

3d Abelian Gauge Theories at the Boundary

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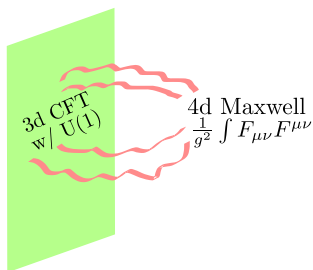
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Lorenzo Di Pietro, Davide Gaiotto, Jingxiang Wu

Overview

3d Abelian Gauge Theories (in their putative IR conformal phase)
as **conformal boundary conditions** for 4d Maxwell



- ▶ Define a **family of conformal boundary conditions** $B(\tau, \bar{\tau})$. Parametrized by the gauge coupling τ .
- ▶ **Decoupling limit**: free Maxwell field in the bulk + 3d CFT with $U(1)$ global symmetry at the boundary
- ▶ **EM duality**: different decoupling with new CFT with 3d gauge fields coupled to $U(1)$

Motivations

“Simple” example of BCFT. Tools from the CFT side (bootstrap) and tools from the gauge-theory side (action of the EM duality on the boundary theories).

CM application: similarity with EFT of graphene [Son], relations with FQHE [Son]

In conjunction w/ 3d dualities: additional computational tool for 3d CFTs alternative to ϵ expansion or large N_f

- ▶ CFT data of $O(2)$ model from perturbation theory around a free Dirac fermion
- ▶ New predictions for large N_f QED₃

Free conformal b.c. for Maxwell on $\mathbb{R}^3 \times \mathbb{R}_+$

Coordinates: $x = (x^a, y \geq 0)$, $F = dA$

$$S = -\frac{i}{8\pi} \int_{y \geq 0} d^4x [\tau(F^-)^2 - \bar{\tau}(F^+)^2], \quad \tau = \frac{\theta}{2\pi} + \frac{2\pi i}{g^2},$$

b.c. obtained from vanishing of the boundary term

$$\delta S_{\partial} \propto \int_{y=0} d^3x \delta A^a (\tau F_{ya}^- - \bar{\tau} F_{ya}^+) \big|_{y=0}.$$

In terms of the **boundary currents**

$2\pi i \hat{J}_a \equiv (\tau F_{ya}^- - \bar{\tau} F_{ya}^+) \big|_{y=0}, \quad 2\pi i \hat{I}_a \equiv (F_{ya}^- - F_{ya}^+) \big|_{y=0}.$

Conformal boundary conditions

$$\begin{array}{lll} \text{Dirichlet} & \hat{J}_a = \text{free} & \hat{I}_a = 0, \\ \text{Neumann} & \hat{J}_a = 0 & \hat{I}_a = \text{free}, \end{array}$$

The BCFT is just a trivial MFT for the non-zero boundary current.

EM duality action on free b.c

S-duality exchanges Dirichlet and Neumann b.c.

More generally: $SL(2, \mathbb{Z})$ duality group $\tau \rightarrow \frac{a\tau+b}{c\tau+d}$, $a, b, c, d \in \mathbb{Z}$
s.t. $ad - bc = 1$ induces action on b.c, since

$$\begin{pmatrix} \hat{J} \\ \hat{I} \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \hat{J} \\ \hat{I} \end{pmatrix}$$

most general free conformal b.c: “ (p, q) ”

$$p\hat{J}_a + q\hat{I}_a = 0, \quad p, q \in \mathbb{Z}$$

The BCFT is just a trivial MFT for the non-zero boundary current.

Requires additional topological dof on the boundary

[\[Witten\]](#)[\[Witten-Gaiotto\]](#)[\[Kapustin-Tikhonov\]](#)

Interacting b.c. for Maxwell on $\mathbb{R}^3 \times \mathbb{R}_+$

Start with a 3d CFT with a $U(1)$ global symmetry at the boundary and 4d Maxwell with Neumann b.c.

Around $\tau = \infty$ couple them by “weakly” gauge $U(1)$ with the boundary value A_a of the 4d gauge field

$$\int_{y=0} d^3x \hat{J}_{\text{CFT}}^a A_a + \text{seagulls} .$$

Due to edge modes: “Modified Neumann b.c”

$$\hat{J}^a = \hat{J}_{\text{CFT}}^a$$

Defines interacting set of correlators at the boundary: 2pt functions of $\hat{J}^a, \hat{I}^a$ are non-trivial functions of $\tau, \bar{\tau}$.

At $\tau = \infty$ $\hat{I}$ decouples $\langle \hat{I} \hat{J} \rangle = 0$ and we recover the original 3d CFT + MFT of $\hat{I}$

A family of BCFTs

Conformal b.c.: make sure this coupling preserves the boundary conformal symmetry:

- ▶ Gauge coupling τ is the coefficient of a bulk operator. Boundary interactions cannot renormalise it (Locality)

As interactions localized on the boundary, τ is **exactly marginal**. Two loop check [Teber] and more general argument based on Ward identities [Herzog-Huang][Dudal-Mizher-Pais]

- ▶ Boundary theory not always critical. Assuming we can set the boundary couplings to their critical values^(*)

⇒ **Continuous family of BCFTs $B(\tau, \bar{\tau})$**

(*) Possible obstructions as we move in τ plane: emergence of a condensate, boundary operator crossing marginality,...

“Integrate” out the bulk: $B(\tau, \bar{\tau})$ is a conformal manifold of 3d CFTs, generically without a stress tensor

Decoupling Limits

Approach a “local” 3d CFT when we decouple the bulk from the boundary

- ▶ The “original” decoupling:

$$\tau \rightarrow \infty, \quad B(\tau, \bar{\tau}) \rightarrow \hat{I} \text{ MFT} + \underbrace{\text{3d CFT}}_{T_{0,1}}$$

- ▶ Cusp on the real axis: $\tau \rightarrow -\frac{q}{p}$, $q, p \in \mathbb{Z}$ (this is $\tau' \rightarrow \infty$)

$$B(\tau, \bar{\tau}) \rightarrow (p\hat{J} + q\hat{I}) \text{ MFT} + \underbrace{\text{3d CFT}'}_{T_{p,q}}$$

In general $T_{p,q}$ can be obtained from $T_{0,1}$ by $SL(2, \mathbb{Z})$ action on 3d CFTs with a $U(1)$ symmetry [Kapustin-Strassler][Witten]

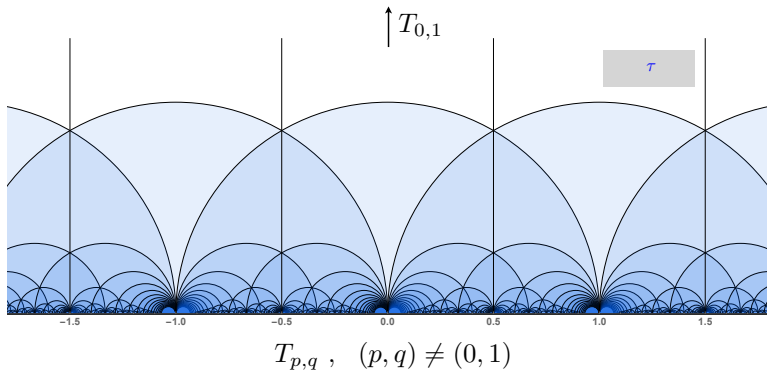
$$S : \tau \rightarrow -\frac{1}{\tau} \quad \text{3d gauging } U(1) \text{ \& adding } U(1)'$$

$$T : \tau \rightarrow \tau + 1 \quad \text{CS contact term for the global } U(1)$$

$\rightsquigarrow T_{p,q}$ is a 3d Abelian gauge theory with a $U(1)$ global symmetry

A Cartoon

In the absence of phase transitions, this conformal manifold of BCFTs interpolates smoothly between data of infinitely many 3d gauge theories $T_{p,q}$ with a $U(1)$ global symmetry.



Application

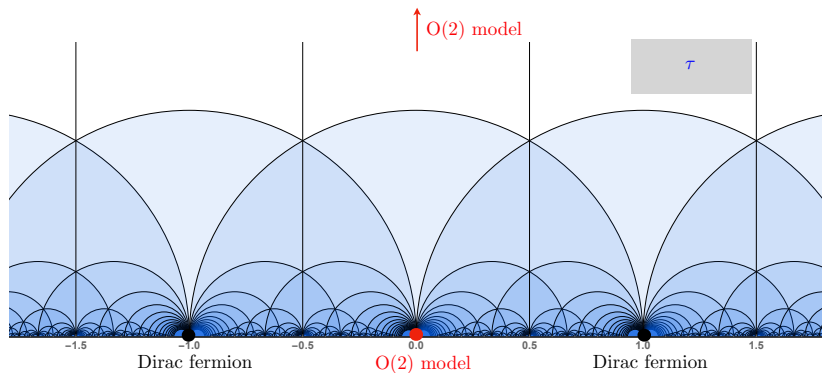
Suppose that $T_{0,1}$ is known. We can compute data in conformal perturbation theory around $\tau = \infty$ and extrapolate to the gauge-theory cusp to compute data of $T_{p,q}$.

Best case scenario: $T_{p,q}$ might be known or (conjectured to be) equivalent to $T_{0,1}$. Then we can perform perturbation theory around the two cusps and extrapolate to other cusps using a **duality improved resummation**.

This best case scenario is realized thanks to web of 3d-3d dualities, e.g. Particle-Vortex duality and its fermionic version

The Best Case scenario

$$U(1) + \phi \longleftrightarrow \phi$$
$$U(1)_{\pm 1} + \phi \longleftrightarrow \psi$$



Web of 3d dualities from different decoupling limits on $B(\tau, \bar{\tau})$

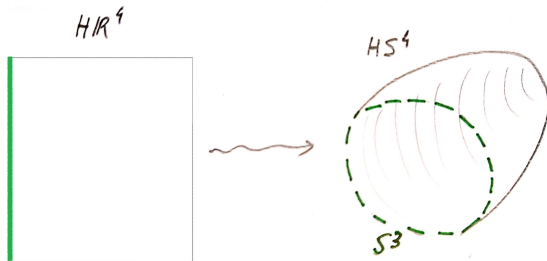
[Wang-Senthil][Seiberg-Senthil-Wang-Witten][Metlitski-Vishwanath][Hsiao-Son]

Observables

- ▶ Scaling Dimensions / OPE coefficients of local operators on the boundary
- ▶ Current Central Charges [[Hsiao-Son](#)][[Teber-Kotikov](#)]
- ▶ Boundary Anomalies [[Herzog-Huang](#)][[Herzog-Huang-Jensen](#)]
- ▶ **Boundary Free Energy** (next slide)
- ▶ Endpoints of Bulk Wilson Lines

Boundary Free Energy

$$F_{\partial} = -\frac{1}{2} \log \frac{|Z_{HS^4}|^2}{Z_{S^4}} = -\text{Re} \log Z_{HS^4} + \frac{1}{2} \log Z_{S^4} \quad [\text{Gaiotto}]$$



Similar to $F_{S^3}^{\text{CFT}}$ (conjecturally) monotonic under boundary RG flow. In our set-up we find

$$\boxed{\frac{\partial F_{\partial}}{\partial \text{Im} \tau} = \frac{\pi}{6} a_{F^2}(\tau, \bar{\tau}) \quad \frac{\partial F_{\partial}}{\partial \text{Re} \tau} = \frac{\pi}{6} i a_{F\tilde{F}}(\tau, \bar{\tau})}$$

Since in a BCFT $\langle \mathcal{O}(\vec{x}, y) \rangle = a_{\mathcal{O}} y^{-\Delta_{\mathcal{O}}}$

$a_{F^2}, a_{F\tilde{F}}$ completely determined by boundary currents 2pt functions via a boundary bootstrap [Liendo-Rastelli-van Rees] reasoning

$$F_{\mu\nu} \quad F_{\rho\sigma} \quad \begin{array}{c} \diagup \\ \diagdown \end{array} \quad \begin{array}{c} 1, F^2, F\tilde{F} \\ \text{wavy line} \end{array} \quad \text{green line} = \begin{array}{c} F_{\mu\nu} \\ \text{line} \end{array} \quad \begin{array}{c} \hat{J}, \hat{I} \\ \text{line} \end{array} \quad \text{green line} + \begin{array}{c} F_{\rho\sigma} \\ \text{line} \end{array} \quad \begin{array}{c} \hat{J}, \hat{I} \\ \text{line} \end{array} \quad \text{green line}$$

Bulk: only $\mathbb{I}, F^2, F\tilde{F}$ with known coefficients (free)

Boundary: two conserved currents $\hat{J}, \hat{I}$ ($SO(4,1)$, eom, Bianchi)

$$a_{F^2} = 3\left(\pi^2 c_{\hat{J}\hat{I}} - \frac{1}{2\pi \text{Im}\tau}\right), \quad a_{F\tilde{F}} = i\frac{3\pi^2}{\text{Im}\tau}(c_{\hat{J}\hat{I}} - \text{Re}\tau c_{\hat{J}\hat{I}}),$$

F_{∂} fixed by current 2pt functions and an initial condition

Boundary free energy and EM duality

The decoupling limit fixes the initial condition

$$F_{\partial} \underset{\tau \rightarrow \infty}{\sim} -\frac{1}{4} \log \underbrace{\left[\frac{2 \operatorname{Im} \tau}{|\tau|^2} \right]}_{\text{4d free vector}} + \underbrace{F_{0,1}^{\text{CFT}}}_{S^3 \text{ part funct}} + \mathcal{O}(|\tau|^{-1}) .$$

Around the gauged cusp $\tau = -q/p$, $\tau' = \frac{p'\tau + q'}{p\tau + q}$

$$F_{\partial} \underset{\tau' \rightarrow \infty}{\sim} -\frac{1}{4} \log \left[\frac{2 \operatorname{Im} \tau'}{|\tau'|^2} \right] + F_{p,q}^{\text{CFT}} + \mathcal{O}(|\tau'|^{-1}) .$$

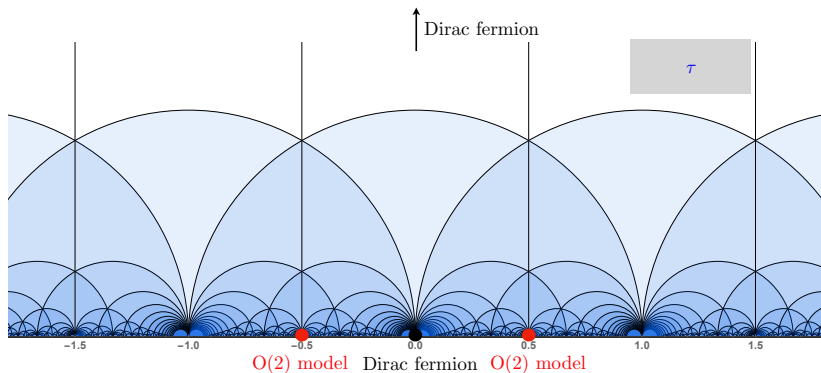
Same singularity in terms of τ' , but **different finite piece**

Compute F_{∂} perturbatively, extrapolate to the gauged cusp,
subtract the free-vector contribution in the new cusp

Application: $O(2)$ model and large N_f QED_3 from free fermions

Computations around the Dirac cusp

From free Dirac to the $O(2)$ model



Shift in τ s.t. the Dirac fermion is T-invariant

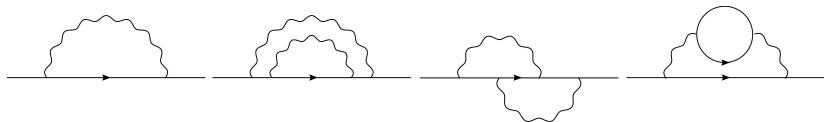
Perturbation theory around $\tau = \infty$ and $\tau = 0$ and extrapolation to $\tau = 1/2$ to get observables of the $O(2)$ model

Anomalous dimension of $\bar{\psi}\psi$

$\bar{\psi}\psi$ is a good operator around the Dirac cusp. It will be related to the energy operator at the of the $O(2)$ model

Dimreg ($d = 3 - 2\epsilon$ with fixed codimension). Non-local photon propagator between two points on the boundary

$$\langle A_a(\vec{p}, 0) A_b(-\vec{p}, 0) \rangle = 2\pi \frac{\text{Im}\tau}{|\tau|^2} \left[\frac{\delta_{ab}}{|\vec{p}|} + \frac{\text{Re}\tau}{\text{Im}\tau} \epsilon_{abc} \frac{p^c}{\vec{p}^2} \right] .$$

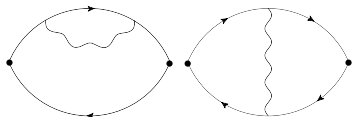


$$\gamma_{\bar{\psi}\psi} = -\frac{8}{3\pi} \frac{\text{Im}\tau}{|\tau|^2} + \frac{36\pi^2 - 32(\text{Im}\tau)^2}{27\pi^2} \frac{1}{|\tau|^4} - \frac{8(\text{Re}\tau)^2}{3|\tau|^4} + \mathcal{O}(|\tau|^{-3})$$

Boundary Free Energy

F_{∂} is completely determined in terms of the current central charges $c_{\hat{\eta}\hat{\eta}}, c_{\hat{\eta}\hat{\jmath}}, c_{\hat{\jmath}\hat{\jmath}}$

The relevant diagrams for NLO were computed by [\[Klebanov-Giombi-Tarnopolsky\]](#), e.g.



$$c_{\hat{\jmath}\hat{\jmath}} = \frac{1}{8\pi^2} + \frac{368 - 45\pi^2}{576\pi^3} \frac{\text{Im}\tau}{|\tau|^2} + \mathcal{O}(|\tau|^{-2}) ,$$

$$F_{\partial} = -\frac{1}{4} \log \left[\frac{2 \text{Im}\tau}{|\tau|^2} \right] + F_{\text{Dirac}} + \frac{\pi}{16} \frac{\text{Im}\tau}{|\tau|^2} \\ + \frac{(368 - 45\pi^2)(\text{Im}\tau)^2 + (144 + 45\pi^2)(\text{Re}\tau)^2}{2304|\tau|^4} + \mathcal{O}(|\tau|^{-3})$$

Extrapolation to $O(2)$

Duality-improved Padé approximant at 2 loops [Beem-Rastelli-Sen-van Rees] manifestly invariant under $\tau \leftrightarrow S\tau$

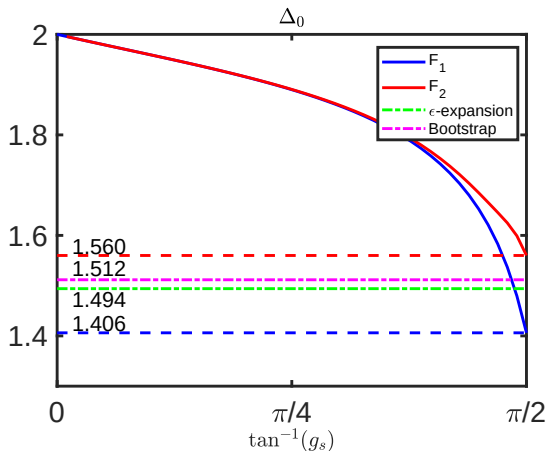
$$F_1(g_s, \theta) = \frac{h_1}{g_s^{-1} + (S \cdot g_s)^{-1} - h_2} ,$$

$$F_2(g_s, \theta) = \frac{h_3 \left(g_s^{-1/2} + (S \cdot g_s)^{-1/2} \right)}{g_s^{-3/2} + (S \cdot g_s)^{-3/2} + h_4 \left(g_s^{-1/2} + (S \cdot g_s)^{-1/2} \right)} .$$

with $g_s = g^2$ and

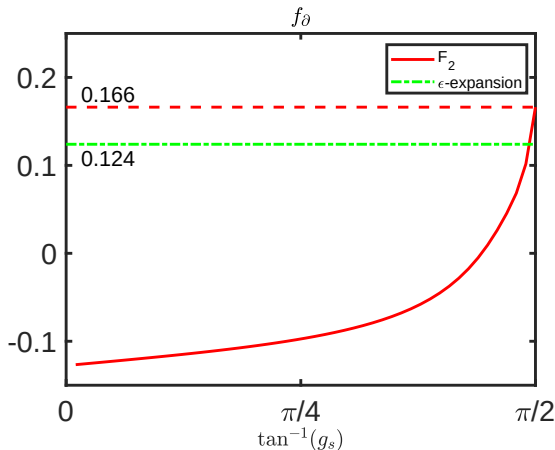
$$S \cdot g_s = \frac{g_s^2 \theta^2 + 16\pi^4}{\pi^2 g_s} .$$

Energy operator of the $O(2)$ model



Comparison with ϵ -expansion and bootstrap predictions (at $\tan^{-1}(g_s) = \pi/2$) [Kos,Poland,Simmons-Duffin, Vichi][Kleinert,Neu,Schulte-Frohlinde,Chetyrkin,Larin]

S^3 Free energy in $O(2)$



Comparison with $4 - \epsilon$ -expansion $\mathcal{O}(\epsilon^5)$ predictions (at $\tan^{-1}(g_s) = \pi/2$)[\[Fei-Giombi-Klebanov-Tarnopolsky\]](#)

Large N_f QED₃

$2N_f$ free Dirac fermions (with same charge) at the boundary and take large N_f with $\lambda = g^2 N_f$ fixed.

Compute F_∂ exactly in the 't Hooft coupling λ

By Witten's $SL(2, \mathbb{Z})$ action, $\lambda \rightarrow \infty$ should correspond to large N_f QED₃.

We recover [\[Klebanov-Pufu-Sachdev-Safdi\]](#) and predict $O(N_f^{-1})$

$$F_{\text{QED}_3} = 2N_f F_{\text{Dirac}} + \frac{1}{2} \log \left(\frac{\pi N_f}{4} \right) + \frac{92 - 9\pi^2}{18\pi^2} \frac{1}{N_f} + \mathcal{O}(N_f^{-2}) .$$

Non-perturbative test of this construction

Conclusions and Directions

- ▶ Considered a continuous family of interacting BCFTs for the free 4d $U(1)$ gauge theory
- ▶ In the absence of phase transition we can approach the data of an infinite family of 3d abelian gauge theories
- ▶ 3d dualities + perturbation theory + improved Padé resummation $\rightarrow$ new computational tool

Some directions

- ▶ More observables of the $O(2)$ model/higher loops
- ▶ Space of b.c. for Maxwell theory? (WIP). Boundary operators crossing marginality?
- ▶ Line operators in the bulk

Thanks!