by considering a 2D linear dilaton background:

$$ds^2 = -dt^2 + dr^2, \quad \Phi = -Qr,$$

1. The string coupling blows up when $r \rightarrow -\infty$.
2. The dimension of perturbative states with momentum p is shifted from $p^2/4$ to $p(p + Q)/4$.

Let's start by considering a 2D linear dilaton background:

$$ds^2 = -dt^2 + dr^2, \quad \Phi = -Qr,$$

1. The string coupling blows up when $r \rightarrow -\infty$.
2. The dimension of perturbative states with momentum p is shifted from $p^2/4$ to $p(p + Q)/4$.

To see this we use the fact that this is one of the simplest CFT we have: r is a free scalar


field $\partial_+ \partial_- r = 0 \quad \longrightarrow \quad r = r_+ + r_-$

Let's start by considering a 2D linear dilaton background:

$$ds^2 = -dt^2 + dr^2, \quad \Phi = -Qr,$$

1. The string coupling blows up when $r \rightarrow -\infty$.
2. The dimension of perturbative states with momentum p is shifted from $p^2/4$ to $p(p + Q)/4$.

To see this we use the fact that this is one of the simplest CFT we have: r is a free scalar

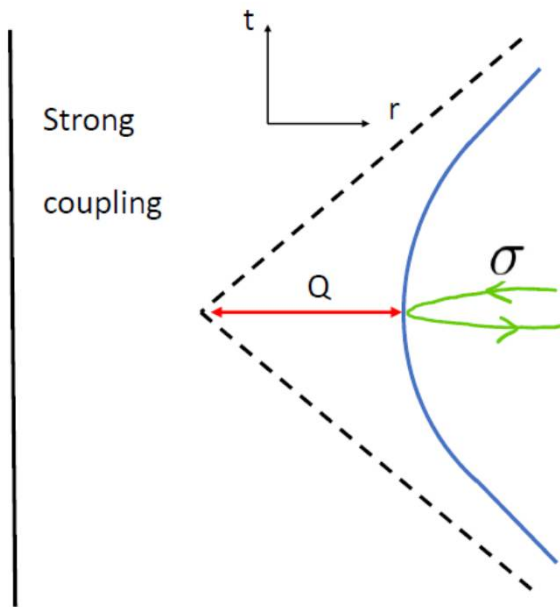
field $\partial_+ \partial_- r = 0$  $r = r_+ + r_-$ only with a non standard energy

momentum tensor $T_{++} = -(\partial_+ r)^2 + Q\partial_+^2 r$

Since now $T_{++} = -(\partial_+ r)^2 + Q\partial_+^2 r$ there are new classical (non-perturbative) solutions [Maldacena, 2003] that describe long strings

Since now $T_{++} = -(\partial_+ r)^2 + Q\partial_+^2 r$ there are new classical (non-perturbative) solutions [Maldacena, 2003] that describe **long strings**

$$r = r_0 + Q \log \left(\frac{1}{2} \left(\cosh \left(\frac{t - t_0}{Q} \right) + \cosh \left(\frac{\sigma}{Q} \right) \right) \right)$$



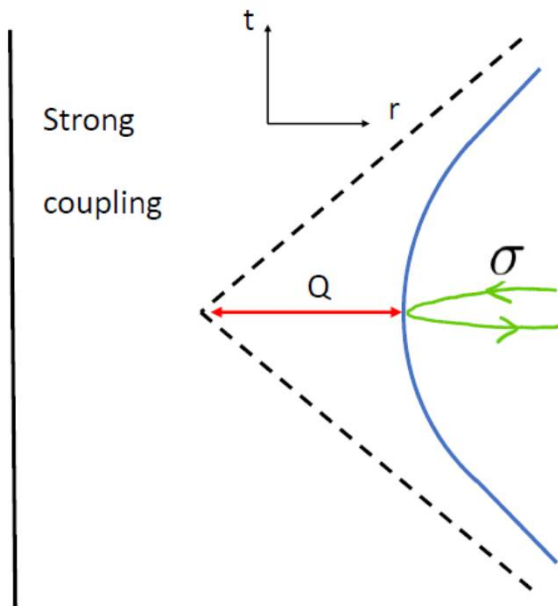
Since now $T_{++} = -(\partial_+ r)^2 + Q\partial_+^2 r$ there are new classical (non-perturbative) solutions [Maldacena, 2003] that describe **long strings**

$$r = r_0 + Q \log \left(\frac{1}{2} \left(\cosh \left(\frac{t - t_0}{Q} \right) + \cosh \left(\frac{\sigma}{Q} \right) \right) \right)$$

They are stable



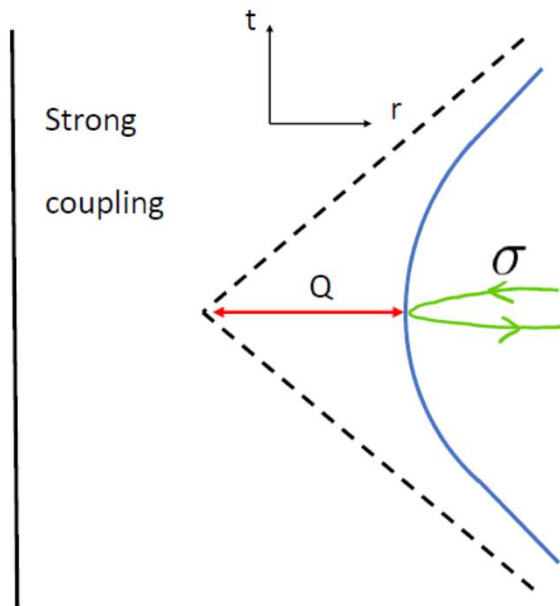
- The string can fold only away from the singularity.



Since now $T_{++} = -(\partial_+ r)^2 + Q\partial_+^2 r$ there are new classical (non-perturbative) solutions [Maldacena, 2003] that describe long strings

$$r = r_0 + Q \log \left(\frac{1}{2} \left(\cosh \left(\frac{t - t_0}{Q} \right) + \cosh \left(\frac{\sigma}{Q} \right) \right) \right)$$

They are stable

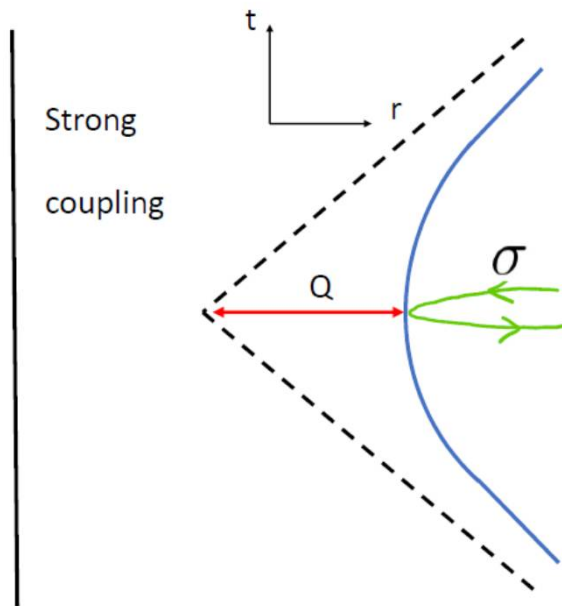


- The string can fold only away from the singularity.
- The tip is a bit massive .

Since now $T_{++} = -(\partial_+ r)^2 + Q\partial_+^2 r$ there are new classical (non-perturbative) solutions [Maldacena, 2003] that describe **long strings**

$$r = r_0 + Q \log \left(\frac{1}{2} \left(\cosh \left(\frac{t - t_0}{Q} \right) + \cosh \left(\frac{\sigma}{Q} \right) \right) \right)$$

They are stable



- The string can fold only away from the singularity.
- The tip is a bit massive .
- The length scale is tiny, of the order of .

The folding is local.

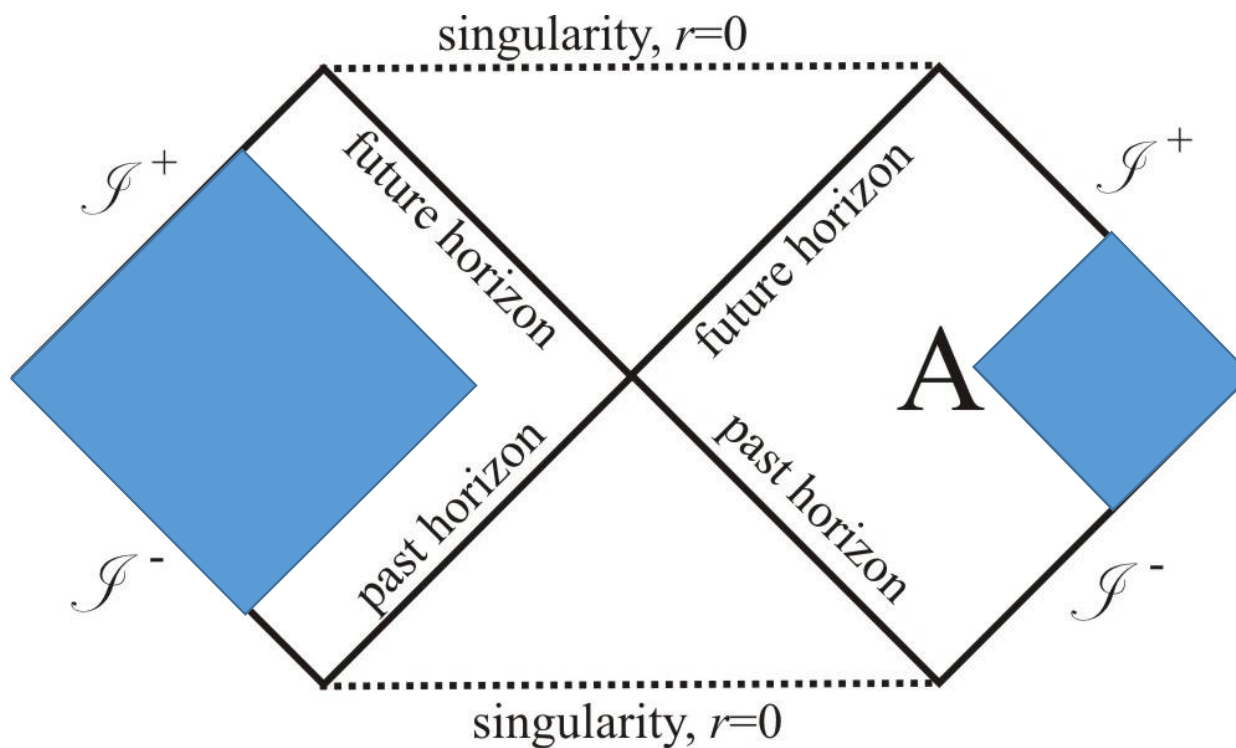
The folding is local.



No short string outside the SL(2)/U(1) BH.

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)}, \quad \Phi(r) = \phi_0 - Qr$$

The string can fold only towards
the weak coupling.



The folding is local.



No short string outside the SL(2)/U(1) BH.

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)}, \quad \Phi(r) = \phi_0 - Qr$$

The string can fold only towards the weak coupling.

What happens beyond the horizon?

The folding is local.



No short string outside the SL(2)/U(1) BH.

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)}, \quad \Phi(r) = \phi_0 - Qr$$

The string can fold only towards the weak coupling.

What happens beyond the horizon?

We have a time like linear dilaton

What happens beyond the horizon?

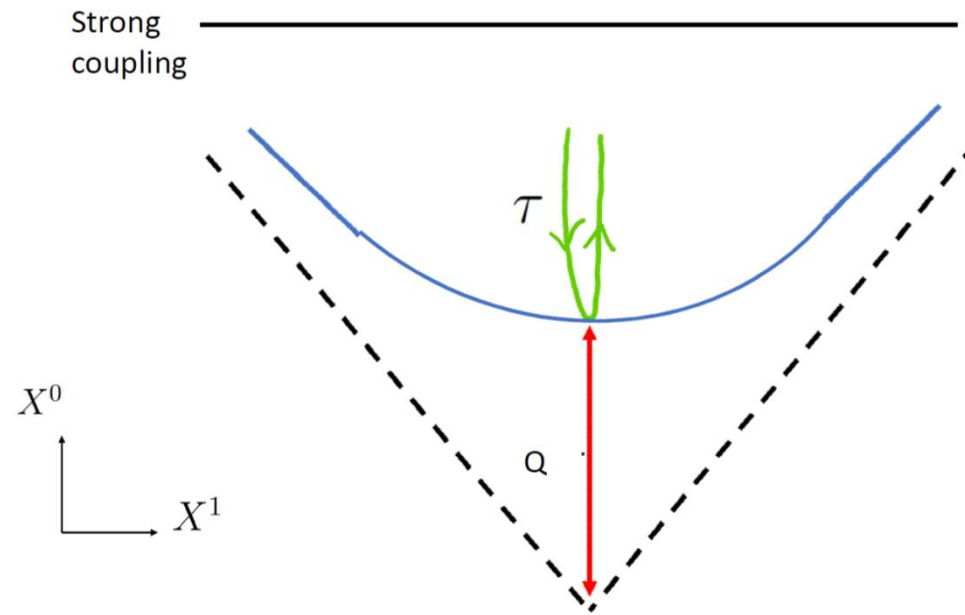
We have a time like linear dilaton $ds^2 = -(dX^0)^2 + (dX^1)^2$, $\Phi = QX^0$.

Now the non-perturbative solutions is $X^0 = x^0 + Q \log \left(\frac{1}{2} \left(\cosh \left(\frac{X^1 - x^1}{Q} \right) + \cosh \left(\frac{\tau}{Q} \right) \right) \right)$.

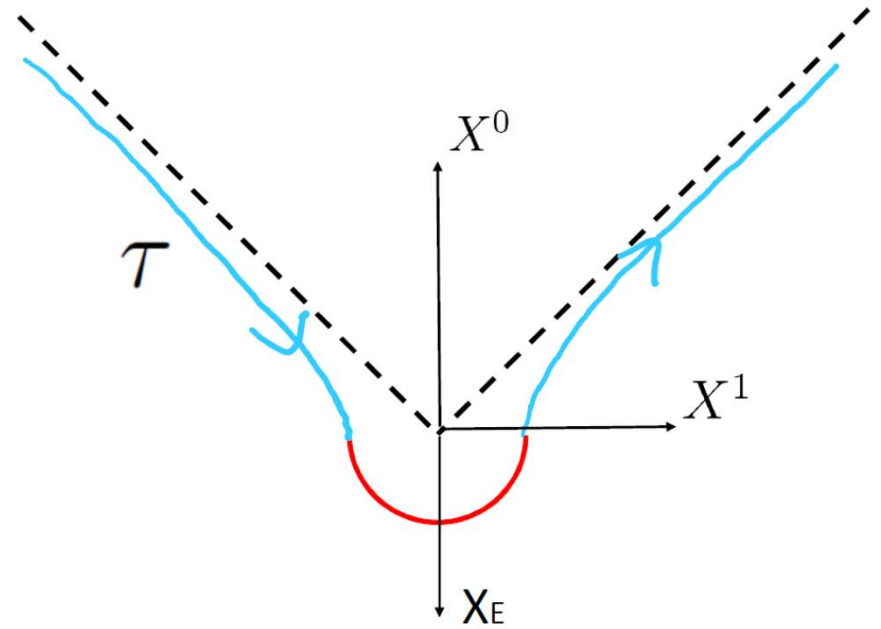
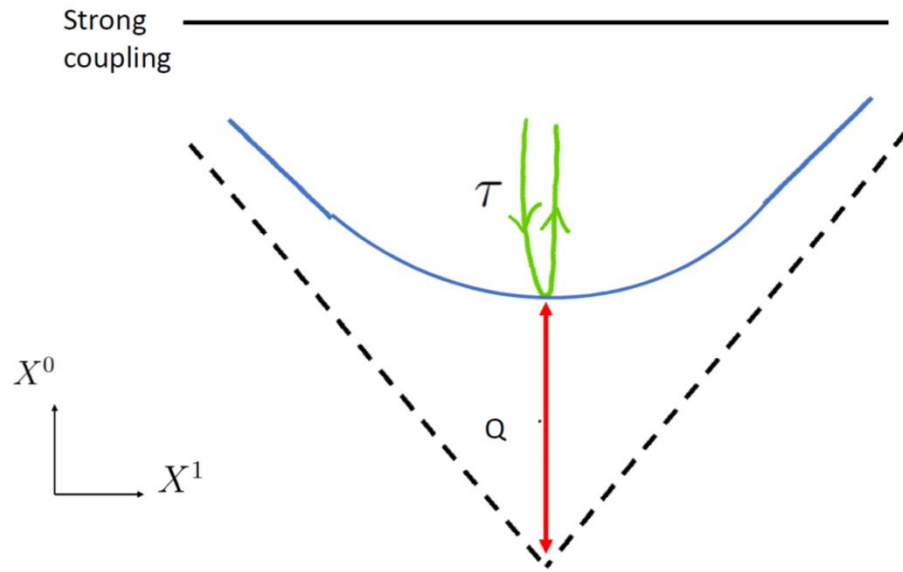
What happens beyond the horizon?

We have a time like linear dilaton $ds^2 = -(dX^0)^2 + (dX^1)^2$, $\Phi = QX^0$.

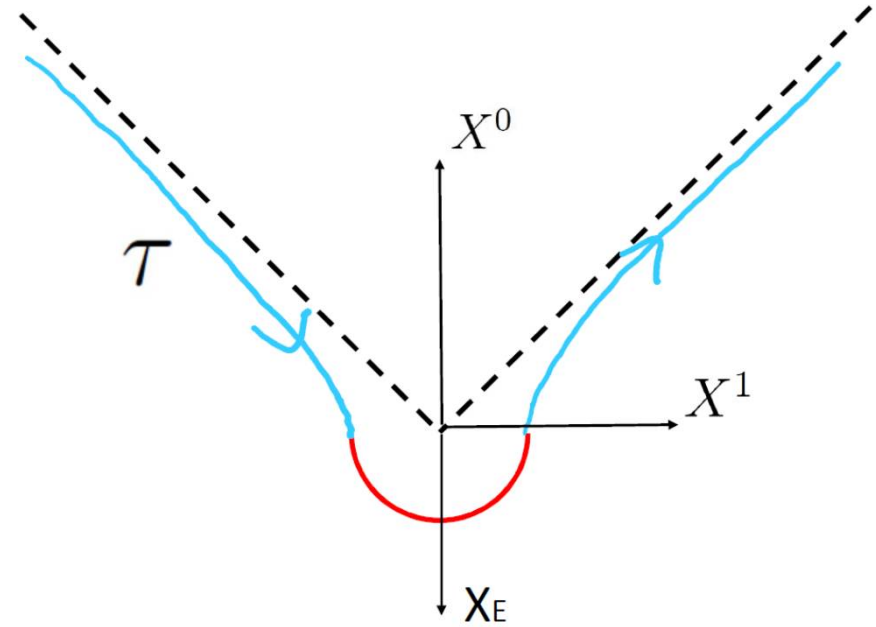
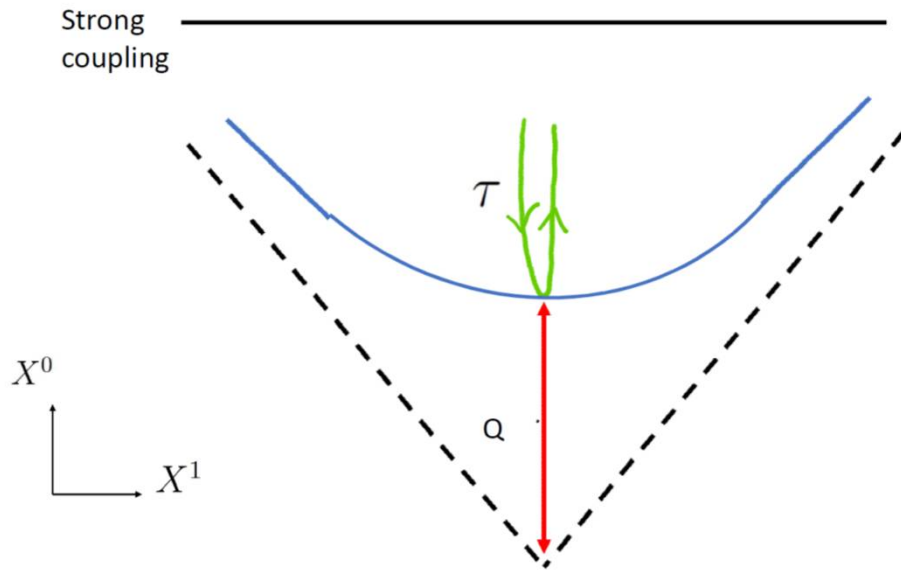
Now the non-perturbative solutions is $X^0 = x^0 + Q \log \left(\frac{1}{2} \left(\cosh \left(\frac{X^1 - x^1}{Q} \right) + \cosh \left(\frac{\tau}{Q} \right) \right) \right)$.



Comparison to the Schwinger mechanism

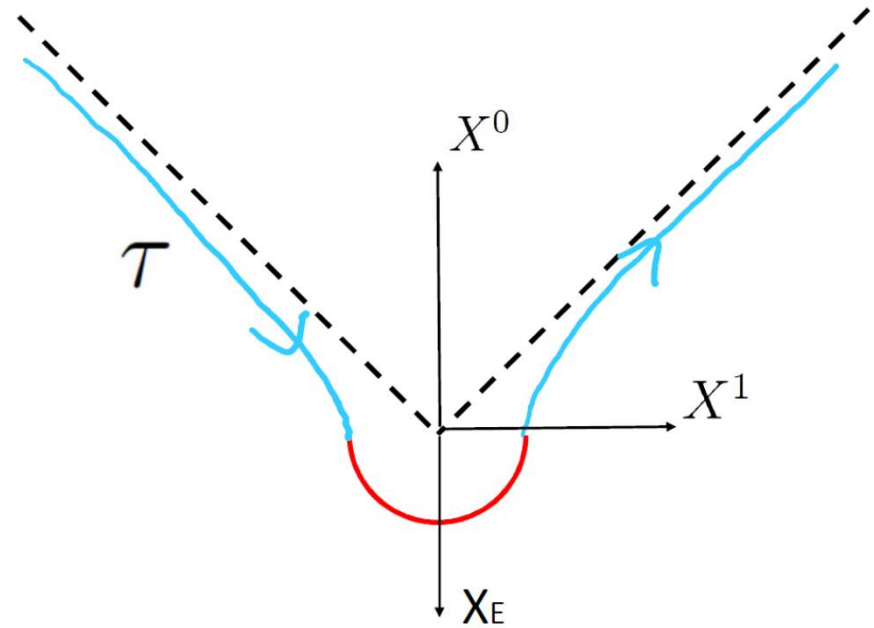
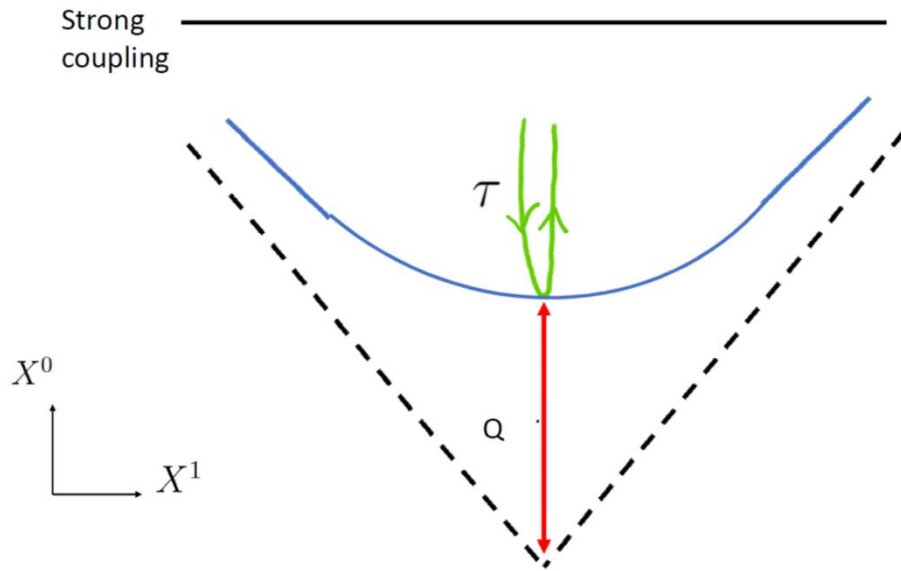


Comparison to the Schwinger mechanism



- Classical solution in Min. space \rightarrow no tunneling.
- Tip is moving faster than light.

Comparison to the Schwinger mechanism



- Classical solution in Min. space \rightarrow no tunneling.
- Tip is moving faster than light.
- Now we have a tachyon at the tip $m^2 = -Q^2$.

What do they do, these folded strings?

What do they do, these folded strings?

In short: they violate the ANEC (in the right way...)

What do they do, these folded strings?

In short: they violate the ANEC (in the right way...)

In details: the string energy momentum tensor is

$$T^{\mu\nu}(x) = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu \right) \delta(x - X(\sigma, \tau))$$

In details: the string energy momentum tensor is

$$T^{\mu\nu}(x) = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu \right) \delta(x - X(\sigma, \tau))$$

which in our case gives

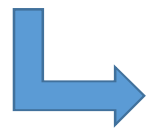
$$T^{00} = \frac{1}{2\pi\alpha'} \frac{-4 \cosh\left(\frac{x-x_0}{Q}\right) + 4e^{\frac{t-t_0}{Q}}}{\sqrt{-1 + \left(2e^{\frac{t-t_0}{Q}} - \cosh\left(\frac{x-x_0}{Q}\right)\right)^2}} \dots$$

In details: the string energy momentum tensor is

$$T^{\mu\nu}(x) = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu \right) \delta(x - X(\sigma, \tau))$$

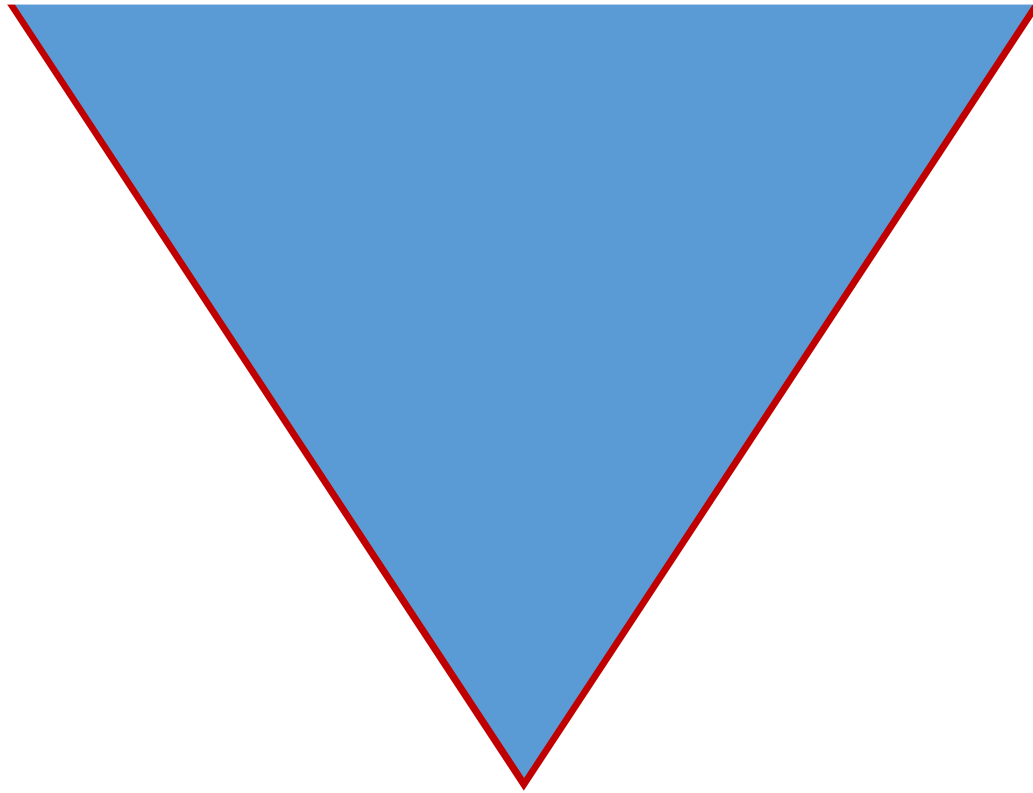
which in our case gives

$$T^{00} = \frac{1}{2\pi\alpha'} \frac{-4 \cosh\left(\frac{x-x_0}{Q}\right) + 4e^{\frac{t-t_0}{Q}}}{\sqrt{-1 + \left(2e^{\frac{t-t_0}{Q}} - \cosh\left(\frac{x-x_0}{Q}\right)\right)^2}}$$



Total energy vanishes

Total energy vanishes



$$T^{00} = \frac{1}{2\pi\alpha'} \frac{-4 \cosh\left(\frac{x-x_0}{Q}\right) + 4e^{\frac{t-t_0}{Q}}}{\sqrt{-1 + \left(2e^{\frac{t-t_0}{Q}} - \cosh\left(\frac{x-x_0}{Q}\right)\right)^2}} \dots$$

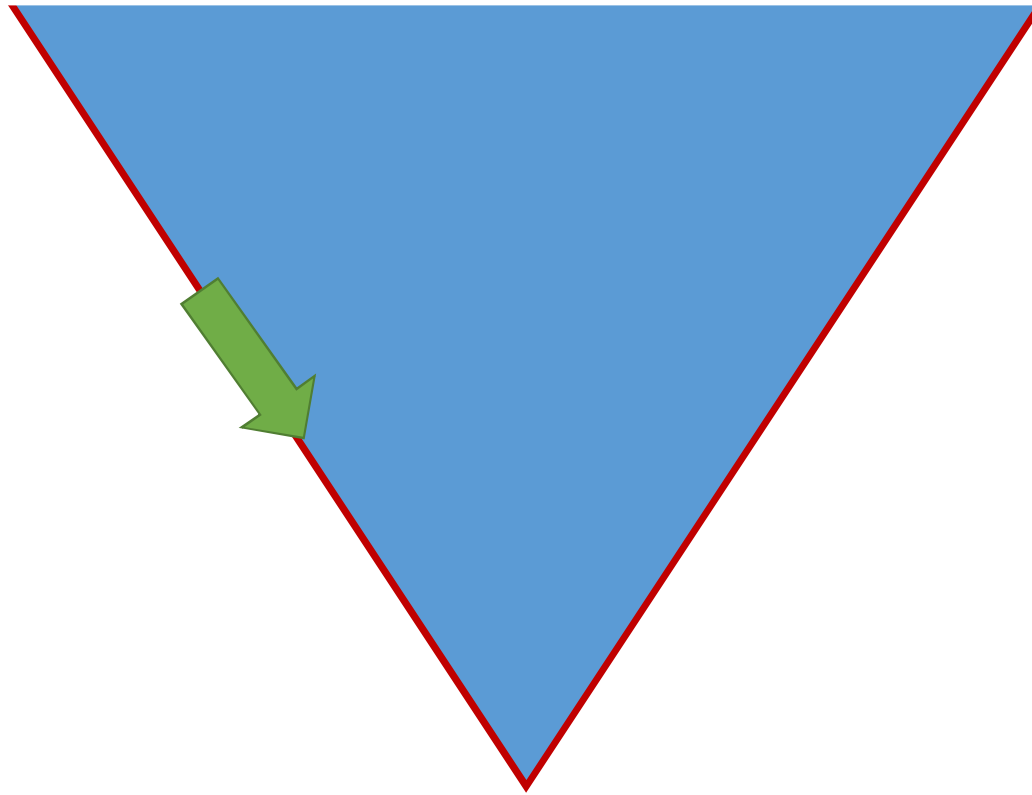


$$T_{uu} = \frac{1}{2\pi\alpha'} \frac{-e^{-\frac{x-x_0}{Q}}}{\sqrt{-1 + \left(2e^{\frac{t-t_0}{Q}} - \cosh\left(\frac{x-x_0}{Q}\right)\right)^2}}$$



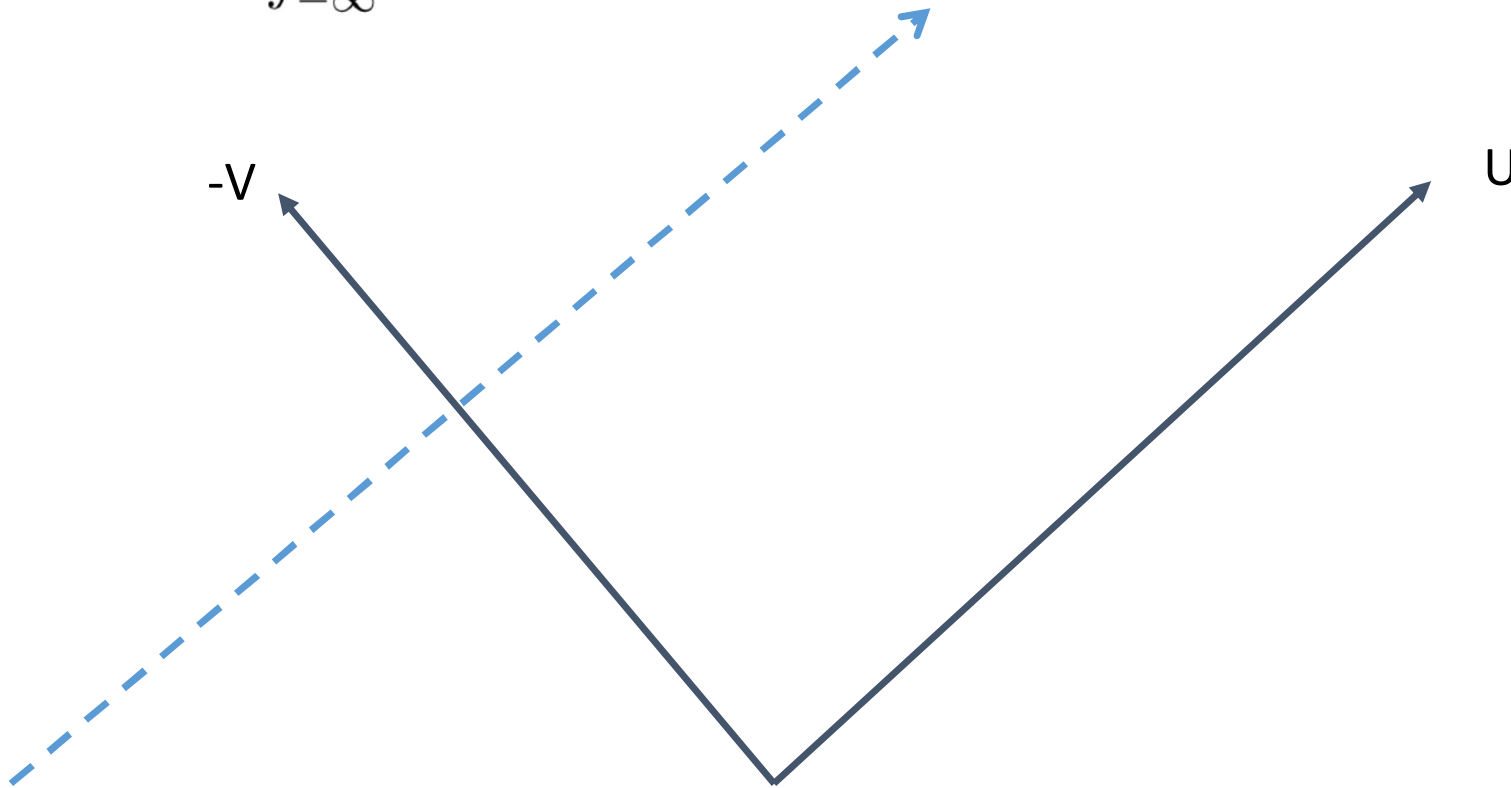
$$T_{uu} < 0$$

$$T_{uu} = \frac{1}{2\pi\alpha'} \frac{-e^{-\frac{x-x_0}{Q}}}{\sqrt{-1 + \left(2e^{\frac{t-t_0}{Q}} - \cosh\left(\frac{x-x_0}{Q}\right)\right)^2}} < 0$$



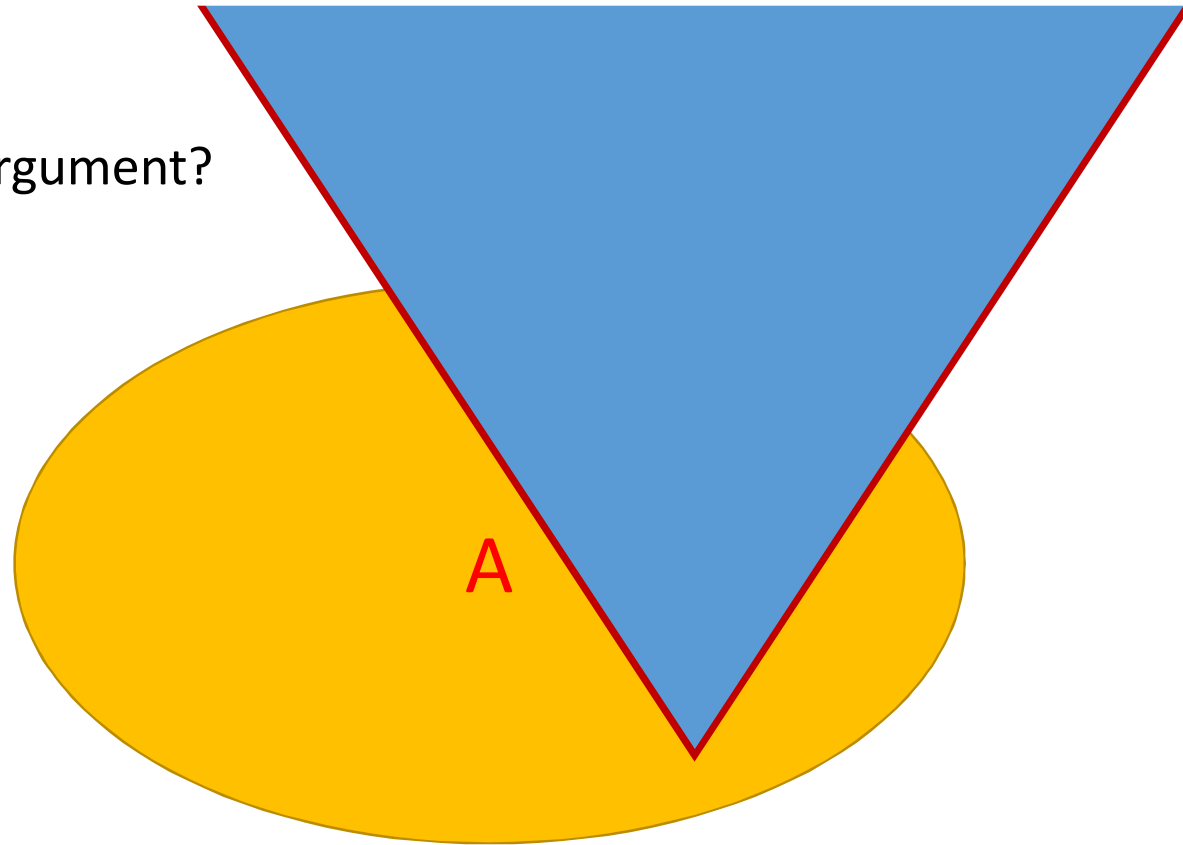
Same with ANEC

$$\int_{-\infty}^{\infty} du T_{uu} = (v - Q \log(4)) < 0$$

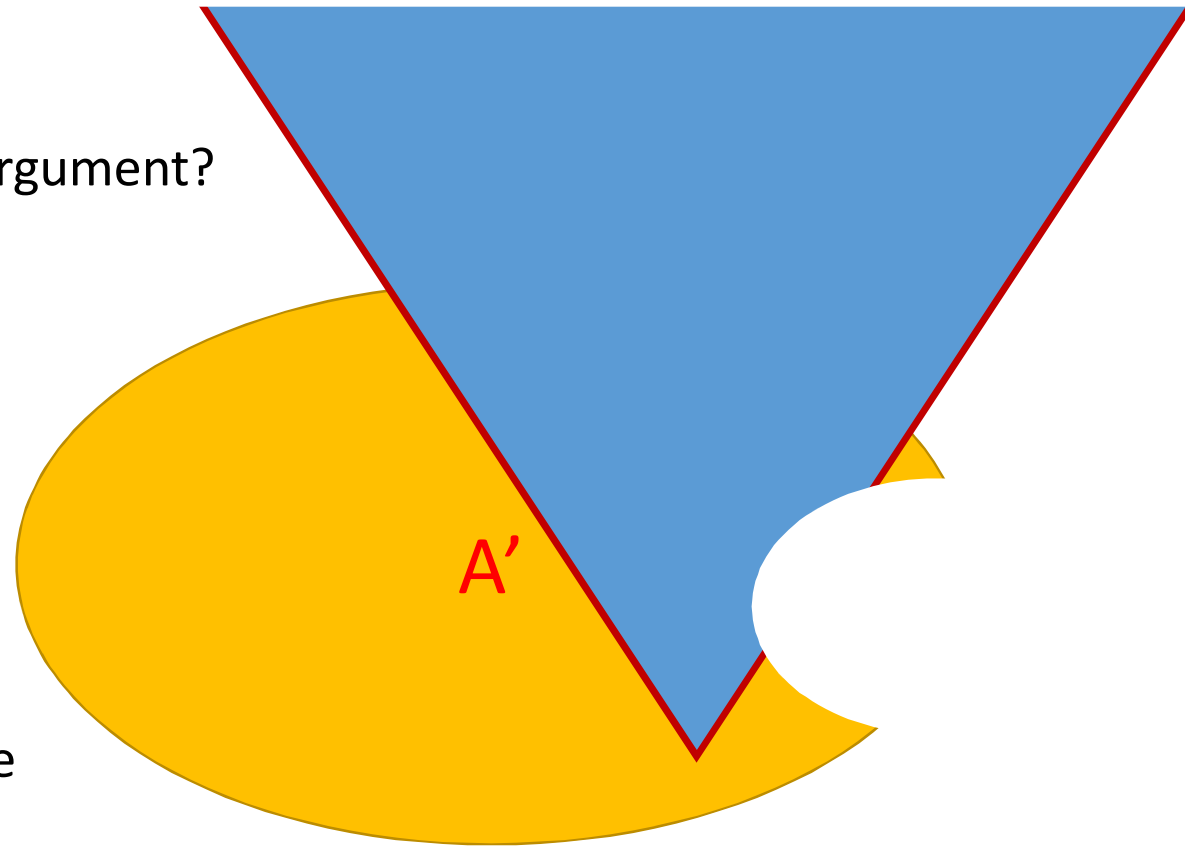


What about the QIT argument?

What about the QIT argument?



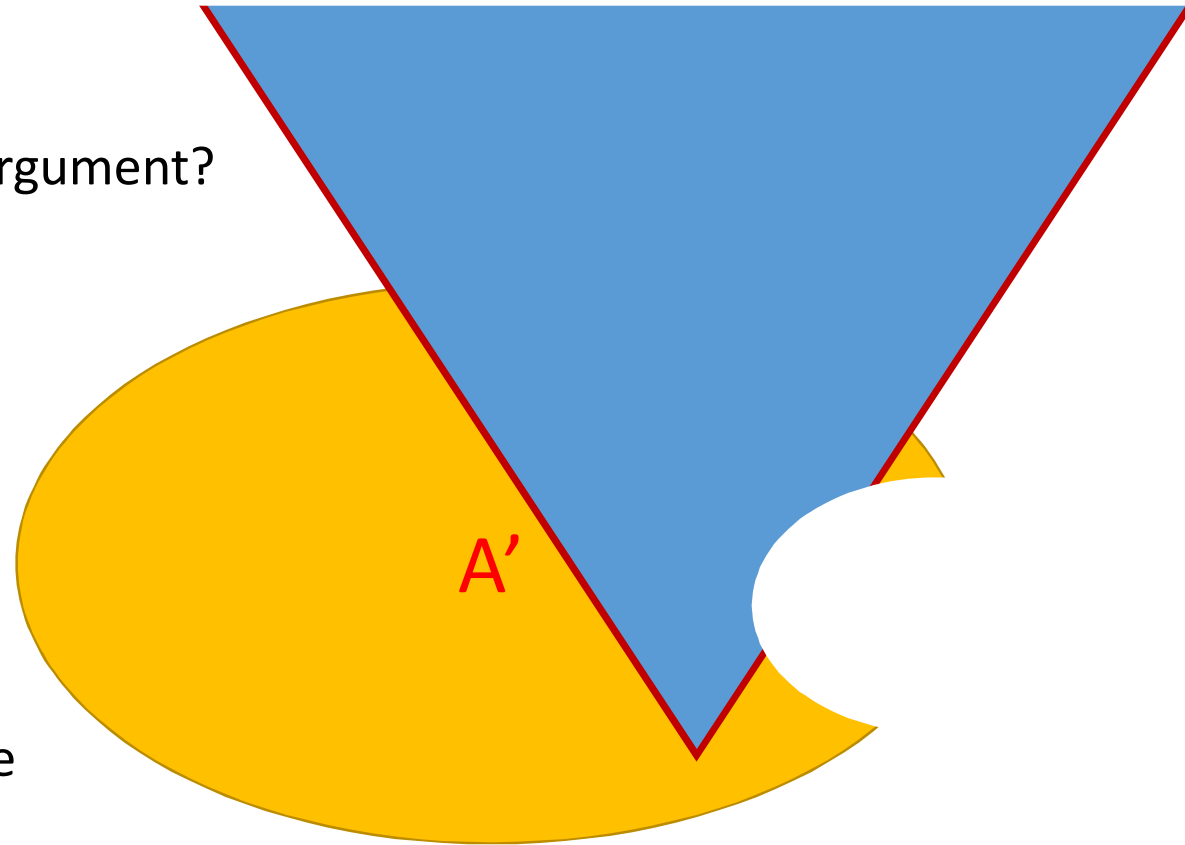
What about the QIT argument?



Still the case that we

have more data in A than in A'.

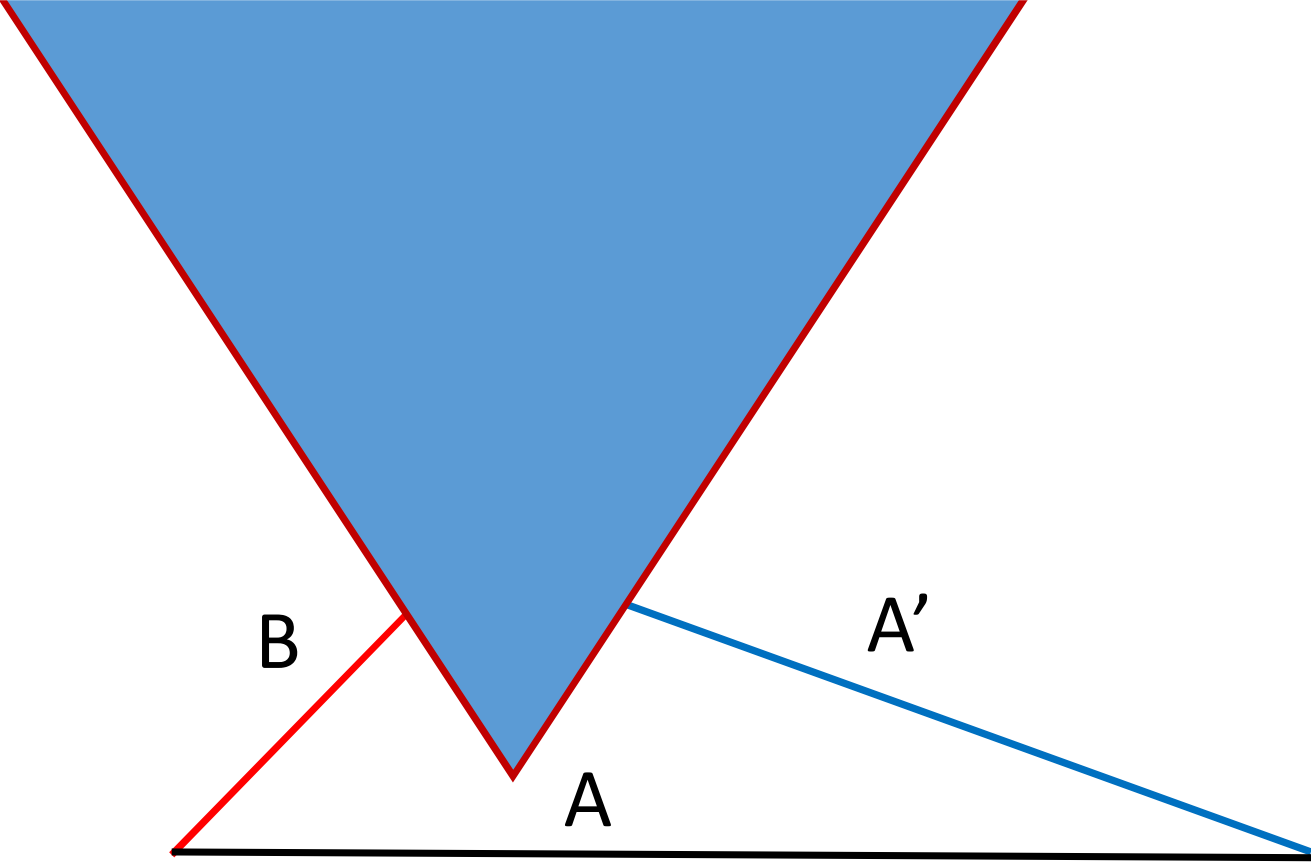
What about the QIT argument?



Still the case that we

have more data in A than in A'.

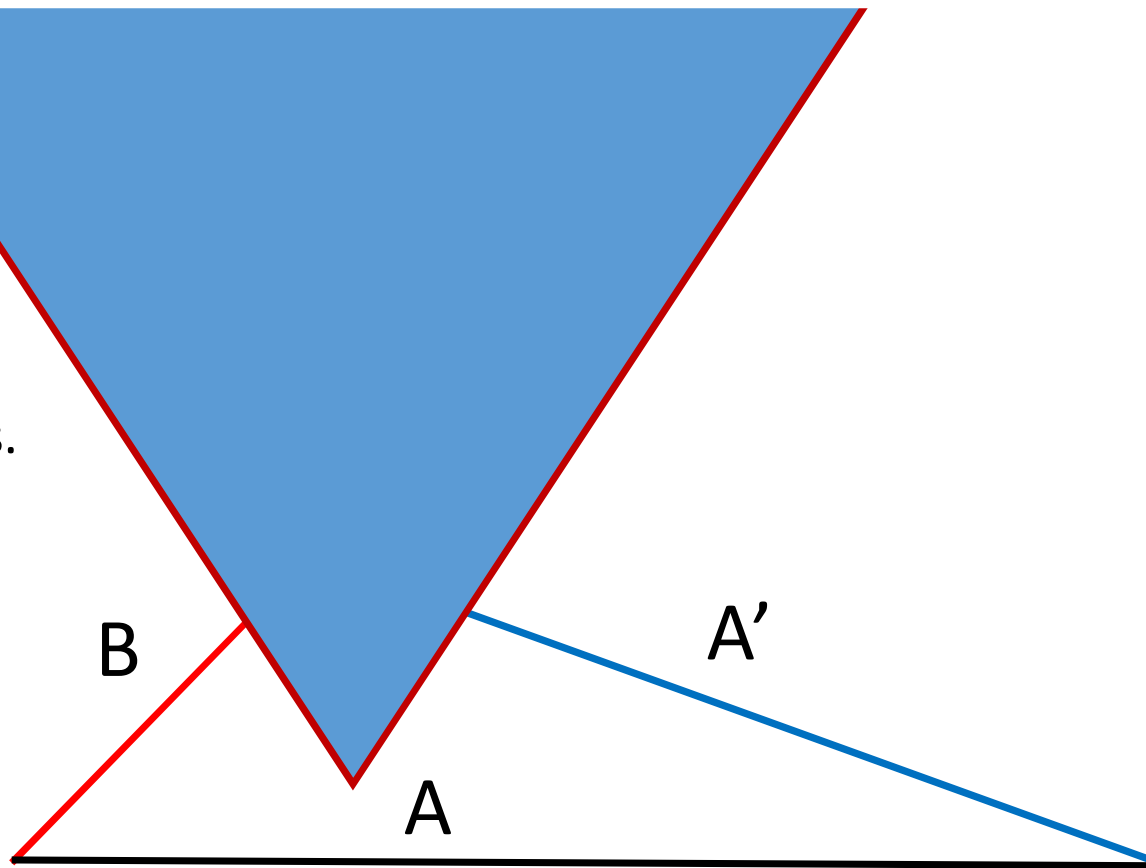
But for the ANEC we needed something a bit different.



A does not know about
the folded string, but A' does.



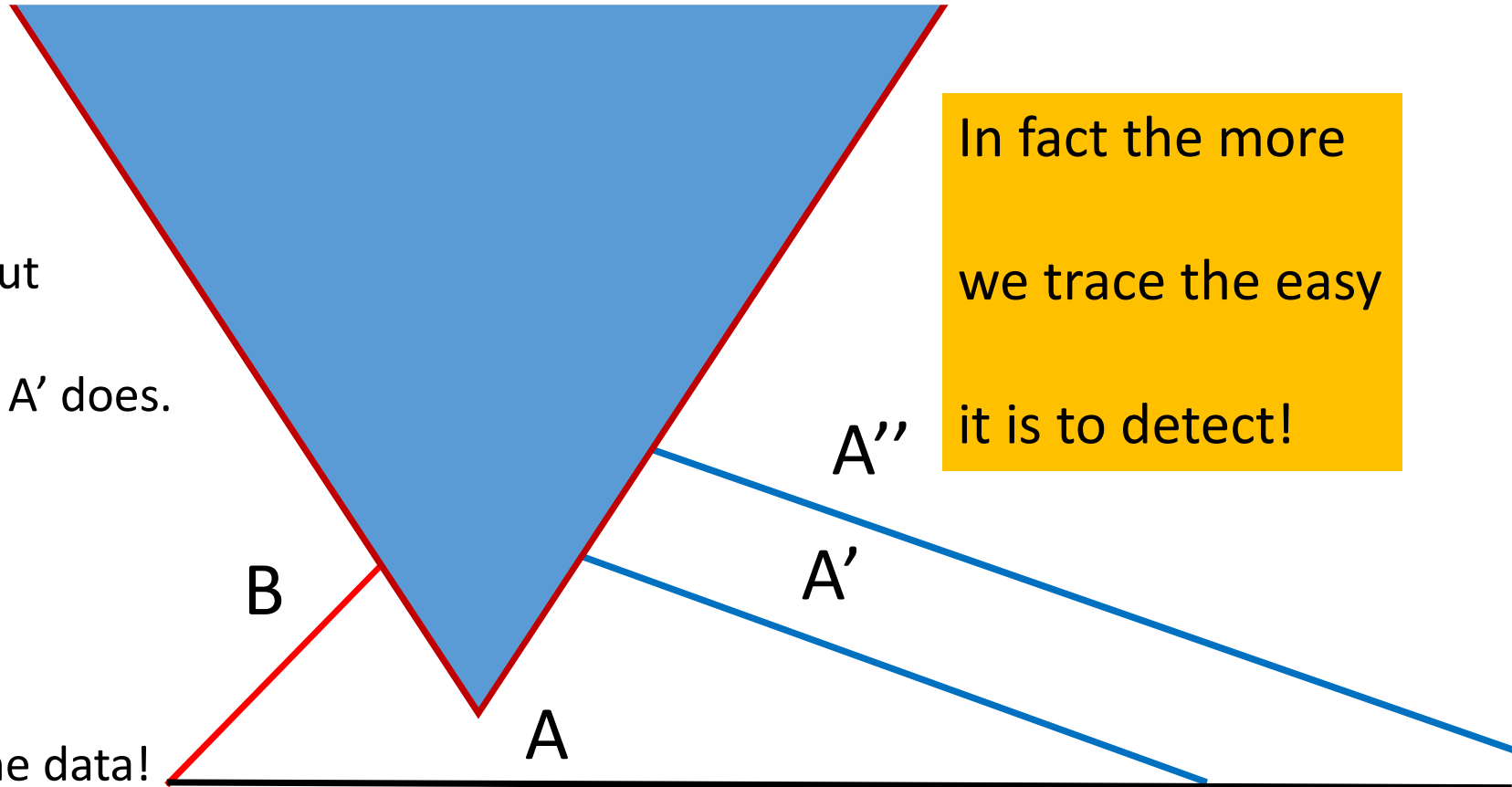
Tracing B increases the data!



A does not know about
the folded string, but A' does.



Tracing B increases the data!

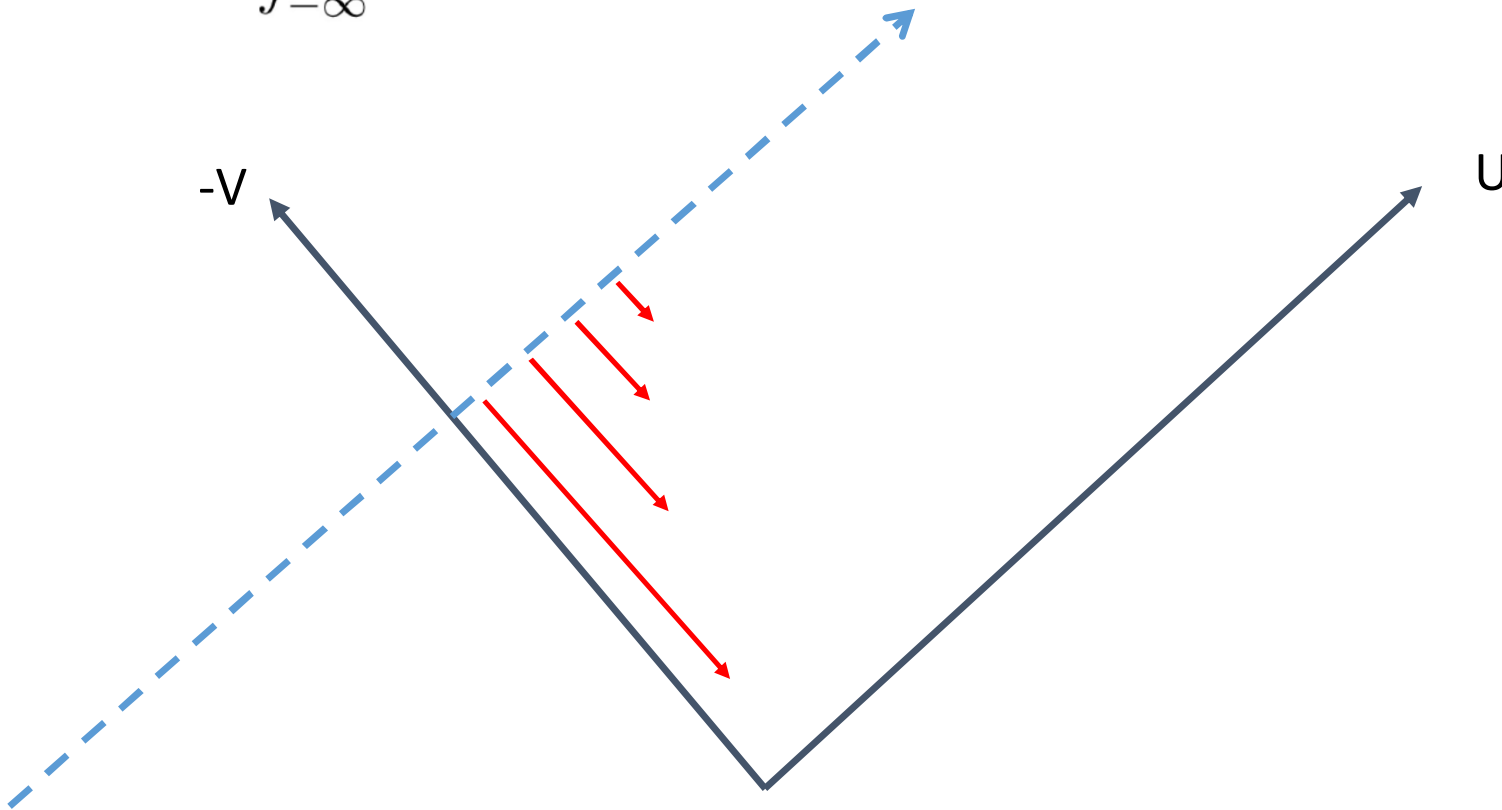


In fact the more
we trace the easy
it is to detect!

What about the causality argument?

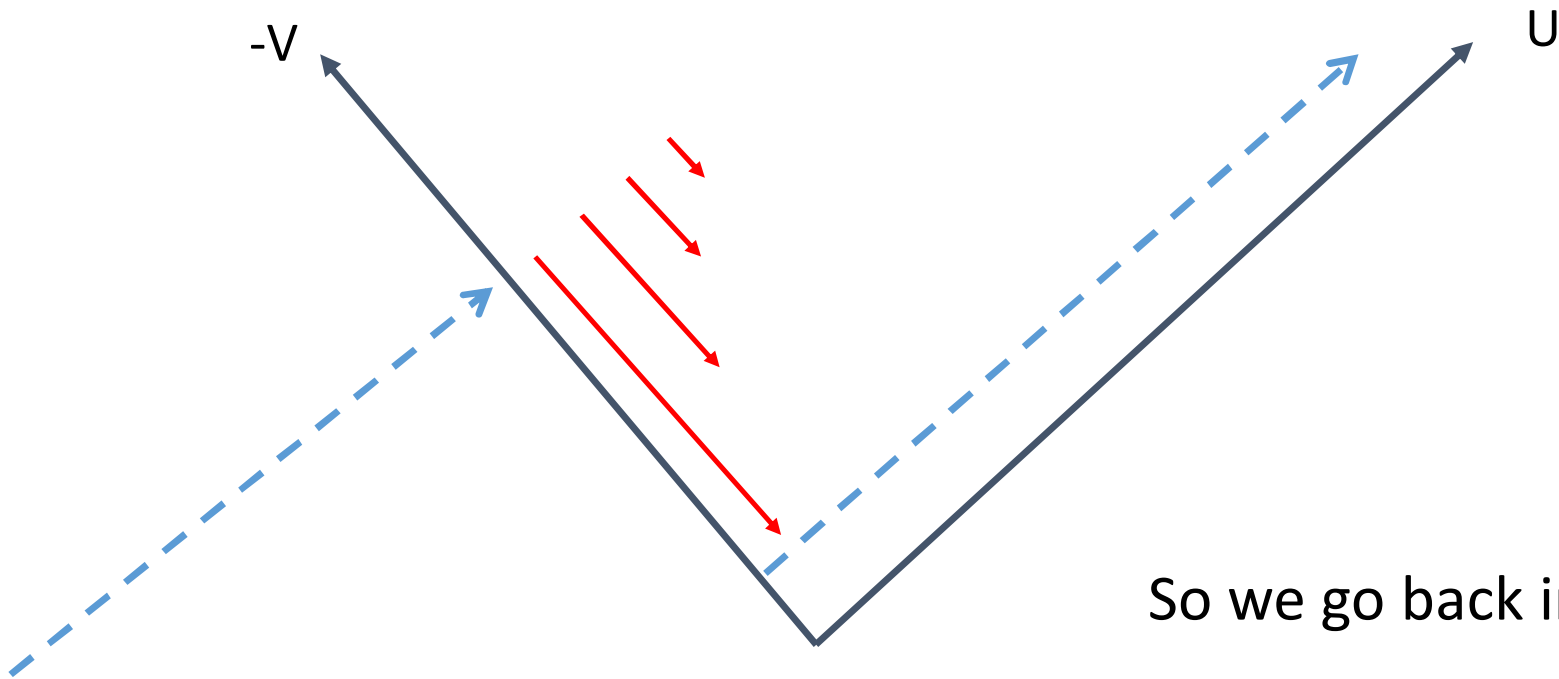
What about the causality argument?

$$\int_{-\infty}^{\infty} du T_{uu} = (v - Q \log(4)) < 0$$



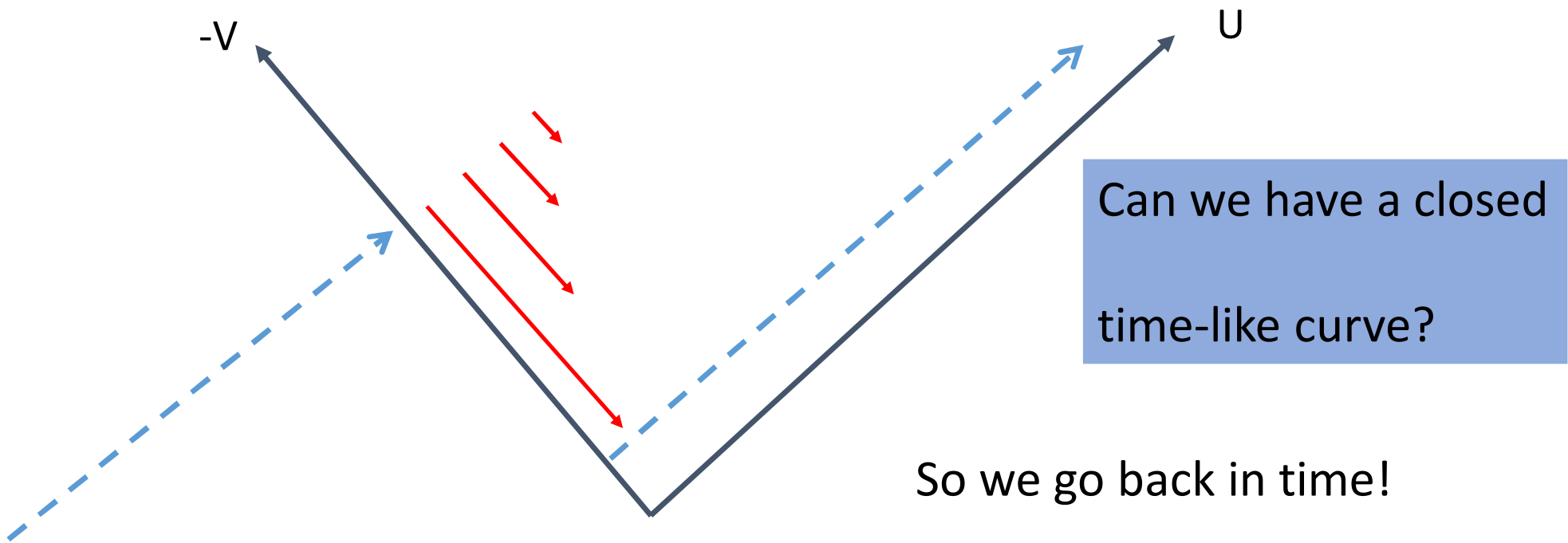
What about the causality argument?

$$\int_{-\infty}^{\infty} du T_{uu} = (v - Q \log(4)) < 0$$



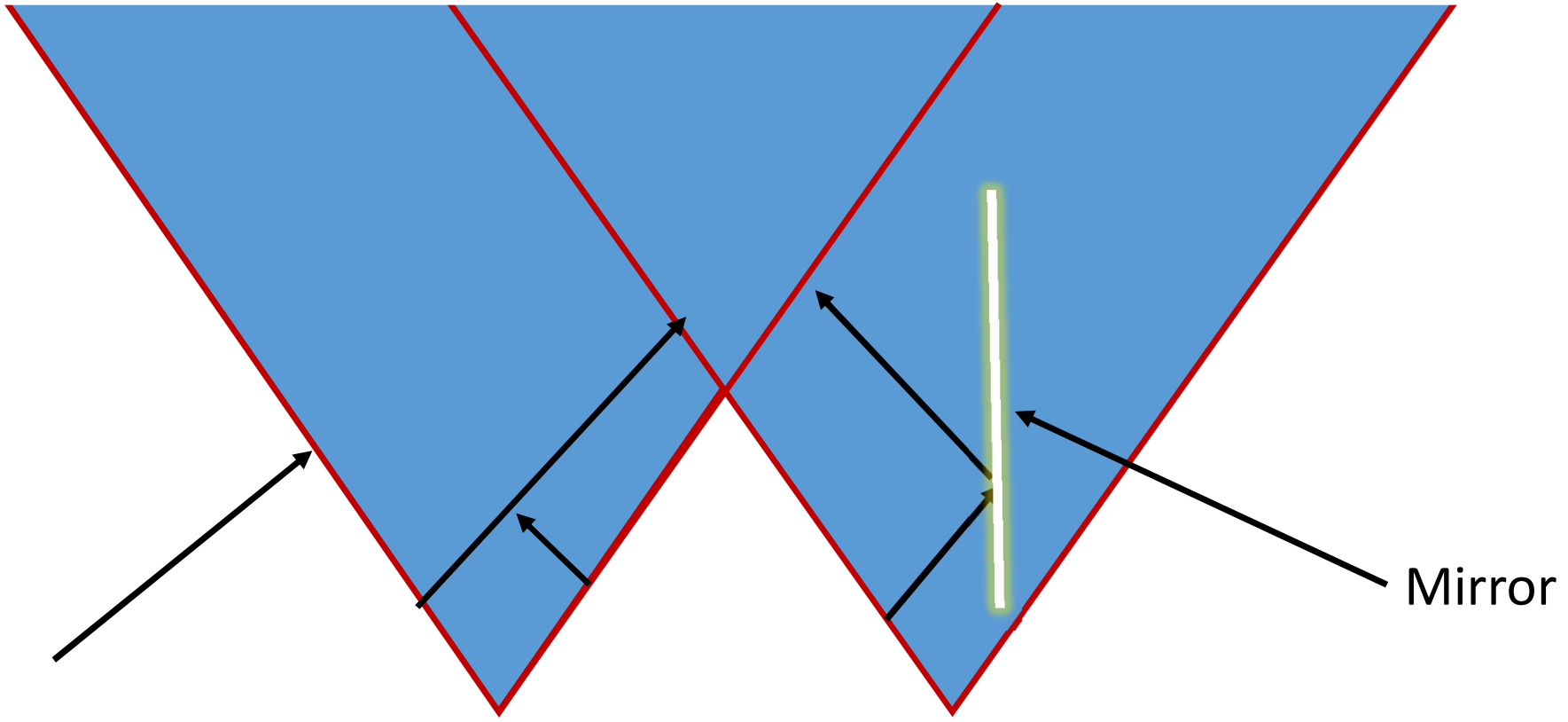
So we go back in time!

$$\int_{-\infty}^{\infty} du T_{uu} = (v - Q \log(4)) < 0$$

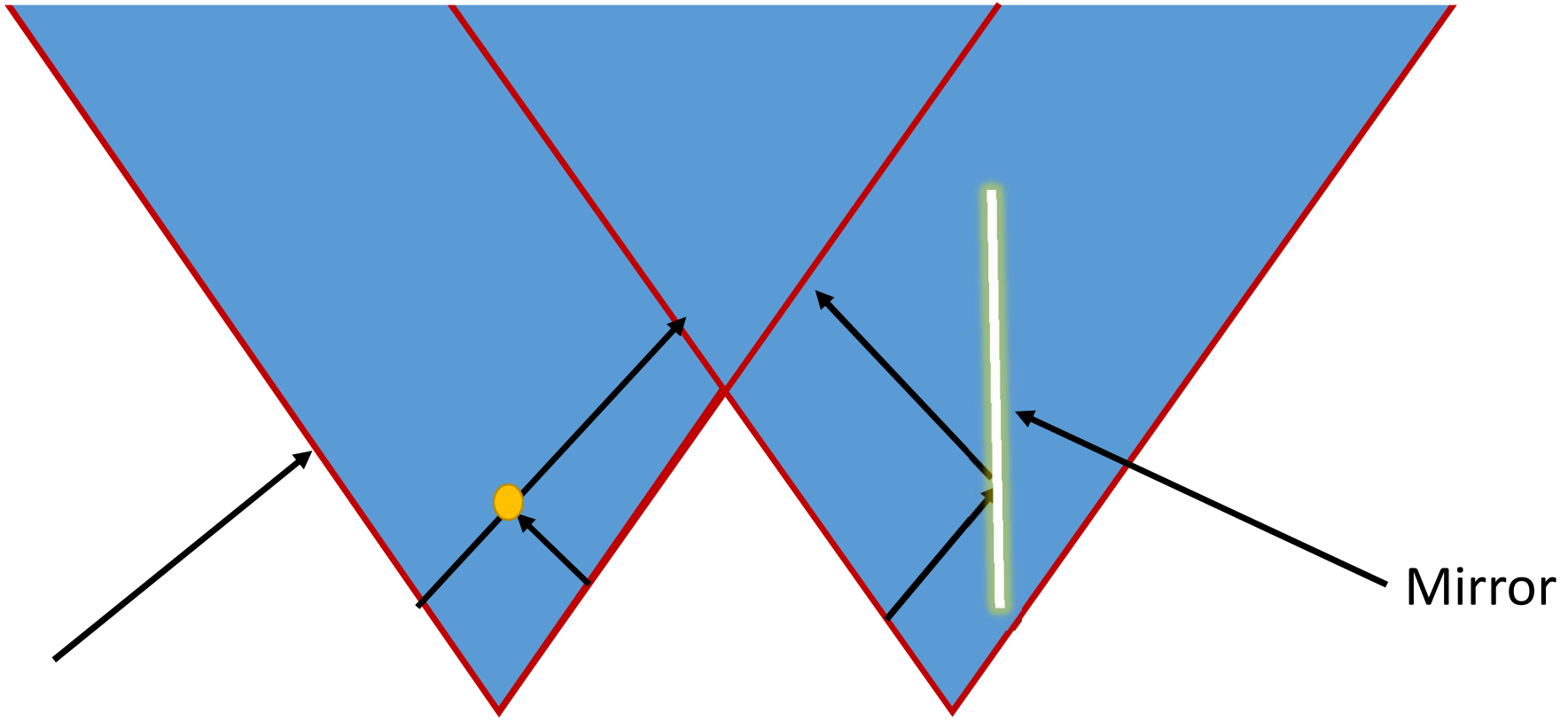


Yes we can

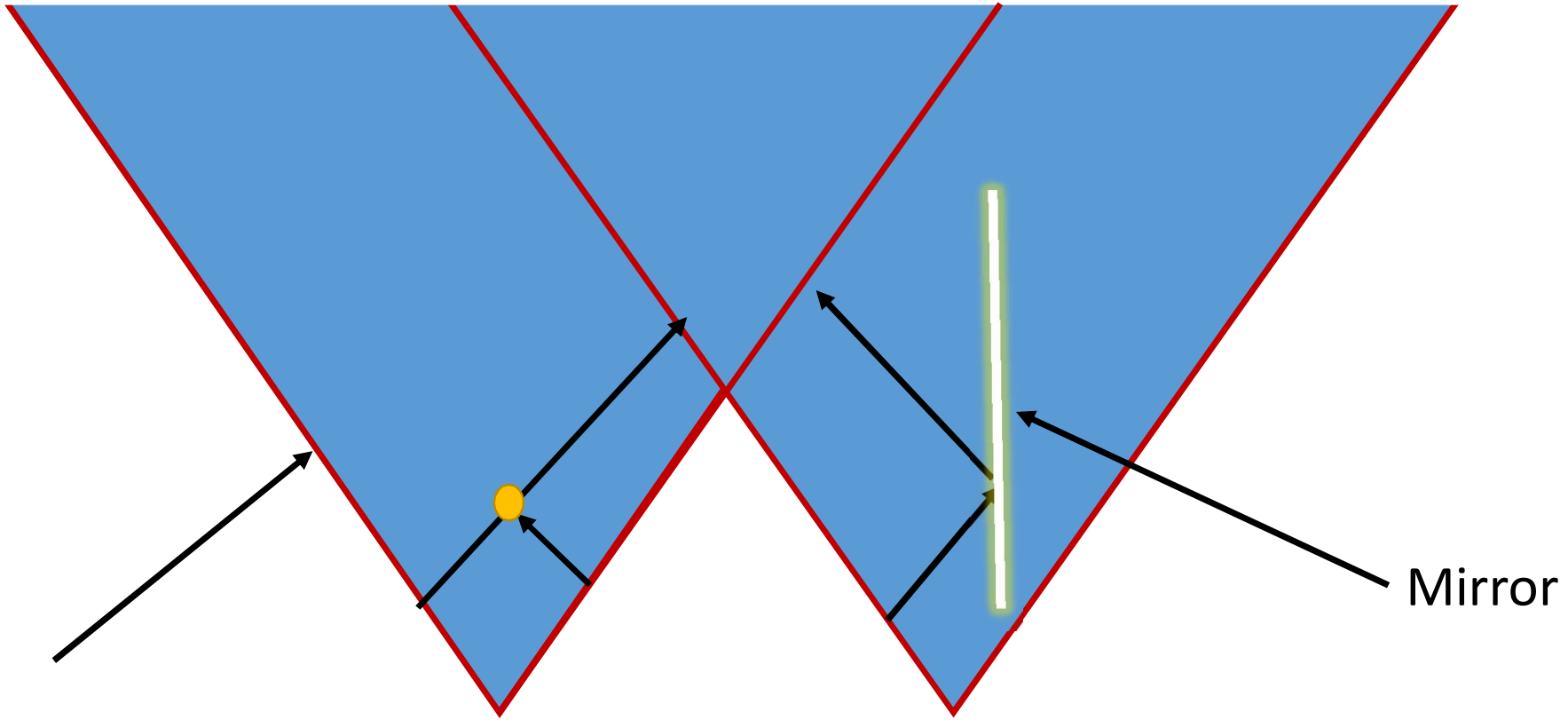
Yes we can



Yes we can



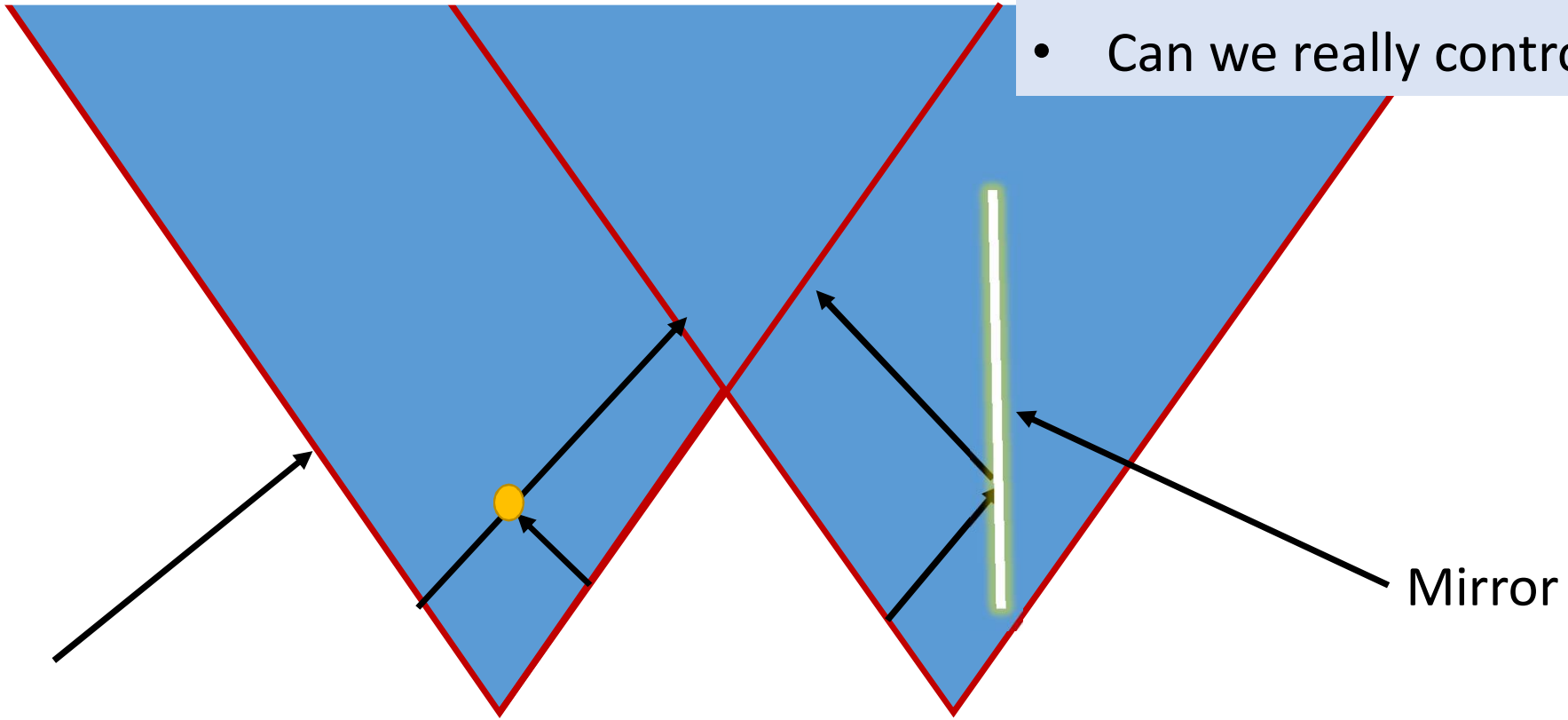
Can we?



Can we?

Issues:

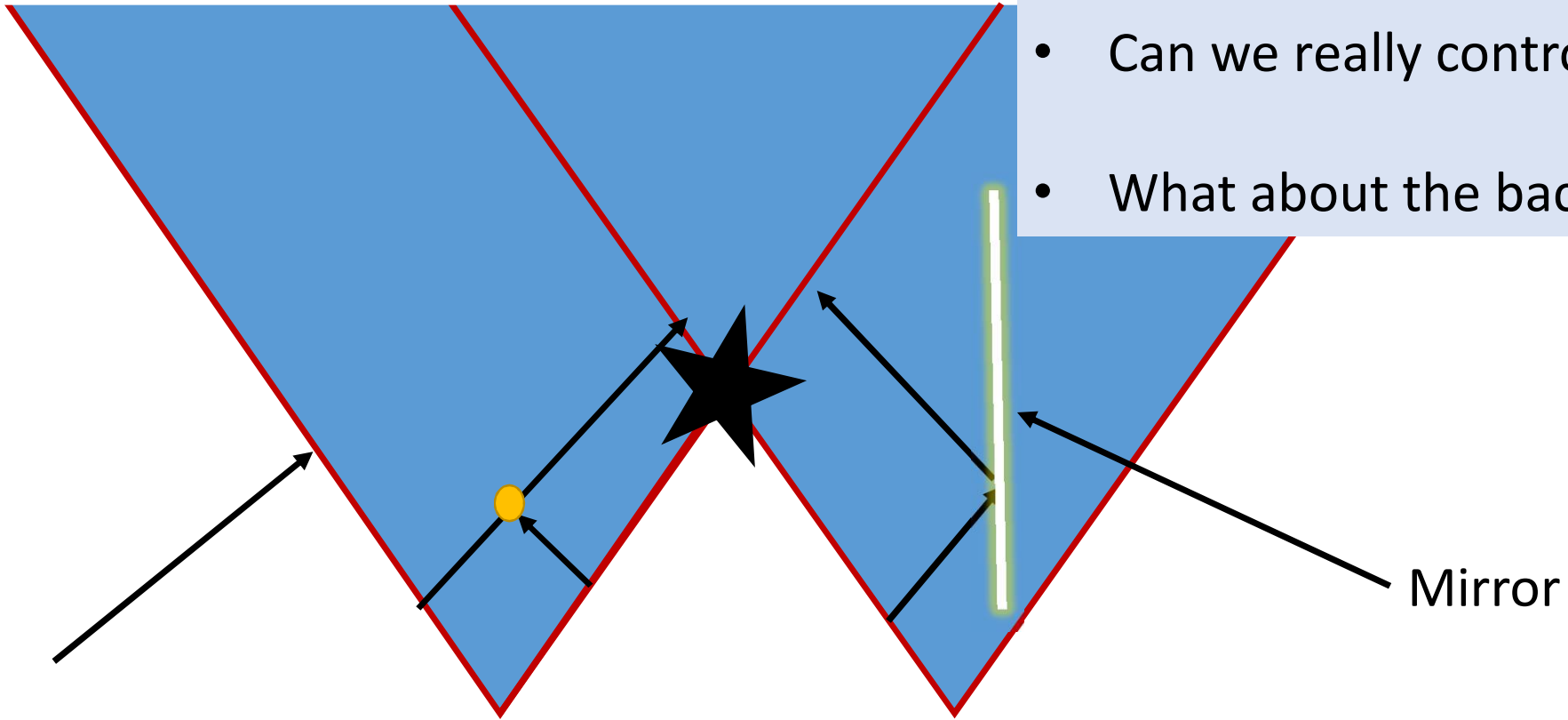
- Can we really control this?



Can we?

Issues:

- Can we really control this?
- What about the backreaction?



What about the argument from Holography?

What about the argument from Holography?

But we don't have a holographic dual of time-like dilaton.

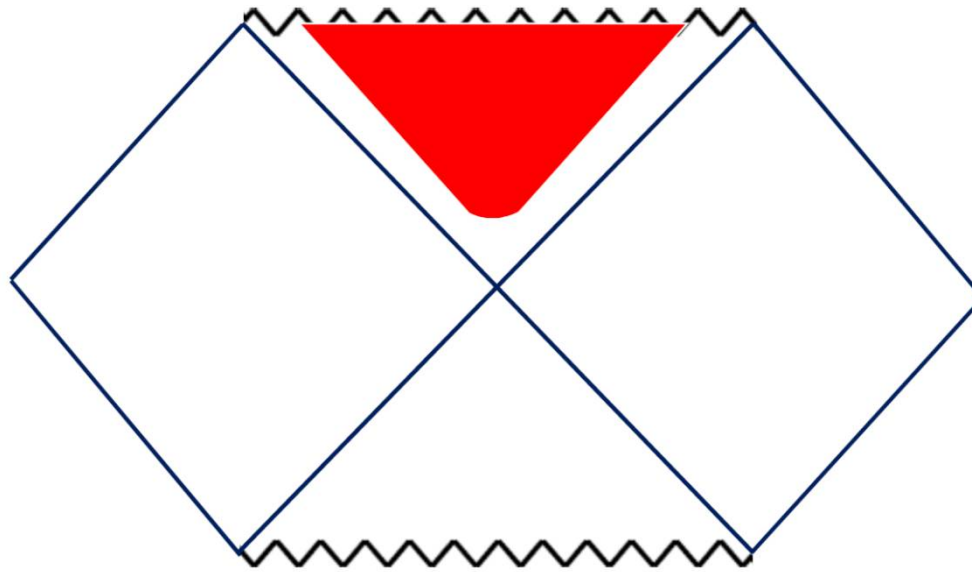
What about the argument from Holography?

But we don't have a holographic dual of time-like dilaton.

We do have time-like dilaton behind the horizon of the $SL(2)/(1)$ BH that is the near horizon of NS5-branes.

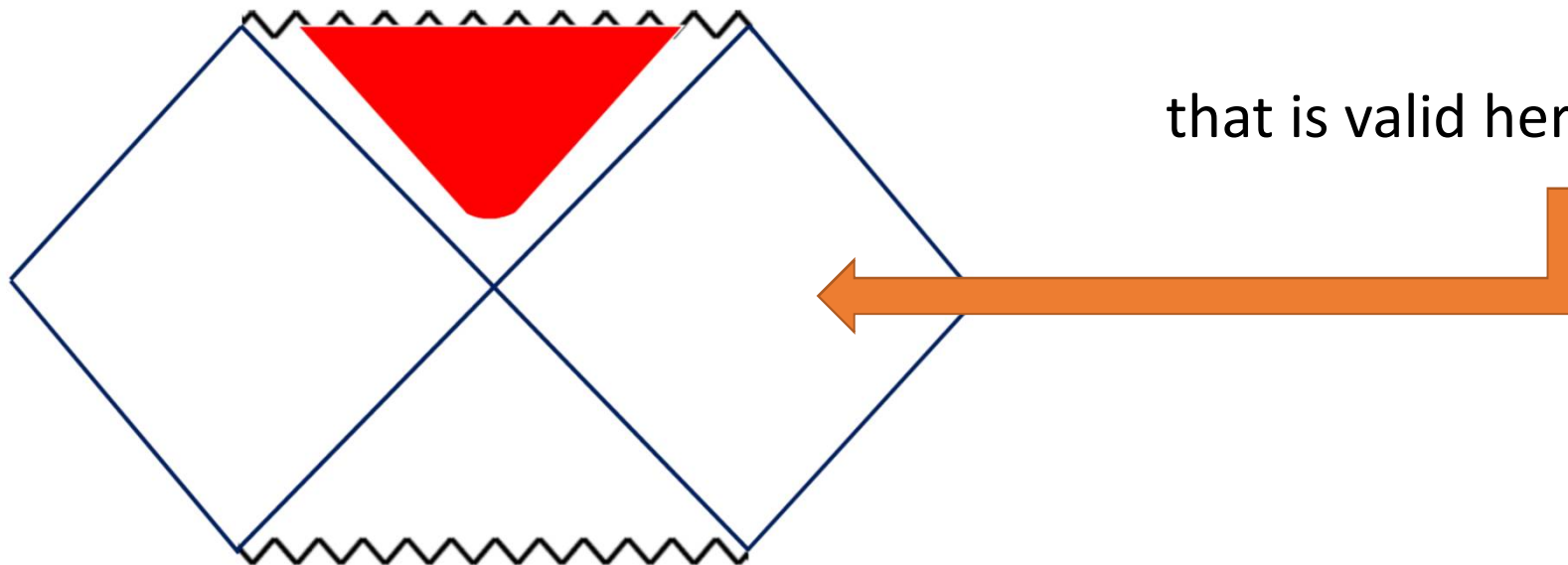


The $SL(2)/U(1)$ BH is not empty, but is filled with folded strings



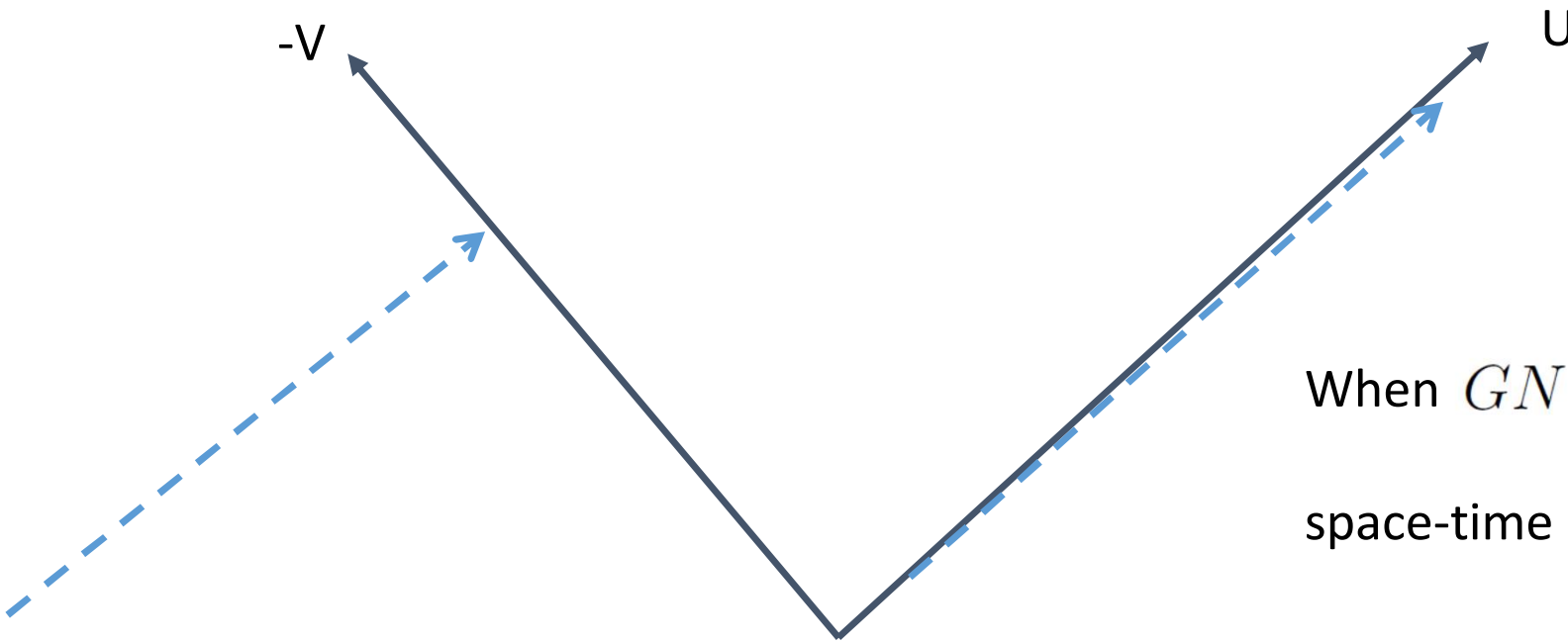


The $SL(2)/U(1)$ BH is not empty, but is filled with folded strings



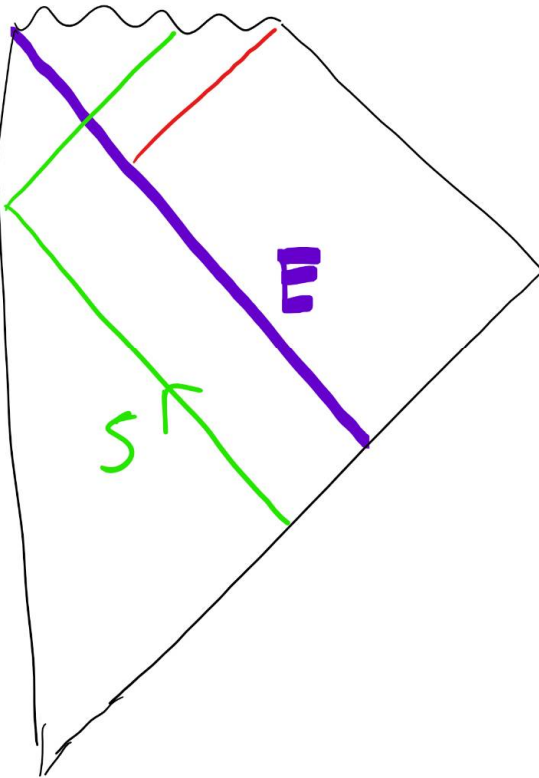
Fine with holography argument
that is valid here.

$$\int_{-\infty}^{\infty} du T_{uu} = (v - Q \log(4)) < 0$$

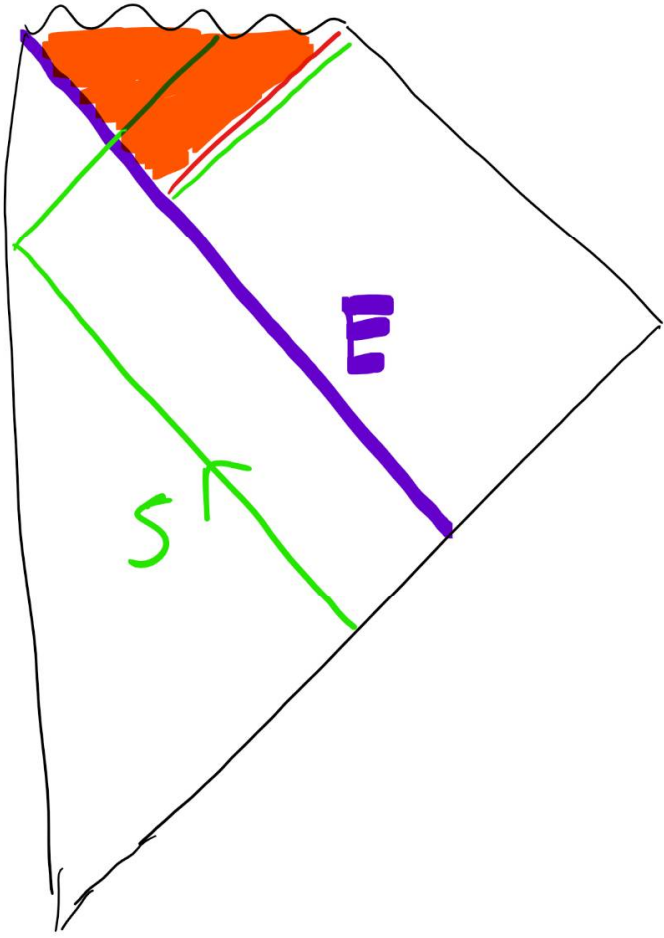


When $GN = 1$ part of
space-time is cloaked.

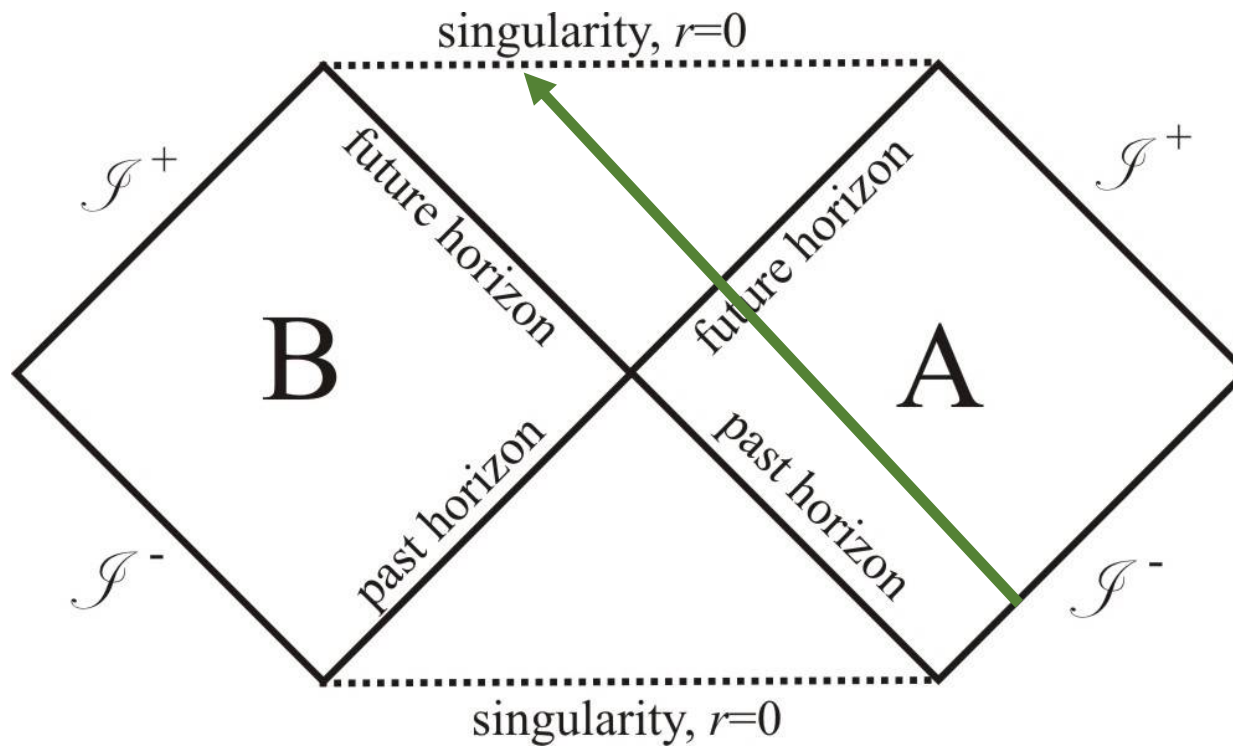
Let's go back to the firewall is not enough. We had this picture



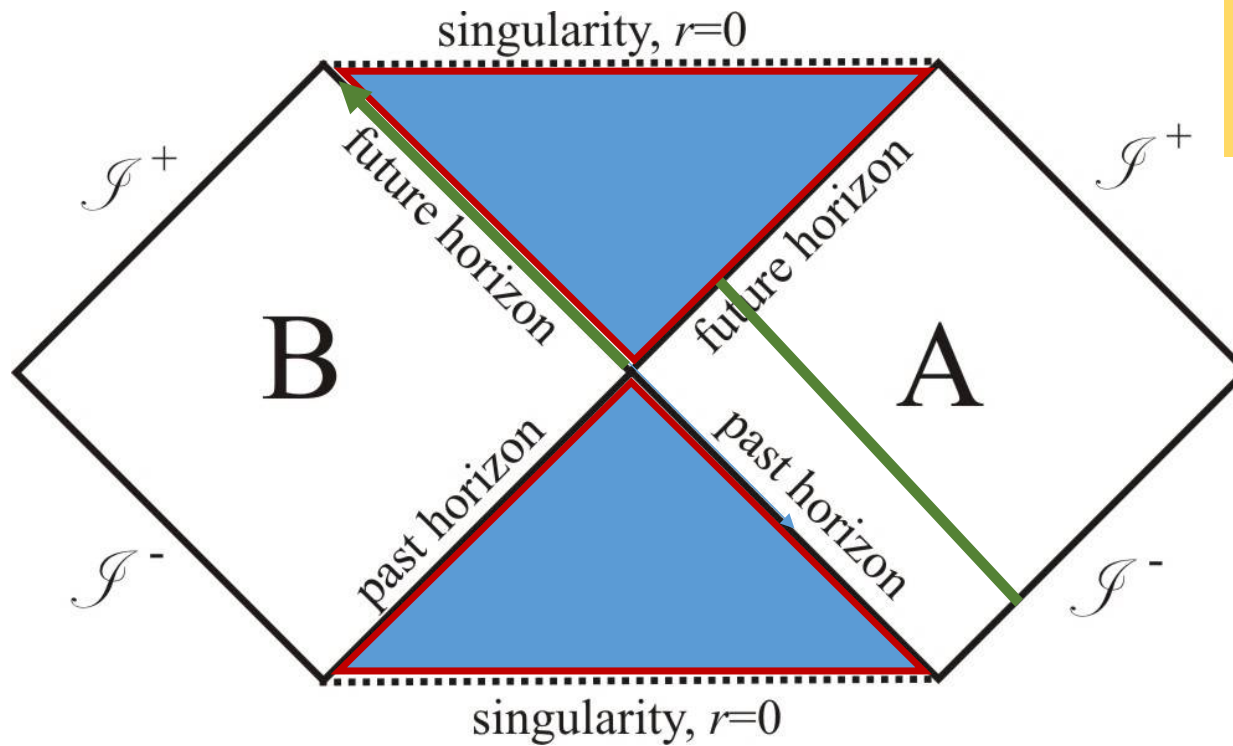
Let's go back to the firewall is not enough. Now we have



Wormhole still nontraversable



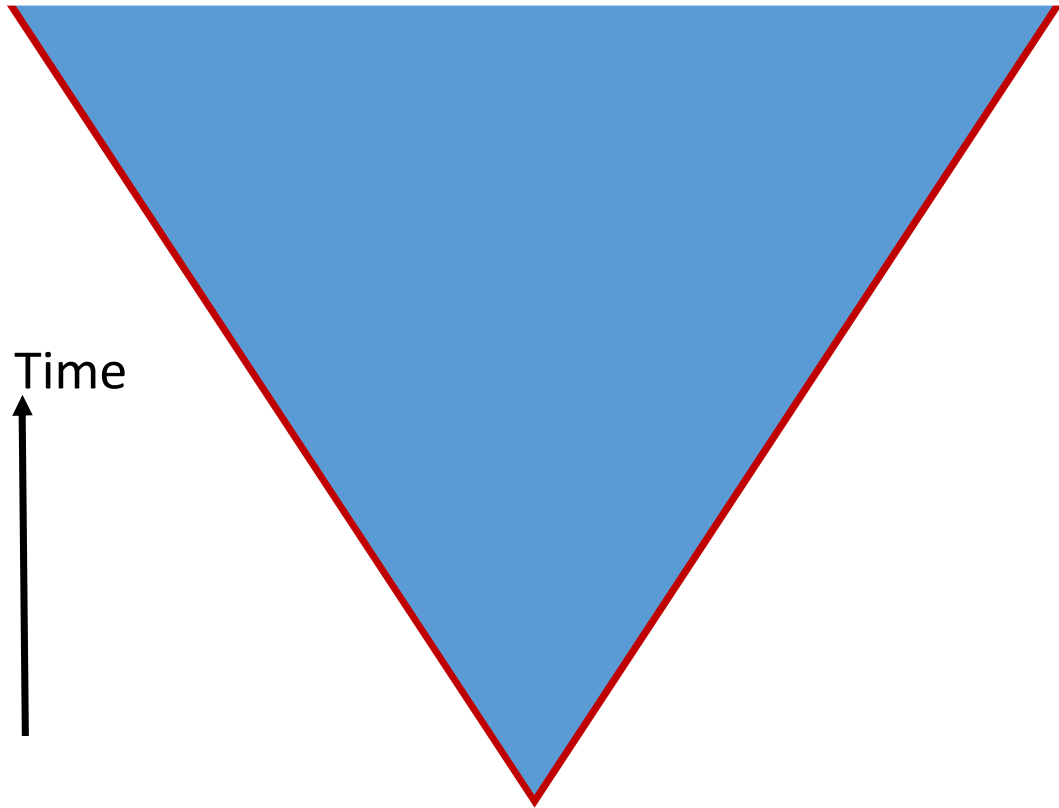
Wormhole still nontraversable



Cannot make it to
the other boundary

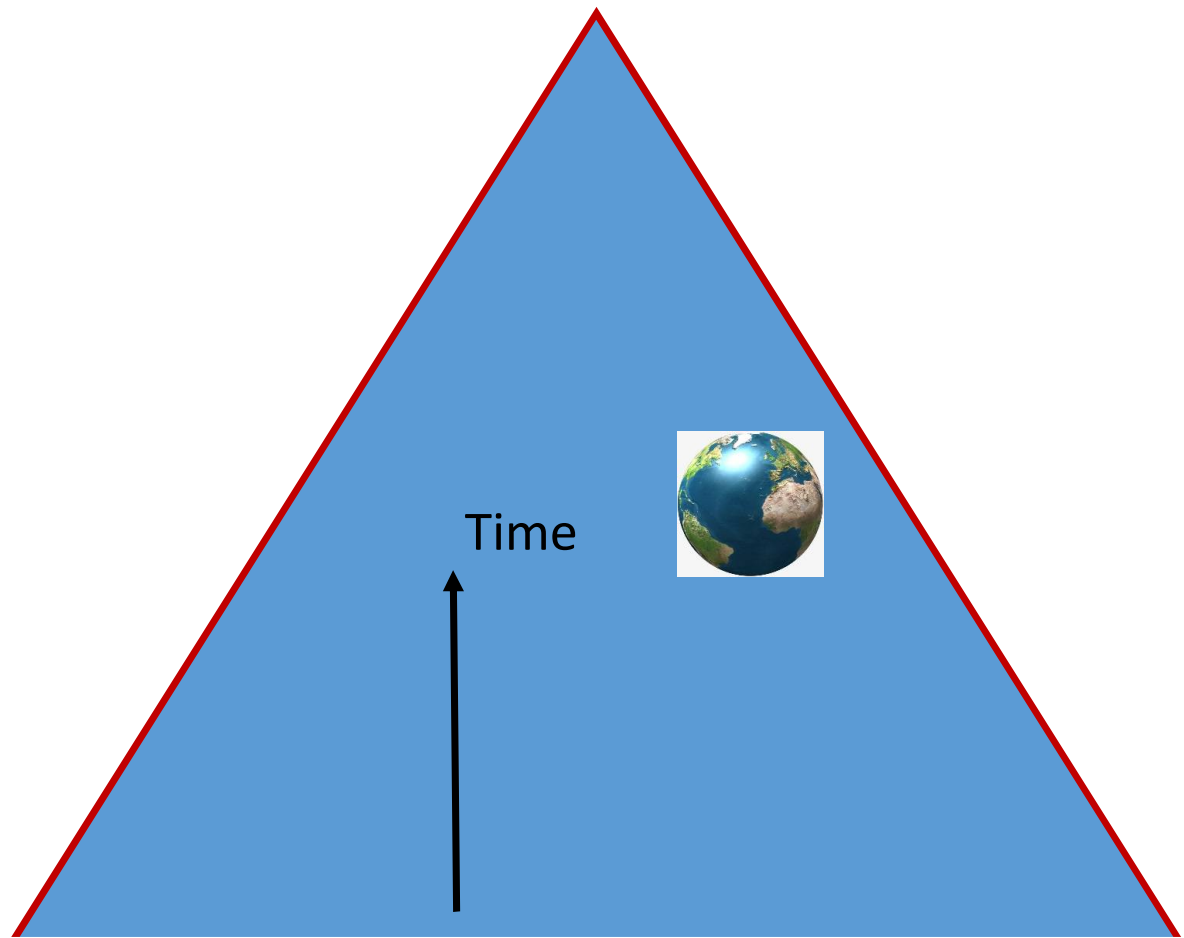
These objects open up new scenarios in cosmology:

For $Q > 0$ we have a new
Universe with a larger
cosmological constant.



These objects open up new scenarios in cosmology:

For $Q < 0$ we might be on our way towards a null singularity.



Conclusion:

- Much of our understanding of BH and cosmology relies on the ANEC.
- In QFT we can even prove it, but string theory includes classical configuration that violates it.
- It should be fun to revisit our understanding in light of that.

Conclusion:

- Much of our understanding of BH and cosmology relies on the ANEC.
- In QFT we can even prove it, but string theory includes classical configuration that violates it.
- It should be fun to revisit our understanding in light of that.

Thank You