A connection between SYK physics and Glassy Dynamics

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SIMONS FOUNDATION

A connection between glassy dynamics and SYK physics

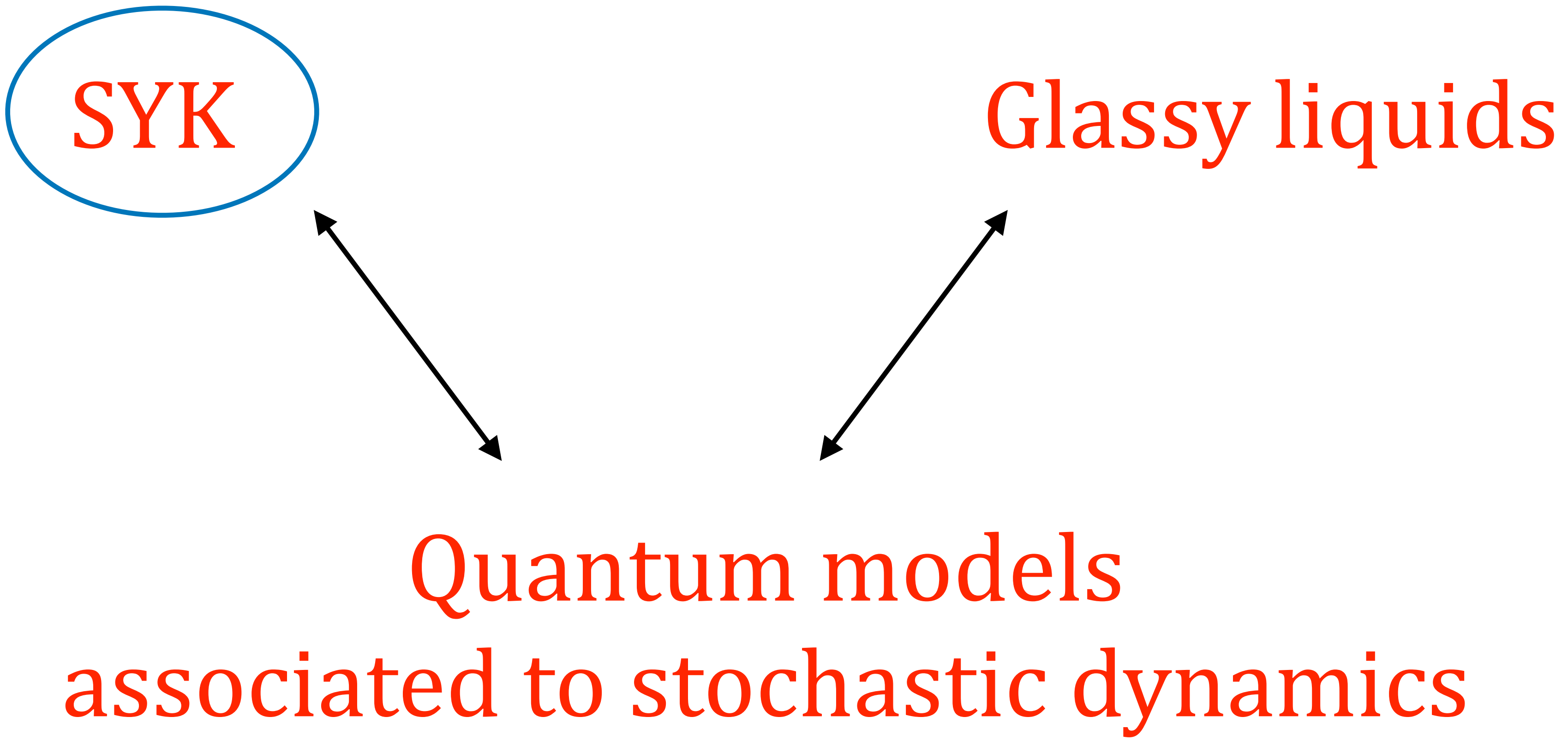
SYK ↔ Quantum models associated to stochastic dynamics ↔ Glassy liquids

SYK

Glassy liquids

Quantum models
associated to stochastic dynamics

A connection between glassy dynamics and SYK physics



SYK Model

Sachdev-Ye ’93
Parcollet-Sachdev-Georges ’01
Kitaev ‘15

$$H = \sum_{1 \leq i < j < k < l \leq N} J_{ijkl} \, \chi_i \chi_j \chi_k \chi_l$$

Majorana fermions $\{\chi_i, \chi_j\} = \delta_{ij}$

Quenched disorder

$$P(J_{ijkl}) \sim \exp\left(-N^3 J_{ijkl}^2 / 12 J^2\right)$$

Studied in several contexts

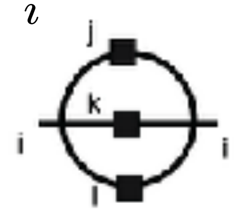
- Related to models of extremal black holes
- Chaos bound: Maximally chaotic
- Non-Fermi liquid behavior

SYK main properties

Kitaev '15
Maldacena, Stanford '16
Polchinski, Rosenhaus '16

- Exactly solvable $G(\tau, 0) = \frac{1}{N} \sum_i \langle T \chi_i(\tau) \chi_i(0) \rangle$

$$\frac{1}{G(\omega)} = -i\omega - \Sigma(\omega) \;, \quad \Sigma(t_1, t_2) =$$



$$\frac{\partial G(t_1 - t_2)}{\partial t_1} + J \int_0^\beta dt\; G(t_1, t)\; G(t, t_2)^3 = -\delta(t_1 - t_2)$$

- Thermodynamics

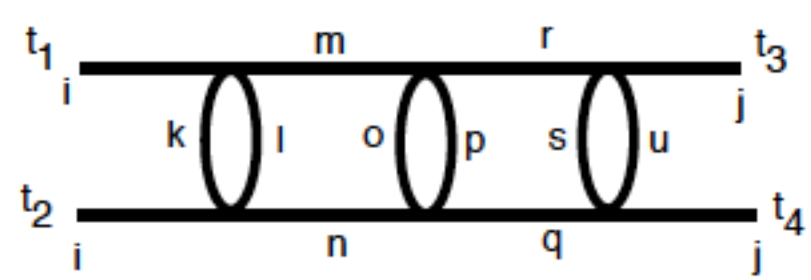
  No ordering: Liquid
  Finite entropy at zero temperature

- Critical at zero temperature

  Gapless & power law behavior in time and in frequency
  Time-scale set by 1/T

- Low energy physics dominated by quasi time-reparametrization invariance

$$G(\tau, \tau') \to [f'(\tau) f'(\tau')]^{1/4} G(f(\tau), f(\tau'))$$



A connection between glassy dynamics and SYK physics

SYK

Glassy liquids

Quantum models
associated to stochastic dynamics

Exactly solvable models of classical glassy liquids

A key example
The p-spin spherical model
(p=4)

$$V = -\sum_{1\le i<j<k<l\le N} J_{ijkl} S_i S_j S_k S_l$$

$$\sum_{i_1=1}^{N} S_{i_1}^2 = N$$

$$i_1 = 1, \cdots, N$$

- Very Rough Energy Landscape

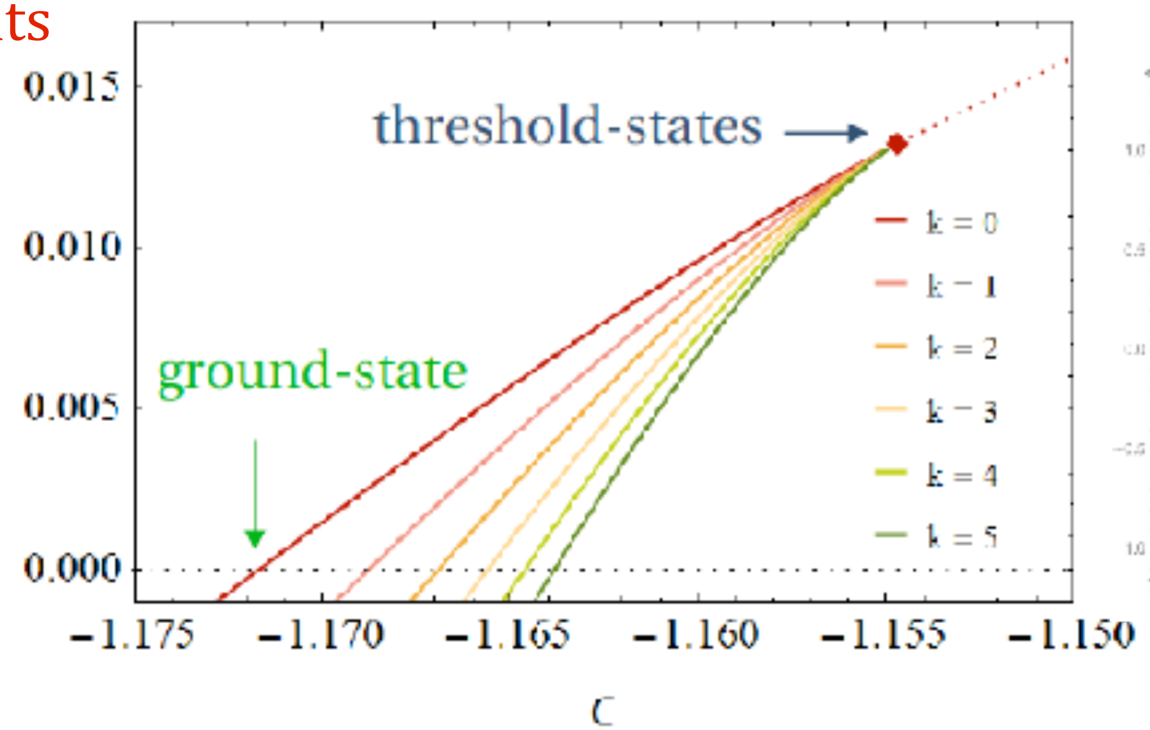
Exponential number of critical points

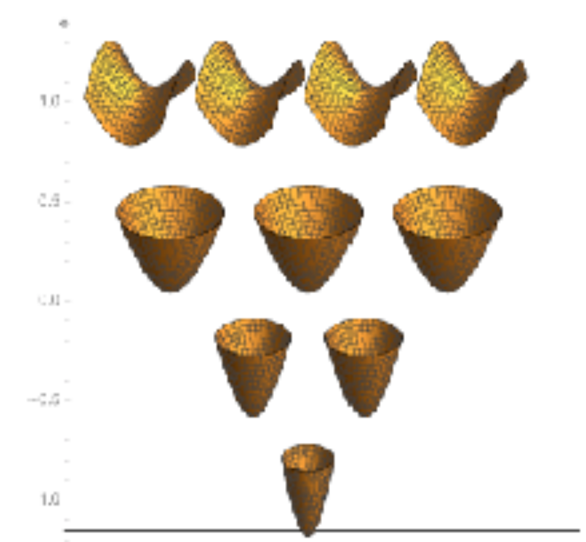
$$\mathcal{N}(e) \sim e^{N\Sigma_k(e)}$$

$\Sigma_k(e)$ Complexity of critical points of index k

Cavagna, Giardina, Parisi (1998)
Auffinger, Ben Arous, Cerny (2013)

$$\Sigma_k(e) = \frac{1}{N} \ln \mathcal{N}(e)$$





Exactly solvable models of classical glassy liquids

- Their stochastic dynamics is exactly solvable by close eqs. on the two point function

$$\dot{S}(t) = -\nabla_S V + \eta(t) \qquad \langle \eta_\alpha(t)\eta_\beta(t')\rangle = 2T\delta(t-t')\delta_{\alpha,\beta}$$

$$C(t,t') = \frac{1}{N}\sum_i \langle S_i(t)S_i(t')\rangle$$

Equilibrium dynamics $\dot{C}(\tau) = -TC(\tau) - \frac{p}{2T}\int_0^\tau du\, C^{p-1}(\tau - u)\dot{C}(u)$

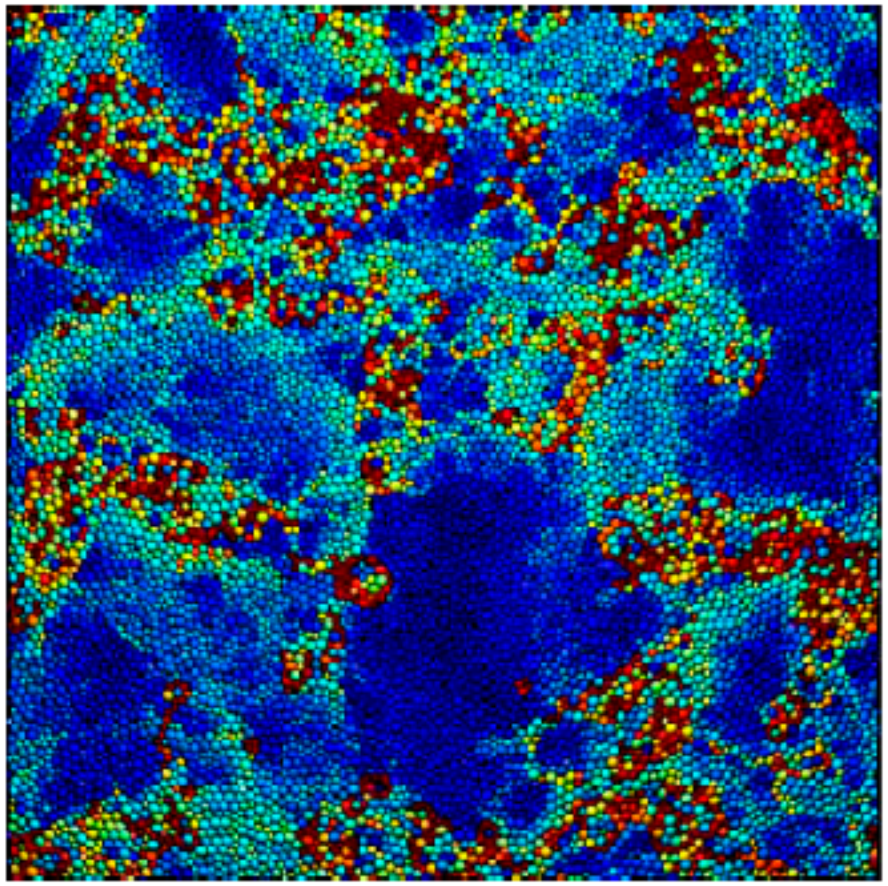
- At Td dynamical glass transition; quenches below Td aging

Gapless relaxation & power law behavior in time

See e.g. Cugliandolo Les Houches

Criticality of glassy dynamics

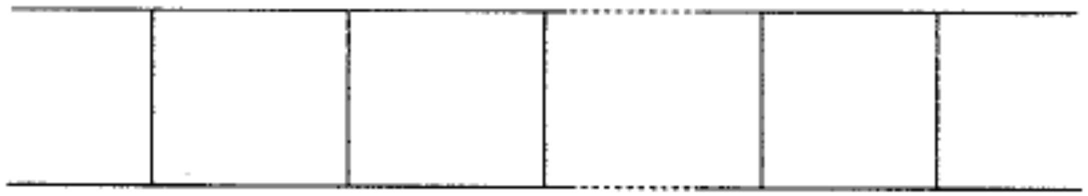
- Dynamical heterogeneity



The critical fluctuations of glassy dynamics

Biroli, Berthier, Bouchaud, Cipelletti, Van Saarloos
Oxford Univ Press 2011

Encoded in ladder diagrams: dominant contribution to 4-point functions



Biroli, Bouchaud '04

Related to quasi time-reparametrization invariance

Chamon, Cugliandolo et al '07

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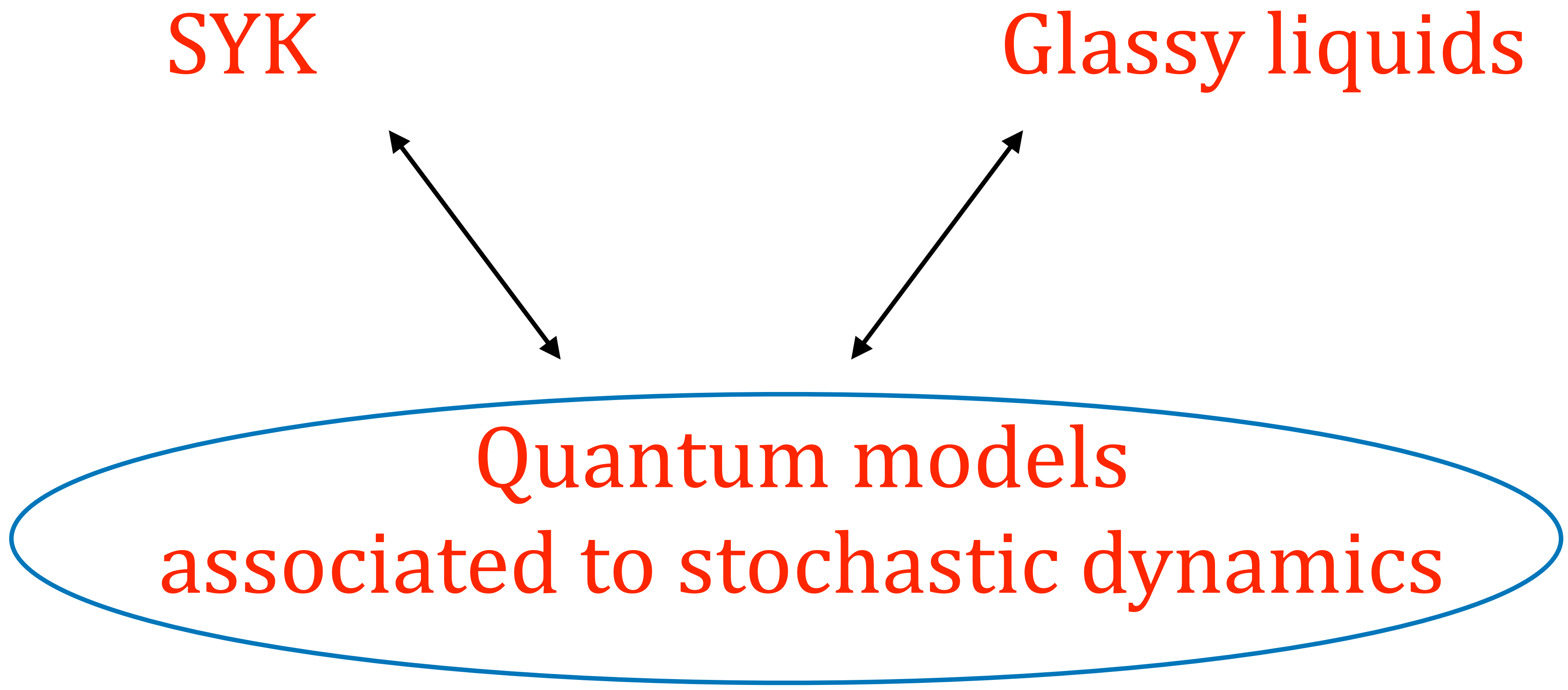

Mapping between stochastic and quantum dynamics

HE: stochastic quantisation
CMT: Rokhsar-Kivelson
SP: quantum-stochastic mapping

Langevin equation $$\dot{q}_i(t) = -\frac{\partial V}{\partial q_i} + \eta_i(t)$$

$$\partial_t P_t(\mathbf{q}) = \sum_i \frac{\partial}{\partial q_i}\left[T_s \frac{\partial}{\partial q_i} + \frac{\partial V}{\partial q_i}\right] P_t(\mathbf{q}) \equiv -H_{\mathrm{FP}} P$$

Quantum Hamiltonian

$$H = \frac{T_s}{2} e^{V/2T_s} H_{\mathrm{FP}} e^{-V/2T_s} = \sum_i \left[-\frac{T_s^2}{2}\frac{\partial^2}{\partial q_i^2} + \underbrace{\frac{1}{8}\left(\frac{\partial V}{\partial q_i}\right)^2 - \frac{T_s}{4}\frac{\partial^2 V}{\partial q_i^2}}_{V_{eff}(\mathbf{q})}\right]$$

- Zero ground-state energy & ground state -> Boltzmann-Gibbs distribution

$$\psi_{GS} = \frac{e^{-\frac{V}{2T_s}}}{\sqrt{Z}}$$

Mapping between stochastic and quantum dynamics

•Mapping between ground state quantum dynamics and stochastic dynamics

$$\frac{1}{N}\sum_i \langle S_i(\tau)S_i(0)\rangle_{clas} = \frac{1}{N}\sum_i \langle S_i(\tau)S_i(0)\rangle_{Qua} = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega)e^{-\omega\tau}$$

$$\frac{1}{2N}\sum_i \langle\{S_i(t),S_i(0)\}\rangle_{Qua,R} = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega)\cos\omega t$$

Henley '04
Biroli, Chamon, Zamponi '08

criticality in stochastic dynamics <-> criticality in quantum dynamics

•Eigenvectors with eigenvalues much smaller than T_q -> metastable states with lifetime much larger than β_q

Gaveau-Schulman '00
Bovier '00
Biroli, Kurchan '01

The quantum model

P-spin spherical model $$V = -\sum_{1\le i<j<k<l\le N} J_{ijkl} S_i S_j S_k S_l$$

$$\sum_{i_1=1}^{N} S_{i_1}^2 = N$$

$$i_1 = 1, \cdots, N$$

Stochastic-Quantum Mapping

$$H = \sum_i -\frac{T_s^2}{2} \frac{\partial^2}{\partial S_i^2} + V_{eff}$$

$$V_{\text{eff}} = \sum_i \left[\frac{1}{8} \left(\frac{\partial V}{\partial S_i} \right)^2 - \frac{T_s}{4} \frac{\partial^2 V}{\partial S_i^2} \right]$$

Quantum model from a parent stochastic glassy dynamics

Exactly solvable by close eqs. on the two point functions

Results

Quantum Model

Parent stochastic glassy dynamics

• Thermodynamics

Finite entropy at zero temperature ← Exponential number of metastable states with exponentially diverging life-time

$$\lim_{\beta_q \to \infty} \lim_{N \to \infty} \ln Tr e^{-\beta_q H} = S = \ln \mathcal{N} = N s_{meta}$$

No ordering: Quantum Liquid ← Metastable states with finite life-time

Results

Quantum Model

Parent stochastic glassy dynamics

- Criticality at zero temperature

  Gapless & power law behavior in time
  cut-off by temperature

← Dynamic criticality of glassy dynamics
Wide distribution of relaxation times

$$\rho_q(\omega) = \rho_c(1/\tau)$$

Results

Quantum Model

Parent stochastic glassy dynamics

•Low energy physics dominated by quasi time-reparametrization invariance ← Quasi time-reparametrization invariance of glassy dynamics

$$t \to h(t)$$

Results

Quantum Model	Parent stochastic glassy dynamics
•Thermodynamics No ordering: Quantum Liquid Finite entropy at zero temperature ←	Exponential number of metastable states with exponentially diverging life-time
•Critical at zero temperature Gapless & power law behavior in time cut-off by temperature ←	Dynamic criticality of glassy dynamics Wide distribution of relaxation times
•Low energy physics dominated by quasi time-reparametrization invariance ←	Quasi time-reparametrization invariance of glassy dynamics

<u>Differences</u>: purely bosonic model, critical exponents, time-reparametrization transformations of correlation functions

Conclusion

Quantum models from parent stochastic glassy dynamics have SYK physics

Realised in many models: p-spin spherical model and many others (quantum spins)

Glassy models without disorder

Witten's '16 model for SYK without disorder

Transition from glassy to simple dynamics

Banerjee, Altman '17 model for transition to non-SYK physics

Open questions

Differences with SYK

Quantum chaotic properties

Effective theory of glassy dynamics & the Schwarzian action for the soft-modes