# Flux Tube S-matrix bootstrap

*LPTHE, Paris — 9/2019*

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talk based on hep-th/1906.08098
w/ A. Guerrieri, A. Hebbar, J. Penedones, P. Vieira

1.- Flux tube effecive action

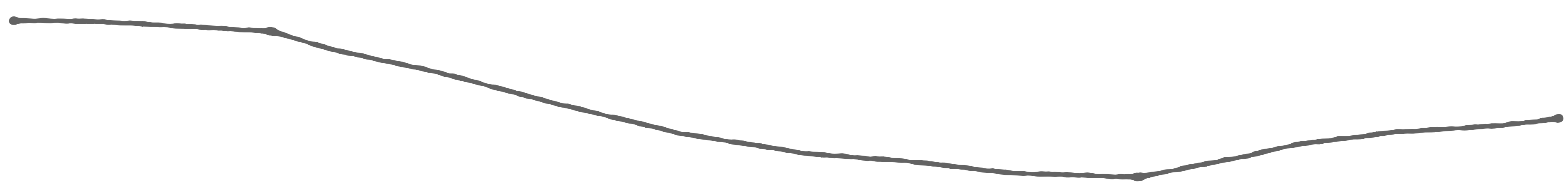
2.- Observables:

2.1 S-matrix of branons, bounds on Wilson coefficients),

2.2 Finite volume Energy spectrum.

3.- **Phenomenology** of flux tubes and YM data.

Set up: a $QFT_D$, gapped, with string like states.



for instance:

- Yang Mills Fluxe Tubes,
- Nielsen-Abrikosov stirngs,
- Domain walls in 3D Ising.


Bulk Poincaré is spontaneously broken,

$$ISO(1, D-1) \to ISO(1,1) \otimes O(D-2)$$

Goldstone modes

$$X^\mu = (\sigma^\alpha, X^i(\sigma))$$


We build the effective action out of

$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

$$K^\mu_{\alpha\beta} = \nabla_\alpha \partial_\beta X^\mu = \partial_\alpha \partial_\beta X^\mu + \cdots$$

Bulk Poincaré is spontane

$$ISO(1, D -$$

Goldstone modes

$$X^\mu = (\sigma^\alpha, X^i(\sigma))$$

$$A = \int d^2\sigma \sqrt{-h} \left[ \ell_s^{-2} + \mathcal{R} + K^2 + \alpha_3\, \ell_s^2 (K^i_{\alpha\beta} K_i^{\alpha\beta})^2 + \beta_3\, \ell_s^2 (K^i_{\alpha\beta} K^{j\,\alpha\beta})^2 + \cdots \right]$$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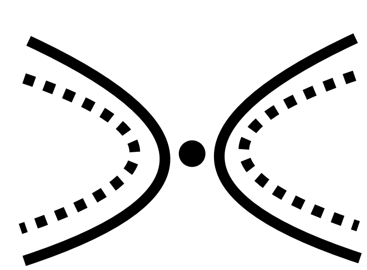

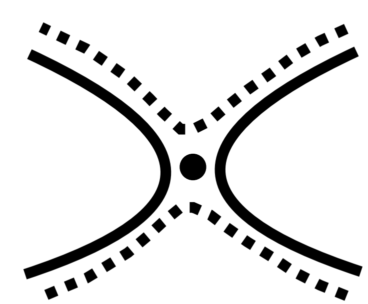

$$\gamma_3 \equiv \alpha_3 - \beta_3$$


## S-matrix

We scatter two massless vectors of O(D-2),

$$\mathbb{S}(s) = \sigma_1(s)\delta_a^b\delta_c^d + \sigma_2(s)\delta_a^c\delta_b^d + \sigma_3(s)\delta_a^d\delta_b^c$$
$$= S_{\mathrm{sing}}(s)\mathbb{P}_{\mathrm{sing}} + S_{\mathrm{sym}}(s)\mathbb{P}_{\mathrm{sym}} + S_{\mathrm{asym}}(s)\mathbb{P}_{\mathrm{asym}}$$

It is convenient to use phase-shifts

$$S_{\mathrm{rep}}(s) \equiv e^{2i\delta_{\mathrm{rep}}(s)}$$

**Unitarity:** for $s > 0$

$$|S_{\text{sing}}|^2 \equiv |(D-2)\sigma_1 + \sigma_2 + \sigma_3| \leq 1$$

$$|S_{\text{asym}}|^2 \equiv |\sigma_2 - \sigma_3| \leq 1$$

$$|S_{\text{sym}}|^2 \equiv |\sigma_2 + \sigma_3| \leq 1$$

$$\begin{aligned} \mathbb{S}(s) &= \sigma_1(s)\delta_a^b\delta_c^d + \sigma_2(s)\delta_a^c\delta_b^d + \sigma_3(s)\delta_a^d\delta_b^c \\ &= S_{\text{sing}}(s)\mathbb{P}_{\text{sing}} + S_{\text{sym}}(s)\mathbb{P}_{\text{sym}} + S_{\text{asym}}(s)\mathbb{P}_{\text{asym}} \end{aligned}$$

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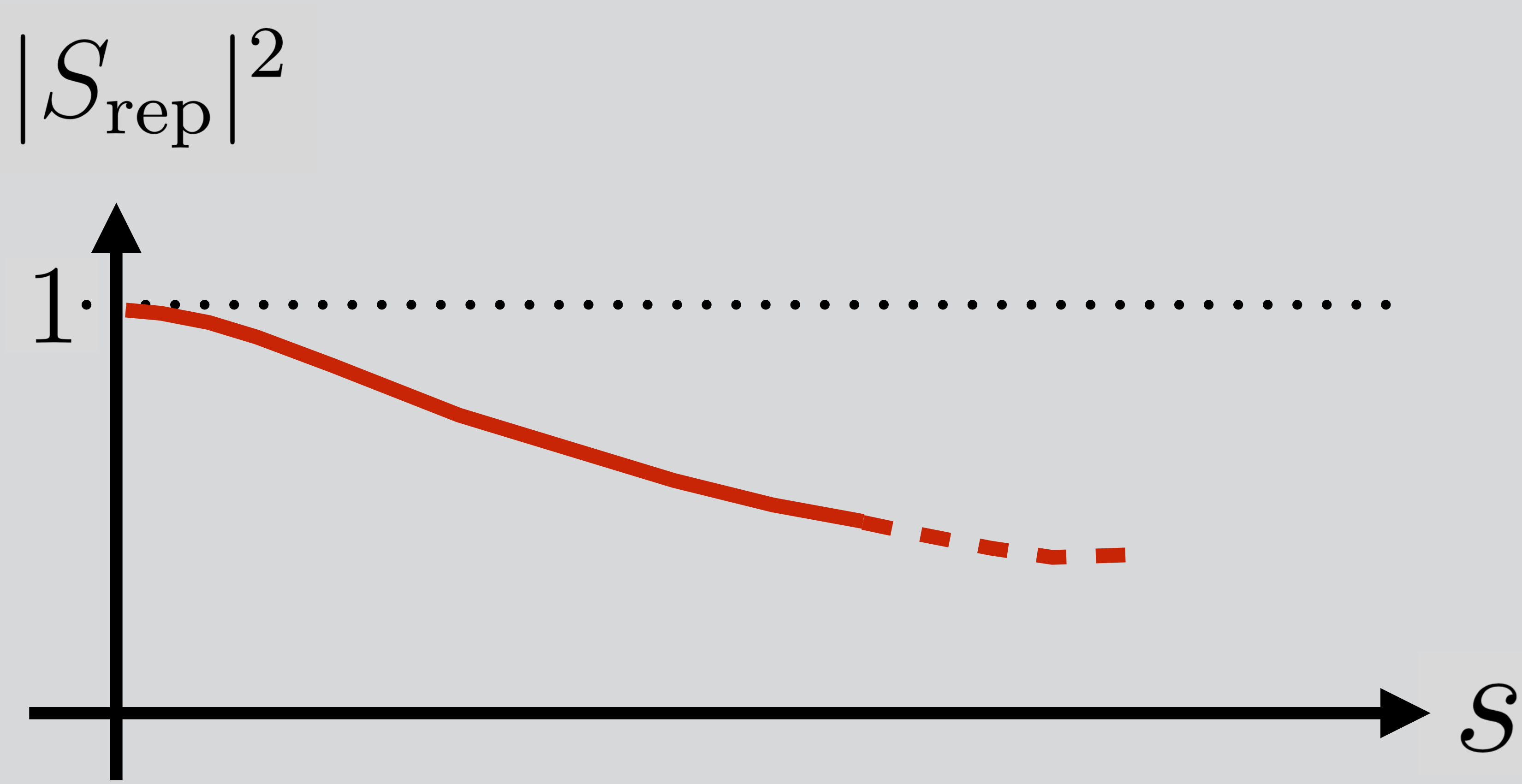
$$\mathbb{S}(s) \quad = \quad \sigma_1(s)\delta^{b}\delta^{d} + \sigma_2(s)\delta^{c}\delta^{d} + \sigma_3(s)\delta^{d}\delta^{c}$$



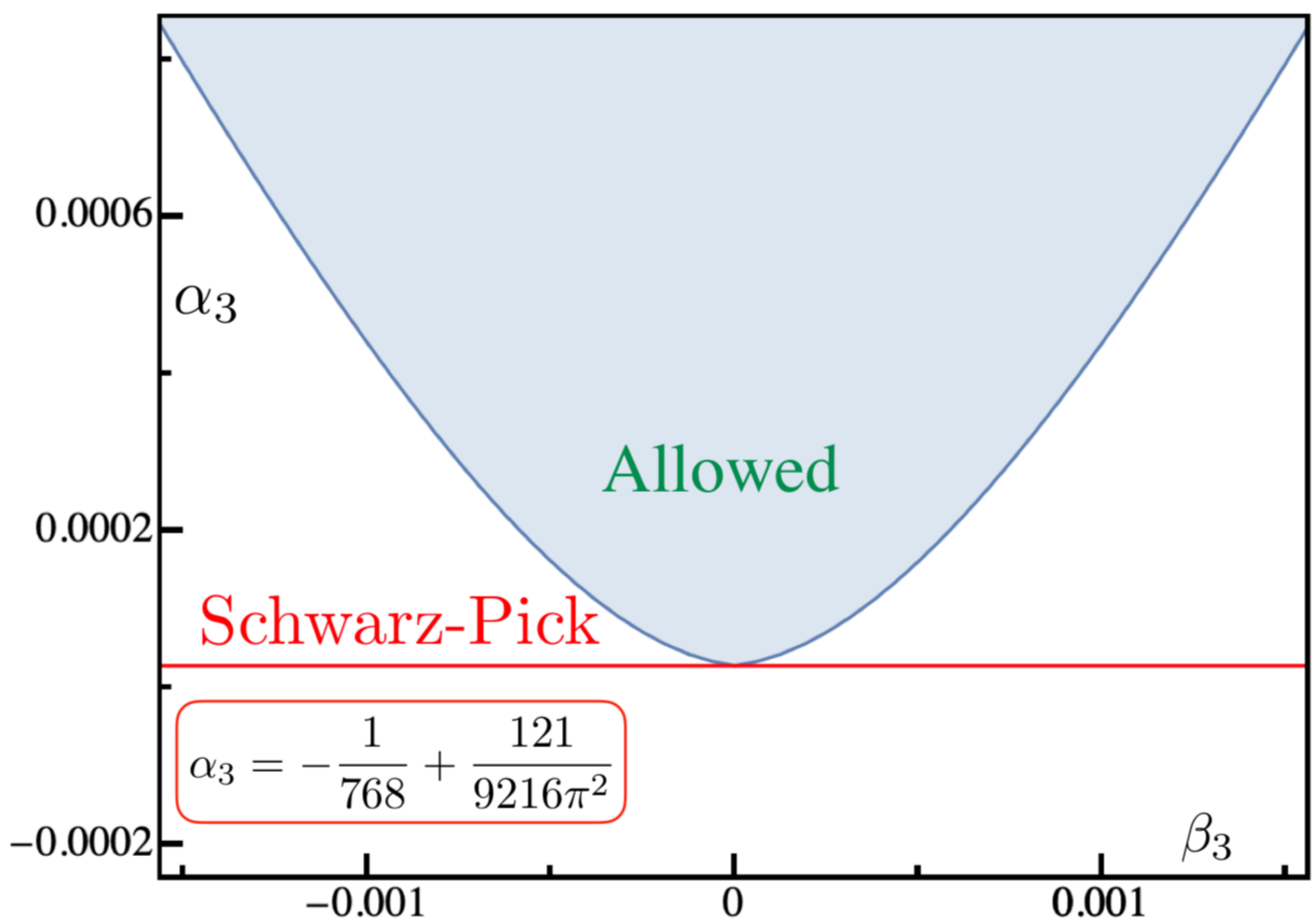
**Phase-shfits,** units $\ell_s = 1$ target Lorentz implies $\alpha_2 = \frac{D-26}{384\pi}$

Dubovsky, Flauger, Gorbenko [1203.1054]

$$2\delta_{sym} = \frac{s}{4} + \alpha_2 s^2 + \alpha_3 s^3 + O(s^4)$$

$$2\delta_{anti} = \frac{s}{4} - \alpha_2 s^2 + (\alpha_3 + 2\beta_3) s^3 + O(s^4)$$

$$2\delta_{sing} = \frac{s}{4} - (D-3)\alpha_2 s^2 + (\alpha_3 - (D-2)\beta_3) s^3 + O(s^4)$$



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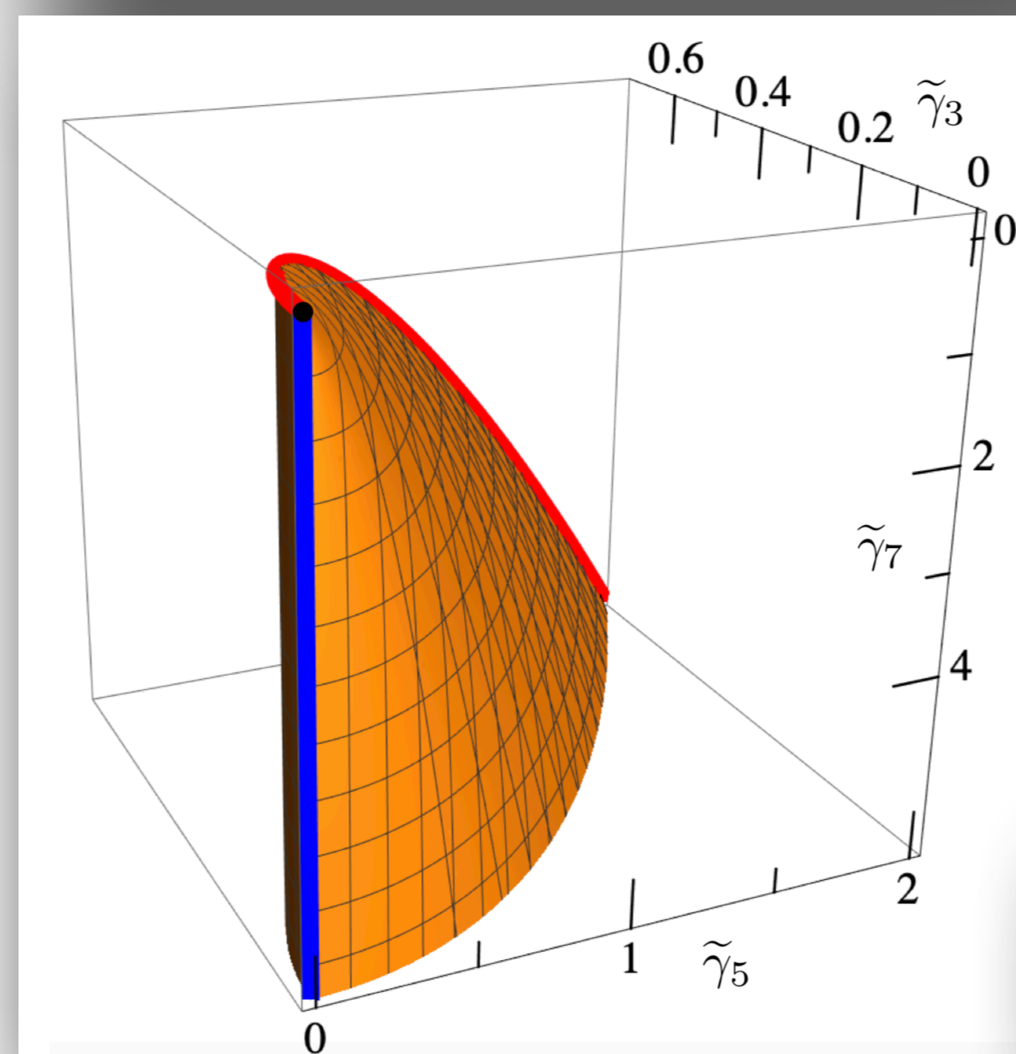
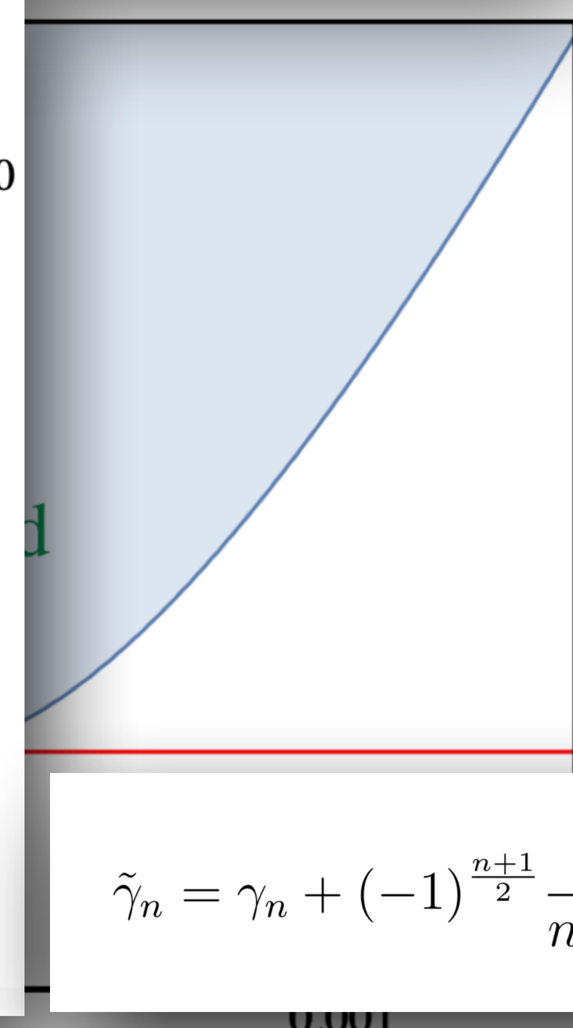
D=3

$$2\delta(s) = \frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7 + i\gamma_8 s^8 + O(s^9)$$

$\gamma_3 \equiv \alpha_3 - \beta_3$



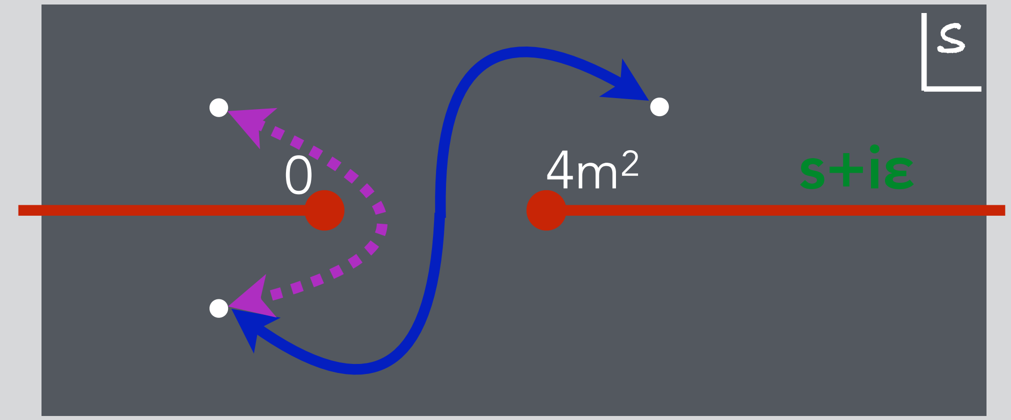




$$\tilde{\gamma}_n = \gamma_n + (-1)^{\frac{n+1}{2}} \frac{1}{n 2^{3n-1}}$$

$\text{Reality } S^*(s) = S(s^*)$

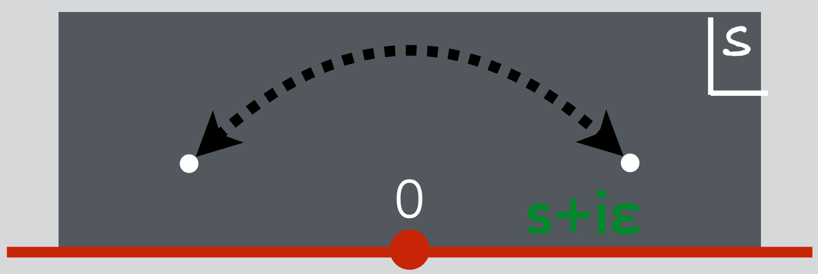
$\text{Crossing } S(4m^2 - s) = S(s)$

$|S(s + i\epsilon)|^2 \leq 1 \text{ for } s > 4m^2$

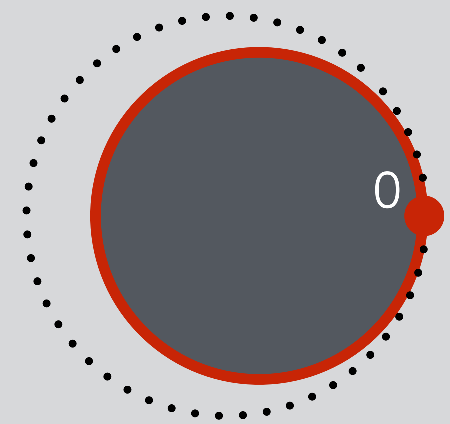



In the massless limit points in the UHP are related by $S(-s^*) = [S(s)]^*$

Unitarity and maximum modulus principle imply




$|S(s)|^2 \leq 1 \text{ for } s \in \text{ UHP}$




−0.001 0 0.001

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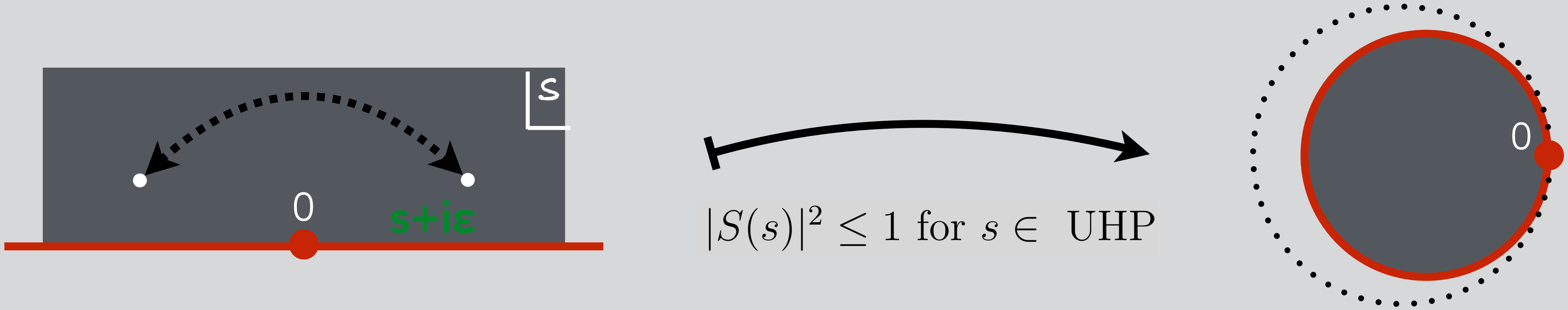


$$|S(s)|^2 \leq 1 \text{ for } s \in \text{ UHP}$$

**Trick!**

$$S^{(1)}(z|w) \equiv \frac{S(z)-S(w)}{1-S(z)\overline{S(w)}} \Big/ \frac{z-w}{z-\overline{w}}$$

Again, unitarity and maximum modulus principle imply

$$|S^{(1)}(z|w)|_{\mathrm{Im} z \geq 0} \leq 1$$

Schwarz-Pick thm.

Expansion around threshold leads to

$$S^{(1)}(ix|iy) = -1 + \left(\tfrac{1}{96} + 8\gamma_3\right) xy + \cdots \geq -1 \;, \quad \gamma_3 \geq -\frac{1}{768}$$

Generalisation to multiple points

$$\{S^{(1)}[S,w](s),\; S^{(2)}[S,w_1,w_2](s) = S^{(1)}[S^{(1)}[S,w],w_2](s), \cdots\}$$

−0.001 0 0.001

**Phase-shfits,** units $\ell_s = 1$ target Lorentz implies $\alpha_2 = \frac{D-26}{384\pi}$

$$2\delta_{sym} = \frac{s}{4} + \alpha_2 s^2 + \alpha_3 s^3 + O(s^4)$$

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$$2\delta_{sing} = \frac{s}{4} - (D-3)\alpha_2 s^2 + (\alpha_3 - (D-2)\beta_3) s^3 + O(s^4)$$

D=3

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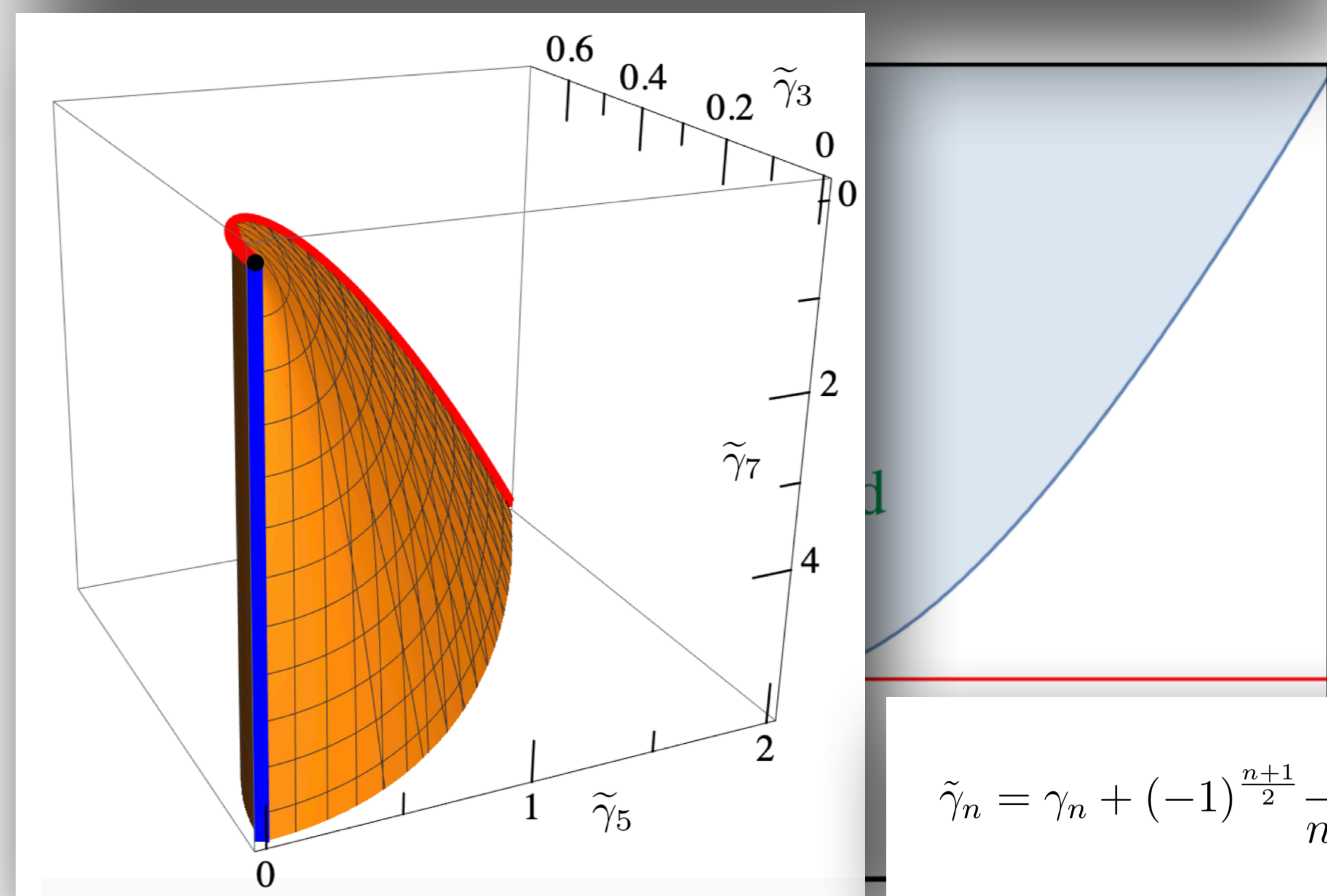


$$\tilde{\gamma}_n = \gamma_n + (-1)^{\frac{n+1}{2}} \frac{1}{n 2^{3n-1}}$$

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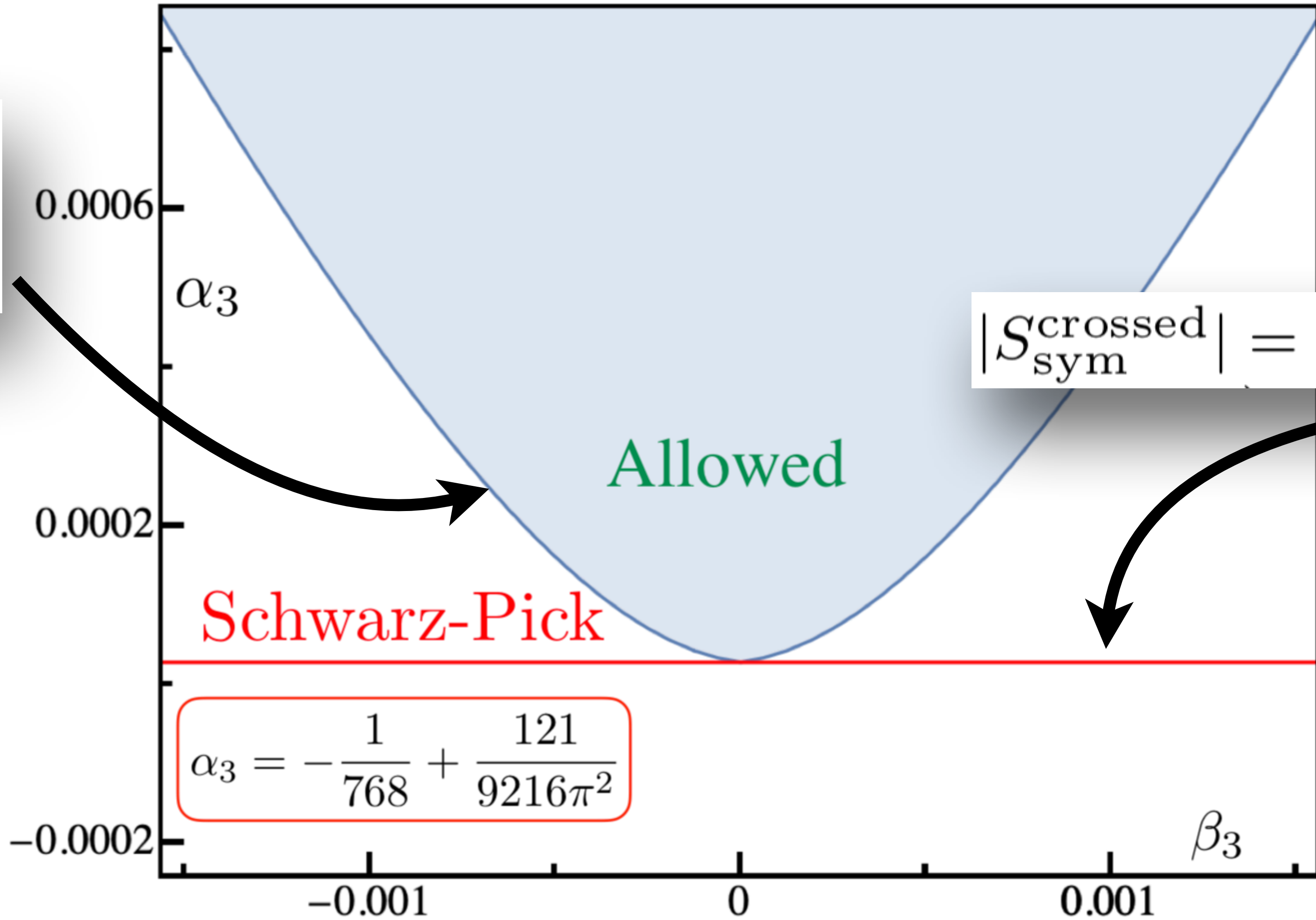
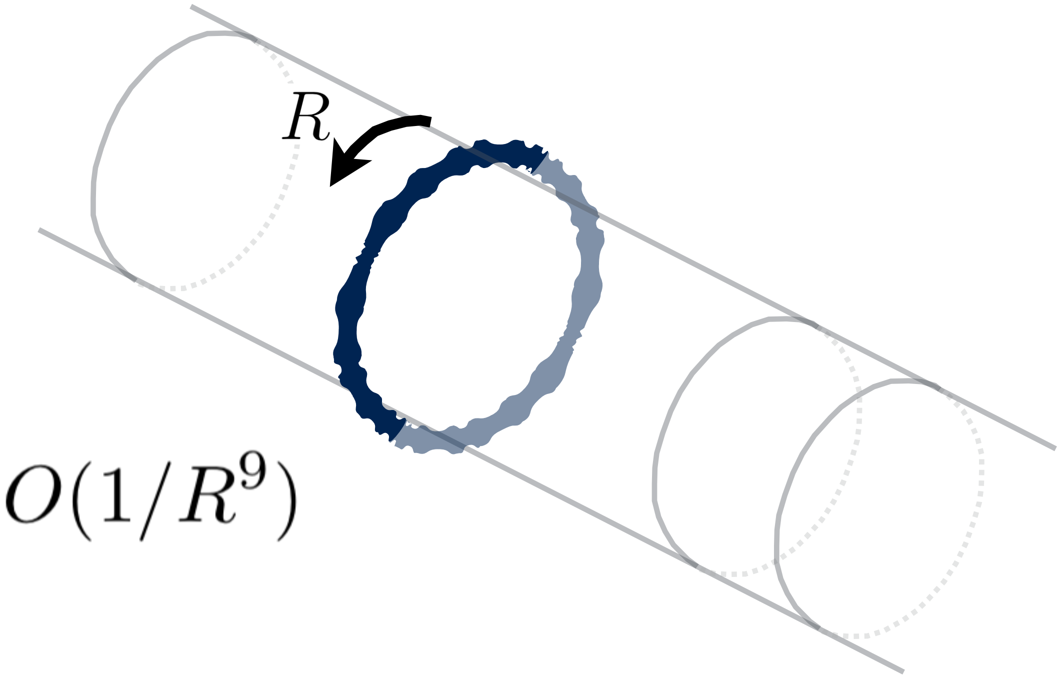
Numerics

$$S_{\text{ansatz}} = \sum_{n=0}^{N_{max}} a_n \chi^n$$

$$|S_{\text{sym}}^{\text{crossed}}| = \tfrac{1}{2}|S_{\text{sing}} + S_{\text{anti}}| \leq 1$$



## Finite volume energy levels




$$E_0(R) = \sqrt{R^2 - \tfrac{\pi}{3}(D-2)} + \frac{\delta(D)}{R^7} + O(1/R^9)$$

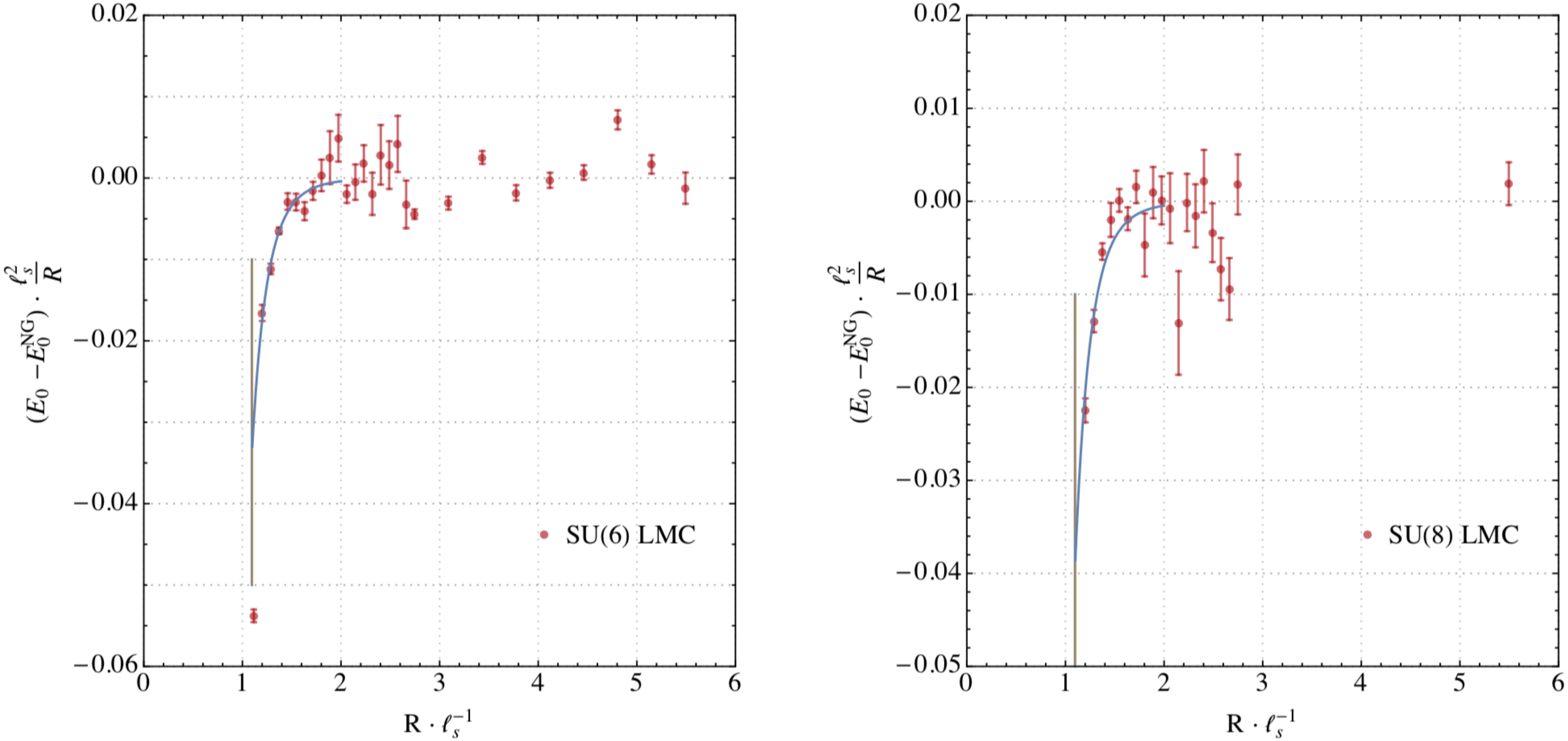
$$\delta(D) = \frac{32\pi^6(2-D)((D-2)\alpha_3 + (D-4)\beta_3)}{225}$$

$$\delta(4) = -\frac{128\pi^6\alpha_3}{225} \le \frac{\pi^6}{1350} - \frac{121\pi^4}{16200}$$

$$\delta(3) = -\frac{32\pi^6\gamma_3}{225} \le \frac{\pi^6}{5400}$$

Also splitting of degeneracies:

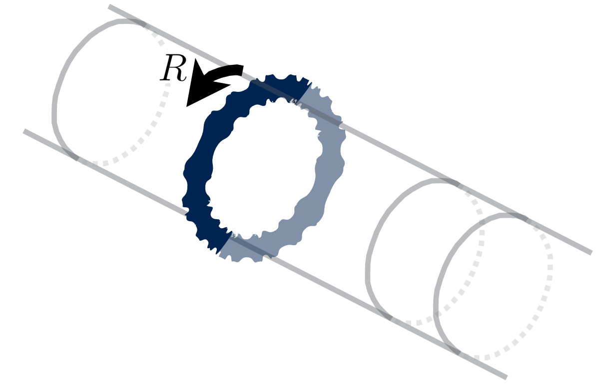
$$E_{2\text{ branons}}(2,-2) - E_{4\text{ branons}}(1,1,-1,-1) = -\frac{2455552\pi^6\gamma_3}{5R^7} + O\left(1/R^9\right) ,$$




Fit to lattice data. With ansatzs $E_0 - E_0^{NG} = 1/R^7 + 1/R^9 + \cdots$.

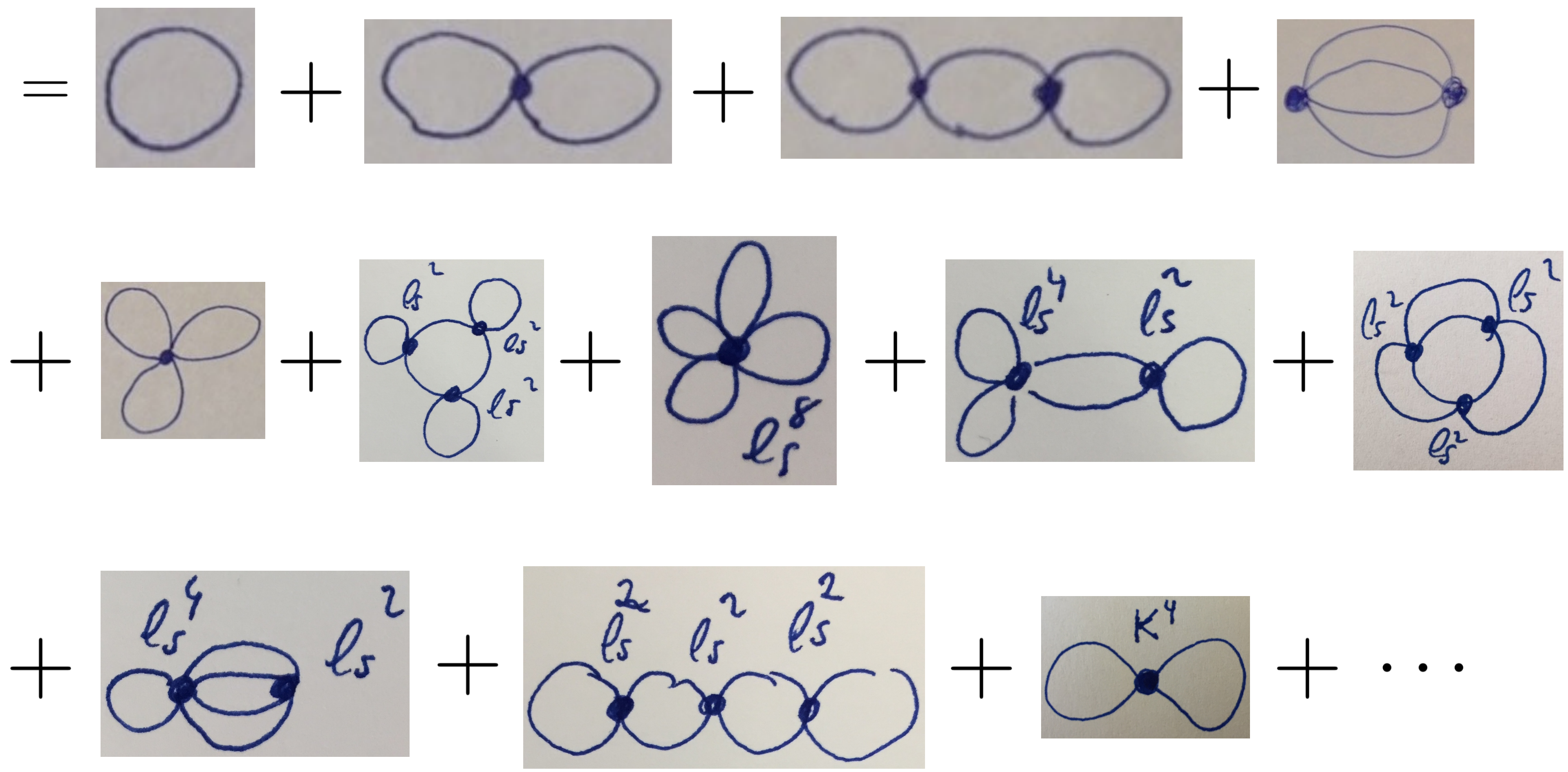
$\delta(3)$ is a small positive number compatible with the bound.
Further precision at large R is needed.
Other systems may saturate the bound.

Data from: Athenodorou, Teper hep/lat:1602.07634




It involves up to 4 loop diagrams…

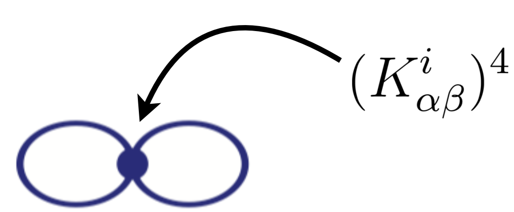
$E_0(R) =$

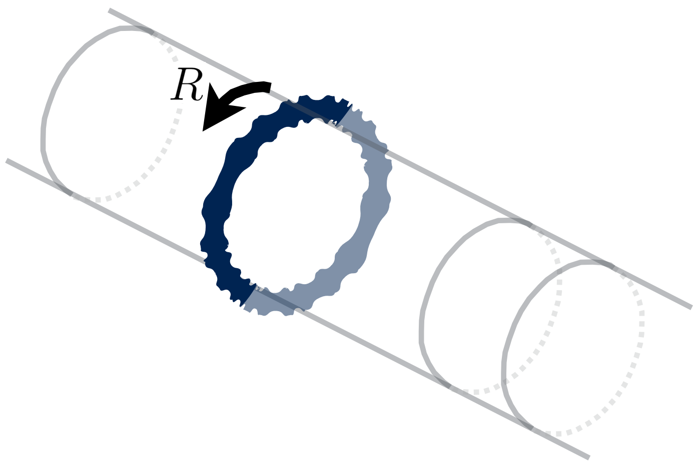


This is not how we did it.

This high order calculation is possible thanks to a trick combining

TBA +






 $= \dfrac{32\pi^6(2-D)((D-2)\alpha_3+(D-4)\beta_3)}{225R^8}$

$$a\,\ell_s^2(K^i_{\alpha\beta}K_i^{\alpha\beta})^2 + b\,\ell_s^2(K^i_{\alpha\beta}K^{j\,\alpha\beta})^2$$

$$A = A_{\text{int}} + A_{\not{\text{int}}}$$

$$(\partial_\mu\partial_\nu X^i)^2\left[(\partial_\rho X^j)^4 - \frac{1}{2}\partial_\rho X^j \partial_\sigma X^j \partial^\rho X^k \partial_\sigma X^k\right]$$

 $= \partial_\nu\partial_\alpha\partial_\beta\Delta_R(0)\,\partial_\nu\partial_\gamma\partial_\beta\Delta_R(0)\,\partial_\alpha\partial_\gamma\Delta_R(0) = 0$

Large R TBA w/

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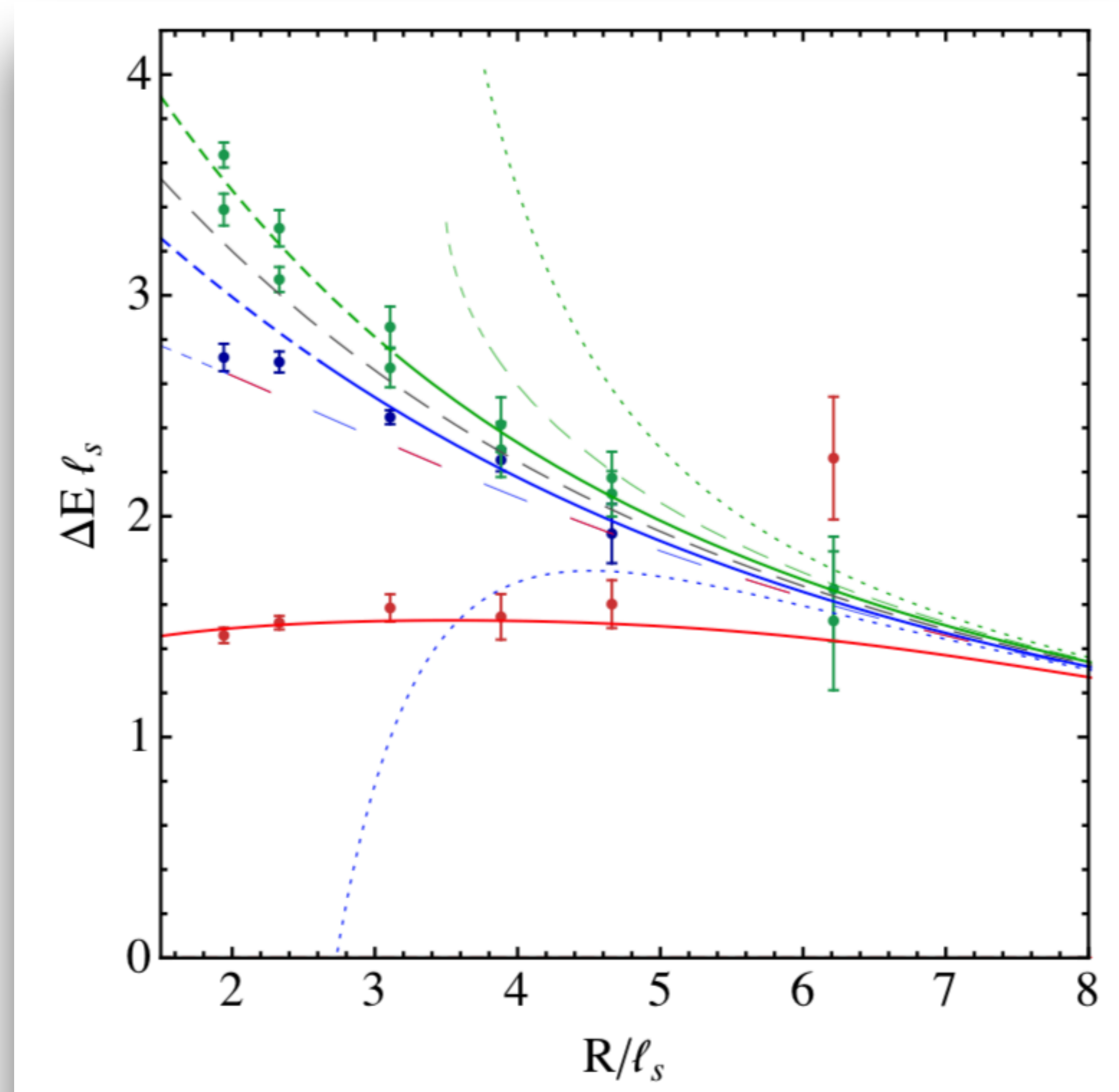
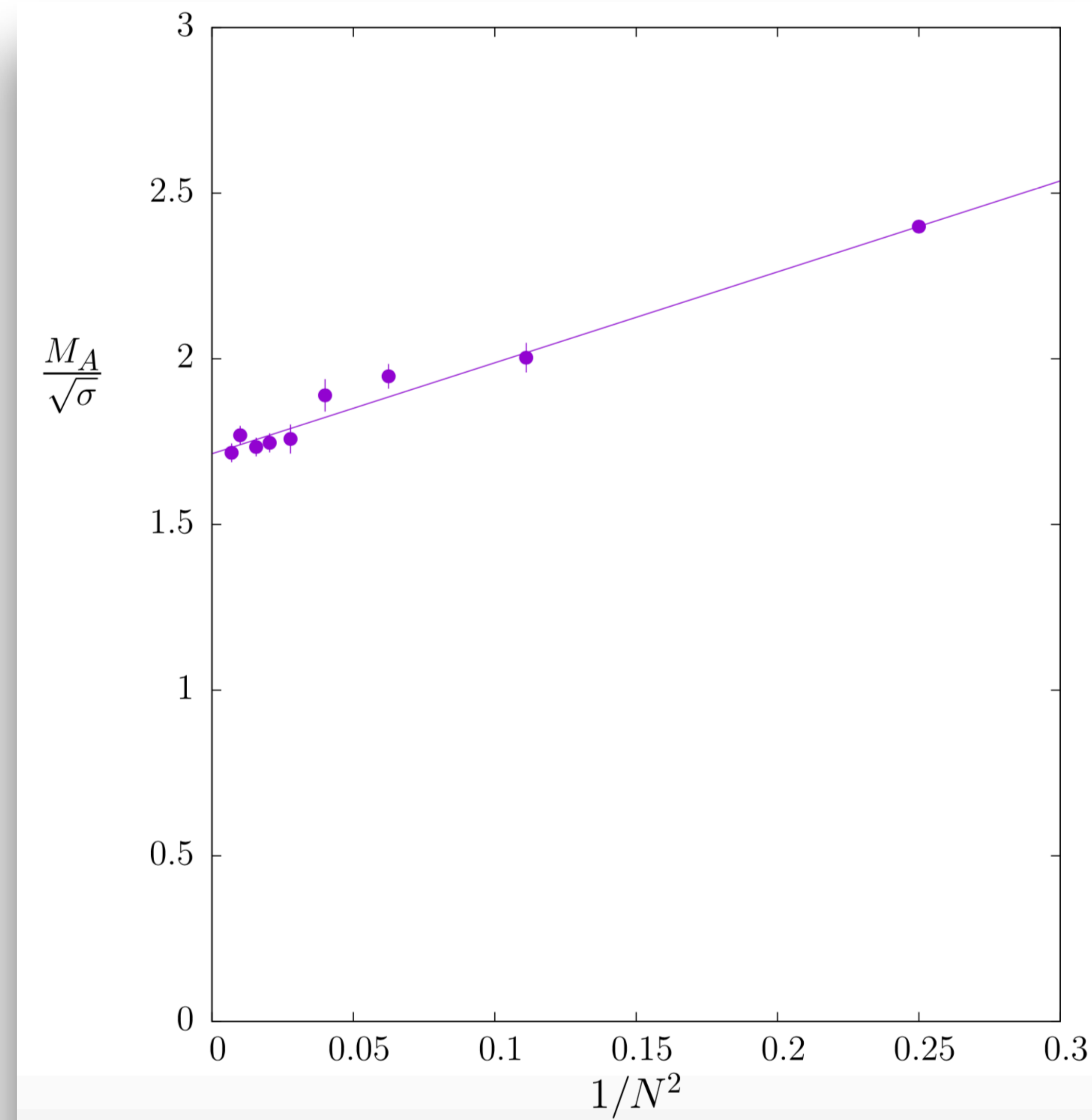
# Flux Tube Phenomenology




Athenodorou, Bringoltz, Teper [1007.4720]
Dubovsky, Flauger, Gorbenko [1301.2325], [1404.0037]
Athenodorou, Teper [1702.03717]

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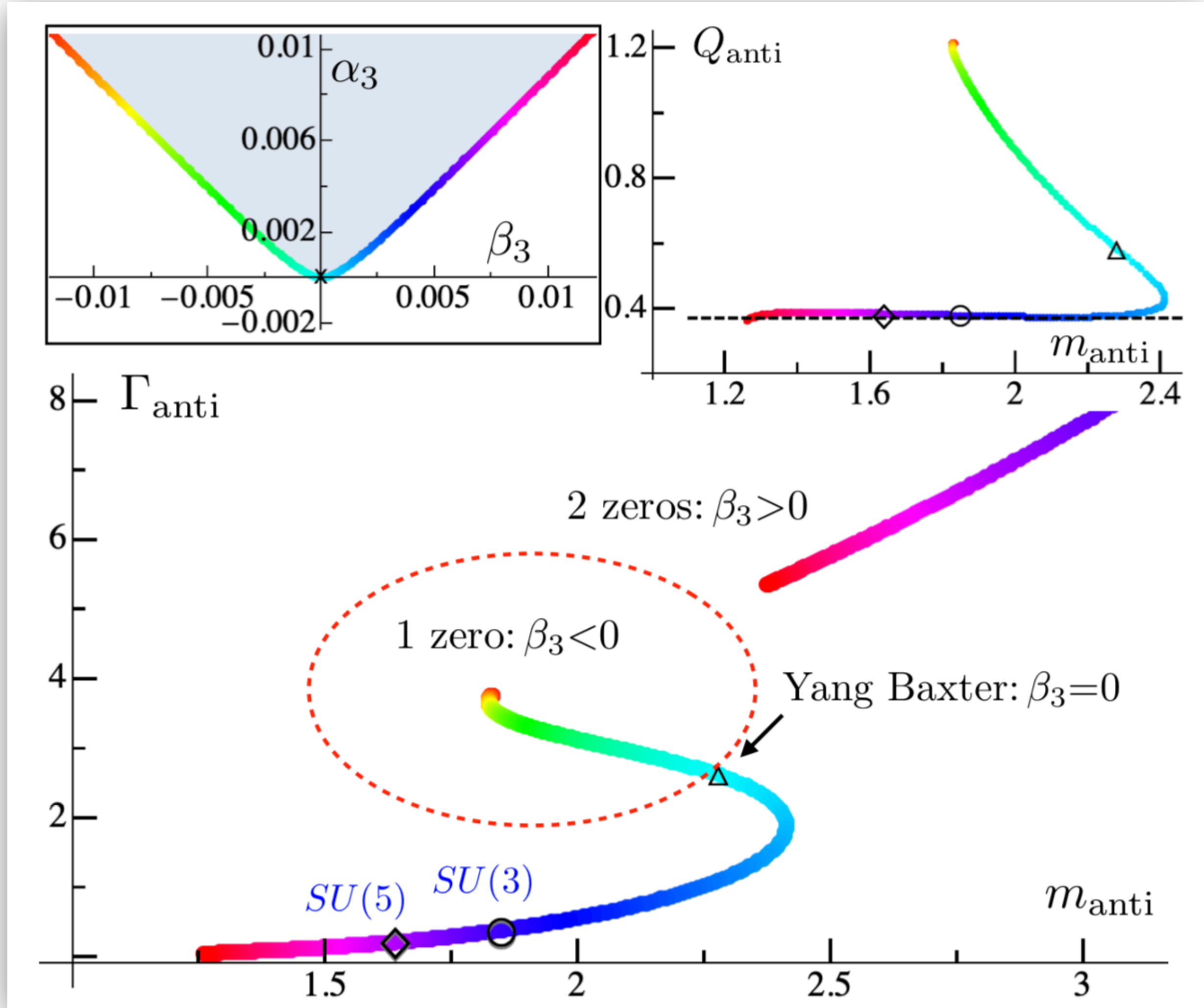


| spectrum $[m, \Gamma]$ | $SU(3)$ | $SU(5)$ |
|---|---|---|
| axion | [1.85, 0.39] | [1.64, 0.22] |
| axion* | [3.25, 8.84] | [2.83, 7.02] |
| symmetron | [2.36, 4.99] | [2.34, 4.54] |
| dilaton | [1.88, 3.37] | [1.84, 3.52] |

## Summary and outlook

- First time optimal bounds on Wilson coefficients are derived.
- Would be nice to apply similar ideas to 4D EFTs.

On the branon scattering

- Derive the D=4 Flux tube line analytically, maybe some theorem for vector valued holomorphic functions?
- Take into account what is known about universal inelasticity.
- Understand better the high energy regime.
- …