Heavy to Light Meson Semileptonic Decays Form Factors

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1 Introduction

• The semileptonic heavy to light meson decays form factor as in the $D, B \to \pi \ell \nu$ decays, is given by the $V - A$ current matrix elements between heavy and light meson state.

• A precise knowledge of these form factors is required for an exclusive determination of the CKM matrix element V_{ub}.

• These form factors are known experimentally from BaBar, Belle and BESIII measurements.

• There have been previous calculations of the form factors in the light-cone sum rule approach(LCSR) and in lattice simulation.

• We note that the expressions for the form factors in the calculations of Ball and Zwicky, PRD 71 014015 (2005) are very long (1 or 2 pages).

• The $B \to \pi$ form factor from lattice simulation and BaBar measurements could be fitted with a BK(Becirevic-Kaidalov) parametrization (PLB 478, 417 (2000)) with $\alpha = 0.52 \pm 0.05 \pm 0.03$:

$$f^+(q^2, \alpha) = \frac{f_0}{(1 - q^2/m^2_{B^*})(1 - \alpha q^2/m^2_{B^*})} \quad (1)$$

• For D meson semileptonic decays, the BaBar, Belle and BESIII measurements show that the $D \to \pi, K$ form factor could be fitted with an effective two-pole :

$$f^+(q^2, \alpha) = \frac{f_0}{(1 - q^2/m^2_{D^*}).(1 - \alpha q^2/m^2_{D^*})} \quad (2)$$

or a single-pole parametrization :

$$f^+(q^2, \alpha) = \frac{f_0}{(1 - q^2/m^2_{\rm pole})} \quad (3)$$

with $m^2_{\rm pole} = (1.906 \pm 0.029 \pm 0.023)$GeV obtained by BaBar and from BESIII $m_{\rm pole} = (1.911 \pm 0.012 \pm 0.004)$GeV for $D \to \pi$ form factor and $m_{\rm pole} = (1.921 \pm 0.010 \pm 0.007)$GeV for $D \to K$ form factor.

• As noted by BESIII in PRD 92, 072012(2015), “The agreement between the extracted values of the pole mass and the expected values

$(M_{D_s^{*+}})$ is extremely poor".

• The problem is to obtain a theoretical expression with this pole-dominance q^2-behaviour for these form factors.

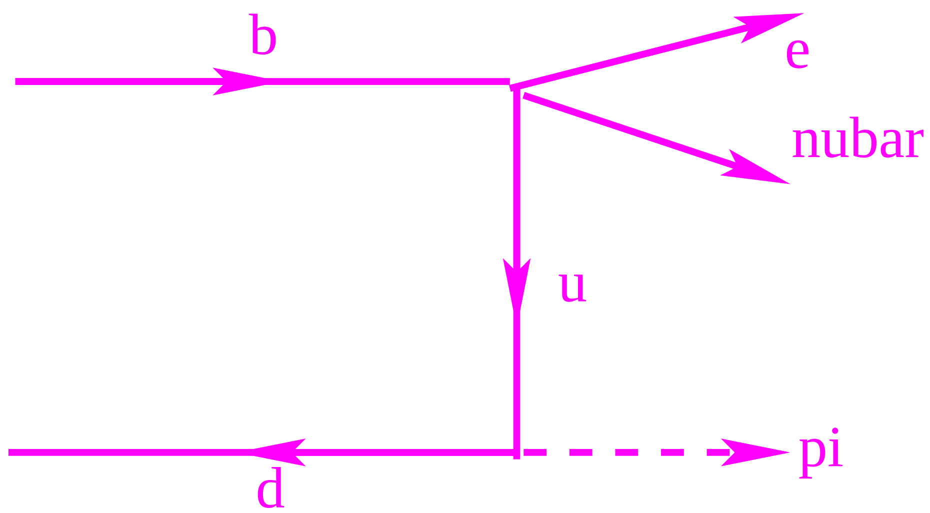

Figure 1: Diagram showing the Born term for the semileptonic B decay form factor

• It is straightforward to obtain the pole dominance term for the form factors: as shown in Figure 1, the form factor for semileptonic decay with pion or light hadron in the final state is, a $c\bar{d} \rightarrow \pi \ell \nu$ and $b\bar{d} \rightarrow \pi \ell \nu$

process, with the π, K meson treated as the Goldstone boson of the chiral $SU(3) \times SU(3)$ symmetry, rather than a two-body bound state which gives a rather small $\bar{B}^0 \to \pi^+ \ell^- \nu$ branching ratio as shown by Suzuki(1988).

• The use of the pion-quark coupling to obtain the heavy to light semileptonic decay amplitudes is similar to the study of strong and radiative decays of light vector mesons with a similar Born term from which the extracted pion-quark coupling consistent with the theoretical value given by the bag model (A. Suzuki and R. K. Bhaduri (1983)) to within 50% .

• Like the two-photon and two-gluon decays of the P-wave $\chi_{c0,2}$ and $\chi_{b0,2}$ charmonium state, the Born term from this process then gives the semileptonic decay amplitudes and the $D \to \pi$ and $B \to \pi$ form factors in terms of the light pseudo scalar meson-quark coupling $g_{\pi qq}$ given by the Goldberger-Treiman relation, the $c\bar{d} \to \pi \ell \nu$ and $b\bar{d} \to \pi \ell \nu$ process then produce the form factors.

• Thus it is possible to obtain the semileptonic decays amplitude with pion in the final state treated as a Goldstone boson of chiral symmetry with the pion-quark coupling given by the Goldberger-Treiman relation, as will be shown in the next section.

2 Effective Lagrangian for $D \to \pi\ell\nu$ and $B \to \pi\ell\nu$

• The $c\bar{c} \to \gamma\gamma$ annihilation in the two-photon decay of $P-$wave charmonium state proceeds through the Born term at the tree level approximation, the reactions $c + \bar{d} \to \pi\ell\nu$ and $b + \bar{d} \to \pi\ell\nu$, can occur through a similar Born term, with the exchange of an u quark which combines with $\bar{d}$ quark to produce a pion in the final state.

• In the weak binding approximation with $b, \bar{d}$ quark taken at rest in the B^0 meson, the effective Lagrangian for $B, D \to \pi\ell\nu$ is given by :

$$O_\mu = \bar{v}(p_d)V_\mu u(p_b) \tag{4}$$

with

$$V_\mu = \frac{1}{i}\left[(-ig_{\pi qq}\gamma_5)i\frac{(\not p - \not p_d + m_u)}{(p-p_d)^2 - m_u^2}(-ig\gamma_\mu)\right] \qquad (5)$$

• In terms of the local 2-quark operator , the vector current matrix element in $B \to \pi\ell\nu$ decays is given by :

$$O_P = \frac{2m_B(\bar{d}\gamma_5 b)p_\mu}{(m_B^2 + m_\pi^2 - q^2)} \qquad (6)$$

showing the appearance of the pole at $q^2 = (m_B^2 + m_\pi^2)$ generated by the Born terms. For $D \to K$ form factor, the pole is at $q^2 = (m_D^2 + m_K^2)$. This result explains the success of the single-pole or two-pole fits of the BaBar and BESIII data as shown below.

• With $m_b + m_d = m_B$, $< 0|\bar{d}\gamma_5 b|B) = m_B f_B$, $< 0|O_P|B >$ then gives us the form factor for $B \to \pi\ell\nu$ decay.

• Using the pion-quark coupling for a constituent quark in the B meson obtained from the Goldberger-Treiman relation(Harada):

$$G_{\pi q} f_\pi = M g_A^Q \ , \quad g_A^Q = 1 \tag{7}$$

$$f_+(0) = \left(\frac{f_B}{f_\pi}\right) \frac{1}{(1 + m_\pi^2/m_B^2)} \tag{8}$$

and similar expression for $D \to K$ form factor.

- With $f^+(q^2)$ at $q^2 = 0$ known from experiment, we have:

$$\begin{aligned} f_+(q^2)_{D\pi} &= \frac{f_+(0)_{D\pi}}{(1 - q^2/(m_D^2 + m_\pi^2))} \\ f_+(q^2)_{DK} &= \frac{f_+(0)_{DK}}{(1 - q^2/(m_D^2 + m_K^2))} \\ f_+(q^2)_{B\pi} &= \frac{f_+(0)_{B\pi}}{(1 - q^2/(m_B^2 + m_\pi^2))} \end{aligned} \tag{9}$$

for the Born term contribution to the form factor.

• The above expressions are essentially the same to those used in the parametrization of the form factors measured at BaBar, Belle and BESIII.

• A remark: As the Born term is of pure kinematic origin, there is no D^{*}, B^{*} pole term in the above expressions.

• This explains the fact that the single-pole fits for $D \to K, \pi$ form factors do not have a D^{*} pole, consistent with the quark propagator pole term.

• For the $B \to \pi$ form factor, the mass difference between B^{*} and B is negligible, to be consistent, $m_{B^{*}}^{2}$ should be replaced by m_{B}^{2} without affecting the BaBar BK fit, making it consistent with the Born term.

• What is new here is that the Born term could generate this pole dominance term at $q^{2} = m_{B}^{2} + m_{\pi}^{2}$ and at $q^{2} = m_{D}^{2} + m_{K}^{2}$ for the $D \to K$ form factor which seems impossible to obtain otherwise.

• There is also possible suppression of the quark-pion coupling due to the off-shell effects of the quark propagator, as the momentum of the

$u-$quark in the Born term gets large for small q^2, the value of $f^+(q^2)$ would be suppressed for small q^2.

• The QCD sum rule calculation of Narison (PLB 283, 384 (1992)) gives $f_+(0) = 0.23 \pm 0.02$, similarly, the value of Ball,Braun, Dosch (PLB 273, 316 (1991)) $f_+(0) = 0.24 \pm 0.025$, and also from Ball (PRD 48, 3190(1993)) $f_+(0) = 0.26 \pm 0.02$. These calculations show that the $B \to \pi$ form factor is strongly suppressed at $q^2 = 0$. The QCDSF Collaboration lattice calculation also shows suppression with $f_+(0) = 0.27 \pm 0.07 \pm 0.07$.

• For $D^0 \to \pi^-$ form factor, the BESIII data give $f_+(0) = 0.6372 \pm 0.0.0080 \pm 0.0044$ and for $D^0 \to K^-$ form factor $f_+(0) = 0.7368 \pm 0.0080 \pm 0.0044$, and similarly for $D^+ \to \pi^0$, $f_+(0) = 0.622 \pm 0.012 \pm 0.003$ and $f_+(0) = 0.725 \pm 0.004 \pm 0.012$ for $D^+ \to \bar{K^0}$ showing no large suppression compared with the $B \to \pi$ form factor.

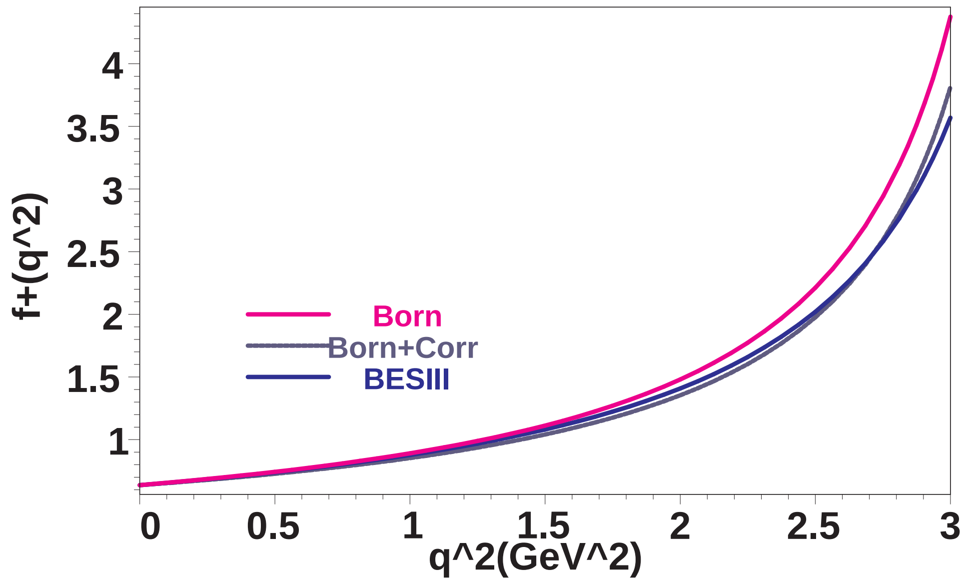

Figure 2: The Born term(upper curve), the Born term with a small polynomial term to fit the $D^0 \to \pi^-$ form factor from BESIII data (lower curve) given in FIG.9 of the BESIII Collaboration: PRD 92, 072012 (2015)

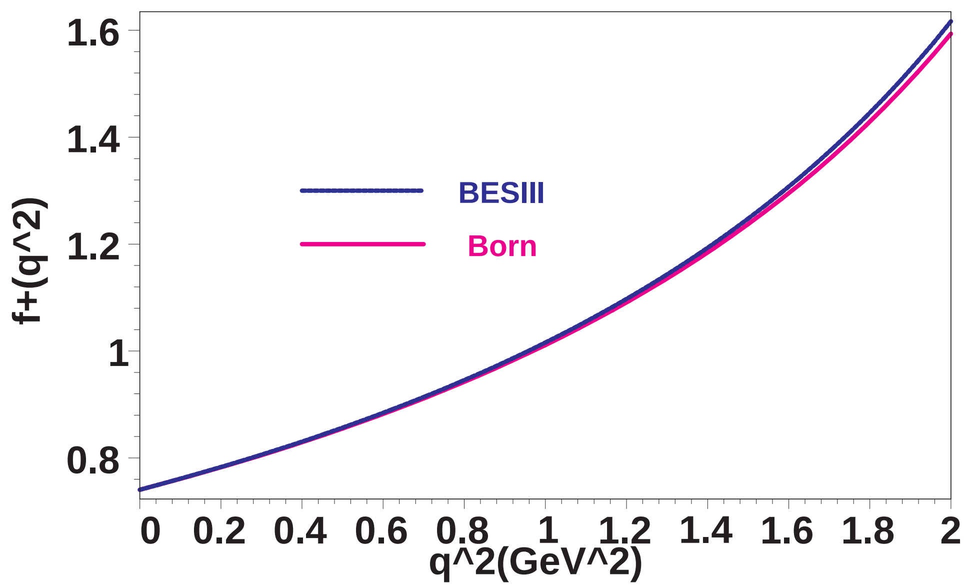

Figure 3: The Born term(upper curve), the Born term with a small polynomial term to fit the $D^+ \to \pi^0$ form factor BESIII data (lower curve)

• As shown in Figure 2 and Figure 3 for $D^0 \to \pi^-$ and $D^+ \to \pi^0$ form factor, the Born term plotted in the upper curve is slightly above the lower curve obtained with the BESIII fit obtained with $f_+^\pi(0) = 0.6372 \pm 0.0080 \pm 0.0044$,

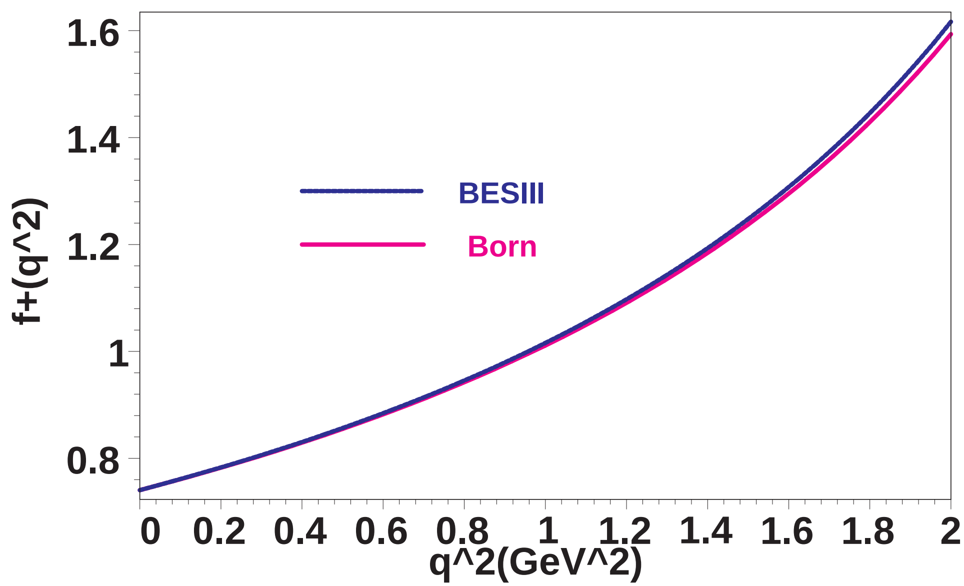

Figure 4: The Born term(lower curve) and the fit to the $D^0 \to K^-$ BESIII measured form factor (upper curve)

For the $D^0 \to K^-$ form factor, in Figure 4, the lower curve(Born term) is in excelllent agreement with the fit to the BESIII data(upper curve) with $f_+^K(0) = 0.7768 \pm 0.0026 \pm 0.0036$, $M_{pole} = 1.921 \pm 0.010 \pm 0.007\text{GeV}$ $V_{cs} = 0.97343 \pm 0.00015$ and $V_{cs} = 0.97343 \pm 0.00015$.

• This good agreement between the two curves in Figure 4 could be explained by the m_K^2 term in the $u-$quark propagator. If we replace the factor $q^2/(m_D^2+m_K^2)$ by $q^2/m_{\rm eff}^2$, with $m_{\rm eff}=\sqrt{m_D^2+m_K^2}$ as the effective mass in the pole term, then $m_{\rm eff}=1.931$GeV, very close to the pole mass of the BESIII fit, $M_{pole}=1.921\pm0.010\pm0.007$GeV.

• Agreement is also found between the Born term and the BESIII fit for the $D^+\to\bar{K}^0$ form factor obtained with
$f_+^\pi(0)=0.7094\pm0.0035\pm0.0111$, $M_{pole}=1.935\pm0.017\pm0.006$GeV shown in Figure 5 , very close to the effective mass $m_{\rm eff}=1.931$GeV in the Born term.

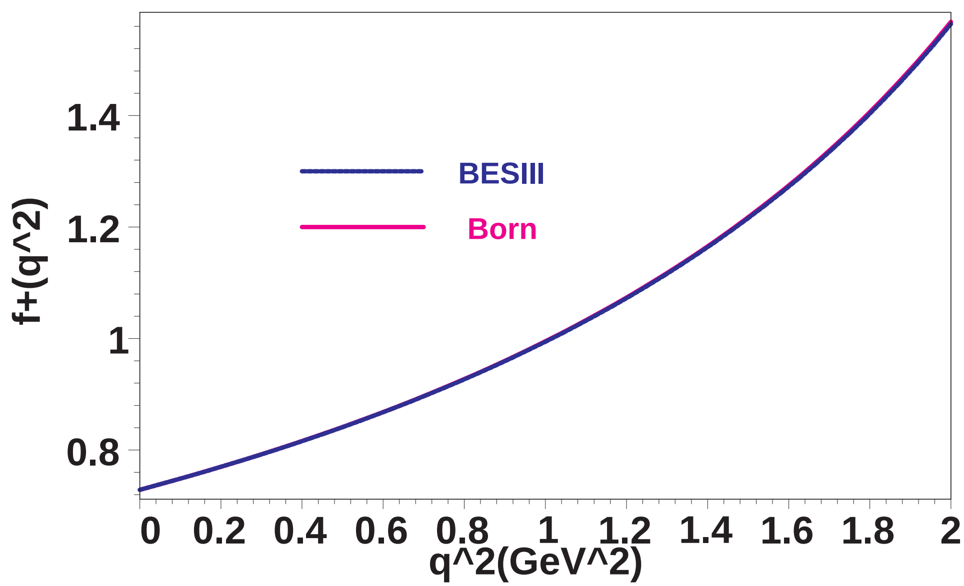

Figure 5: The Born term(lower curve) and the fit to the $D^+ \rightarrow \bar{K^0}$ BESIII measured form factor (upper curve)

• This dependence on m_K^2 in both $D^0 \rightarrow K^-$ and $D^+ \rightarrow \bar{K^0}$ form factor shows evidence for the dominance of the Born term for the $D \rightarrow K$ semileptonic decay form factors.

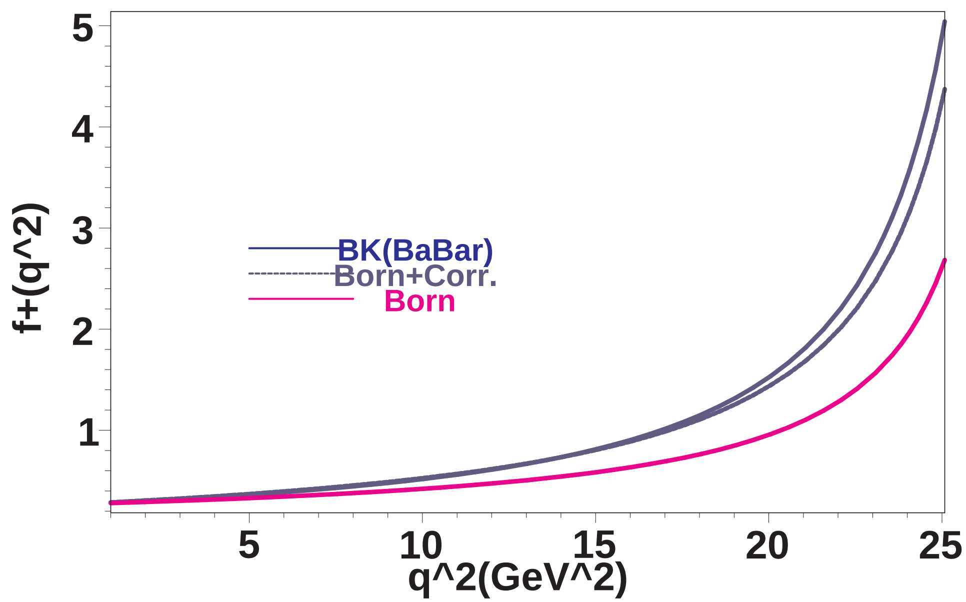

Figure 6: The Born term(lower curve), the Born+Corr plot has a small polynomial term added (middle curve) and the BaBar data (upper curve) represented by the BK fit for the $B \to \pi$ form factor

• There is also a correction to the $B \to \pi$ form factor Born term to compensate for a suppression at large q^2 induced by $f_+(0)_{B\pi}$ mentioned above. Thus with these corrections included, the middle curve of Figure 6 is now in agreement with data and almost coincides with the lower curve obtained with a BK fitted to the BaBar data.

• The $D \to K, \pi$ and $B \to \pi$ form factors with the Born term as the main contribution, and assuming the same correction term for $D^0 \to \pi^-$ and $D^+ \to \pi^0$ form factor, are now in agreement with data and are given by :

$$f_+(q^2)_{D\pi} = \frac{f_+(0)_{D\pi}(1 - 0.15q^2/(m_D^2 + m_\pi^2))}{(1 - q^2/(m_D^2 + m_\pi^2))}$$
$$f_+(q^2)_{DK} = \frac{f_+(0)_{DK}}{(1 - q^2/(m_D^2 + m_K^2))}$$
$$f_+(q^2)_{B\pi} = \frac{f_+(0)_{B\pi}(1 + 0.70q^2/(m_B^2 + m_\pi^2))}{(1 - q^2/(m_B^2 + m_\pi^2))} \tag{10}$$

3 Conclusion

• In conclusion, we have shown that, the tree-level Born term for the process $c + \bar{d} \to \pi\ell\nu$ and $b + \bar{d} \to \pi\ell\nu$ in semileptonic decays of a heavy meson to a light meson in the final state is found to describe rather well the q^2-dependence of the $D \to \pi$,$D \to K$ and $B \to \pi$ form factors.

• The $D^0 \to K^-$ and $D^+ \to \bar{K}^0$ form factors show possible evidence for the K mass term in the q^2-dependence generated by this Born term.

• Note added: While preparing this talk, I found a previous paper by Riazuddin, T. A. Al-Aithan and Amjad Hussain Shah Gilani, Int. J. Mod. Phys. A17, 4927 (2002) (hep-ph/0007164v2 (2000)) in which a B-meson pole q^2-dependence term $1/(1 - q^2/m_B^2)$ for the $B \to \pi$ form factor is also obtained.