Cosmology from the boundary

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based on: N. Arkani-Hamed, P.B., A. Postnikov – 1709.02813;
N. Arkani-Hamed, P.B. – 1811.01125;
P.B. – 1811.02515;
N. Arkani-Hamed, P.B. – work in progress

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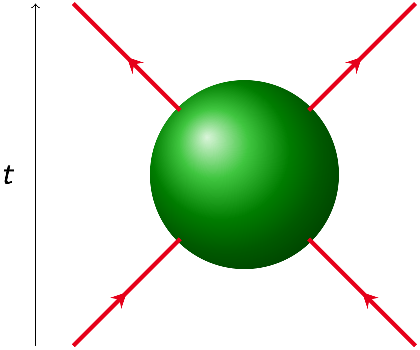
Observables and boundary data

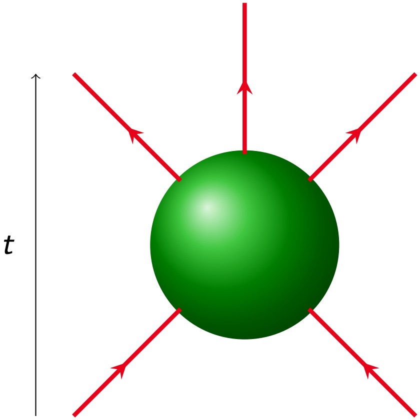
Observables and boundary data

In flat space-time



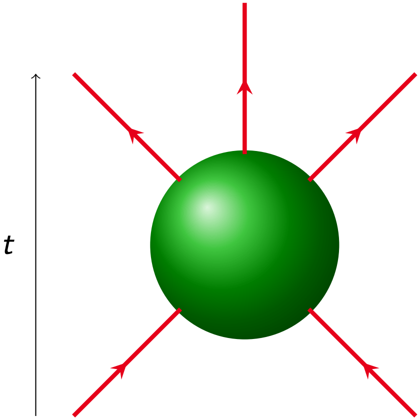
Observables and boundary data

In flat space-time



Observables and boundary data

In flat space-time



Experiment repeated ∞ times

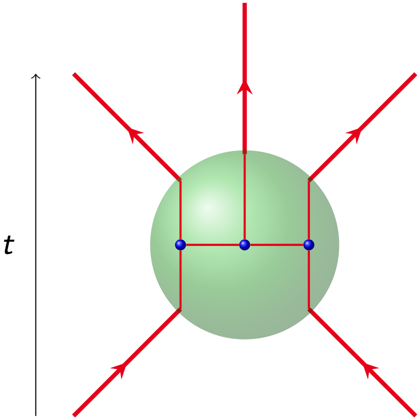
S-Matrix

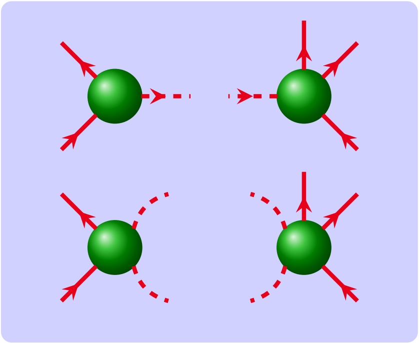
Probability of scattering processes

Relative frequencies

Observables and boundary data

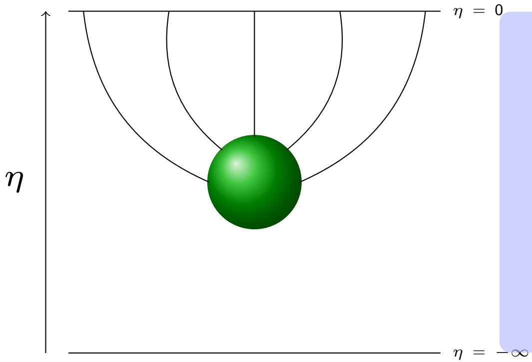
In flat space-time





Observables and boundary data

In cosmology



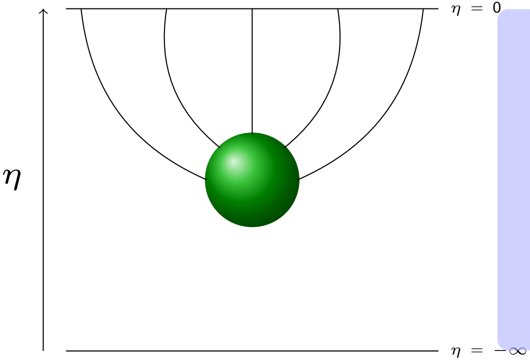
Observations at present time

Accelerated expansion

Observables and boundary data

In cosmology

No Q.M. observable!



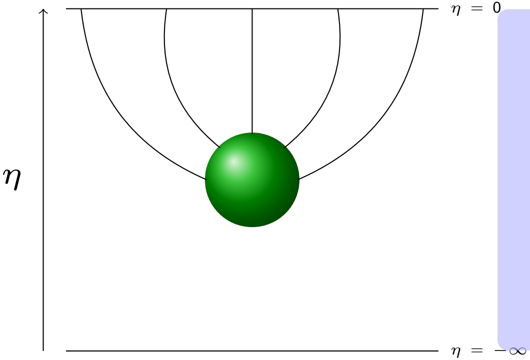
Observations at present time

Accelerated expansion

Observables and boundary data

In cosmology

Late-time correlators



Observations at present time

Spatial averages

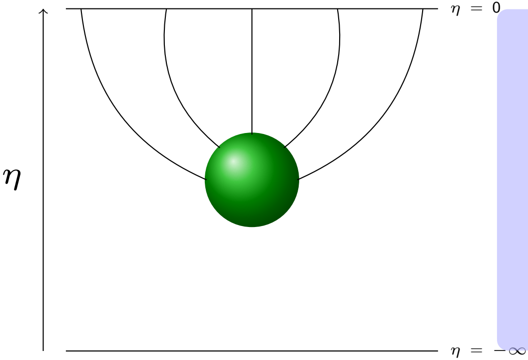
Accelerated expansion

Universe ∞ large

Observables and boundary data

In cosmology

Wavefunction of the universe



Observations at present time

Spatial averages

Accelerated expansion

Universe ∞ large

Observables and boundary data

In cosmology: Late-time correlators

$$\langle \frac{d\rho}{\rho}(\vec{x}_1)\, \frac{d\rho}{\rho}(\vec{x}_2) \rangle$$

Observables and boundary data

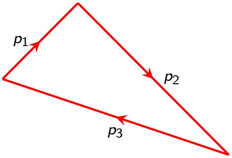
In cosmology: Late-time correlators

$$\langle \frac{d\rho}{\rho}(\vec{p}_1)\,\frac{d\rho}{\rho}(\vec{p}_2)\rangle = \delta^{(3)}(\vec{p}_1 + \vec{p}_2)\,\frac{10^{-10}}{|\vec{p}|^3}$$

In cosmology: Late-time correlators

$$\langle \frac{d\rho}{\rho}(\vec{p}_1)\,\frac{d\rho}{\rho}(\vec{p}_2)\rangle \;=\; \delta^{(3)}\left(\vec{p}_1+\vec{p}_2\right)\frac{10^{-10}}{|\vec{p}\,|^3}$$

$$\langle \prod_{i=1}^{3}\frac{d\rho}{\rho}(\vec{p}_i)\rangle \;=\; \delta^{(3)}\left(\sum_{i=1}^{3}\vec{p}_i\right)\,f_3(p_i)$$



Observables and boundary data

In cosmology: Late-time correlators

$$\langle \frac{d\rho}{\rho}(\vec{p}_1)\,\frac{d\rho}{\rho}(\vec{p}_2)\rangle \;=\; \delta^{(3)}\left(\vec{p}_1+\vec{p}_2\right)\frac{10^{-10}}{|\vec{p}|^3}$$

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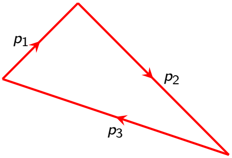


The wavefunction of the universe $\Psi[\phi]$ encodes these correlations

Observables and boundary data

- Limited control on the correlators:

  Interest for: non-gaussianities

  prediction of existence of particles during inflation..

- Consistency conditions not completely understood
- Not general rules for extracting fundamental physics
- Which are the invariant properties they ought to satisfy

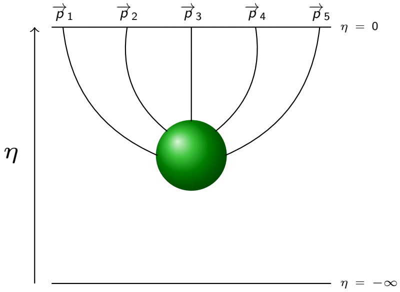
  in order to be consistent with a unitary evolution

  in cosmological space-times?

What can we say?

Observables and boundary data

The wavefunction of the universe $\Psi[\phi]$ encodes the correlations



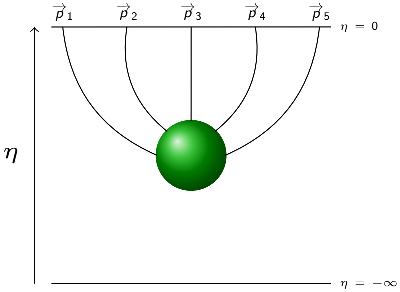
$$\Psi[\phi_0] = \int D\varphi \, e^{iS[\phi_0+\varphi]}$$

$$\begin{cases} \phi_0 = \text{free solution} \\ \varphi(\eta = 0) = 0 \\ \varphi(\eta \longrightarrow -\infty) = 0 \end{cases}$$

$$\langle \prod_{i=1}^{n} \phi(p_i) \rangle = \mathcal{N} \int D\phi \prod_{i=1}^{n} \phi(p_i) \, |\Psi[\phi]|^2$$

Observables and boundary data

The wavefunction of the universe $\Psi[\phi]$ encodes the correlations



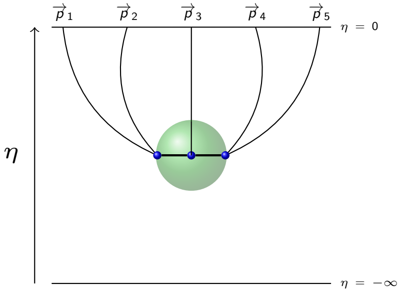
$$\Psi[\phi_0] = \int D\varphi \, e^{iS[\phi_0+\varphi]}$$

$$\begin{cases} \phi_0 = \phi(p) e^{iE\eta}, \; E \equiv |\vec{p}| \\ \varphi(\eta = 0) = 0 \\ \varphi(\eta \longrightarrow -\infty) = 0 \end{cases}$$

$$S[\phi] = \int_{-\infty}^{0} d\eta \int d^d x \left[\frac{1}{2} (\partial\phi)^2 - \sum_{k\geq 3} \frac{\lambda(\eta)}{k!} \phi^k \right], \quad \lambda(\eta) = \int_0^{\infty} d\varepsilon \, e^{i\epsilon\eta} \tilde{\lambda}_k(\varepsilon)$$

Observables and boundary data

The wavefunction of the universe $\Psi[\phi]$ encodes the correlations



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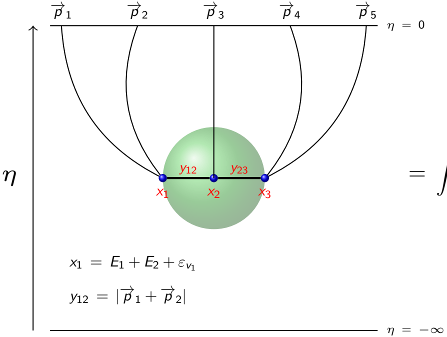
General properties of the wavefunction

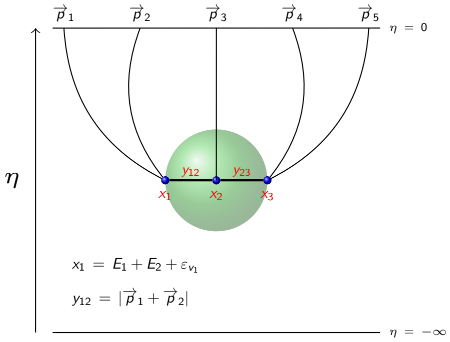


$$= \int_{-\infty}^{0} \prod_j \left[d\eta_j \, e^{i x_j \eta_j} \right] \prod_{e_{ij}} G(y_{e_{ij}}, \eta_i, \eta_j)$$

$$G(\eta_i, \eta_j) = \frac{1}{y} \left[e^{-iy(\eta_i - \eta_j)} \vartheta(\eta_i - \eta_j) + e^{iy(\eta_i - \eta_j)} \vartheta(\eta_j - \eta_j) - e^{iy(\eta_i + \eta_j)} \right]$$

Observables and boundary data

General properties of the wavefunction



- Function of sum of energies
- Singularities $\equiv$ sum of energies corresponding to each subgraph

$$\psi_3 = \frac{x_1 + y_{12} + 2x_2 + y_{23} + x_3}{(x_1+x_2+x_3)(x_1+y_{12})(x_1+x_2+y_{23})(y_{12}+x_2+y_{23})(y_{12}+x_2+x_3)(y_{23}+x_3)}$$

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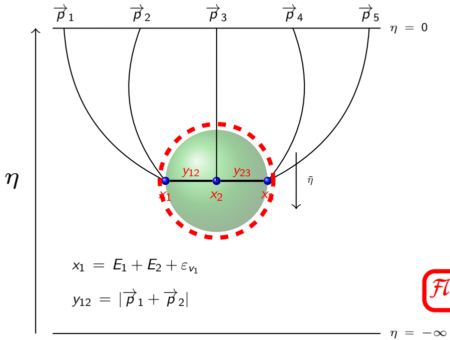
Observables and boundary data

General properties of the wavefunction



- Function of sum of energies
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$$\sim \frac{\mathcal{A}_3}{x_1 + x_2 + x_3}$$

Flat-space scattering amplitude!

$$\psi_3 = \frac{x_1 + y_{12} + 2x_2 + y_{23} + x_3}{(x_1 + x_2 + x_3)(x_1 + y_{12})(x_1 + x_2 + y_{23})(y_{12} + x_2 + y_{23})(y_{12} + x_2 + x_3)(y_{23} + x_3)}$$

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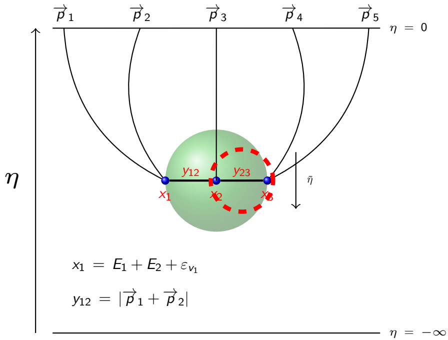
Observables and boundary data

General properties of the wavefunction



- Function of sum of energies
- Singularities $\equiv$ sum of energies corresponding to each subgraph

$$\sim \frac{\hat{\psi} \times \mathcal{A}_2}{y_{12} + x_2 + x_3}$$

Factorization: $\hat{\psi} \otimes A$

$$\psi_3 = \frac{x_1 + y_{12} + 2x_2 + y_{23} + x_3}{(x_1 + x_2 + x_3)(x_1 + y_{12})(x_1 + x_2 + y_{23})(y_{12} + x_2 + y_{23})(y_{12} + x_2 + x_3)(y_{23} + x_3)}$$

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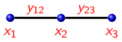
Observables and boundary data

General properties of the wavefunction



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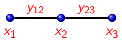
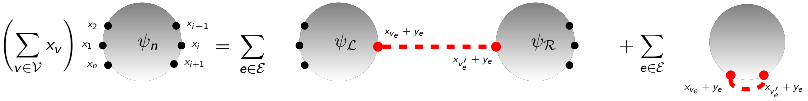
Observables and boundary data

General properties of the wavefunction

A boundary recursive formula

- Function of sum of energies
- Singularities $\equiv$ sum of energies corresponding to each subgraph





Observables and boundary data

General properties of the wavefunction

A boundary recursive formula

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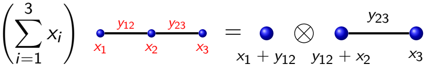
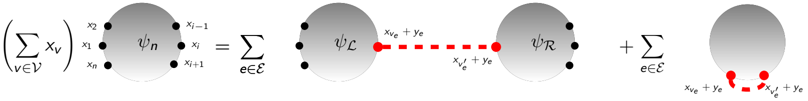


$$\frac{1}{x_1+y_{12}} \otimes \frac{1}{(y_{12}+x_2+x_3)(y_{12}+x_2+y_{23})(y_{23}+x_3)}$$



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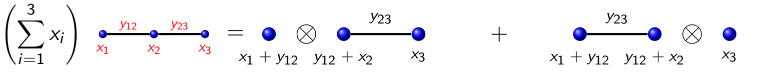
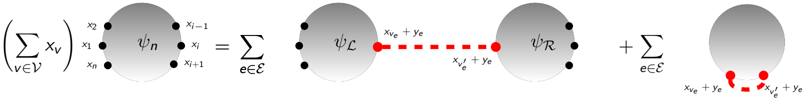


$$\frac{1}{x_1+y_{12}} \otimes \frac{1}{(y_{12}+x_2+x_3)(y_{12}+x_2+y_{23})(y_{23}+x_3)} + \frac{1}{(x_1+x_2+y_{12})(x_1+y_{12})(y_{12}+x_2+y_{23})} \otimes \frac{1}{(y_{23}+x_3)}$$



In this talk

The wavefunction of the universe from first principles:
Cosmological Polytopes

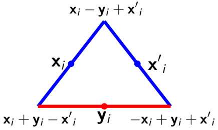
Form cosmology to flat-space physics and back

Taking averages:
Late-time correlators from the Cosmological Polytopes

Cosmology from the boundary:

The Wavefunction of the Universe

Cosmological Polytopes



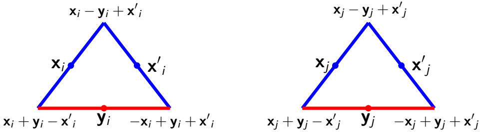
n_e triangles

$$\mathbb{P}^{3n_e - 1}$$

$$\mathcal{Y} = \sum_{v \in \mathcal{S}} x_v \mathbf{X}_v + \sum_{e \in \mathcal{S}} y_e \mathbf{Y}_e$$

$$\{\mathbf{x}_i - \mathbf{y}_i + \mathbf{x}'_i,\, \mathbf{x}_i + \mathbf{y}_i - \mathbf{x}'_i,\, -\mathbf{x}_i + \mathbf{y}_i + \mathbf{x}'_i\}_{i=1}^{n_e}$$

Cosmological Polytopes



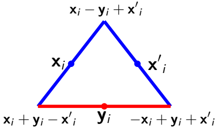
2 triangles

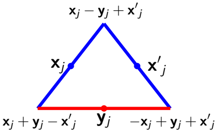
$$\mathbb{P}^5$$

$$\mathcal{Y} = \sum_{v \in \mathcal{S}} x_v \mathbf{X}_v + \sum_{e \in \mathcal{S}} y_e \mathbf{Y}_e$$

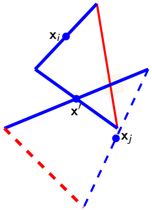
$$\{\mathbf{x}_i - \mathbf{y}_i + \mathbf{x}'_i,\, \mathbf{x}_i + \mathbf{y}_i - \mathbf{x}'_i,\, -\mathbf{x}_i + \mathbf{y}_i + \mathbf{x}'_i,\, \mathbf{x}_j - \mathbf{y}_j + \mathbf{x}'_j,\, \mathbf{x}_j + \mathbf{y}_j - \mathbf{x}'_j,\, -\mathbf{x}_j + \mathbf{y}_j + \mathbf{x}'_j\}$$

Cosmological Polytopes



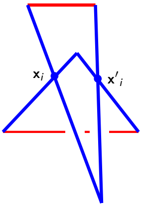


2 triangles



$\mathbf{x'}_i = \mathbf{x'}_j \equiv \mathbf{x}$

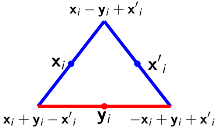
$\mathbb{P}^4$

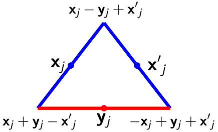


$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x'}_i = \mathbf{x'}_j$

$\mathbb{P}^3$

Cosmological Polytopes



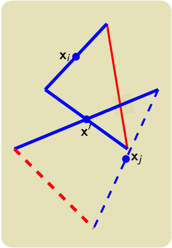


2 triangles

$$x'_i = x'_j \equiv x' \Rightarrow \mathbb{P}^4$$

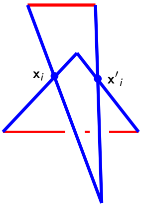
$$\mathcal{Y} = (x_1, y_1, x', x_2, y_2)$$

$$\{x_i - y_i + x',\ x_i + y_i - x',\ -x_i + y_i + x',\ x_j - y_j + x',\ x_j + y_j - x',\ -x_j + y_j + x'\}$$



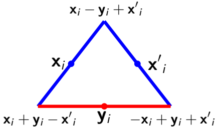
$x'_i = x'_j \equiv x$

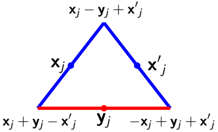
$\mathbb{P}^4$



$x_i = x_j, \quad x'_i = x'_j$

$\mathbb{P}^3$



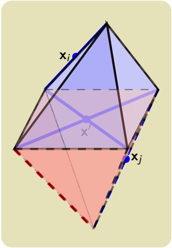


2 triangles

$$\mathbf{x}'_i = \mathbf{x}'_j \equiv \mathbf{x}' \Rightarrow \mathbb{P}^4$$

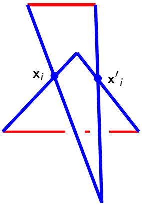
$$\mathcal{Y} = (x_1, y_1, x', x_2, y_2)$$

$$\{\mathbf{x}_i - \mathbf{y}_i + \mathbf{x}',\ \mathbf{x}_i + \mathbf{y}_i - \mathbf{x}',\ -\mathbf{x}_i + \mathbf{y}_i + \mathbf{x}',\ \mathbf{x}_j - \mathbf{y}_j + \mathbf{x}',\ \mathbf{x}_j + \mathbf{y}_j - \mathbf{x}',\ -\mathbf{x}_j + \mathbf{y}_j + \mathbf{x}'\}$$



$\mathbf{x}'_i \mathbf{x}_i = \mathbf{x}'_j \mathbf{x}_j = \mathbf{x}$

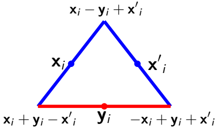
$\mathbb{P}^4$

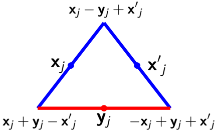


$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x}'_i = \mathbf{x}'_j$

$\mathbb{P}^3$

Cosmological Polytopes



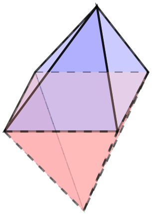


2 triangles

$$\left.\begin{aligned} x_i &= x_j \equiv x \\ x'_i &= x'_j \equiv x' \end{aligned}\right\} \Rightarrow \mathbb{P}^3$$

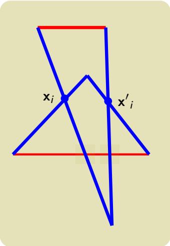
$$\mathcal{Y} = (x, y_1, x', y_2)$$

$$\{x - y_i + x',\ x + y_i - x',\ -x + y_i + x',\ x - y_j + x',\ x + y_j - x',\ -x + y_j + x'\}$$



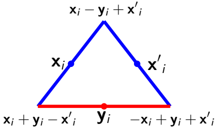
$x'_i = x'_j$

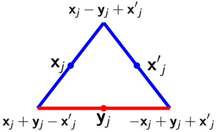
$\mathbb{P}^4$



$x_i = x_j, \quad x'_i = x'_j$

$\mathbb{P}^3$



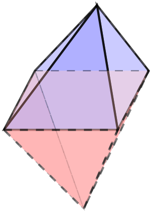


2 triangles

$$\left.\begin{array}{l} \mathbf{x}_i = \mathbf{x}_j \equiv \mathbf{x} \\ \mathbf{x'}_i = \mathbf{x'}_j \equiv \mathbf{x'} \end{array}\right\} \Rightarrow \mathbb{P}^3$$

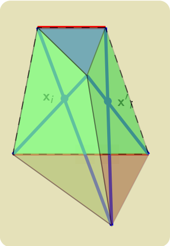
$$\mathcal{Y} = (x, y_1, x', y_2)$$

$$\{\mathbf{x} - \mathbf{y}_i + \mathbf{x'},\ \mathbf{x} + \mathbf{y}_i - \mathbf{x'},\ -\mathbf{x} + \mathbf{y}_i + \mathbf{x'},\ \mathbf{x} - \mathbf{y}_j + \mathbf{x'},\ \mathbf{x} + \mathbf{y}_j - \mathbf{x'},\ -\mathbf{x} + \mathbf{y}_j + \mathbf{x'}\}$$



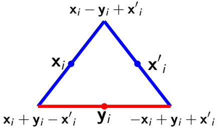
$\mathbf{x'}_i = \mathbf{x'}_j$

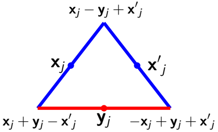
$\mathbb{P}^4$



$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x'}_i = \mathbf{x'}_j$

$\mathbb{P}^3$



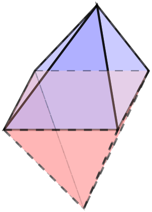


2 triangles

$\mathbb{P}^5$

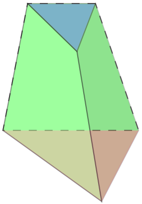
$$\mathcal{Y} = \sum_{v \in \mathcal{S}} x_v \mathbf{X}_v + \sum_{e \in \mathcal{S}} y_e \mathbf{Y}_e$$

$$\Omega = \int \prod_{k=1}^{\nu} \frac{dc_k}{c_k - i\varepsilon_k} \delta^{(N)} \left(\mathcal{Y} - \sum_{k=1}^{\nu} c_k \mathbf{V}^{(k)} \right)$$



$\mathbf{x'}_i = \mathbf{x'}_j$

$\mathbb{P}^4$



$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x'}_i = \mathbf{x'}_j$

$\mathbb{P}^3$

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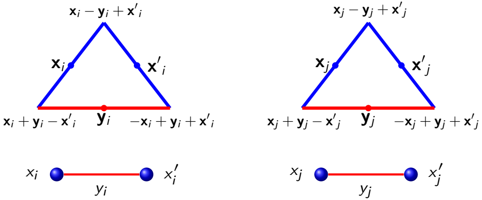
Cosmological Polytopes

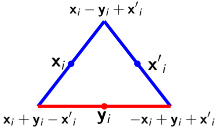


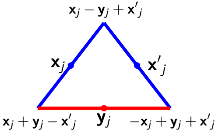
2 triangles

$$\mathbb{P}^5 \equiv \mathbb{P}^{n_v+n_e-1}$$

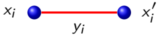
$$\mathcal{Y} = \sum_{v \in n_v} x_v \mathbf{X}_v + \sum_{e \in n_e} y_e \mathbf{Y}_e$$

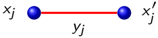
Cosmological Polytopes



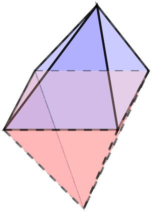


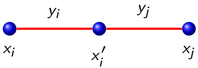
2 triangles





$$\mathcal{Y} = \sum_{v \in n_v} x_v \mathbf{X}_v + \sum_{e \in n_e} y_e \mathbf{Y}_e$$





$$\mathbb{P}^4 \equiv \mathbb{P}^{n_v + n_e - 1}$$

$$\mathbf{x}'_i = \mathbf{x}'_j$$

$\mathbb{P}^4$

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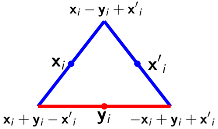
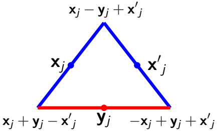
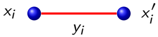
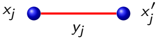
Cosmological Polytopes



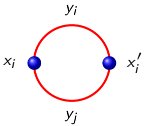


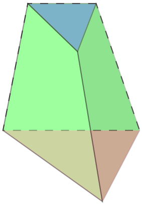
2 triangles





$$\mathcal{Y} = \sum_{v \in n_v} x_v \mathbf{X}_v + \sum_{e \in n_e} y_e \mathbf{Y}_e$$





$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x}'_i = \mathbf{x}'_j$

$$\mathbb{P}^3 \equiv \mathbb{P}^{n_v + n_e - 1}$$

$\mathbb{P}^3$

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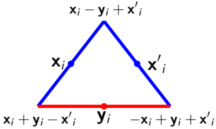
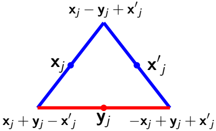
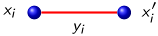
Cosmological Polytopes

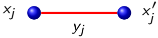




2 triangles

$$\mathbb{P}^{n_v + n_e - 1}$$





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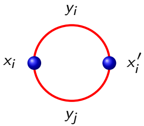
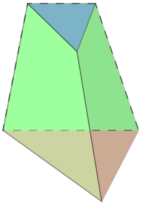




$$\Omega = \frac{\prod_{v,e} dx_v dy_e}{\text{Vol}\{GL(1)\}} \Psi_{\mathcal{G}}(x_v, y_e)$$

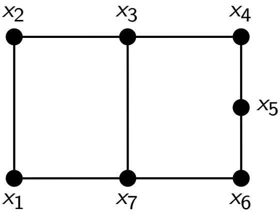
$$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x'}_i = \mathbf{x'}_j$$

$$\mathbb{P}^3$$

Cosmological Polytopes: The Face Structure

Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow$ 0

$\mathcal{W} \cdot \mathbf{V} = 0$



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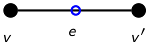
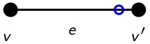
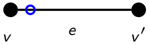


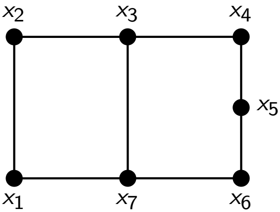
$$\mathcal{W} \cdot (\mathbf{x}_v + \mathbf{x}_{v'} - \mathbf{y}_e) = 0$$



$$\mathcal{W} \cdot (-\mathbf{x}_v + \mathbf{x}_{v'} + \mathbf{y}_e) = 0$$



$$\mathcal{W} \cdot (\mathbf{x}_v - \mathbf{x}_{v'} - \mathbf{y}_e) = 0$$



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Cosmological Polytopes: The Face Structure

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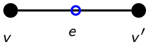
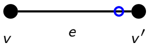
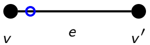


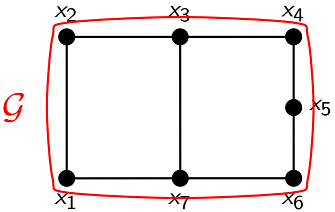
$$\mathcal{W}\cdot(\mathbf{x}_v+\mathbf{x}_{v'}-\mathbf{y}_e) = 0$$



$$\mathcal{W}\cdot(-\mathbf{x}_v+\mathbf{x}_{v'}+\mathbf{y}_e) = 0$$



$$\mathcal{W}\cdot(\mathbf{x}_v-\mathbf{x}_{v'}-\mathbf{y}_e) = 0$$



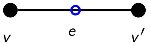
$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

Scattering Facet

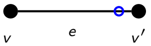
Cosmological Polytopes: The Face Structure

Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow 0$

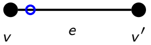
$$\mathcal{W} \cdot \mathbf{V} = 0$$



$\mathcal{W} \cdot (\mathbf{x}_v + \mathbf{x}_{v'} - \mathbf{y}_e) = 0$



$\mathcal{W} \cdot (-\mathbf{x}_v + \mathbf{x}_{v'} + \mathbf{y}_e) = 0$



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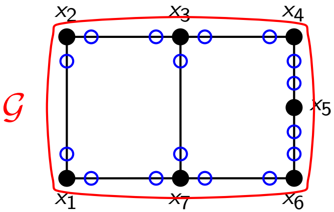

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

Scattering Facet

Vertices $(2n_e)$

$$\{\mathbf{x}_i - \mathbf{x}_j + \mathbf{y}_{ij},$$

$$-\mathbf{x}_i + \mathbf{x}_j + \mathbf{y}_{ij}\}$$

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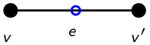
Cosmological Polytopes: The Face Structure

Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow 0$

$$\mathcal{W} \cdot \mathbf{V} = 0$$



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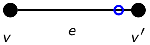
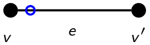
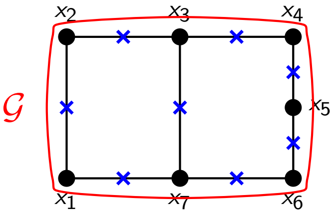


$\mathcal{W} \cdot (-\mathbf{x}_v + \mathbf{x}_{v'} + \mathbf{y}_e) = 0$



$\mathcal{W} \cdot (\mathbf{x}_v - \mathbf{x}_{v'} - \mathbf{y}_e) = 0$



$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

Scattering Facet

Vertices $(2n_e)$

$$\{\mathbf{x}_i - \mathbf{x}_j + \mathbf{y}_{ij},$$

$$-\mathbf{x}_i + \mathbf{x}_j + \mathbf{y}_{ij}\}$$

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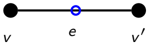
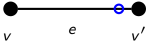
Cosmological Polytopes: The Face Structure

Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow$ 0

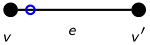
$$\mathcal{W}\cdot\mathbf{V} \;=\; 0$$



$$\mathcal{W}\cdot(\mathbf{x}_v+\mathbf{x}_{v'}-\mathbf{y}_e) \;=\; 0$$



$$\mathcal{W}\cdot(-\mathbf{x}_v+\mathbf{x}_{v'}+\mathbf{y}_e) \;=\; 0$$



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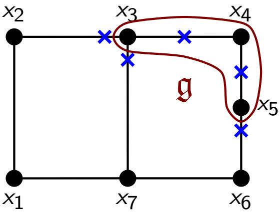


$$y_{23}+y_{37}+y_{56}+\sum_{j=3}^{5}x_j \;\longrightarrow\; 0$$

Cosmological Polytopes & Wavefunction: A dictionary

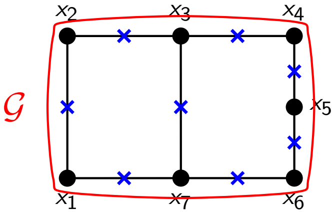
Cosmological Polytope $\mathcal{P}$	Universe Wavefunction Ψ
Canonical form	Ψ
Triangulations	Representations for Ψ
Boundaries	Residues of Ψ
Volume preserving transformations	Symmetries of Ψ

Cosmology from the boundary:

The Flat-Space Physics

Scattering Facet: Emergent Unitarity

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$



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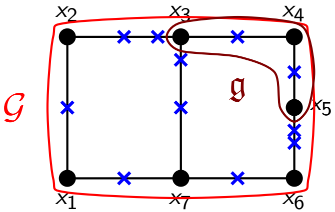
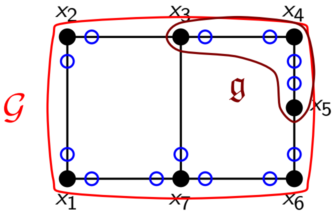


$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$

Scattering Facet: Emergent Unitarity

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$



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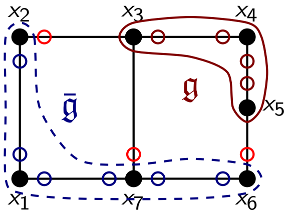

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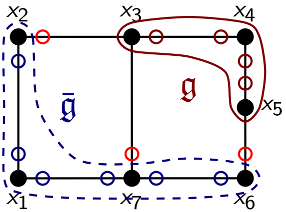


$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$

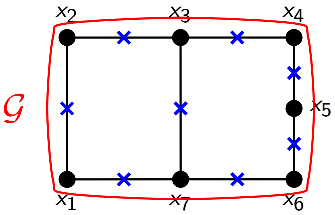
$\circ \implies$ *Energy flow!*

$$\Omega = \left(\prod_{e \in \mathcal{E}} \frac{1}{2y_e} \right) \mathcal{A}[\mathfrak{g}] \times \mathcal{A}[\bar{\mathfrak{g}}]$$

cutting rules!

Scattering Facet: Emergent Lorentz Invariance

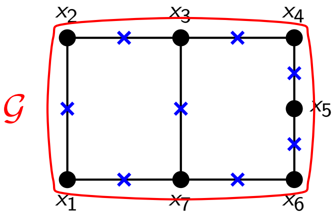
$$\sum_{j=1}^{7} x_j \longrightarrow 0$$



Scattering Facet: Emergent Lorentz Invariance

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

$$\Omega \sim \int_{\mathbb{R}^N} \prod_{j=1}^{\nu} \frac{dc_j}{c_j - i\varepsilon_j} \delta^{(N)}\left(\mathcal{Y} - \sum_{j=1}^{\nu} c_j \mathbf{V}^{(j)}\right)$$



$$\mathbb{P}^{N-1}, \qquad N \equiv n_e + n_v - 1$$

$$\nu \equiv n_e + n_v - 1 + L$$

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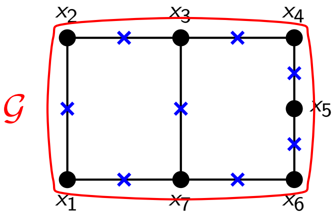

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$$\sim \int \prod_{j=1}^{L} \frac{dc_j}{\left(c_j - \frac{y_{e_j}}{2}\right)^2 - \left(\frac{y_{e_j}}{2} - i\varepsilon_j\right)^2} \times$$

$$\times \prod_{s=1}^{n_e - L} \frac{1}{\left(\sum_r \sigma_{r_s} c_r - \frac{\mathfrak{y}_s}{2}\right)^2 - \left(\frac{y_s}{2} - i\varepsilon_s\right)^2}$$

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$$\mathcal{I} \sim \int \prod_{j=1}^{L} d\overrightarrow{l}^{(j)} dl_0^{(j)} \frac{1}{(l_0^{(j)})^2 - (|\overrightarrow{l}^{(j)}| - i\varepsilon_j)^2} \prod_{s=1}^{n_e - L} \frac{1}{\left(\sum_r \sigma_{r_s} l_0^{(j)} - \mathfrak{p}_s\right)^2 - \left(|\overrightarrow{P}_s| - i\varepsilon_s\right)^2}$$

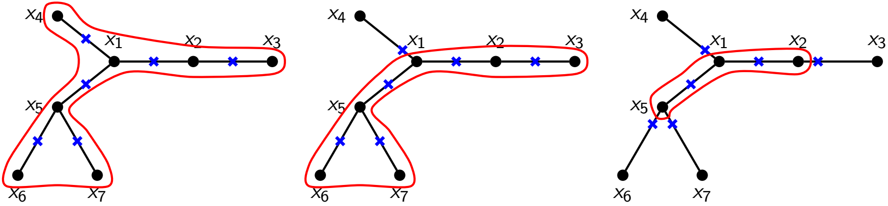
Lorentz invariant loop integrand!

$c_j \sim l_0,$

$\Omega - i\varepsilon \sim \text{Feynman} - i\varepsilon$

From the flat-space S-matrix to the wavefunction

Isomorphic facets | Simplices



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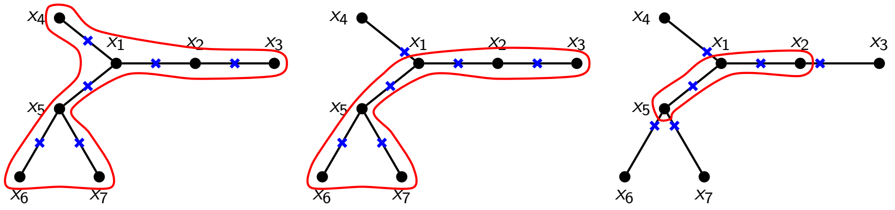


$$\Omega = (-1)^{\dim\{\mathcal{E}^{\text{ext}}\}} \mathcal{A}$$

Flat-space scattering amplitude from other facets!

Flat-space scattering amplitude as residues of several poles!

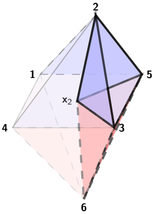
Combinatorial automorphisms map these facets into the scattering one

Symmetries in energy space $x_I \longleftrightarrow y_I$

From the flat-space S-matrix to the wavefunction

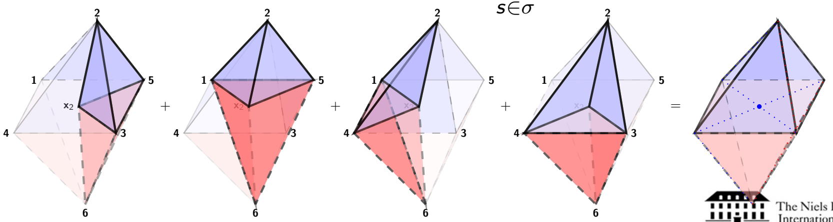
- Data: flat-space amplitude $\mathcal{A}_{\mathcal{G}}^{\text{tree}}$ + combinatorial automorphisms σ
- Representative of the wavefuntion:

$$\hat{\Psi}^{\text{tree}}_{\mathcal{A}_{\mathcal{G}}} = \frac{\mathcal{A}^{\text{tree}}}{\sum_k x_k}$$



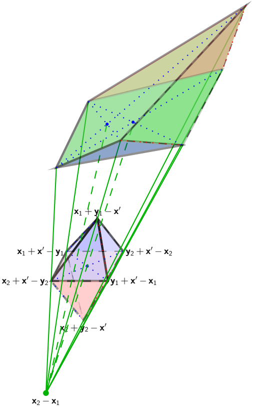
Scattering facet
+ extra vertex $\mathbf{x}_2$

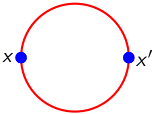
- Images of $\hat{\Psi}_{\mathcal{A}_{\mathcal{G}}}$ under σ. Then: $\Psi = \sum_{s \in \sigma} \mathfrak{Im}_{\sigma} \left\{ \hat{\Psi}_{\mathcal{A}_{\mathcal{G}}} \right\}$

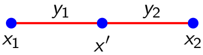


From the flat-space S-matrix to the wavefunction

- Loops from trees: Projection through the cone $\mathbf{x}_i - \mathbf{x}_j$







$\mathbf{x}_2$

Cosmology from the boundary:

The Late-Time Correlators

Taking averages

$$\Psi[\phi] \quad \Longrightarrow \quad \langle \prod_{i=1}^{n} \phi(p_i) \rangle = \mathcal{N} \int D\phi \prod_{i=1}^{n} \phi(p_i) \, |\Psi[\phi]|^2 \equiv \left(\prod_{k=1}^{n} \frac{1}{E_k} \right) \mathcal{C}_{\mathcal{G}}$$

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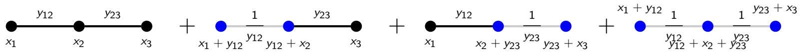

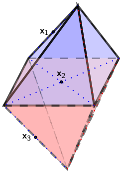

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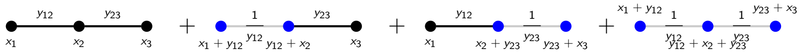
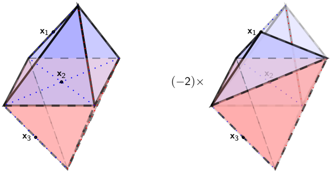
$\mathcal{C}_\mathcal{G} =$ 



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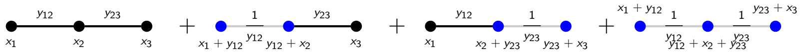

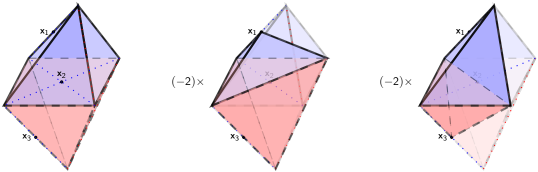

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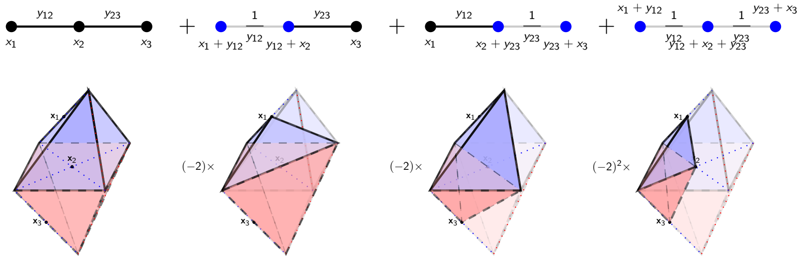
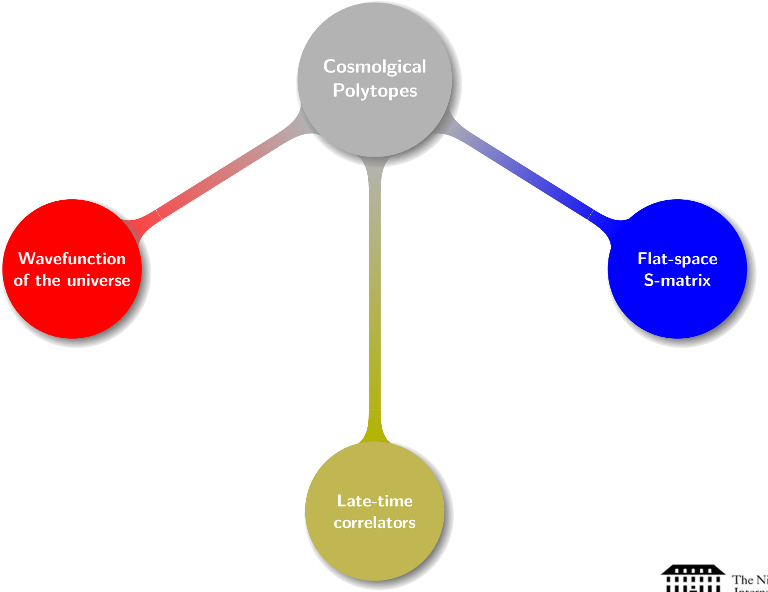


Conclusion

- Our knowledge of cosmological observables is quite primitive, both conceptually and computationally.
- Hints from amplitudes: Symmetries + Control on the analytic structure; New mathematical structures..
- Cosmological polytopes as a combinatorial def of Ψ
- Triangulations $\Longleftrightarrow$ representations of Ψ (Feynman, OFPT..)
- Face structure $\Longleftrightarrow$ Singularity structure.
- It contains the flat-space S-matrix, with unitarity manifesting in the vertex structure of its facets
- Rules for Ψ which *do not* refer to Lorentz invariance & unitarity but contains a Lorentz-invariant & unitary object.
- Rules for extracting the late-time correlators from $\mathcal{P}$

Conclusion



Cosmolgical Polytopes
Wavefunction of the universe
Flat-space S-matrix
Late-time correlators