Effect of non-eikonal corrections on two particle correlations

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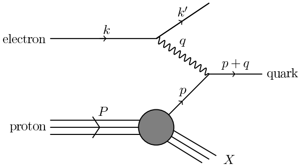
Outline

- eikonal scattering in pA collisions
- relaxing the eikonal approximation: finite width target
- from pA to pp
- non-eikonal single and double inclusive gluon production in pp
- azimuthal harmonics from non-eikonal double inclusive gluon production in pp

High energy scattering in QCD

"hard" scattering	**"soft" scattering**
→ large momentum exchange	→ small momentum exchange
→ weakly coupled	→ strongly coupled
→ perturbative	→ non-perturbative

DIS in QCD :



Three Lorentz invariant quantities :

1. $q^2 = -Q^2 \equiv$ virtuality of the incoming photon
2. $x = \frac{Q^2}{2P \cdot Q} \equiv$ longitudinal momentum fraction carried by the parton
3. $s \simeq 2P \cdot Q \equiv$ energy of the colliding $\gamma - p$ system

increasing the energy ($s = Q^2/x$) of the system:

Bjorken limit fixed x, $Q^2 \to \infty$

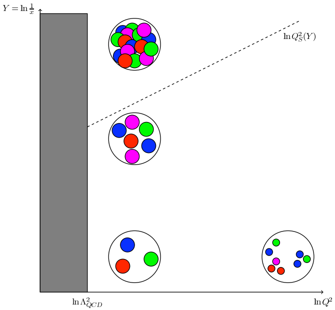
- density of partons decreases.
- system becomes more dilute!
- evolution is given by DGLAP.

Regge-Gribov limit fixed Q^2, $x \to 0$

- density of partons increases.
- system becomes dense!
- causes saturation !

Color Glass Condensate (CGC)

High energy scattering in QCD:



- Regge-Gribov limit : $x \to 0$
- at small $x \to$ **saturation**!
  - $Q_s \equiv$ saturation scale
    $\equiv \alpha_s \times$ (gluon density per unit area)
  - Q_s is a measure of the strength of the gluon interaction processes that may occur when the gluon density becomes large.
  - $Q_s \gg \Lambda_{QCD} \Rightarrow$ weak coupling
    methods can still be applied !

[McLerran, Venugopalan - hep-ph/9309289 / hep-ph/9311205]
In the saturation regime the prescription of scattering process: Color Glass Condensate (CGC)

CGC description of a process: "effective degrees of freedom" with respect to a cut off Λ^+

- fast partons : $k^+ > \Lambda^+$ $\to$ described by color sources: $J^\mu(x) = \delta^{\mu+}\rho(x^-, x_\perp)$
- slow partons: $k^+ < \Lambda^+$ $\to$ described by color fields $A^\mu(x)$

interaction between fast and slow partons: $\int d^4x J^\mu(x) A_\mu(x)$

Dilute-Dense Scattering within CGC

- *dilute-dense scattering* : saturated target / CGC formalism
  - can be applied to: DIS on A , pA collisions, forward particle production in pp.

High energy pA scattering within the CGC :

- Semi-classical approximation :
  - dense target $\equiv$ classical background field $\mathcal{A}_a^\mu(x) = O\left(\frac{1}{g}\right)$ at weak coupling g
  - dilute projectile $\equiv$ color charge $J_a^\mu(x) = O(g)$
- Eikonal approximation:
  - take the high energy limit $s \to \infty$.
  - drop power-suppressed contributions.

In the semi-classical approximation, the eikonal limit can be obtained by either boosting the projectile or the target or both...

Dilute-Dense Scattering

Boosting the target:

$$A_a^\mu(x) \mapsto \begin{cases} \gamma_t\, A_a^-\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ \frac{1}{\gamma_t} A_a^+\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ A_a^i\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \end{cases}$$

- $A_a^- \gg A_a^i \gg A_a^+$ in a generic gauge
- in the light-cone gauge:

$A_a^\mu(x) = \delta^{\mu-}\delta(x^+)A_a^-(\mathbf{x})$

target is localized at $x^+ = 0$

independent of x^-

Boosting the projectile :

$$J_a^\mu(x) \mapsto \begin{cases} \frac{1}{\gamma_p}\, J_a^-\left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ \gamma_p\, J_a^+\left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ J_a^i\left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \end{cases}$$

- $J_a^+ \gg J_a^i \gg J_a^-$
- slow x^+ dependence due to Lorentz time dilation

$J_a^\mu(x) \propto \delta^{\mu+}\delta(x^-)\rho^a(\mathbf{x})$

projectile is localized at $x^- = 0$

Corrections beyond eikonal accuracy

At the level of the background field, the eikonal approximation amounts to

1. $\mathcal{A}^{\mu}_{a}(x) \simeq \delta^{\mu -}\mathcal{A}^{-}_{a}(x)$
2. $\mathcal{A}^{\mu}_{a}(x) \simeq \mathcal{A}^{\mu}_{a}(x^{+}, \mathbf{x})$
3. $\mathcal{A}^{\mu}_{a}(x) \propto \delta(x^{+})$

Relaxing any of these approximations will give correction to the strict eikonal limit! Three sources of corrections to eikonal approximation:

1. other components of the target background field $\mathcal{A}^{\mu}_{a}(x)$
2. dynamics of the target : x^{-} dependence of $\mathcal{A}^{\mu}_{a}(x)$
3. Finite width L^{+} of the target along x^{+}

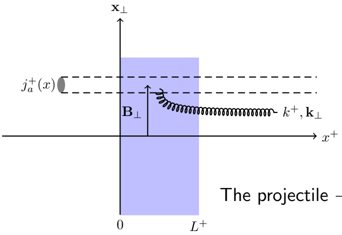
When the target is a large nucleus, the dominant contribution beyond the eikonal accuracy is obtained by relaxing the 3rd approximation because of the $A^{1/3}$ nuclear enhancement of the finite width target!

$$\mathcal{A}^{\mu} = \delta^{\mu -}\delta(x^{+})\mathcal{A}^{-}(\mathbf{x}) \rightarrow \mathcal{A}^{\mu} = \delta^{\mu -}\mathcal{A}^{-}(x^{+}, \mathbf{x})$$

Finite width target: relaxing the eikonal approximation

[T.A., N. Armesto, G. Beuf, M. Martinez, C.A. Salgado - 2014]
[T.A., N. Armesto, G. Beuf, A. Moscoso - 2015]

Consider a finite width target :



The target $\rightarrow \mathcal{A}^\mu(x) \equiv \delta^{\mu -} \mathcal{A}_a^-(x^+, \mathbf{x})$

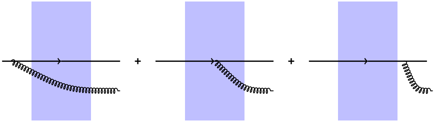
The projectile $\rightarrow j_a^\mu(x) \propto \delta^{\mu +} \delta(x^-)\, \rho^b(\mathbf{x} - \mathbf{B})$

The single inclusive gluon cross section for pA:

$$(2\pi)^3 \, (2k^+) \, \frac{d\sigma}{dk^+ \, d^2\mathbf{k}} = \int d^2\mathbf{B} \sum_{\lambda \, \text{phys.}} \left\langle \left\langle |\mathcal{M}_\lambda^a(\underline{k}, \mathbf{B})|^2 \right\rangle_p \right\rangle_A$$

gluon production amplitude

Properties of the background retarded gluon propagator

$\mathcal{M}^a_\lambda(\underline{k}, \mathbf{B}) =$ 

at LO in g, LSZ reduction formula $\Rightarrow \mathcal{M}^a_\lambda(\underline{k}, \mathbf{B}) = \varepsilon^{\mu *}_\lambda \int d^4x\, e^{ik\cdot x}\, \Box_x A^a_\mu(x)$

power counting : dense target $\Rightarrow \mathcal{A}^-_a(x^+, \mathbf{x}) = \mathcal{O}(1/g)$

dilute projectile $\Rightarrow j^+_a(\underline{x}) = \mathcal{O}(g)$

perturbative expansion of the classical field : $A^\mu_a(x) = \mathcal{A}^\mu_a(x) + a^\mu_a(x) + \mathcal{O}(g^3)$

$\downarrow$

$-i \int d^4y\, G^{\mu -}_R(x,y)_{ab}\, j^+_b(y)$

In the LC gauge : $A^+_a = 0$ & $\varepsilon^{+*}_\lambda = 0 \Rightarrow \varepsilon^{\mu *}_\lambda A^a_\mu(x) = -\varepsilon^{i*}_\lambda a^i_a$

$$\mathcal{M}^a_\lambda(\underline{k}, \mathbf{B}) = \varepsilon^{i*}_\lambda\, (2k^+) \lim_{x^+\to+\infty} \int d^2\mathbf{x} \int dx^-\, e^{ik\cdot x} \int d^4y\, G^{i-}_R(x,y)_{ab}\, j^+_b(y)$$

$G^{\mu\nu}_R(x,y)_{ab}$ is the background retarded gluon propagator

[F. Gelis, R. Venugopalan - 2004 / Y. Mehtar-Tani - 2007]

Properties of the background retarded gluon propagator

$\mathcal{A}_a^-(x^+,\mathbf{x})$ is independent of the x^-, so it is convenient to introduce the 1-d Fourier transform of $G_R^{\mu\nu}(x,y)_{ab}$

$$G_R^{\mu\nu}(x,y)_{ab} = \int \frac{\mathrm{d}k^+}{2\pi}\, e^{-ik^+(x^- - y^-)}\, \frac{1}{2(k^+ + i\epsilon)}\, \mathcal{G}_{k^+}^{\mu\nu}(\underline{x};\underline{y})_{ab}$$

Conveniently, the $(i-)$ component of the the background retarded propagator can be written in terms of the scalar background propagator:

$$G_R^{i-}(x,y)_{ab} = \int \frac{\mathrm{d}k^+}{2\pi}\, e^{-ik^+(x^- - y^-)}\, \frac{i}{2(k^+ + i\epsilon)^2}\, \mathcal{G}_{k^+}^{i-}(\underline{x};\underline{y})_{ab}$$

$$\boxed{\mathcal{G}_{k^+}^{i-}(\underline{x};\underline{y})^{ab} = \frac{i}{k^+ + i\epsilon}\, \partial_{\mathbf{y}^i}\, \mathcal{G}_{k^+}^{ab}(\underline{x};\underline{y})}$$

$\mathcal{G}_{k^+}^{ab}(\underline{x};\underline{y})$ satisfies the scalar Green's eq. whose solution can be written formally as a path integral

$$\mathcal{G}_{k^+}^{ab}(\underline{x};\underline{y}) = \theta(x^+ - y^+) \int_{\mathbf{z}(y^+)=\mathbf{y}}^{\mathbf{z}(x^+)=\mathbf{x}} \mathcal{D}\mathbf{z}(z^+)\, \exp\left[\frac{ik^+}{2}\int_{y^+}^{x^+} dz^+\, \dot{\mathbf{z}}^2(z^+)\right] \mathcal{U}^{ab}\Big(x^+, y^+, \big[\mathbf{z}(z^+)\big]\Big)$$

with the Wilson line

$$\mathcal{U}^{ab}\Big(x^+, y^+, \big[\mathbf{z}(z^+)\big]\Big) = \mathcal{P}_+ \exp\left\{ig \int_{y^+}^{x^+} dz^+\, T\cdot\mathcal{A}^-\Big(z^+, \mathbf{z}(z^+)\Big)\right\}^{ab}$$

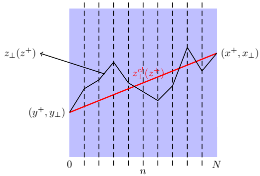
following the Brownian trajectory $\mathbf{z}(z^+)$.

AIM: Perform an eikonal expansion of $\mathcal{G}_{k^+}^{ab}(\underline{x};\underline{y})$.

Expanding the background propagator

(i) discretize the background propagator.

(ii) Perturbative expansion around free classical path:



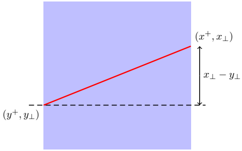
eikonal limit : $\frac{k^+}{(x^+ - y^+)} \gg Q_\perp^2$ in the problem

$\mapsto$ large k^+ limit (classical free path!)

$\Rightarrow$ perturbative expansion around the free classical path:

$\mathbf{z}_n = \mathbf{z}_n^{\text{cl}} + \mathbf{u}_n$ with $\mathbf{z}_n^{\text{cl}} = \mathbf{y} + \frac{n}{N}(\mathbf{x} - \mathbf{y})$

(iii) Expansion around the initial transverse position:



The first expansion is performed for fixed initial and final positions.

In the large k^+ limit , the result has to be re-expanded since $\mathbf{z}^{\text{cl}}(z^+) - \mathbf{y}$ is small at each step.

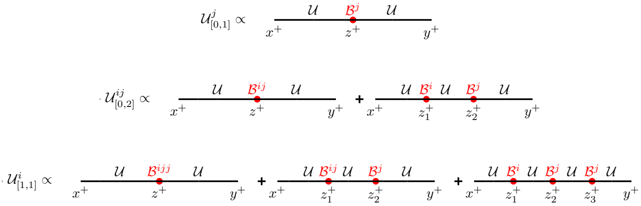
Background scalar propagator at NNE accuracy

After all:

$$\int d^2\mathbf{x}\; e^{-i\mathbf{k}\cdot\mathbf{x}}\; \mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+-y^+)\; e^{-i\mathbf{k}\cdot\mathbf{y}}\; e^{-ik^-(x^+-y^+)}\Bigg\{\mathcal{U}(x^+,y^+,\mathbf{y})$$
$$+\frac{(x^+-y^+)}{k^+}\bigg[\mathbf{k}^i\,\mathcal{U}^i_{[0,1]}(x^+,y^+,\mathbf{y}) + \frac{i}{2}\,\mathcal{U}_{[1,0]}(x^+,y^+,\mathbf{y})\bigg]$$
$$+\frac{(x^+-y^+)^2}{(k^+)^2}\bigg[\mathbf{k}^i\mathbf{k}^j\mathcal{U}^{ij}_{[0,2]}(x^+,y^+;\mathbf{y}) + \frac{i}{2}\mathbf{k}^i\mathcal{U}^i_{[1,1]}(x^+,y^+;\mathbf{y}) - \frac{1}{4}\mathcal{U}_{[2,0]}(x^+,y^+;\mathbf{y})\bigg]\Bigg\}^{ab}$$

- $\mathcal{U}(x^+,y^+,\mathbf{y}) \equiv$ standard Wilson lines that appears only at the eikonal level as expected.
- $\mathcal{U}_{[\alpha,\beta]}(x^+,y^+,\mathbf{y}) \equiv$ decorated Wilson lines that only appears beyond eikonal accuracy.
- The subscripts
  - α stands for the order of the expansion around the classical path.
  - β stands for the order of the expansion around the initial transverse position.

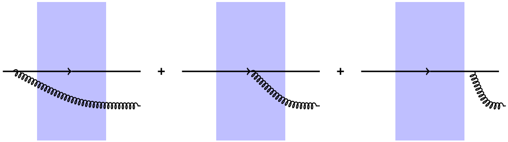
Structure of the decorated Wilson lines



with

$$
\begin{aligned}
\mathcal{B}^{i}(z^+,\mathbf{y}) &\equiv igT\cdot\partial_{\mathbf{y}^i}\mathcal{A}^-(z^+,\mathbf{y}),\\
\mathcal{B}^{ij}(z^+,\mathbf{y}) &\equiv igT\cdot\partial_{\mathbf{y}^i}\partial_{\mathbf{y}^j}\mathcal{A}^-(z^+,\mathbf{y}),\\
\mathcal{B}^{ijl}(z^+,\mathbf{y}) &\equiv igT\cdot\partial_{\mathbf{y}^i}\partial_{\mathbf{y}^j}\partial_{\mathbf{y}^l}\mathcal{A}^-(z^+,\mathbf{y}),
\end{aligned}
$$

Total amplitude at next-to-eikonal accuracy



strict eikonal term!

$$\overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k},\mathbf{q}) = i\,\varepsilon^{i*}_{\lambda}\int d^2\mathbf{y}\; e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{y}}\Bigg\{2\left[\frac{\mathbf{k}^i}{\mathbf{k}^2}-\frac{\mathbf{q}^i}{\mathbf{q}^2}\right]\mathcal{U}(L^+,0;\mathbf{y})$$

$$+\frac{L^+}{k^+}\left[\delta^{ij}-2\frac{\mathbf{q}^i}{\mathbf{q}^2}\mathbf{k}^j\right]\mathcal{U}^j_{[0,1]}(L^+,0;\mathbf{y}) - i\frac{L^+}{k^+}\frac{\mathbf{q}^i}{\mathbf{q}^2}\mathcal{U}_{[1,0]}(L^+,0;\mathbf{y}) + O\left(\left(\frac{L^+}{k^+}\,\partial^2_{\perp}\right)^2\right)\Bigg\}^{ab}$$

Next-to-eikonal corrections!!!

Squared amplitude at next-to-eikonal accuracy

By defining the new operators as

$$S_A(\mathbf{r},\mathbf{b}) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;\mathbf{b}-\tfrac{\mathbf{r}}{2}\right)\mathcal{U}\left(L^+,0;\mathbf{b}+\tfrac{\mathbf{r}}{2}\right)\right]\right\rangle_A$$

$$\mathcal{O}^j_{[0,1]}(\mathbf{r},\mathbf{b}) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;\mathbf{b}-\tfrac{\mathbf{r}}{2}\right)\mathcal{U}^j_{[0,1]}\left(L^+,0;\mathbf{b}+\tfrac{\mathbf{r}}{2}\right)\right]\right\rangle_A$$

$$\mathcal{O}_{[1,0]}(\mathbf{r},\mathbf{b}) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;\mathbf{b}-\tfrac{\mathbf{r}}{2}\right)\mathcal{U}_{[1,0]}\left(L^+,0;\mathbf{b}+\tfrac{\mathbf{r}}{2}\right)\right]\right\rangle_A$$

Square of the reduced amplitude then reads

$$\frac{1}{N_c^2-1}\sum_{\lambda\,\mathrm{phys.}}\left\langle \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})^\dagger\,\overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})\right\rangle_A = \frac{1}{\mathbf{k}^2\,\mathbf{q}^2}\int_{\mathbf{b},\mathbf{r}} e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}\Big\{4\,(\mathbf{k}-\mathbf{q})^2 S_A(\mathbf{r},\mathbf{b})$$
$$+\frac{L^+}{k^+}\Big[f(\mathbf{k}^i,\mathbf{q}^i)\underbrace{\big[\mathcal{O}^j_{[0,1]}(\mathbf{r},\mathbf{b})+\mathcal{O}^j_{[0,1]}(-\mathbf{r},\mathbf{b})\big]} + g(\mathbf{k},\mathbf{q})\underbrace{\big[\mathcal{O}_{[1,0]}(\mathbf{r},\mathbf{b})-\mathcal{O}_{[1,0]}(-\mathbf{r},\mathbf{b})\big]}\Big]\Big\}$$

Vanish upon integration over **b** due to rotational symmetry

$$k^+\frac{d\sigma}{dk^+\,d^2\mathbf{k}} = \frac{1}{\mathbf{k}^2}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\,\varphi_p(\mathbf{q})\,(\mathbf{k}-\mathbf{q})^2\int_{\mathbf{b},\mathbf{r}} e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}\,S_A(\mathbf{r},\mathbf{b}) + O\left(\left(\frac{L^+}{k^+}\partial_\perp^2\right)^2\right)$$

Well known $k_\perp$ factorization formula!!

The first correction appears at NNEik order.

Dilute target limit and the modified Lipatov vertex

[T.A., A. Dumitru - 2015]

go from pA → pp:

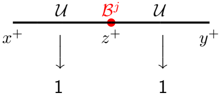
- *dilute limit of the target:*

expand the standard & decorated Wilson lines to first order in the background field.

- *Standard Wilson line*: $U_{ab}(x) \approx 1 + igT^c_{ab} \int_{x^+ q} e^{iqx} A^-_c(x^+, q)$

- *the first decorated Wilson line*:

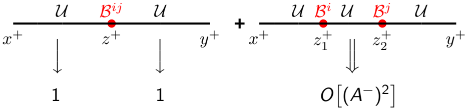
$\mathcal{U}^j_{[0,1]} \propto$



$$U^{i,ab}_{[0,1]}(x^+, y^+, y) = \int_{y^+}^{x^+} dz^+ \frac{z^+ - y^+}{x^+ - y^+} \left[igT^e_{ab} \partial_{y_i} A^{-,e}(z^+, y) \right]$$

- *the second decorated Wilson line*:

$\mathcal{U}^{ij}_{[0,2]} \propto$



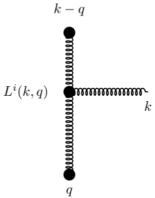
Dilute target limit and the modified Lipatov vertex

[T.A., A. Dumitru - 2015]

- summing up all the NEik and NNEik terms in the dilute target limit, one gets

$$\mathcal{M} \propto \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right]\left\{1 + i\frac{k^2}{2k^+}x^+ - \frac{1}{2}\left(\frac{k^2}{2k^+}x^+\right)^2\right\}$$

- $O(1)$ term $\rightarrow$ eikonal Lipatov vertex.



$$L^i(k,q) = \frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}$$

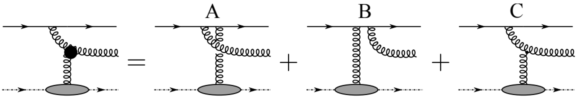
- we get NEik and NNEik corrections to the Lipatov vertex.

- the form suggests exponentiation. However, we do not know the corrections beyond NNEik accuracy!

Dilute target limit and the modified Lipatov vertex

[P. Agostini, T.A., N. Armesto - 2019]

- calculate the diagrams by keeping the phase $e^{ik^- x^+}$ which is taken to be 1 in the eikonal limit.



The total amplitude reads

$$i(\mathcal{M}_A + \mathcal{M}_B + \mathcal{M}_C) \propto \int \frac{d^2q}{(2\pi)^2} L^i(k,q) e^{ik^- x_1^+} A_a^-(k^-, q) e^{-iq\cdot x_1}$$

with $L^i(k,q)$ is the standard Lipatov vertex

$$\boxed{L^i(k,q) = \frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}}$$

and the non-eikonal Lipatov vertex being

$$\boxed{L^i_{\mathrm{NE}}(\underline{k}, q; x^+) = \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right] e^{ik^- x^+}}$$

$k^- = \frac{k^2}{2k^+}$

[U. A. Wiedemann - 2000 / Y. Mehtar-Tani, C. A. Salgado, K. Tywoniuk - 2011]

Noneikonal single inclusive gluon production

Single inclusive gluon production in pA collisions (eikonal accuracy):

$$\frac{d\sigma}{d^2k d\eta} \propto \int_{zx\bar{z}y} e^{ik(z-\bar{z})} A^i(x-z)\, A^i(\bar{z}-y) \left\langle \rho^a(x)\rho^b(y) \right\rangle_P \left\langle [U_z - U_x]^{ac} [U^\dagger_{\bar{z}} - U^\dagger_y]^{cb} \right\rangle_T$$

- *projectile averaging:* in x-space $\rightarrow \langle \rho^a(x)\rho^b(y)\rangle_P = \delta^{ab}\mu^2(x,y)$
  in p-space $\rightarrow \langle \rho^a(k)\rho^b(p)\rangle_P = \delta^{ab}\mu^2(k,p) = \delta^{ab}\, T\left(\frac{k-p}{2}\right) F\left[(k+p)R\right]$

T $\rightarrow$ transverse momentum dependent distribution of the color charge densities
F $\rightarrow$ soft form factor which is peaked when its argument vanihes

Single inclusive gluon production in pp collisions (eikonal accuracy):

- *dilute target limit* $\rightarrow U_{ab}(x) \approx 1 + igT^c_{ab} \int_{x^+ q} e^{iqx} A^-_c(x^+, q)$

$$\left.\frac{d\sigma}{d^2k d\eta}\right|_{\rm dilute} \propto \int_{x_1^+ x_2^+ q_1 q_2} L^i(k,q_1)\, L^i(k,q_2)\, \mu^2\left[k-q_1, k-q_2\right] \left\langle A^-_c(x_1^+, q_1) A^-_{\bar{c}}(x_2^+, q_2) \right\rangle_T$$

- *go from eikonal to non-eikonal:* $L^i(k,q) \rightarrow L^i_{\rm NE}(\underline{k}, q; x^+)$

$\underline{k} \equiv (k^+, k)$

Noneikonal single inclusive gluon production

target averaging:

• Adopt a modified expression for the correlator of two target fields:

Since the target has finite longitudinal length, the target fields can be located at two different longitudinal positions. We consider a generalization of the MV model in which the two color fields are located at different longitudinal positions.

$$\left\langle A^-_c(x_1^+, q_1) A^-_{\bar{c}}(x_2^+, q_2)\right\rangle_T = \delta^{c\bar{c}} n(x_1^+) \frac{1}{2\lambda^+} \Theta\big(\lambda^+ - |x_1^+ - x_2^+|\big)(2\pi)^2 \delta^{(2)}(q_1 - q_2)|a(q_1)|^2$$

• $\lambda^+ \equiv$ color correlation length in the target ($\lambda^+ \ll L^+$)

• $n(x^+) \equiv$ 1-d target density along longitudinal direction

($n(x^+) = n_0$ for $0 \leq x^+ \leq L^+$ and 0 elsewhere)

• $a(q) \equiv$ functional form of the potential in p-space

It is Yukawa type $\rightarrow |a(q)|^2 = \frac{\mu_T^2}{(q^2+\mu_T^2)^2}$ with μ_T is Debye screening mass.

In the limit $\lambda^+ \rightarrow 0$ together with a constant potential $|a(q)|^2$ and constant 1-d target density, the correlator goes to standard MV model one.

Noneikonal single inclusive gluon production

When we plug this back in the X-section we get

$$\frac{d\sigma}{d^2k\,d\eta}\bigg|_{\rm dilute}^{\rm NE} \propto \int_q |a(q)|^2 \mu^2\big[k-q, q-k\big] L^i(k,q)L^i(k,q)\, n_0\, \frac{1}{2\lambda^+} \int_0^{L^+} dx_1^+ \int_{x_1^+-\lambda^+}^{x_1^+-\lambda^+} dx_2^+ \, e^{i\frac{k^2}{2k^+}(x_1^+-x_2^+)}$$

- The NE Lipatov vertex is incorporated in the phase.
- The θ-function in the correlator provides the integration limits.
- The 1-d target density is taken to be constant for $0 \leq x_1^+ \leq L^+$.
- integration over x_1^+ gives a factor of $(n_0 L^+)$ which corresponds to number of scattering centers in inside the finite length L^+. Since in the dilute target limit we only take into account a single scattering in the amplitude and c.c. amplitude, this factor can be set to 1.

After all said and done:

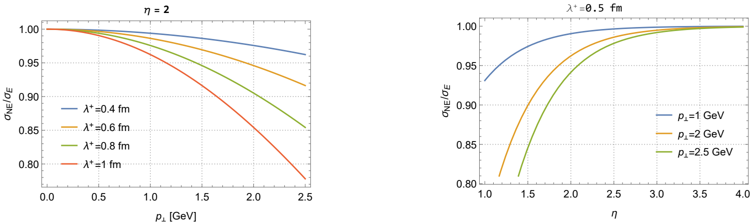
$$\frac{d\sigma}{d^2k d\eta}\Big|^{\rm NE}_{\rm dilute} \propto \mathcal{G}_1^{\rm NE}(k^-;\lambda^+) \int_q \mu^2\big[k-q, q-k\big] L^i(k,q) L^i(k,q) \big|a(q)\big|^2$$

the function that encodes the non-eikonal effects

$$\boxed{\mathcal{G}_1^{\rm NE}(k^-;\lambda^+) = \frac{1}{k^-\lambda^+}\sin(k^-\lambda^+)}$$

in the eikonal limit:

$$\lim_{(k^-\lambda^+)\to 0} \mathcal{G}_1^{\rm NE}(k^-;\lambda^+) = 1$$



$(N_c = 3,\ \mu_T = 0.2$ GeV, $\mu^2(k,q) = \delta^{(2)}(k+q)$ with a projectile size $S_\perp = 4GeV^{-2}.)$

Double inclusive gluon production and glasma graphs

Same procedure can be adopted to calculate the double inclusive gluon production.

The double inclusive gluon production X-section for dilute-dense scattering:

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2\eta_2} \propto \int_{z_1 z_2 x_1 x_2 \bar{z}_1 \bar{z}_2 y_1 y_2} e^{ik_1(z_1-\bar{z}_1)+ik_2(z_2-\bar{z}_2)} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)$$
$$\times \big\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\big\rangle_P \Big\langle [U_{z_1}-U_{x_1}]^{a_1c}[U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}]^{cb_1}[U_{z_2}-U_{x_2}]^{a_2d}[U^\dagger_{\bar{z}_2}-U^\dagger_{y_2}]^{db_2}\Big\rangle_T$$

- *projectile averaging:* pair wise Wick contraction:

$$\big\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\big\rangle_P = \big\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\big\rangle_P\big\langle \rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\big\rangle_P + \big\langle \rho^{a_1}_{x_1}\rho^{b_1}_{y_1}\big\rangle_P\big\langle \rho^{a_2}_{x_2}\rho^{b_2}_{y_2}\big\rangle_P + \big\langle \rho^{a_1}_{x_1}\rho^{b_2}_{y_2}\big\rangle_P\big\langle \rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\big\rangle_P$$

- *projectile averaging:* use the same two color charge correlator:

in x-space $\rightarrow \langle\rho^a(x)\rho^b(y)\rangle_P = \delta^{ab}\mu^2(x,y)$

in p-space $\rightarrow \langle\rho^a(k)\rho^b(p)\rangle_P = \delta^{ab}\mu^2(k,p) = \delta^{ab}\, T\left(\frac{k-p}{2}\right) F\big[(k+p)R\big]$

Double inclusive gluon production and glasma graphs

- *dilute target limit* $\rightarrow$ $U_{ab}(x) \approx 1 + igT^c_{ab}\int_{x^+q} e^{iqx}\, A^-_c(x^+, q)$
- *go from eikonal to non-eikonal:* $L(k,q) \rightarrow L_{\rm NE}(\underline{k}, q; x^+)$
- *target averaging:* pair wise Wick contraction

$$\Big\langle A^-_a(x^+_1,q_1)A^-_b(x^+_2,q_2)A^-_c(x^+_3,q_3)A^-_d(x^+_4,q_4)\Big\rangle_T = \Big\langle A^-_a(x^+_1,q_1)A^-_b(x^+_2,q_2)\Big\rangle_T\Big\langle A^-_c(x^+_3,q_3)A^-_d(x^+_4,q_4)\Big\rangle_T$$
$$+\Big\langle A^-_a(x^+_1,q_1)A^-_d(x^+_4,q_4)\Big\rangle_T\Big\langle A^-_c(x^+_3,q_3)A^-_b(x^+_2,q_2)\Big\rangle_T + \Big\langle A^-_a(x^+_1,q_1)A^-_c(x^+_3,q_3)\Big\rangle_T\Big\langle A^-_b(x^+_2,q_2)A^-_d(x^+_4,q_4)\Big\rangle_T$$

- *target averaging:* use the same two field correlator:

$$\boxed{\big\langle A^-_c(x^+_1,q_1)A^-_{\bar c}(x^+_2,q_2)\big\rangle_T = \delta^{c\bar c}\, n(x^+_1)\frac{1}{2\lambda^+}\Theta\big(\lambda^+ - |x^+_1 - x^+_2|\big)(2\pi)^2\delta^{(2)}(q_1-q_2)|a(q_1)|^2}$$

The dilute limit with non-eikonal corrections:

$$\frac{d\sigma}{d^2k_1\, d\eta_1\, d^2k_2\eta_2}\bigg|^{\rm NE}_{\rm dilute} \propto \int_{q_1 q_2} |a(q_1)|^2|a(q_2)|^2\,\mathcal{G}^{\rm NE}_1(k^-_1;\lambda^+)\,\mathcal{G}^{\rm NE}_1(k^-_2;\lambda^+)\bigg\{I^{(0)}_{\rm 2tr} + \frac{1}{N_c^2-1}\Big[I^{(1)}_{\rm 2tr} + I^{(1)}_{\rm 1tr}\Big]\bigg\}$$

In our set up:

- $k_1 - q_1$ and $k_2 - q_2$: momenta of the two gluons in the projectile.
- k_1 and k_2: momenta of the two gluons in the final state.
- q_1 and q_2: momenta transferred from the target to the projectile during the interaction.

In such a set up:

- (forward/backward) Bose enhancement of the gluons in the projectile $\Rightarrow F\left[|(k_1 - q_1) \mp (k_2 - q_2)|R\right]$
- (forward/backward) HBT correlations of the final state gluons $\Rightarrow F\left[|k_1 \mp k_2|R\right]$
- (forward/backward) Bose enhancement of the gluons in the target $\Rightarrow F\left[|q_1 \mp q_2|R\right]$

Glasma graphs / two particle correlations

identification of the terms:

$$I^{(0)}_{2\,\mathrm{tr}} = \Big(\mu^2\big[k_1 - q_1, q_1 - k_1\big]\, L^i(k_1, q_1) L^i(k_1, q_1)\Big)\;\Big(\mu^2\big[k_2 - q_2, q_2 - k_2\big]\; L^j(k_2, q_2) L^j(k_2, q_2)\Big)$$

- Square of the single inclusive production / uncorrelated production.

$$I^{(1)}_{2\,\mathrm{tr}} = \Big\{\mathcal{G}_2^{\mathrm{NE}}(k_1^-, k_2^-; L^+)\mu^2\big[k_1 - q_1, q_2 - k_1\big]\,\mu^2\big[k_2 - q_2, q_1 - k_2\big] \times L^i(k_1, q_1) L^i(k_1, q_2)\, L^j(k_2, q_2) L^j(k_2, q_1)\Big\} + (\underline{k}_2 \to -\underline{k}_2)$$

- $\underline{k} \equiv (k^+, k)$
- $\mu^2[k_1 - q_1, q_2 - k_1] \propto F\big[|q_1 - q_2|R\big] \Rightarrow$ Bose enhancement of the target gluons.
- A new function appears that accounts for non-eikonal effects:

$$\boxed{\mathcal{G}_2^{\mathrm{NE}}(k_1^-, k_2^-; L^+) = \left\{\frac{2}{(k_1^- - k_2^-)L^+}\sin\left[\frac{(k_1^- - k_2^-)}{2}L^+\right]\right\}^2}$$

- in the eikonal limit:

$$\lim_{L^+\to 0} \mathcal{G}_2^{\mathrm{NE}}(k_1^-, k_2^-; L^+) = 1$$

identification of the terms:

$$I^{(1)}_{1\,\mathrm{tr}} = \Big\{ \mu^2\big[k_1 - q_1, q_2 - k_2\big]\, \mu^2\big[k_2 - q_2, q_1 - k_1\big]\, L^i(k_1, q_1) L^i(k_1, q_1)\, L^j(k_2, q_2) L^j(k_2, q_2)$$

$$+\, \mathcal{G}_2^{\mathrm{NE}}(k_1^-, k_2^-; L^+) \Big(\mu^2\big[k_1 - q_1, q_1 - k_2\big]\, \mu^2\big[k_2 - q_2, q_2 - k_1\big] + \frac{1}{2}\mu^2\big[k_1 - q_1, k_2 - q_2\big]\, \mu^2\big[q_2 - k_1, q_1 - k_2\big] \Big)$$

$$\times\, L^i(k_1, q_1) L^i(k_1, q_2)\, L^j(k_2, q_1) L^j(k_2, q_2) \Big\} + (\underline{k}_2 \to -\underline{k}_2)$$

- $\mu^2\big[k_1 - q_1, q_2 - k_2\big] \propto F\big[|(k_1 - q_1) - (k_2 - q_2)|R\big] \Rightarrow$ Bose enhancement of the projectile gluons (forward peak).
- $\mu^2\big[k_1 - q_1, q_1 - k_2\big] \propto F\big[|k_1 - k_2|R\big] \Rightarrow$ HBT correlations of the produced gluons.
- $\mu^2\big[k_1 - q_1, k_2 - q_2\big] \propto F\big[|(k_1 - q_1) + (k_2 - q_2)|R\big] \Rightarrow$ Bose enhancement of the projectile gluons (backward peak).

The nature of $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$

In the double inclusive production X-section:

- certain terms are accompanied by $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$
- and their mirror images given by $(\underline{k}_2 \to -\underline{k}_2)$ are accompanied by $\mathcal{G}_2^{\rm NE}(k_1^-, -k_2^-; L^+)$.

$$\boxed{\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+) = \left\{ \frac{2}{\big(k_1^- - k_2^-\big) L^+} \sin\left[\frac{\big(k_1^- - k_2^-\big)}{2} L^+ \right] \right\}^2}$$

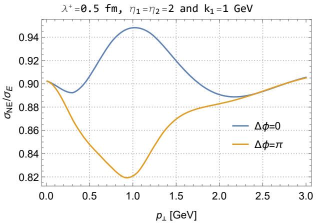
- $k^- = k^2/2k^+$
- $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$ is not symmetric under $(\underline{k}_2 \to -\underline{k}_2)$!!

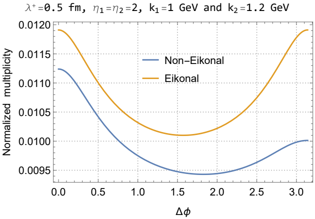
In certain kinematics the behavior of $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$ differs completely from $\mathcal{G}_2^{\rm NE}(k_1^-, -k_2^-; L^+)$:

- in the region where $k_1^- \sim k_2^-$ we get

$$\boxed{\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+) \gg \mathcal{G}_2^{\rm NE}(k_1^-, -k_2^-; L^+)}$$

• *This asymmetry created by the non-eikonal effects immediately reminds the asymmetry between the forward and backward peaks of the ridge structure observed in two particle production.*





• $L^+ = 6\,fm$ and $N_c = 3$

• $\mu_T = 0.2$ GeV

• translational invariance: $\mu^2(k, q) = \delta^{(2)}(k + q)$ with a projectile size $S_\perp = 4GeV^{-2}$.

• regulate the denominators that give rise to infrared divergencies by substituting the corresponding squared transverse momenta $l^2 \to l^2 + \mu_P$ where we have used the numerical value $\mu_P = 0.2$ GeV.

X-section is completely symmetric with respect to $\Delta\phi = \pi/2$ in the eikonal case, while an asymetric behavior is seen for the non-eikonal case.

Accidental symmetry of the CGC

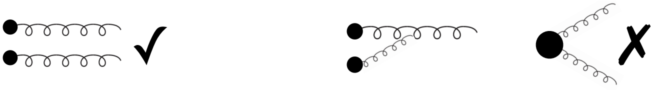
Eikonal double inclusive X-section is symmetric under $(k_2 \to -k_2)$!

$$\frac{d\sigma}{d^3k_1 d^3k_2} \propto \int_{q_1 q_2} \left\{ d(q_1)d(q_2)\left[I_0 + \frac{1}{N_c^2-1} I_1 + \frac{1}{(N_c^2-1)^2} I_2 \right] + (k_2 \to -k_2) \right\} + O\left(\frac{1}{Q_s S_\perp}\right)$$

symmetry under $(k_2 \to -k_2)$: "**accidental symmetry of the CGC**" $\Rightarrow$ vanishing odd harmonics

• *breaking the accidental symmetry with the density corrections to the projectile:*

[Kovner, Lublinsky, Skokov - arXiv:1612.07790] / [Kovchegov, Skokov - arXiv:1802.08166]



$$\frac{dN^{\text{even, odd}}(\mathbf{k}_\perp)}{d^2kdy} = \frac{1}{2}\left(\frac{dN(\mathbf{k}_\perp)}{d^2kdy}\left[\rho_p, \rho_t\right] \pm \frac{dN(-\mathbf{k}_\perp)}{d^2kdy}\left[\rho_p, \rho_t\right] \right)$$

$\Rightarrow$ non-vanishing odd harmonics.

• **Non-eikonal double inclusive X-section:**

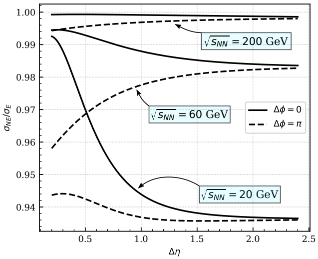
$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 \eta_2}\Big|^{\text{NE}}_{\text{dilute}} \propto \int_{q_1 q_2} \left\{ \left[f(k_1, q_1, k_2, q_2) + \mathcal{G}_2^{\text{NE}}(k_1^-, k_2^-; L^+)\, g(k_1, q_1, k_2, q_2) \right] + (\underline{k}_2 \to -\underline{k}_2) \right\}$$

non-eikonal corrections seem to be breaking the accidental symmetry!!

[P. Agostini, T.A., N. Armesto - 2019]

Can we generate non-zero odd harmonics from the non-eikonal corrections?

• The difference between the peaks at $\Delta\phi = 0$ and at $\Delta\phi = \pi$ is a sign of generating non-zero odd harmonics.

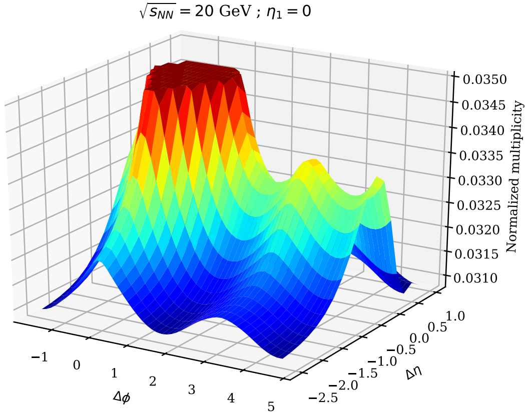


• $L^+ = 6$ fm in the rest frame and we scale it with the γ factor for different energies.
• $\mu_T = 0.4$ GeV and $\mu_P = 0.2$ GeV (these are the values that maximize v_3).
• $\eta_1 = 0 \to \Delta\eta = \eta_2$ & $k_1 = 1$ GeV and $k_2 = 1.2$ GeV.

With increasing energy the difference between the peaks gets smaller $\to$ non-eikonal corrections gets smaller.

the asymmetry exists in an interval of roughly two units of rapidity.

odd-harmonics from the non-eikonal corrections?



- The difference between the peaks is max for $\Delta\eta = 0$ (max. v_3 as well).
- The difference between the peaks vanishes after two units of rapidity.

Azimuthal harmonics

$$\left.\frac{dN}{d^2\mathbf{k}_1 d\eta_1 d^2\mathbf{k}_2 d\eta_2}\right|_{\rm dilute}^{\rm NE} \equiv N(k_1, k_2, \Delta\phi) = a_0(k_1, k_2) + \sum_{n=1}^{\infty} a_n(k_1, k_2)\cos(n\Delta\phi)$$

Following the literature

$$N(k_1, k_2, \Delta\phi) = a_0(k_1, k_2)\left[1 + \sum_{n=1}^{\infty} 2V_{n\Delta}(k_1, k_2)\cos(n\Delta\phi)\right]$$

where

$$2V_{n\Delta}(k_1, k_2) = \frac{a_n(k_1, k_2)}{a_0(k_1, k_2)} = 2\,\frac{\int_0^{\pi} N(k_1, k_2, \Delta\phi)\,\cos(n\Delta\phi)\,d\Delta\phi}{\int_0^{\pi} N(k_1, k_2, \Delta\phi)\,d\Delta\phi}$$

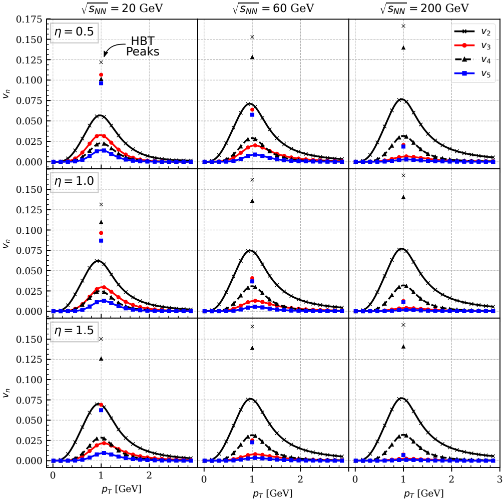
- We set $k_1 = p_T^{ref}$ and $k_2 = p_T$. Then, the azimuthal harmonics are defined as

$$v_n(p_T) = \frac{V_{n\Delta}(p_T, p_T^{ref})}{\sqrt{V_{n\Delta}(p_T^{ref}, p_T^{ref})}}$$

[T. Lappi, B. Schenke, S. Schlichting, R. Venugopalan - 2015]

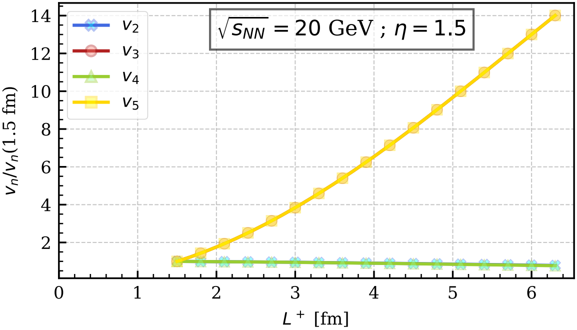
Azimuthal harmonics from non-eikonal production

- $p_T^{ref} = k_1 = 1$ GeV
- $\eta_1 = \eta_2 = \eta$



A side remark:

even harmonics do not depend on L^+ but odd harmonics do.



Summary / Remarks / Discussions

$$\gamma \sim \sqrt{S_{NN}}/2 \qquad \& \qquad \textit{eikonal parameter} : (p_T\, L^+\, e^{-\eta})$$

- At LHC energies $\sqrt{S_{NN}} > 2$ TeV $\Rightarrow \gamma \simeq 1000 \Rightarrow L^+ \simeq 10^{-2}\ \mathrm{GeV}^{-1}$
  - LHC small p_T (0-3 GeV): (i) vanishing odd harmonics
    (ii) $\mathcal{G}_2^{\mathrm{NE}} \to 1 \Rightarrow$ Non-eikonal expressions $\to$ eikonal ones.
  - LHC high p_T (3-10 GeV): Does $\mathcal{G}_2^{\mathrm{NE}} \to 1? \Rightarrow$ Non-eikonal terms might be still important.
- At RHIC energies $\sqrt{S_{NN}} < 200$ GeV $\Rightarrow \gamma < 100 \Rightarrow L^+ > 0.3\ \mathrm{GeV}^{-1}$
  - RHIC small p_T (0-3 GeV): (i) difference between the peaks
    (ii) non-vanishing odd harmonics
  - RHIC high p_T (3-10 GeV): (i) no difference between the peaks.
    (ii) $\mathcal{G}_2^{\mathrm{NE}}(k_1^-, k_2^-, L^+) \to \mathcal{G}_2^{\mathrm{NE}}(k_1^-, -k_2^-, L^+)$
    (iii) vanishing odd harmonics

Summary / Remarks / Discussions

- *With the change of the azimuthal angle from $\Delta\phi = 0$ to $\Delta\phi = \pi$ the magnitude of the non-eikonal parameter is changing $\rightarrow$ breaks the accidental symmetry of the CGC and generates non-zero odd harmonics.*

- Other corrections to the eikonal limit may carry a similar effect:

  • including the transverse component of the background field will bring k^+ dependence.

  • the dynamics of the target: x^- dependence of the target field ??

- *Non-eikonal effects alone can not explain the odd-harmonics HOWEVER there is a contribution originating from these effects for certain kinematic region.*