Rencontres théoriciennes

Jussieu, Paris

Double-copy structures and universality in string tree-level interactions

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Oliver Schlotterer (Uppsala University)

based on work in collaboration T. Azevedo,

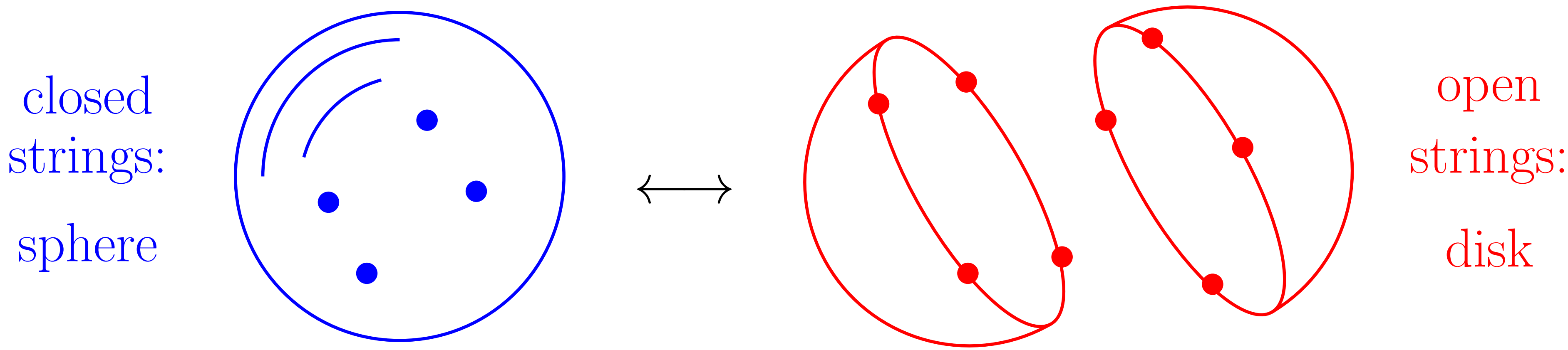
M. Chiodaroli, Y. Huang, H. Johansson, C. Wen

06.02.2020

Introduction: Double copy – from string to field theory

Birth of double copy: KLT relations among string amplitudes at tree-level

$$\mathcal{M}^{4\,\mathrm{pt}}_{\mathrm{closed}}(\alpha') \;=\; \bar{\mathcal{A}}_{\mathrm{open}}(1,2,4,3;\alpha')\,\sin\left(\frac{\pi\alpha'}{2}k_1\cdot k_2\right)\mathcal{A}_{\mathrm{open}}(1,2,3,4;\alpha')\ .$$



[Kawai, Lewellen, Tye 1986]

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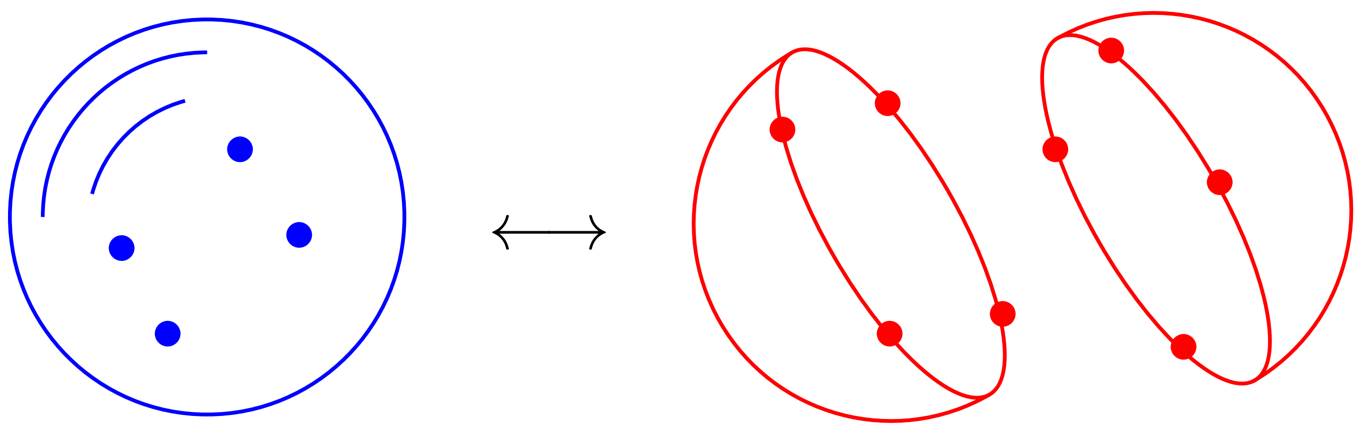
closed strings: sphere



open strings: disk

[Kawai, Lewellen, Tye 1986]

Field-theory limit $\alpha'\to 0$: relate gravity to double copy of gauge theories:

$$M^{4\,\mathrm{pt}}_{\mathrm{SUGRA}} \;=\; \bar{A}_{\mathrm{SYM}}(1,2,4,3)\,k_1\cdot k_2\,A_{\mathrm{SYM}}(1,2,3,4)\;\equiv\;\bar{A}_{\mathrm{SYM}}\otimes_{\mathrm{KLT}}A_{\mathrm{SYM}}\,.$$

Will refer to operation $\otimes_{\mathrm{KLT}}$ at $\alpha'\to 0$ as **field-theory double copy**.

Introduction: Double copy – from string to field theory

At n points, more combinatorics and $(n-3)!$-element BCJ bases

$$\left\{ A_{\rm SYM}(1,\rho(2,3,\ldots,n-2),n-1,n) \ , \quad \text{permutation } \rho \in S_{n-3} \right\}$$

[Bern, Carrasco, Johansson 0805.3993]

$$M^{n\text{ pt}}_{\rm SUGRA} = \sum_{\rho,\tau \in S_{n-3}} \bar A_{\rm SYM}(1,\rho,n,n-1) \, \underbrace{S(\rho|\tau)_1}_{(n-3)! \times (n-3)! \text{ KLT matrix, entries are } \sim (k_i \cdot k_j)^{n-3}} \, A_{\rm SYM}(1,\tau,n-1,n)$$

[Bern, Dixon, Perelstein, Rozowsky 1998]

[Bjerrum-Bohr, Damgaard, Feng, Sondergaard, Vanhove 2010]

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e.g. 2×2 terms at 5 points with

$$S(\rho(2,3)|\tau(2,3))_1 = \begin{pmatrix} (k_1\cdot k_2)(k_{1+2}\cdot k_3) & (k_1\cdot k_2)(k_1\cdot k_3) \\ (k_1\cdot k_2)(k_1\cdot k_3) & (k_1\cdot k_3)(k_{1+3}\cdot k_2) \end{pmatrix}$$

Shorthand for KLT formulae: $\boxed{M_{\rm SUGRA} = \bar A_{\rm SYM} \otimes_{\rm KLT} A_{\rm SYM}}$

Key result: Massless tree amplitudes in various string theories

Field-theory double copy $\otimes_{\mathrm{KLT}} \Rightarrow$ web of relations for string amplitudes

$\rightarrow$ representations of the flavour (field theory) $\otimes_{\mathrm{KLT}}$ (stringy building block)

$\otimes_{\mathrm{KLT}}$	SYM		
Z-theory	open superstring		

- "Z-theory" $\leftrightarrow$ α'-dependent disk integrals (over moduli space $\mathcal{M}_{0,n}$)

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sv(open superstring)	closed superstring	heterotic string (grav)	
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Outline

I. Open superstrings as a field-theory double copy

[Mafra, OS, Stieberger 1106.2645; Broedel, OS, Stieberger 1304.7267]

II. Closed superstrings from single-valued open superstrings

[OS, Stieberger 1205.1516; Stieberger 1310.3259]

III. Bosonic strings from $(DF)^2 +$ YM field theory

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

IV. Heterotic strings and $(DF)^2 +$ YM $+ \phi^3$ field theory

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

V. Universal string interactions

[Huang, OS, Wen 1602.01674]

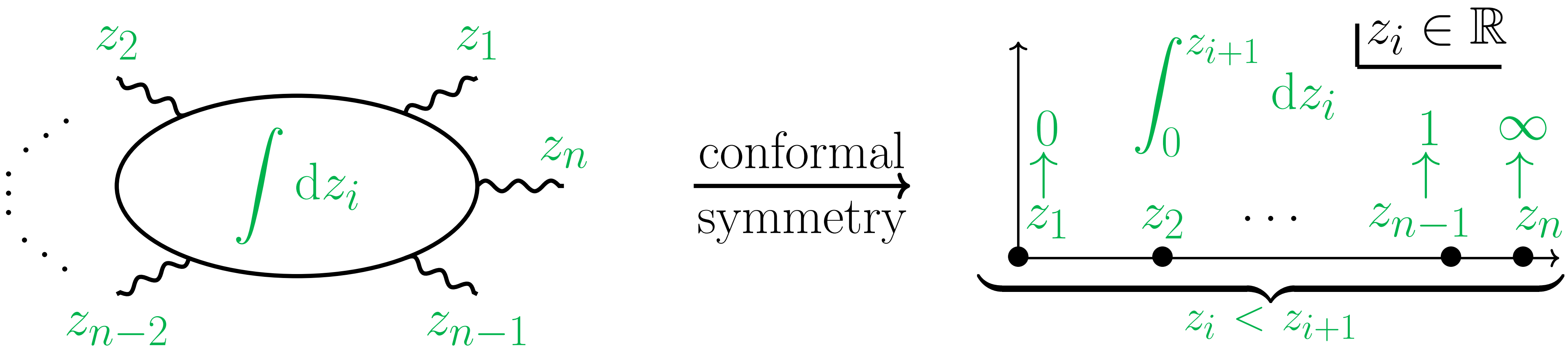
I. Open superstrings as a field-theory double copy

Color ordered n-point trees of open superstring (with $\sigma = \sigma(2,3,\ldots,n{-}2)$)

$$\boxed{\mathcal{A}^{\text{open}}_{\text{super}}(1,\sigma,n{-}1,n;\alpha') \; = \; \sum_{\tau \in S_{n-3}} F_{\sigma}{}^{\tau}(\alpha') \, A_{\text{SYM}}(1,\tau,n{-}1,n)}$$

[Mafra, OS, Stieberger 1106.2645, 1106.2646]

- all polarizations in BCJ basis of 10-dim SYM amplitudes $A_{\text{SYM}}(\ldots)$
- all α' in $(n{-}3)! \times (n{-}3)!$ disk integrals $F_{\sigma}{}^{\tau}(\alpha')$



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$$F_\sigma{}^\tau(\alpha') \ = \int\limits_{0<z_{\sigma(2)}<z_{\sigma(3)}<\ldots<z_{\sigma(n-2)}<1} \mathrm{d}z_2 \ldots \mathrm{d}z_{n-2} \prod_{i<j}^{n-1} |z_i{-}z_j|^{s_{ij}}\, \tau\Big\{ \prod_{l=2}^{n-2} \sum_{m=1}^{l-1} \frac{s_{lm}}{z_l - z_m} \Big\}$$

- at $n=4$ points, permutations trivialize & recover Veneziano amplitude

$$F_2{}^2(\alpha') \ = \int_0^1 \mathrm{d}z_2\, |z_2|^{s_{12}}\, |1{-}z_2|^{s_{23}}\, \frac{s_{12}}{z_2} \ = \ \frac{\Gamma(1+s_{12})\Gamma(1+s_{23})}{\Gamma(1+s_{12}+s_{23})}$$

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- $\tau \in S_{n-3}$ acts on k_j and z_j enclosed in $\{\ldots\}$ with $j = 2,3,\ldots,n{-}2$
- recover $\mathcal{A}^{\rm open}_{\rm super} \rightarrow A_{\rm SYM}$ from field-theory limit $F_\sigma{}^\tau(\alpha') \ = \ \delta^\tau_\sigma + \mathcal{O}(\alpha'^2)$

I. Open superstrings as a field-theory double copy

Color ordered n-point trees of open superstring (with $\sigma = \sigma(2,3,\ldots,n-2)$)

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A KLT formula in disguise involving disk integrals (with $z_{ij} \equiv z_i - z_j$)

$$Z_\sigma\big(\rho(1,2,\ldots,n)\big) \equiv (2\alpha')^{n-3} \int_{z_{\sigma(i)}<z_{\sigma(i+1)}} \frac{{\rm d}z_1 \ \ldots \ {\rm d}z_n}{{\rm vol}\,{\rm SL}_2(\mathbb{R})} \frac{\prod_{i<j}^n |z_{ij}|^{s_{ij}}}{\rho(z_{12}z_{23}\ldots z_{n-1,n}z_{n,1})}$$

Permutation $\rho = \rho(1,2,\ldots,n)$ acts on cyclic denominator $(z_{12}z_{23}\ldots z_{n,1})^{-1}$

... and integrand of $F_\sigma{}^\tau$ is $\sum_\rho S(\rho|\tau)_1 \rho(z_{12}z_{23}\ldots z_{n,1})^{-1}$ w. KLT matrix

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$\Longrightarrow$ field-theory double copy $\boxed{\mathcal{A}^{\rm open}_{\rm super}(\sigma) = Z_\sigma \otimes_{\rm KLT} A_{\rm SYM}}$

Integrals Z_σ dubbed "Z-theory amplitudes" **[Carrasco, Mafra, OS 1608.02569]**

II. Closed superstrings from single-valued open superstrings

α'-expansion of $F_\sigma{}^\tau$ & $\mathcal{A}^{\rm open}_{\rm super}$ involves multiple zeta values (MZVs)

$$\zeta_{n_1,n_2,\ldots,n_r} \equiv \sum_{0<k_1<k_2<\ldots<k_r}^{\infty} k_1^{-n_1} k_2^{-n_2} \ldots k_r^{-n_r} \ , \qquad n_r \geq 2$$

[Terasoma 2002 & Brown 2006]

Schematically ($s_{ij} \equiv 2\alpha' k_i \cdot k_j$),

$$\mathcal{A}^{\rm open}_{\rm super}(\sigma) = \overbrace{\Big(\mathbb{1} + \zeta_2\,(s_{ij})^2 + \zeta_3\,(s_{ij})^3 + \mathcal{O}(\alpha'^4)\Big)_\sigma{}^\tau}^{F_\sigma{}^\tau(\alpha')} A_{\rm SYM}(\tau)$$

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At four points, for instance, only Riemann zeta values ζ_k,

$$\begin{aligned} \mathcal{A}^{\text{open}}_{\text{super}}(1,2,3,4) &= \frac{\Gamma(1+s_{12})\Gamma(1+s_{23})}{\Gamma(1+s_{12}+s_{23})} A_{\text{SYM}}(1,2,3,4) \\ &= \exp\Big(\sum_{k=2}^{\infty} \frac{\zeta_k}{k} (-1)^k \left[s_{12}^k + s_{23}^k - (s_{12}+s_{23})^k \right] \Big) A_{\text{SYM}}(1,2,3,4) \\ &= \Big(1 - \zeta_2 \, s_{12}s_{23} - \zeta_3 \, s_{12}s_{13}s_{23} + \mathcal{O}(\alpha'^4)\Big) A_{\text{SYM}}(1,2,3,4) \end{aligned}$$

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Polynomial structure in s_{ij} at n points can be determined to any order

[Brödel, OS, Stieberger, Terasoma 1304.7304 & Mafra, OS 1609.07078]

- explicit results at $n \leq 7$ points available for download

  `http://wwwth.mpp.mpg.de/members/stieberg/mzv/index.html`

- at $n \geq 5$ points, can't avoid multiple arguments ($\zeta_{3,5}$, $\zeta_{3,7}$, $\zeta_{3,3,5}$ etc.)

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Define single-valued projection **sv** of MZVs via their polylogarithm origin

[Schnetz 1302.6445 & Brown 1309.5309]

$$\begin{tikzcd}
\text{MZVs} \arrow[r, dashed, "\mathbf{sv}"'] & \text{single-valued MZVs} \\
\begin{array}{c}\text{polylogarithms}\\ G(\{\text{word in } 0,1\};z)\end{array} \arrow[u, "z\to 1"] \arrow[r, "\begin{array}{c}\text{Francis Brown 2004:}\\ \exists\ \text{procedure to}\end{array}", "\text{cancel monodromies}"'] & \begin{array}{c}\text{single-valued polylog's}\\ G_{\text{SV}}(\{\text{word in } 0,1\};z)\end{array} \arrow[u, "z\to 1"']
\end{tikzcd}$$

e.g. $\; G(1;z) \; = \; \log(1-z) \quad \longrightarrow \quad G_{\text{SV}}(1;z) = \log|1-z|^2$

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$$\mathbf{sv}(\zeta_{2k}) \ = \ 0\, , \qquad \mathbf{sv}(\zeta_{2k+1}) \ = \ 2\,\zeta_{2k+1}\, , \qquad \mathbf{sv}(\zeta_{3,5}) \ = \ -10\,\zeta_3\,\zeta_5\, , \qquad \text{etc.}$$

II. Closed superstrings from single-valued open superstrings

In principle, [Kawai, Lewellen, Tye 1986] determine closed-superstring trees as

$$\mathcal{M}^{\text{closed}}_{\text{super}}(\alpha') \sim \bar{\mathcal{A}}^{\text{open}}_{\text{super}}(\ldots;\alpha') \prod \sin\left(\frac{\pi\alpha'}{2} k_i \cdot k_j\right) \mathcal{A}^{\text{open}}_{\text{super}}(\ldots;\alpha')\,,$$

however, this obscures cancellations among MZVs (e.g. $\alpha'^{2k}\zeta_{2k}$ drop out).

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Instead, simplify to field-theory double copy @ $\sin\big(\frac{\pi\alpha'}{2} k_i \cdot k_j\big) \rightarrow k_i \cdot k_j$

$$\boxed{\mathcal{M}^{\text{closed}}_{\text{super}}(\alpha') \; = \; \bar{A}_{\text{SYM}} \otimes_{\text{KLT}} \mathbf{sv}\, \mathcal{A}^{\text{open}}_{\text{super}}(\alpha')}$$

[OS, Stieberger 1205.1516 & Stieberger 1310.3259]

Schematically, by $\mathbf{sv}(\zeta_{2k}) \; = \; 0$ and $\mathbf{sv}(\zeta_{2k+1}) \; = \; 2\,\zeta_{2k+1}$,

$$\mathcal{A}^{\text{open}}_{\text{super}}(\sigma) \; = \; \Big(\mathbb{1} \, + \, \zeta_2\,(s_{ij})^2 \, + \, \zeta_3\,(s_{ij})^3 \, + \, \mathcal{O}(\alpha'^4)\Big)_\sigma{}^\tau \, A_{\text{SYM}}(\tau)$$

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At $n=4$, for instance, **sv** maps Veneziano- to Virasoro–Shapiro amplitude

$$\mathcal{A}^{\text{open}}_{\text{super}}(1,2,3,4) = \frac{\Gamma(1+s_{12})\Gamma(1+s_{23})}{\Gamma(1-s_{13})} A_{\text{SYM}}(1,2,3,4)$$

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- emergence of $\mathbf{sv}\, \mathcal{A}^{\rm open}_{\rm super}(\alpha')$ was conjectural until recently ...

... and several approaches to an all-order proof can be found in

[OS, Schnetz 1808.00713 & Brown, Dupont 1810.07682

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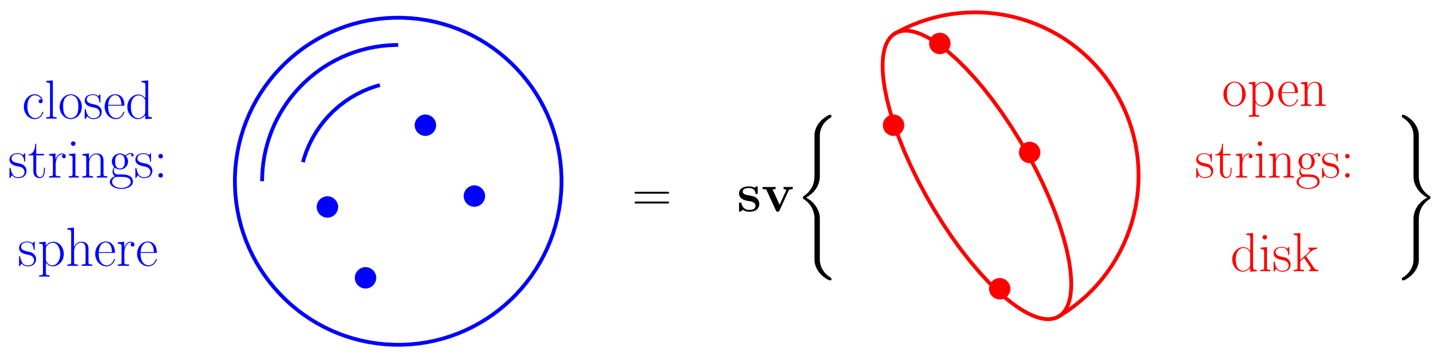


II. Closed superstrings from single-valued open superstrings

Closed strings as a field-theory double copy

$$\boxed{\mathcal{M}^{\text{closed}}_{\text{super}}(\alpha') = \bar{A}_{\text{SYM}} \otimes_{\text{KLT}} \mathbf{sv}\, \mathcal{A}^{\text{open}}_{\text{super}}(\alpha')}$$

[OS, Stieberger 1205.1516 & Stieberger 1310.3259]

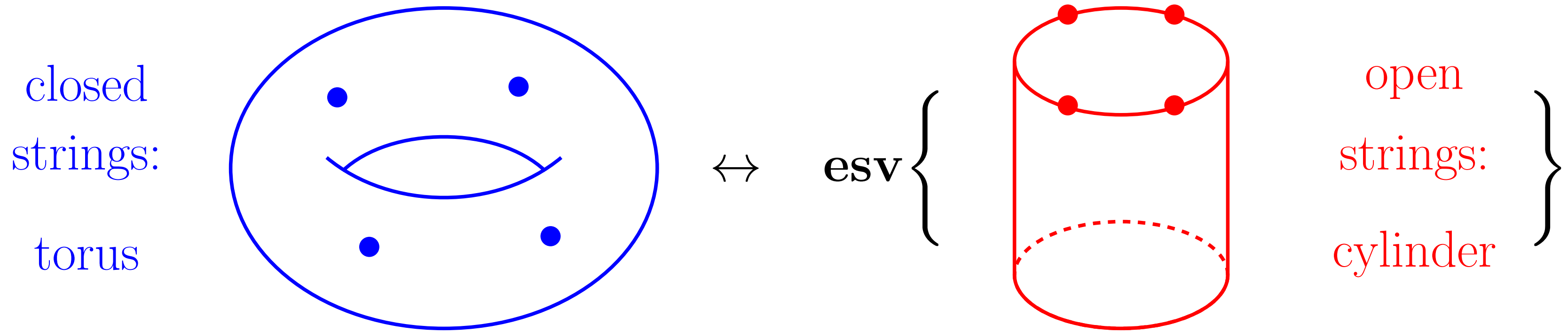
- emergence of $\mathbf{sv}\, \mathcal{A}^{\text{open}}_{\text{super}}(\alpha')$ was conjectural until recently ...

  ... and several approaches to an all-order proof can be found in

  [OS, Schnetz 1808.00713 & Brown, Dupont 1810.07682 & Vanhove, Zerbini 1812.03018 & Brown, Dupont 1910.01107]

- $\exists$ first 1-loop echos of elliptic **sv**-map "**esv**" from open to closed strings

  [Brödel, OS, Zerbini 1803.00527 & Gerken, Kleinschmidt, OS 1811.02548]



III. Bosonic strings from $(DF)^2 + \mathrm{YM}$ field theory

Open **bosonic** string: can still expand n-point trees via integrals $F_\sigma{}^\tau$

$$\mathcal{A}^{\rm open}_{\rm super}(1,\sigma,n{-}1,n;\alpha') \ = \ \sum_{\tau \in S_{n-3}} F_\sigma{}^\tau(\alpha')\, A_{\rm SYM}(1,\tau,n{-}1,n)$$

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α'-dependent kinematic factors $B(\ldots;\alpha')$, e.g. ($s_{ij} \equiv k_i{\cdot}k_j$ from now on)

tachyon pole

$$B(1,2,3;\alpha') \ = \ A_{\text{YM}}(1,2,3) \ - \ 4\alpha'\,(e_1\cdot k_2)(e_2\cdot k_3)(e_3\cdot k_1)$$

$$B(1,2,3,4;\alpha') \ = \ A_{\text{YM}}(1,2,3,4) \ - \ 4\alpha' s_{13}\Big\{\Big[\frac{f_{12}f_{34}}{s_{12}^2(1{-}2\alpha' s_{12})} + \text{cyc}(2,3,4)\Big] - \frac{g_1g_2g_3g_4}{s_{12}^2s_{13}^2s_{23}^2}\Big\}$$

with $f_{ij} \equiv s_{ij}(e_i{\cdot}e_j) - (k_i{\cdot}e_j)(k_j{\cdot}e_i)$ and $g_i \equiv (k_{i-1}{\cdot}e_i)s_{i,i+1} - (k_{i+1}{\cdot}e_i)s_{i-1,i}$.

[Huang, OS, Wen 1602.01674]

Is there a field-theory interpretation of these $B(\ldots;\alpha')$ with BCJ rel's?

Open **bosonic** string: can still expand n-point trees via integrals $F_\sigma{}^\tau$

$$\mathcal{A}^{\rm open}_{\rm bos}(1,\sigma,n{-}1,n;\alpha') \ = \ \frac{\alpha'}{2} \sum_{\tau \in S_{n-3}} F_\sigma{}^\tau(\alpha')\, A_{(DF)^2+{\rm YM}}(1,\tau,n{-}1,n;\alpha')$$

"$(DF)^2 + \mathrm{YM}$" gauge theory $\Longrightarrow$ kin. factors $B \rightarrow \frac{\alpha'}{2} A_{(DF)^2+{\rm YM}}$

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

$$\mathcal{L}_{(DF)^2+{\rm YM}} \equiv \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 \ - \ \frac{1}{4} m^2 (F^a_{\mu\nu})^2 \ - \ \frac{1}{3} f^{abc} F^{a\nu}_\mu F^{b\lambda}_\nu F^{c\mu}_\lambda$$

$$+ \frac{1}{2}(D_\mu \varphi^\alpha)^2 \ - \ \frac{1}{2} m^2 (\varphi^\alpha)^2 \ + \ \frac{1}{2} C^{\alpha ab} \varphi^\alpha F^a_{\mu\nu} F^{b\,\mu\nu} \ + \ \frac{1}{3!} d^{\alpha\beta\gamma} \varphi^\alpha \varphi^\beta \varphi^\gamma$$

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- kin. operator $(\partial^4{-}m^2\partial^2)A^2 \;\Rightarrow\;$ 2 gluon modes: $\left(\begin{smallmatrix}\text{massless}\\ \text{physical}\end{smallmatrix}\right)\oplus\left(\begin{smallmatrix}\text{massive}\\ \text{ghost}\end{smallmatrix}\right)$

- massive-ghost scalar φ^α: index $\alpha \leftrightarrow$ real representation of gauge group

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- set $m^2 = -\dfrac{1}{\alpha'} \ \Rightarrow \$ required tachyon poles $A_{(DF)^2+{\rm YM}} \sim \dfrac{1}{1{-}2\alpha' k_i{\cdot}k_j}$

- for external gluons, Clebsch Gordans $C^{\alpha a b}$ & $d^{\alpha\beta\gamma}$ conspire to $\prod f^{abc}$

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Brief history of $(DF)^2 + {\rm YM}$ theory **[Johansson, Nohle 1707.02965]**

- massless version $\mathcal{L}_{(DF)^2} \equiv \lim_{m\rightarrow 0} \mathcal{L}_{(DF)^2+{\rm YM}}$ (i.e. "$\alpha' \rightarrow \infty$")

  $\Rightarrow$ conformal supergravity as double copy $\mathcal{M}_{\rm CSG} = \bar{A}_{\rm SYM} \otimes_{\rm KLT} A_{(DF)^2}$

- mass parameter of $\mathcal{L}_{(DF)^2+{\rm YM}}$ preserves BCJ relations of $A_{(DF)^2+{\rm YM}}$

Open **bosonic** string: can still expand n-point trees via integrals $F_\sigma{}^\tau$

$$\mathcal{A}^{\rm open}_{\rm bos}(1,\sigma,n{-}1,n;\alpha') \;=\; \frac{\alpha'}{2}\sum_{\tau\in S_{n-3}} F_\sigma{}^\tau(\alpha')\, A_{(DF)^2+{\rm YM}}(1,\tau,n{-}1,n;\alpha')$$

Since $F_\sigma{}^\tau = \sum_\rho Z_\sigma(\rho)S(\rho|\tau)$ signal KLT formula in disguise,

$\Longrightarrow$ field-theory double copy $\boxed{\;\mathcal{A}^{\rm open}_{\rm bos}(\sigma) \;=\; Z_\sigma \otimes_{\rm KLT} A_{(DF)^2+{\rm YM}}\;}$

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Corollaries by recycling the underlying open-string CFT correlator:

- via **sv** projection of MZVs: $\boxed{\ \mathcal{M}^{\rm closed}_{\rm bos} \ = \ \mathbf{sv}\, \bar{\mathcal{A}}^{\rm open}_{\rm bos} \otimes_{\rm KLT} A_{(DF)^2+{\rm YM}}\ }$
- heterotic strings (gravity): can put SUSY on either side of double copy

$$\mathcal{M}^{\rm het}_{\rm grav} \ = \ \left\{ \begin{array}{ll} \mathbf{sv}\, \bar{\mathcal{A}}^{\rm open}_{\rm super} \otimes_{\rm KLT} A_{(DF)^2+{\rm YM}} & : \ \text{SUSY on } \mathbf{sv}\text{(string) side} \\ \mathbf{sv}\, \bar{\mathcal{A}}^{\rm open}_{\rm bos} \otimes_{\rm KLT} A_{\rm SYM} & : \ \text{SUSY on field-theory side} \end{array} \right.$$

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

IV. Heterotic strings and $(DF)^2 + \text{YM} + \phi^3$ field theory

Now incorporate gauge sector & gauge/gravity couplings of heterotic string

- single-trace gluon amplitudes: $\mathcal{A}^{\text{het}}_{\text{s.tr.}}(1,2,\ldots,n) = \textbf{sv}\,\mathcal{A}^{\text{open}}_{\text{super}}(1,\ldots,n)$

[Stieberger, Taylor 1401.1218]

- various mixed gauge/gravity & double-trace amplitudes reduced to $\mathcal{A}^{\text{het}}_{\text{s.tr.}}$

[OS 1608.00130]

Suggests double-copy structure in color-dressed het. amplitudes $\mathcal{M}^{\text{het}}_{\substack{\text{gauge}\\ \oplus\text{grav}}}$,

$$\mathcal{M}^{\text{het}}_{\substack{\text{gauge}\\ \oplus\text{grav}}} \sim A_{??} \otimes_{\text{KLT}} \textbf{sv}\,\mathcal{A}^{\text{open}}_{\text{super}}$$

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n gravitons
k gluons

n gluons
k scalars

adds spin one and
carries the SUSY

As an amplitude $A_{??} \supset$ colored scalars "s" & uncolored vectors "g" e.g.

$$A_{??}(1_s, 2_s, 3_g) \sim \delta^{A_1 A_2}\,(e_3 \cdot k_1)\ .$$

Further examples of $A_{??}$ (with colored scalar "s" & uncolored vectors "g")

$$A_{??}(1_s,2_s,3_s,4_s) \;=\; \overbrace{\frac{\bar f^{A_1A_2B}\,\bar f^{BA_3A_4}}{2s_{12}}+\frac{\bar f^{A_2A_3B}\,\bar f^{BA_4A_1}}{2s_{23}}}^{\text{single trace}}+\overbrace{s_{13}\Big\{\frac{2\alpha'\delta^{A_1A_2}\delta^{A_3A_4}}{s_{12}(1-2\alpha's_{12})}+\text{cyc}(2,3,4)\Big\}}^{\text{double trace}}$$

$$A_{??}(1_s,2_g,3_s,4_g) \;=\; \delta^{A_1A_3}\Big\{\frac{(e_2\cdot k_1)(e_4\cdot k_3)}{s_{12}}+\frac{(e_2\cdot k_3)(e_4\cdot k_1)}{s_{14}}+(e_2\cdot e_4)+\frac{2\alpha' f_{24}}{1-2\alpha's_{24}}\Big\}$$

…

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$(DF)^2$+YM+ϕ^3

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suggest extension of $(DF)^2$+YM by bi-adjoint scalars $\phi=\sum_{a,A}\phi^{aA}t^a\otimes\bar t^A$

$$\begin{aligned}\mathcal{L}_{(DF)^2+\mathrm{YM}+\phi^3} \;\equiv\; & \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 \;-\; \frac{1}{4}m^2(F^a_{\mu\nu})^2 \;-\; \frac{1}{3}f^{abc}F^{a\nu}_{\mu}F^{b\lambda}_{\nu}F^{c\mu}_{\lambda}\\ &+\;\frac{1}{2}(D_\mu\varphi^\alpha)^2 \;-\; \frac{1}{2}m^2(\varphi^\alpha)^2 \;+\; \frac{1}{2}C^{\alpha ab}\varphi^\alpha F^a_{\mu\nu}F^{b\,\mu\nu} \;+\; \frac{1}{3!}d^{\alpha\beta\gamma}\varphi^\alpha\varphi^\beta\varphi^\gamma\\ &+\;\frac{1}{2}D_\mu\phi^{aA}D^\mu\phi^{aA} \;+\; \frac{\lambda}{3!}f^{abc}\bar f^{ABC}\phi^{aA}\phi^{bB}\phi^{cC} \;+\; \frac{1}{2}\varphi^\alpha\phi^{aA}\phi^{bA}C^{\alpha ab}\,\Big|_{m^2=-\frac{1}{\alpha'}}\end{aligned}$$

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$(DF)^2$+YM+ϕ^3

$$A_{\cancel{??}}(1_s,2_g,3_s,4_g) \;=\; \delta^{A_1A_3}\Big\{\frac{(e_2\cdot k_1)(e_4\cdot k_3)}{s_{12}}+\frac{(e_2\cdot k_3)(e_4\cdot k_1)}{s_{14}}+(e_2\cdot e_4)+\frac{2\alpha' f_{24}}{1-2\alpha' s_{24}}\Big\}$$

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- only A^a_μ,φ^α have tachyonic mode @ $m^2=-\alpha'^{-1}$, the ϕ^{aA} are massless

- only color order w.r.t. $\mathrm{Tr}(t^at^b\ldots)$, not w.r.t. $\mathrm{Tr}(\bar t^A\bar t^B\ldots)$

$$A_{(DF)^2+\mathrm{YM}+\phi^3}(1,2,\ldots,n) \;\equiv\; M_{(DF)^2+\mathrm{YM}+\phi^3}\Big|_{\mathrm{Tr}(t^{a_1}t^{a_2}\ldots t^{a_n})}$$

All mixed gluon/graviton amplitudes of heterotic string from $(DF)^2$+YM+ϕ^3,

$$\mathcal{M}^{\rm het}_{\substack{\rm gauge\\ \oplus \rm grav}} \sim A_{(DF)^2+{\rm YM}+\phi^3} \otimes_{\rm KLT} \textbf{sv}\, \mathcal{A}^{\rm open}_{\rm super}$$

n gravitons
k gluons

n gluons
k scalars

adds spin one and
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The $(DF)^2$+YM+ϕ^3-theory is unique once we impose BCJ relations and

- limiting behaviour as $\alpha' \to 0$ and $\alpha' \to \infty$
- relative normalization of het. single-trace vs. double-trace amplitudes

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

V. Universal string interactions

Compare α'-dependence in open-superstring vs. bosonic-string amplitudes

$$\mathcal{A}^{\rm open}_{\rm super}(1,\sigma,n{-}1,n;\alpha') \ = \ \sum_{\tau\in S_{n-3}} F_\sigma{}^\tau(\alpha') \ \overbrace{A_{\rm SYM}(1,\tau,n{-}1,n)}^{\text{independent on } \alpha'}$$

$$\mathcal{A}^{\rm open}_{\rm bos}(1,\sigma,n{-}1,n;\alpha') \ = \ \sum_{\tau\in S_{n-3}} F_\sigma{}^\tau(\alpha') \ \underbrace{B(1,\tau,n{-}1,n;\alpha')}_{\text{rational fct. of } \alpha'}$$

α'-expansion of disk integrals $F_\sigma{}^\tau(\alpha')$ is "uniformly transcendental"

$$w^{\rm th} \text{ order in } \alpha' \quad \longleftrightarrow \quad \zeta_{n_1,n_2,\ldots,n_r} \text{ of weight } w = n_1+n_2+\ldots+n_r$$

α'-expansion of $B(\ldots;\alpha') = \frac{\alpha'}{2}A_{(DF)^2+{\rm YM}}(\ldots;\alpha')$: no MZVs

$$\sum_{m=0}^{\infty}(2\alpha' s_{12})^m$$

$$B(1,2,3;\alpha') \ = \ A_{\rm YM}(1,2,3) \ - \ 4\alpha'\,(e_1\cdot k_2)(e_2\cdot k_3)(e_3\cdot k_1)$$

$$B(1,2,3,4;\alpha') \ = \ A_{\rm YM}(1,2,3,4) \ - \ 4\alpha' s_{13}\Big\{\Big[\frac{f_{12}f_{34}}{s_{12}^2(1{-}2\alpha' s_{12})} + {\rm cyc}(2,3,4)\Big] - \frac{g_1g_2g_3g_4}{s_{12}^2 s_{13}^2 s_{23}^2}\Big\}$$

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$$\mathcal{A}^{\rm open}_{\rm super}(\ldots;\alpha') \ = \ \left(\sum_{w=0}^{\infty}(\alpha')^w \mathrm{MZV}_w\right)\Big(A_{\rm YM} + \text{gauginos}\Big)$$

identical $(\alpha')^0$ order for ext. bosons

$$\mathcal{A}^{\rm open}_{\rm bos}(\ldots;\alpha') \ = \ \left(\sum_{w=0}^{\infty}(\alpha')^w \mathrm{MZV}_w\right)\Big(A_{\rm YM} + \sum_{m=1}^{\infty}(\alpha')^m B_m\Big)$$

- no MZVs: $B(1,\ldots,n;\alpha') \ = \ A_{\rm YM}(1,\ldots,n)+\sum_{m=1}^{\infty}(\alpha')^m B_m(1,\ldots,n)$

- universal kinematics $A_{\rm YM}$ @ "leading transcendentality": MZV_w at $(\alpha')^w$

$\Rightarrow$ leading-transcendentality interactions @ $(\alpha')^w(\mathrm{MZV})_w$ are universal

to open superstring & bosonic string e.g. $\alpha'^2\zeta_2\mathrm{Tr}(F^4)$ & $\alpha'^3\zeta_3\mathrm{Tr}(D^2F^4{+}F^5)$

[Huang, OS, Wen 1602.01674]

V. Universal string interactions

Compare α'-dependence in open-superstring vs. bosonic-string amplitudes

$$\mathcal{A}^{\rm open}_{\rm super}(\ldots;\alpha') \ = \ \Big(\sum_{w=0}^{\infty} (\alpha')^w {\rm MZV}_w \Big) \Big(A_{\rm YM} + {\rm gauginos} \Big)$$

identical $(\alpha')^0$ order for ext. bosons

$$\mathcal{A}^{\rm open}_{\rm bos}(\ldots;\alpha') \ = \ \Big(\sum_{w=0}^{\infty} (\alpha')^w {\rm MZV}_w \Big) \Big(A_{\rm YM} + \sum_{m=1}^{\infty} (\alpha')^m B_m \Big)$$

- no MZVs: $B(1,\ldots,n;\alpha') \ = \ A_{\rm YM}(1,\ldots,n) + \sum_{m=1}^{\infty} (\alpha')^m B_m(1,\ldots,n)$
- universal kinematics $A_{\rm YM}$ @ "leading transcendentality": ${\rm MZV}_w$ at $(\alpha')^w$
- non-universal $\alpha'^w B_{m\geq 1} \ \leftrightarrow \ {\rm MZV}_{w-m}$ "subleading transcendentality"

$\Rightarrow$ operators @ $(\alpha')^w ({\rm MZV})_{m<w}$ only exist for bos. strings e.g. $\alpha' {\rm Tr}(F^3)$

[Huang, OS, Wen 1602.01674]

V. Universal string interactions

Compare α'-dependence in gravitational amplitudes

does not alter weights, e.g. $\mathbf{sv}(\zeta_3) = 2\zeta_3$

$$\mathcal{M}^{\text{closed}}_{\text{super}}(\alpha') \sim \Big(\bar{A}_{\text{YM}} + \text{gauginos}\Big)\, \mathbf{sv}\Big(\sum_{w=0}^{\infty} (\alpha')^w \text{MZV}_w\Big) \Big(A_{\text{YM}} + \text{gauginos}\Big)$$

$$\mathcal{M}^{\text{het}}_{\text{grav}}(\alpha') \sim \Big(\bar{A}_{\text{YM}} + \sum_{m=1}^{\infty} (\alpha')^m \bar{B}_m\Big)\, \mathbf{sv}\Big(\sum_{w=0}^{\infty} (\alpha')^w \text{MZV}_w\Big) \Big(A_{\text{YM}} + \text{gauginos}\Big)$$

$$\mathcal{M}^{\text{closed}}_{\text{bos}}(\alpha') \sim \Big(\bar{A}_{\text{YM}} + \sum_{m=1}^{\infty} (\alpha')^m \bar{B}_m\Big)\, \mathbf{sv}\Big(\sum_{w=0}^{\infty} (\alpha')^w \text{MZV}_w\Big) \Big(A_{\text{YM}} + \sum_{m=1}^{\infty} (\alpha')^m B_m\Big)$$

universal   no MZVs   universal   no MZVs

$\Rightarrow$ leading-transcendentality interactions $\alpha'^w \text{MZV}_w \times D^{2n} R^m$

are universal to type-II superstrings, heterotic & bosonic strings

e.g. $\alpha'^3 \zeta_3 R^4$ & $\alpha'^5 \zeta_5 (D^4 R^4 + D^2 R^5 + R^6)$ & $\alpha'^6 \zeta_3^2 (D^6 R^4 + \ldots)$

[Huang, OS, Wen 1602.01674]

V. Universal string interactions

Compare α'-dependence in gravitational amplitudes

does not alter weights, e.g. $\mathbf{sv}(\zeta_3) = 2\zeta_3$

$$\mathcal{M}^{\text{closed}}_{\text{super}}(\alpha') \sim \Big(\bar{A}_{\text{YM}} + \text{gauginos}\Big)\, \mathbf{sv}\Big(\sum_{w=0}^{\infty} (\alpha')^w \text{MZV}_w\Big) \Big(A_{\text{YM}} + \text{gauginos}\Big)$$

$$\mathcal{M}^{\text{het}}_{\text{grav}}(\alpha') \sim \Big(\bar{A}_{\text{YM}} + \sum_{m=1}^{\infty} (\alpha')^m \bar{B}_m\Big)\, \mathbf{sv}\Big(\sum_{w=0}^{\infty} (\alpha')^w \text{MZV}_w\Big) \Big(A_{\text{YM}} + \text{gauginos}\Big)$$

$$\mathcal{M}^{\text{closed}}_{\text{bos}}(\alpha') \sim \Big(\underbrace{\bar{A}_{\text{YM}}}_{\text{universal}} + \sum_{m=1}^{\infty} (\alpha')^m \underbrace{\bar{B}_m}_{\text{no MZVs}}\Big)\, \mathbf{sv}\Big(\sum_{w=0}^{\infty} (\alpha')^w \text{MZV}_w\Big) \Big(\underbrace{A_{\text{YM}}}_{\text{universal}} + \sum_{m=1}^{\infty} (\alpha')^m \underbrace{B_m}_{\text{no MZVs}}\Big)$$

@ subleading transcendentality $\alpha'^w \text{MZV}_{m<w}$, operators only

only exist for bosonic or heterotic strings e.g. $\alpha' R^2$ & $\alpha'^2 R^3$

[Huang, OS, Wen 1602.01674]

VI. Conclusions & Outlook

- generated all massless tree amplitudes of superstrings, bosonic strings and heterotic strings from field-theory double-copy "$\otimes_{\text{KLT}}$"

$$\left(\begin{array}{c} \text{string} \\ \text{amplitudes} \end{array} \right) = \left(\begin{array}{c} \text{gauge theory: possibly} \\ \text{with } (DF)^2/\phi^3 \text{ extension} \end{array} \right) \otimes_{\text{KLT}} \left(\begin{array}{c} \text{disk integrals or} \\ \textbf{sv}\text{(open strings)} \end{array} \right)$$

$\otimes_{\text{KLT}}$	SYM	$(DF)^2$+YM	$(DF)^2$+YM+ϕ^3
Z-theory	open superstring	open bosonic string	comp(open bos. string)
sv(open superstring)	closed superstring	heterotic string (grav)	het. string (gauge/grav)
sv(open bos. string)	heterotic string (grav)	closed bosonic string	comp(closed bos. string)

VI. Conclusions & Outlook

- generated all massless tree amplitudes of superstrings, bosonic strings and heterotic strings from field-theory double-copy "$\otimes_{\text{KLT}}$"

$$\begin{pmatrix} \text{string} \\ \text{amplitudes} \end{pmatrix} = \begin{pmatrix} \text{gauge theory: possibly} \\ \text{with } (DF)^2/\phi^3 \text{ extension} \end{pmatrix} \otimes_{\text{KLT}} \begin{pmatrix} \text{disk integrals or} \\ \textbf{sv}(\text{open strings}) \end{pmatrix}$$

- corollary: highest-transcendentality interactions $(\alpha')^w(\text{MZV})_w \times D^{2n} R^m$ @ tree level are universal to type-II, heterotic, and bosonic string theories

  $\longrightarrow$ conditio sine qua non in quantum gravity?

- also at one loop: evidence for double-copy structure of open superstrings

  $\longrightarrow$ generic property at loop level? simplify higher-genus calculations?

[Mafra, OS 1711.09104 & 1812.109{69,70,71}]