KU LEUVEN

Bulk Reconstruction Beyond the Entanglement Wedge

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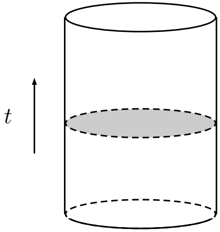
Outline

1. Introduction: Subregion Duality in AdS/CFT
2. Minimal Boundary-Anchored Surfaces
3. Bulk Metric Reconstruction
4. Discussion: Implications for Subregion Duality
5. Summary

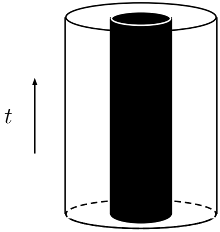
The AdS/CFT correspondence

(certain) states in *(certain)* conformal field theories *(in the right limit)*	$\leftrightarrow$	asymptotically locally Anti de Sitter geometry *(alAdS)*

Examples:



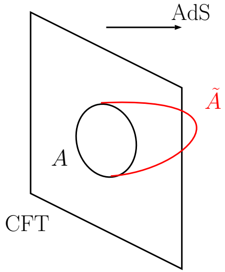
$$|0\rangle$$



$$\rho = \frac{1}{Z}\sum_n e^{-\beta E_n}|n\rangle\langle n|$$

The Ryu-Takayanagi formula

$$S(\rho_A) = \frac{\text{area}(\tilde{A})}{4G} + O(1)$$

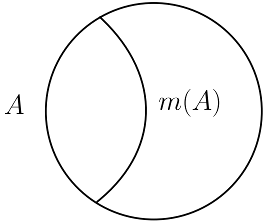


$\tilde{A} \equiv m(A) \equiv$ minimal surface homologous to A

Note: We'll stick to the static case throughout, except for briefly at the end.

The entanglement wedge

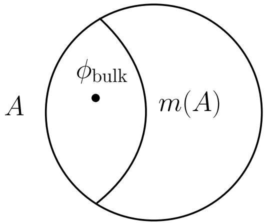
Definition: The *entanglement wedge* of A is the bulk region interior to $A \cup m(A)$.



$\rightarrow$ subregion-subregion duality

Evidence for subregion duality*: HKLL

- *Hamilton, Kabat, Lifschytz, Lowe (2006); hep-th/0606141*
- *Morrison (2014); 1403.3426*
- *others/follow-ups*

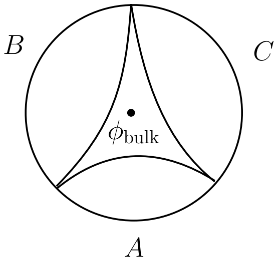


$$\phi_{\textsf{bulk}}(x, z) = \int_A dx' \; K(z, x|x')\mathcal{O}_{\textsf{boundary}}(x')$$

*for operators in the *causal wedge*

Evidence for subregion duality*: quantum error correction

- *Almheiri, Dong, Harlow (2014); 1411.7041*
- *Dong, Harlow, Wall (2016); 1601.05416*



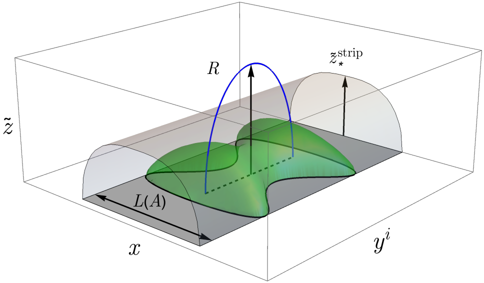
c.f. qutrit code

$$|\tilde{0}\rangle = \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right)$$
$$|\tilde{1}\rangle = \frac{1}{\sqrt{3}}\left(|012\rangle + |120\rangle + |201\rangle\right)$$
$$|\tilde{2}\rangle = \frac{1}{\sqrt{3}}\left(|021\rangle + |102\rangle + |210\rangle\right)$$

*for operators in the *entanglement wedge*

Tension with subregion duality

- In 3D gravity, there is only one type of minimal surface
- In higher dimensions, structure is richer



Observation: different $\dim$ minimal surfaces probe different bulk regions

$\rightarrow$ let's exploit this observation

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Please forget about holography now.

Bulk reach of boundary-anchored minimal surfaces

The setup:

- static $d+1$ dim'l asymptotically AdS space-time
- Poincaré patch,

$$\mathrm{d}s^2 = \frac{1}{\tilde{z}^2}\left(-f(\tilde{z})\mathrm{d}t^2 + \mathrm{d}x_i \mathrm{d}x^i + h(\tilde{z})\mathrm{d}\tilde{z}^2\right)$$

- Boundary at $\tilde{z}=0$, $f(\tilde{z}), g(\tilde{z}) \to 1$ as $\tilde{z} \to 0$
- Look at single space-like slice

B_k: simply-connected, k-dim'l submanifold in the boundary of the slice, $1 \leq k \leq d-1$, $\partial B_k \neq \emptyset$

$m(B_k)$: minimal k-surface with $\partial m(B_k) = \partial B_k$

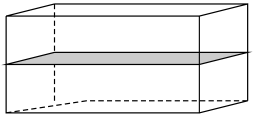
Q: deepest bulk reach $\tilde{z}_*$ of $m(B_k)$?

Example: pure AdS, strip

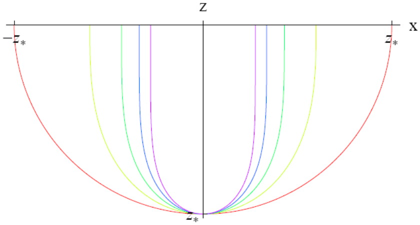
- *Hubeny (2012); 1203.1044*

$x \in [-L/2, L/2]$ and $y^i \in \mathbb{R}$, $i = 2, \ldots, k$

$$\tilde{z}_*^{\text{strip}} = L \frac{(k-1)}{\sqrt{\pi}} \frac{\Gamma(\frac{2k-1}{2k-2})}{\Gamma(\frac{k}{2k-2})}$$



e.g. 2d strip, 3d boundary



Example: pure AdS, ball

- *Hubeny (2012); 1203.1044*

$\sum_{i=1}^{k}(x^i)^2 \leq R$

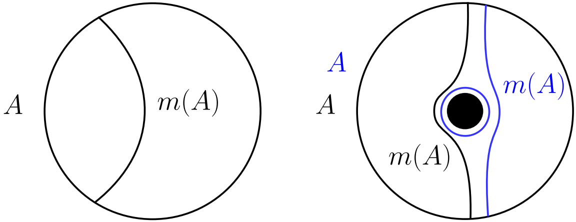
$$\tilde{z}_*^{\text{ball}} = R$$

k-wedges

Definition:

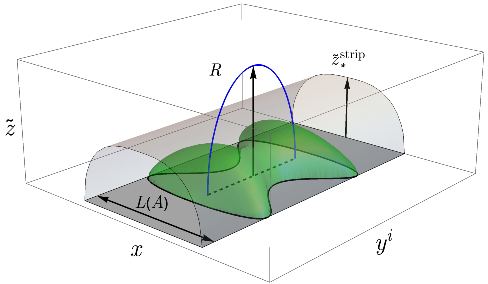
- A: simply-connected, $(d-1)$-dimensional subregion in boundary of static aAdS$_{d+1}$ slice
- *k-wedge* of A, $W_k(A)$: set of all points that lie on a k-dimensional minimal surface $m(B_k)$ for at least one simply-connected submanifold $B_k \subseteq A$, where $1 \leq k \leq d-1$.

Eg: $W_{d-1}(A) \equiv$ entanglement wedge of A (if no entanglement shadows.)



k-wedges reach beyond the entanglement wedge

- Ok, not too surprising!
- But, can construct situations where k-wedges probe parametrically beyond the entanglement wedge



- $d-2$ directions y^i
- shown: "peanut", $(d-1)$-dim'l strip, $(d-2)$-dim'l disk

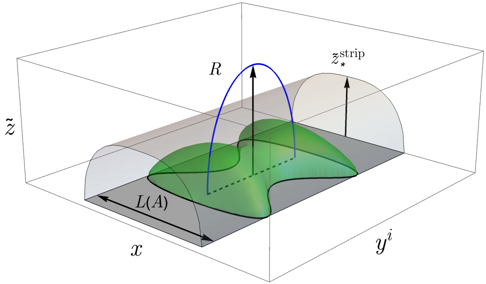
k-wedges reach beyond the entanglement wedge

Exploit entanglement wedge nesting
- *Akers, Koeller, Leichenauer, Levine (2016); 1610.08968*

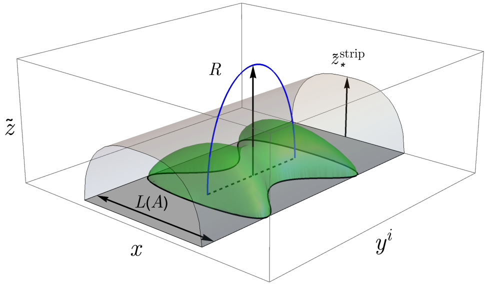
Theorem: If $A \subset A'$, then $W_E(A) \subset W_E(A')$.

$\Rightarrow$ $\tilde{z}^{\text{peanut}} \leq z^{\text{strip}}$

- by choosing A appropriately, can inscribe k-balls with arbitrarily large R



Notes



- Can always push $W_{d-1}(A)$ arbitrarily close to boundary
- Subleading corrections known if you really want (Hubeny)

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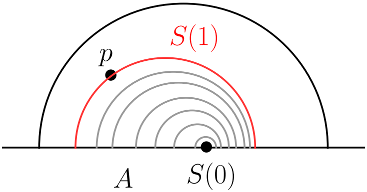
The BCFK Theorem

- *Bao, Cao, Fischetti, Keeler (2019); 1904.04834*

In a nutshell: $W_2(A)$ is metric-reconstructible

Theorem:

- Let $d \geq 3$
- A: $(d-1)$-dim'l topological ball
- Suppose for each $p \in W_2(A)$, $\exists$ continuous family $m(S(\lambda))$
  - $\lambda \in [0,1]$
  - $S(\lambda) \subset A$, topological disks
  - $S(0) \equiv$ single point in A
  - $S(1) = B : p \in m(B)$



Then, knowledge of the areas of all minimal 2-surfaces whose anchors lie in A guarantees a unique reconstruction of the bulk metric in $W_2(A)$.

Please remember holography now.

2-surface areas from boundary operators

Example: Wilson loops

- *Maldacena (1998); hep-th/9803002*
- *Drukker, Gross, Ooguri (1999); hep-th/9904191*

- Specify loop as boundary of some disk $B \subset A$

$$\langle W(\partial B)\rangle_{\rho_A} = \int_{\sigma \sim B} \mathcal{D}\sigma \; e^{-\sqrt{\lambda} S[\sigma]}$$

Large-N, large-λ:

- $S[\sigma]$ evaluates to (Legendre transform of) area of σ
- Saddle point approximation:

$$\langle W(\partial B)\rangle_{\rho_A} = e^{-\sqrt{\lambda}\,\mathcal{A}[m(B)]}$$

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Recap

- In $d \geq 4$, can construct boundary subregions A where $W_2(A)$ extends parametrically far beyond $W_{d-1}(A)$ (*entanglement wedge*)
- If there is *any* boundary operator localized to A which lets you learn the areas of minimal 2-surfaces $m(B) \subset W_2(A)$ (e.g. *Wilson loop expectation values*), then BCFK $\Rightarrow$ learn the bulk metric in $W_2(A)$

The Alternatives:

1. In general, no boundary ops. in A calculated by minimal 2-surface areas.
2. ρ_A knows about more than just the entanglement wedge.

Option 1: No boundary operators reveal 2-surface areas

Or, "No boundary operators *localized to* A reveal *sufficiently large* 2-areas"

E.g. Wilson loop expectation values

- Cases where $\langle W \rangle$ does reveal minimal 2-surface areas, like pure AdS, must be "accidents"
- c.f. analyticity
  - $\rightarrow$ know $g_{\mu\nu}$ in any open neighbourhood of any point, know $g_{\mu\nu}$ everywhere
- Either $\langle W \rangle$ breaks down at the mildest non-analyticity
  - $\rightarrow$ e.g. gravitational wave with discontinuous 3$^{\text{rd}}$ derivative
- Or $\langle W \rangle$ fails for sufficiently large loops

Note: applies to any operator that reveals minimal 2-areas

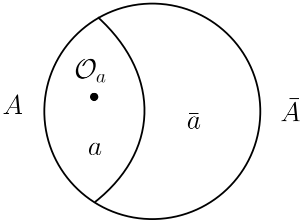
Option 2: Breakdown of subregion duality

Or, ρ_A knows about more than just the entanglement wedge.

Q: How is it still consistent with the error correction picture?
(*It's a theorem, after all...*)

Recall Dong/Harlow/Wall's "AdS/CFT as Quantum Error Correction"

$$\mathcal{H}_{\text{code}} \subset \mathcal{H}_{\text{CFT}}$$
$$\mathcal{H}_{\text{CFT}} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$$
$$\mathcal{H}_{\text{code}} = \mathcal{H}_a \otimes \mathcal{H}_{\bar{a}}$$



Entanglement Wedge Reconstruction:

"any $\mathcal{O}_a$ on $\mathcal{H}_a$ represented by $\mathcal{O}_A$ on $\mathcal{H}_A$"

AdS/CFT as QEC

Theorem (Dong, Harlow, Wall)

- Let $\mathcal{H}$: $\dim \mathcal{H} < \infty$, $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$, $\mathcal{H}_{\text{code}} \subset \mathcal{H}$
- Suppose $\mathcal{O}, \mathcal{O}^\dagger : \mathcal{H}_{\text{code}} \to \mathcal{H}_{\text{code}}$

If: $\forall\ |\phi\rangle, |\psi\rangle \in \mathcal{H}_{\text{code}}$, $\exists\ \mathcal{H}_{\text{code}} = \mathcal{H}_a \otimes \mathcal{H}_{\bar{a}}$ such that $\mathcal{O} = \mathcal{O}_a \otimes I_{\bar{a}}$

And If:

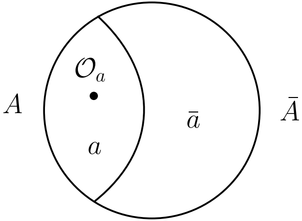
$$\begin{aligned} \rho_{\bar{A}} &= \mathrm{Tr}_A\,|\phi\rangle\langle\phi| \quad & \sigma_{\bar{A}} &= \mathrm{Tr}_A\,|\psi\rangle\langle\psi| \\ \rho_{\bar{a}} &= \mathrm{Tr}_a\,|\phi\rangle\langle\phi| \quad & \sigma_{\bar{a}} &= \mathrm{Tr}_a\,|\psi\rangle\langle\psi| \end{aligned}$$

satisfy $\rho_{\bar{a}} = \sigma_{\bar{a}} \Rightarrow \rho_{\bar{A}} = \sigma_{\bar{A}}$

Then:

1. For any $X_{\bar{A}} : \mathcal{H}_{\bar{A}} \to \mathcal{H}_{\bar{A}}$ and any $|\phi\rangle \in \mathcal{H}_{\text{code}}$, $\langle\phi|[\mathcal{O}, X_{\bar{A}}]|\phi\rangle = 0$
2. $\exists\ \mathcal{O}_A : \mathcal{H}_A \to \mathcal{H}_A$ such that $\mathcal{O}_A|\phi\rangle = \mathcal{O}|\phi\rangle\ \forall\ |\phi\rangle \in \mathcal{H}_{\text{code}}$

No conflict with Dong/Harlow/Wall

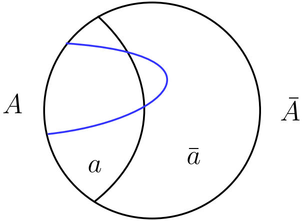


Subregion duality à la DHW $\leftrightarrow$ sufficiently small algebra of bulk operators on a can be represented on A

An apparent puzzle regarding complementary recovery

For any $X_{\bar{A}} : \mathcal{H}_{\bar{A}} \to \mathcal{H}_{\bar{A}}$ and any $|\phi\rangle \in \mathcal{H}_{\mathrm{code}}$, $\langle\phi|[\mathcal{O}, X_{\bar{A}}]|\phi\rangle = 0$

Untrue for, e.g., boundary Wilson loops?

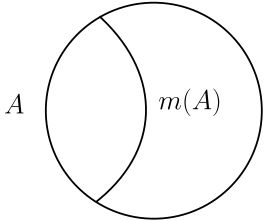


Two observations:

1. $\langle W \rangle \sim$ area not an operator equation
2. states with different metrics in $\bar{a}$ not in same $\mathcal{H}_{\mathrm{code}}$

Boundary reconstruction?

Given $W_E(A)$, what do you learn about ρ_A?



Metric reconstruction from other data

- Can you reconstruct the metric from other minimal surfaces?
- What can you reconstruct in $W_k(A)$ for $k \neq 2$?

Example: Boundary-anchored geodesics

- Metric reconstruction known as *boundary rigidity problem* in applied maths community
  - → Not proven for $A \subset$ aAdS boundary

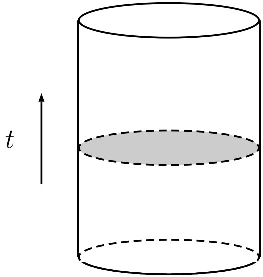
- Holographically: calculated by 2-point functions in the geodesic approximation

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \sim e^{-|\mathrm{geod}(x,y)|}$$

Covariant generalization?

Or, can you play the same games in non-static geometries?

E.g. BCFK still applies covariantly.



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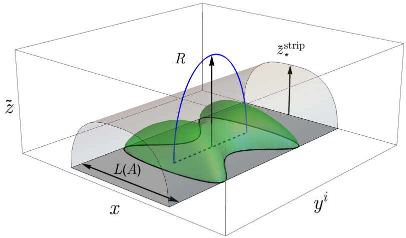
Bulk reconstruction beyond the entanglement wedge

1. Given slice of static aAdS$_{d+1}$, can construct subregions A where $W_2(A)$ extends parametrically far beyond A's entanglement wedge.
2. Spectrum of minimal 2-surfaces areas encodes bulk metric in $W_2(A)$
   - BCFK theorem
3. there are settings where minimal 2-surface areas are given by boundary operator $\langle \cdot \rangle$ localized to A
   - e.g. Wilson loops

$\therefore$ can construct examples where ρ_A reveals bulk metric beyond its entanglement wedge

Alternatives and follow-up questions



1 No boundary operators in A reveal too large minimal 2-surface areas.
2 ρ_A knows about more than just the entanglement wedge.

So, which is it?
What does the entanglement wedge encode?
Other minimal surface data / non-static?