Anomalous supersymmetry

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Outline

Introduction

- Quantum anomalies form a cornerstone of modern theoretical physics.
- Anomalies in global symmetries are a feature of the theory and they lead to observable effects.
- For example, the famous axial anomaly is directly linked with the π^0 decay.
- Anomalies in local symmetries render the theory inconsistent.
- Such anomalies must be cancelled.

- In particular, anomalous global symmetries cannot be coupled to corresponding gauge theories.

Supersymmetry

- Supersymmetry has been a dominant topic in theoretical physics since its discovery in the beginning of 70s.
- Are there anomalies in supersymmetry?
- Such anomalies could have implications ...
- ... from the quantum consistency of supergravities ... to supersymmetric phenomenology ...

References

- George Katsianis, Ioannis Papadimitriou, KS and Marika Taylor
  - *Anomalous supersymmetry*, 1902.06715
  - *Computation of supersymmetric anomalies*, to appear
- Ioannis Papadimitriou
  - *Supersymmetry anomalies in $\mathcal{N} = 1$ conformal supergravity*, 1902.06717
  - *Supersymmetry anomalies in new minimal supergravity* , 1904.00347
- Ioannis Papadimitriou, *Supercurrent anomalies in 4d SCFTs*, 2017

Outline

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Anomalies: basics

- Anomalies represent a breaking of a symmetry at the quantum level.
- Anomalies appear either because of
  - (0 because of symmetry)/(0 because of UV infinities)
  - cancellation of infinities requires counterterms that necessarily break some symmetries.
- There are sometimes spurious or trivial anomalies – such anomalies can be removed by local counterterms.

Prime example: triangle anomaly

- Consider a massless fermion

$$S = \int d^4x(i\bar{\psi}\gamma^i\partial_i\psi)$$

- Associated with the symmetries

$$\psi \to e^{ia}\psi \qquad \psi \to e^{ib\gamma_5}\psi$$

are the vector current V^i and the axial current J^i, respectively.

Ward identities: elementary fields

- Conservation of the current gives rise to Ward identities:

$$\partial_\mu \left\langle j^\mu(x) \prod_{i=1}^{n} O_i(x_i) \right\rangle = \sum_{i=1}^{n} \delta^4(x - x_i) \left\langle \delta O_i(x_i) \prod_{j \neq i}^{n} O_j(x_j) \right\rangle$$

- When O_i are elementary fields (*i.e.* ψ or $\bar{\psi}$) these correlators saturate at tree-level, and one can readily verify that the Ward identities are satisfied both for the axial and the vector current.

➠ The axial and vector current are conserved inside correlators of elementary fields.

Ward identities: composite operators

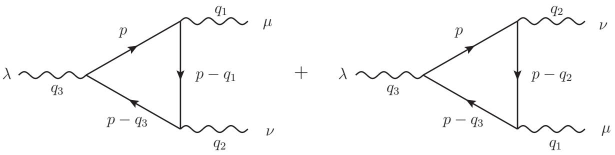
- Correlation functions of multiple number of composite operators, such as conserved currents, have additional short distance singularities and require separate discussion.
- In $d = 4$ and for symmetry currents the issue appears first with 3-point functions.
- The Ward identities for the 3-point function of three currents (in momentum space) read:

$$q_{1i}\langle V^i(q_1)V^j(q_2)J^k(q_3)\rangle = 0$$

and

$$q_{3k}\langle V^i(q_1)V^j(q_2)J^k(q_3)\rangle = 0$$

1-loop computation



- One can show (using a shift of the integration variable) that the "naive" Ward identities are satisfied.
- Power counting suggests that the Feynman diagrams are linearly divergent and if one imposes a momentum cut-off the integral depends on the *momentum routing*.
- The integral is finite but ambiguous: it exhibits a 0/0 structure.

The chiral anomaly

- Requiring conservation of the vector current leads to an anomaly in the axial current:

$$q_{3i} \left\langle V^j(q_1) V^k(q_2) J^i(q_3) \right\rangle = -\frac{1}{2\pi^2} \epsilon^{jkmn} q_{2m} q_{1n}$$

- Introducing sources A^i, B^i for the vector and axial currents,

$$S \to S + \int d^4x (A_i V^i + B_i J^i)$$

one finds that the anomaly can be expressed as

$$\partial_i \langle J^i(x) \rangle = \frac{1}{(4\pi)^2} \epsilon^{jklm} F_{jk} F_{lm}(x)$$

where $F_{ij} = \partial_i A_j - \partial_j A_i$

Partition function

- The partition function in the presence of sources, A, B is given by

$$Z[A,B] = e^{iW[A,B]} = \int [D\psi] e^{iS + i\int d^4x (A_i V^i + B_i J^i)}$$

- The partition function encodes correlators of the currents

$$\langle V^i(x_1) \cdots J^j(x_n) \rangle \sim \left. \frac{\delta^n Z}{\delta A_i(x_1) \cdots B_j(x_n)} \right|_{A=B=0}$$

Partition function and anomaly

- The anomaly implies that the partition function transforms with with a phase under the anomalous transformation

$$Z[A, B + db] = Z[A, B]e^{-i \int d^4 x b \mathcal{A}}$$

or equivalently

$$\delta_b W = -\int d^4 x b \mathcal{A}, \qquad \text{and} \qquad \partial_i \langle J^i \rangle = \mathcal{A}$$

- For free fermions

$$\mathcal{A} \sim \epsilon^{jklm} F_{jk} F_{lm}$$

Moving the anomaly around

- One can change the symmetry which is anomalous by adding a local term to the action,

$$\tilde{Z}[A,B] = Z[A,B] \exp\left(i\alpha_c \int d^4x \epsilon^{ijkl} B_i A_j F_{kl} \right)$$

- With appropriate choice of the coefficient α_c one may arrange to cancel the axial anomaly.
- Since the new term is not invariant under $A \to A + d\alpha$ the vector Ward identity will now be violated.

Compensators

- One can restore gauge invariance in the partition function by introducing a new background field, a "compensator" or "gauge-away" field ϕ,

$$Z[A, B, \phi] = Z[A, B] e^{i \int d^4x \phi \mathcal{A}}$$

which transforms as follows

$$\delta\phi = b$$

- One may use the gauge invariance to gauge away ϕ, returning to the original theory, so the new theory is equivalent to the original one.

Gauge-away fields and anomalies

- In the presence of ϕ the partition function is gauge invariant,

$$\delta_b W[A, B, \phi] = 0$$

- However, this does not imply that there is no anomaly. Indeed, the new term does not change the triangle diagram.
- ➠ Gauge invariance in the presence of "gauge-away" fields does not imply absence of anomalies.

Outline

Anomalies in Supersymmetry

- The discussion in this talk will be focused on four dimensional $\mathcal{N} = 1$ SCFT theories. We will mention generalizations at the end.
- For such theories, there is a well-known superconformal anomaly.
- The current supermultiplet consists of:
  1. The stress tensor T_{ij}
  2. The supercurrent Q^i
  3. The R symmetry current J^i
- These satisfy the standard conservation equations

$$\partial^i T_{ij} = 0, \qquad \partial_i Q^i = 0, \qquad \partial_i J^i = 0.$$

- In addition, superconformal transformations are generated by

$$T_i^i = 0, \qquad \gamma_i Q^i = 0.$$

Superconformal Anomalies

- At the quantum level superconformal transformations are anomalous.
- Bosonic currents:

$$T_i^i = \frac{c}{16\pi^2} W^2 - \frac{a}{16\pi^2} E - \frac{c}{6\pi^2} F^2,$$
$$\nabla_i J^i = \frac{c-a}{24\pi^2} P + \frac{5a-3c}{27\pi^2} F\tilde{F},$$

W=Weyl, E=Euler density, P=Pontryagin density

- $\gamma_i Q^i$ is also anomalous.
- In fact $(\nabla_i J^i, \gamma_i Q^i, T_i^i)$ form an $\mathcal{N}=1$ chiral multiplet.

Outline

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Early work: SUSY vs superconformal anomaly

- In anomalies one often has a choice which symmetry to preserve and which to have anomalous.
  - Axial anomalies: vector or axial symmetry?
  - Trace anomalies: Diffeomorphisms ($\partial_i T^{ij} \neq 0$) or trace ($T_i^i \neq 0$)?
- In supereconformal theories:
  SUSY anomaly ($\partial_i Q^i \neq 0$) or superconformal anomaly ($\gamma_i Q^i \neq 0$)

  ... and if the model is a gauge theory one also has the option to keep $\partial_i Q^i = \gamma_i Q^i = 0$ but break gauge invariance.

  [Abbott, Grisaru, Schnitzer (1977)] ...

Gauge anomaly and susy anomaly

- SYM theory in the Wess-Zumino gauge has a SUSY anomaly if it has a gauge anomaly. [Itoyama, Nair (1985)] ...

- If the theory is consistent (no gauge anomalies) there is no susy anomaly.

Gauge anomaly and susy anomaly

- A vector multiplet V is a real multiplet ($V = V^\dagger$) consisting of

$$V = (C, \chi, \bar{\chi}, M, \bar{M}, A_\mu, \lambda, \bar{\lambda}, D)$$

and has a gauge invariance

$$V \to V + \frac{i}{2}(\Lambda - \Lambda^\dagger) \qquad \bar{D}_{\dot{\alpha}}\Lambda = 0.$$

- This gauge invariance can be used to gauge away $C, \chi, \bar{\chi}, M, \bar{M}$: these are the "gauge-away" fields.

  Setting these fields to zero is called the "Wess-Zumino" gauge.
- In the WZ gauge, the vector multiplet reduces to physical fields $(A_\mu, \lambda, \bar{\lambda}, D)$, where D is the auxiliary field.

Gauge anomaly and susy anomaly

- Consider the case the theory has a gauge anomaly:

$$\delta_\Lambda W[V] = i \int d^4x d^2\theta \Lambda W^\alpha W_\alpha + h.c. \qquad (W^\alpha = -\frac{1}{4}\bar{D}^2 D V)$$

- In the WZ gauge, susy transformations are a combination of the original susy transformation and a compensating gauge transformations in order to remain in WZ gauge.

  This transfers the anomaly from gauge transformations to SUSY.

Compensators

- Let $\Lambda_0 = iC - 2\theta\chi - \theta^2 M$ the gauge transformation that sets to zero the gauge-away fields:

$$V_{WZ} = V + \frac{1}{2}(\Lambda_0 - \Lambda_0^\dagger)$$

Then [Kuzenko etal (2019)]

$$W_{WZ}[V_{WZ}] = W[V] + \left(i \int d^4x d^2\theta \Lambda_0 W^\alpha W_\alpha + h.c. \right)$$

- As $W[V]$ is manifestly supersymmetric, one may be tempted to conclude that the last term acts as a counterterm that restores susy.
- However, it is clear from its form that the "gauge-away" fields act as compensators.
- Invariance of W in the presence of gauge-away fields does not imply absence of anomalies.

"No-go" theorems

In series of papers, summarised in the monograph “Renormalized Supersymmetry” by Piguet and Sibold, it was established that

- ➢ renormalization of correlators of elementary fields does not induce a supersymmetry anomaly to all orders in perturbation theory.
- ➢ The supercurrent is conserved inside correlation functions of elementary fields.

Anomaly candidates

One may solve the Wess-Zumino consistency condition/use BRST methods to look for anomaly candidates.

- This was done for anomalies that may appear in correlators of elementary fields [Piquet, Sibold], as in the previous slide.
- For SQFTs coupled to dynamical (on-shell) SUGRAs
  - In superspace [Bonora, Pasti, Tonin (1985)] [Buchbinder, Kuzenko (1986)] ... [Bonora, Giaccari (2013)]
  - In components [Brandt (1994) (1997)]
- For the problem of interest we would need to solve the WZ consistency conditions in a background of off-shell $\mathcal{N} = 1$ conformal SUGRA.

  The SUGRA fields are external sources that couple to conserved currents.

Outline

Supersymmetry anomaly

- It was thus a big surprise when a holographic computation produced a supersymmetric anomaly [Papadimitriou (2017)],

$$\nabla_i \langle Q^i \rangle \sim \epsilon^{iskl} F_{sk} A_l (\gamma_{ij} - 2 g_{ij}) \gamma^{jpq} \nabla_p \Psi_q$$

where Ψ_q is a gravitino (source for the supercurrent)).
- This implies that there should be an anomaly on flat background first appearing in 4-point functions:

$$q_{1i} \langle Q^i(q_1) \bar{Q}^j(q_2) J^k(q_3) J^l(q_4) \rangle \neq 0.$$

Holographic computation [Papadimitriou (2017)]

- Given a bulk action, holographic renormalization leads to the Ward identities and anomalies of the dual QFT [Henningson, KS (1998)], [de Haro, Solodukhin, KS (2000)].
- Consider $\mathcal{N} = 2$ 5d gauged SUGRA and turn on sources for the supergravity multiplet (including the gravitino).
- Holographic renormalization then provides the complete set of anomalies for CFTs with $c = a$ at leading order as $N \to \infty$.

Holographic anomalies

The holographic calculation gives:

$$T_i^i + \cdots = c(W^2 - E) + \cdots \equiv \mathcal{A}_W$$

$$\nabla_i J^i + \cdots = c\epsilon^{jklm} F_{jk} F_{lm} \equiv \mathcal{A}_R$$

$$\nabla_i Q^i + \cdots = c\epsilon^{jklm} F_{kl} A_m (\gamma_{jn} - 2g_{jn}) \gamma^{npq} \nabla_p \Psi_q \equiv \mathcal{A}_Q$$

$$\gamma_i Q^i - \cdots = c\epsilon^{jklm} F_{kl} A_m \Psi_j + \cdots \equiv \mathcal{A}_S$$

Here the ellipses on the lhs denote standard contact term contributions to the Ward identity and the ones on the rhs that there are additional terms in the anomalies. (I also omit numerical factors).

Should we have expected this?

- A standard path integral derivation of the Ward identity leads to

$$\frac{\partial}{\partial x_1^i}\langle Q^i(x_1)\bar{Q}^j(x_2)J^k(x_3)J^l(x_4)\rangle = \delta(x_1 - x_2)\langle \delta\bar{Q}^j(x_2)J^k(x_3)J^l(x_4)\rangle + \delta(x_1 - x_3)\langle \bar{Q}^j(x_2)\delta J^k(x_3)J^l(x_4)\rangle + \cdots$$

  where δ is a supersymmetric variation.
- Now use [Ferrara, Zumino (1975)]

$$\delta\bar{Q}^i = \bar{\alpha}\left(-2\gamma_j T^{ij} + \frac{1}{2}\epsilon^{ijkl}\gamma_j\partial_k J_l + i\gamma_5 \partial\!\!\!/ J^i - i\gamma_5\gamma^i\partial_j J^j\right)$$

- Inserting in the Ward identity one finds some of the same correlators responsible for the R-anomaly.
- This suggests that supersymmetry is always anomalous when the R-symmetry is anomalous.

Wess-Zumino consistency condition

- Symmetry variations form an algebra,

$$[\delta_i, \delta_j] = f_{ij}^k \delta_k$$

  in general with field dependent structure constants.
- The Wess-Zumino consistency condition comes from acting with the commutator on the generating functional of CFT connected graphs:

$$[\delta_i, \delta_j] W = f_{ij}^k \delta_k W$$

- Having an anomaly means

$$\delta_i W = \int e \epsilon_i \mathcal{A}_i$$

  and the WZ condition is then

$$\int d^4x \ \left(\delta_i(e\epsilon_j \mathcal{A}_j) - \delta_j(e\epsilon_i \mathcal{A}_i) - f_{ij}^k e\epsilon_k \mathcal{A}_k\right) = 0$$

Wess-Zumino consistency condition

- Consider the commutator of R-symmetry (with parameter θ) with Q-supersymmetry (with parameter ε):

$$\int d^4x\big(\delta_\varepsilon(e\ \theta\mathcal{A}_R) - \delta_\theta(e\ \varepsilon\mathcal{A}_Q)\big) = 0\,.$$

- Using the explicit form of $\mathcal{A}_R$ it is easy to see that $\delta_\varepsilon\mathcal{A}_R \neq 0$ and the WZ consistency condition requires that $\mathcal{A}_Q \neq 0$.

⇒ The WZ condition imply that SUSY must be anomalous.

Wess-Zumino consistency condition: solution

$$\mathcal{A}_Q = -\frac{(5a-3c)i}{27\pi^2}\tilde{F}^{\mu\nu}A_\mu\gamma^5\gamma^\kappa\left(D_\kappa\psi_\nu - D_\nu\psi_\kappa - \frac{i}{2}\gamma^5\epsilon_{\kappa\nu}{}^{\rho\sigma}D_\rho\psi_\sigma\right)$$
$$+\frac{(a-c)}{6\pi^2}\left(\nabla_\mu(A_\rho\tilde{R}^{\mu\nu\rho\sigma})\gamma_{(\nu}\psi_{\sigma)} - \frac{1}{4}F_{\mu\nu}\tilde{R}^{\mu\nu\rho\sigma}\gamma_\rho\psi_\sigma\right)$$

$$\mathcal{A}_S = \frac{(5a-3c)}{6\pi^2}\tilde{F}^{\mu\nu}\left(D_\mu - \frac{2i}{3}A_\mu\gamma^5\right)\psi_\nu + \frac{ic}{6\pi^2}F^{\mu\nu}\left(\gamma_\mu{}^{[\sigma}\delta_\nu^{\rho]} - \delta_\mu^{[\sigma}\delta_\nu^{\rho]}\right)\gamma^5 D_\rho\psi_\sigma$$
$$+\frac{3(2a-c)}{4\pi^2}P_{\mu\nu}g^{\mu[\nu}\gamma^{\rho\sigma]}D_\rho\psi_\sigma + \frac{(a-c)}{8\pi^2}\left(R^{\mu\nu\rho\sigma}\gamma_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}g^{\mu[\nu}\gamma^{\rho\sigma]}\right)D_\rho\psi_\sigma$$

Remarks

- This confirms the anomaly when $a = c$ and shows that there are additional terms when $a \neq c$.
- The new terms imply that is an additional (flat space) anomalous correlator when $a \neq c$:

$$q_{1i}\langle Q^i(q_1)\bar{Q}^j(q_2)J^k(q_3)T^{lm}(q_4)\rangle \neq 0$$

- This is again as expected based on the susy variation of the supercurrent:

$$\frac{\partial}{\partial x_1^i}\langle Q^i(x_1)\bar{Q}^j(x_2)J^k(x_3)T^{lm}(x_4)\rangle = \delta(x_{12})\langle\delta\bar{Q}^j(x_2)J^k(x_3)T^{lm}(x_4)\rangle + \cdots$$

$$\delta\bar{Q}^i = \bar{\alpha}\left(-2\gamma_j T^{ij} + \frac{1}{2}\epsilon^{ijkl}\gamma_j\partial_k J_l + i\gamma_5 \partial\!\!\!/ J^i - i\gamma_5\gamma^i\partial_j J^j\right)$$

- The anomaly is now related to the anomalous $\langle TTJ\rangle$ correlator, encoded in the Pontraygin term in

$$\nabla_i J^i = \frac{c-a}{24\pi^2}P + \frac{5a-3c}{27\pi^2}F\tilde{F}.$$

Can we remove the anomaly using a counterterm?

- The commutator of two supersymmetry transformations is

$$[\delta_\varepsilon, \delta_{\varepsilon'}] = \delta_\xi + \delta_\lambda + \delta_\theta$$

δ_ε susy variation, δ_ξ diffeo, δ_λ local Lorentz, δ_θ R-symmetry.

- Suppose we add a counterterm such that $\mathscr{W}_{\rm ren} = \mathscr{W} + \mathscr{W}_{\rm ct}$ is non-anomalous, i.e. $\delta_\varepsilon \mathscr{W}_{\rm ren} = 0$.
- Using the commutator of two supersymmetry variations, we find

$$(\delta_\xi + \delta_\lambda + \delta_\theta)\mathscr{W}_{\rm ren} = 0 \quad \Rightarrow \quad (\delta_\xi + \delta_\lambda)\mathscr{W}_{\rm ren} \neq 0\,, \qquad (1)$$

since $\delta_\theta \mathscr{W}_{\rm ren} = \mathcal{A}_R \neq 0$.

➠ It follows that if one wishes to preserve supersymmetry $\mathscr{W}_{\rm ct}$ must break diffeomorphisms and/or local Lorentz transformations.

Compensators

- It was argued in [Kuzenko etal (2019)] that one can link this anomaly to the WZ gauge for the background supergravity multiplet, similarly to the discussion of the vector multiplet.
- From the perspective of the QFT, one need not introduce any other sources beyond the ones needed to compute the correlators of interest.
- The supergravity fields in the WZ gauge couple to the gauge invariant operators (T_{ij}, Q^i, J^i).
- The gauge-away fields may be considered as compensators.
- Invariance of W in the presence of gauge-away fields does not imply absence of anomaly.

Implications in non-trivial backgrounds

- The anomaly implies that the SUSY variation of the supercurrent is anomalous in a non-trivial background [Papadimitriou (2017)(2019)]:

$$\delta_\varepsilon Q^i = \cdots + \frac{\delta}{\delta\bar{\psi}_i}\int d^4x\bar{\varepsilon}\mathcal{A}_Q$$

  where the dots are the standard terms.
- This has implication about the notion of 'BPS' states of the SCFT on a rigid supersymmetric background.
- It also has implications on localizations computations.

Outline

Anomaly in free field theory

Consider the simplest setup:

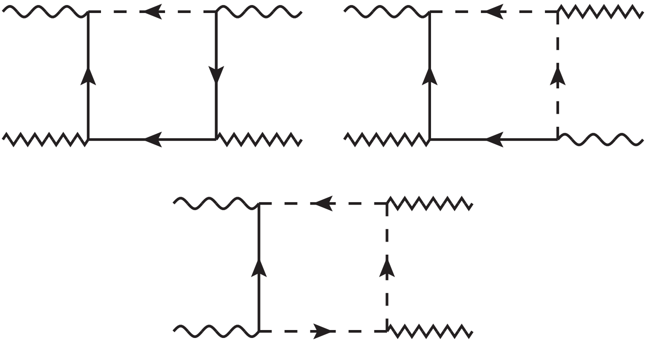
- Free Wess-Zumino massless model,

$$S = \int d^4x \left((\partial_i \phi)(\partial^i \phi^*) + \frac{1}{2} \bar{\chi} \gamma^i \partial_i \chi \right)$$

- In this theory the (connected) 4-point function $\langle Q^i(q_1) \bar{Q}^j(q_2) J^k(q_3) J^l(q_4) \rangle$ and $\langle Q^i(q_1) \bar{Q}^j(q_2) J^k(q_3) T^{lm}(q_4) \rangle$ are given by one-loop diagrams.

  - Here we focus on $\langle Q^i(q_1) \bar{Q}^j(q_2) J^k(q_3) J^l(q_4) \rangle$.

Feynman diagrams



Computation

- Like in the case of triangle anomalies, the box diagrams involve integrals that by power counting are linearly divergent.
- Formal manipulations (involving shifts of integration variables) show that naively the SUSY Ward identity is satisfied.
- Using momentum cut-off, one finds that there is a momentum routing ambiguity.
- We fix these ambiguities by requiring the R-symmetry anomaly is standard:
  1. The R-symmetry anomaly does not receive a contribution from 4-point functions, i.e. $\partial_k \langle Q^i(x_1)\bar{Q}^j(x_2)J^k(x_3)J^l(x_4)\rangle$ is non-anomalous.
  2. The triangle R-symmetry anomaly is correctly reproduced.
- We are also computing the same anomaly using Pauli-Villars regularization.

Results

- The free field computation exactly reproduces the SUSY anomaly, with the correct values of a and c for our model, $c = 2a = 1/24$.
- The same result holds for

$$q_{1i} \langle Q^i(q_1) \bar{Q}^j(q_2) J^k(q_3) T^{lm}(q_4) \rangle$$

- A similar computation exactly reproduces the $\gamma^i Q_i$ anomaly.

Generalisations

- ➢ It is straightforward to see that the same results hold if we consider the free massive WZ model.
- ➠ The Ward identities acquire new terms but the anomaly remains the same.
- ➢ In general we expect the anomaly to be present in all SQFTs that in the UV have an anomalous R-symmetry.

- ➢ Similar results hold for $\mathcal{N} = 1$ SQFTs with anomalous $U(1)$ flavour symmetries [Papadimitriou (2019)].
- ➠ There are 5 additional anomaly coefficients in the presence of anomalous flavour symmetries.

Outline

Coupling to SUGRA

- CFTs with anomalous R-symmetry cannot be coupled to quantum supergravity, unless there is a cancellation mechanism of the susy anomaly that involves SUGRA fields.
- There are indications that there is such a cancellation mechanism in "old-minimal supergravity" *[in progress]*
- Relevant earlier work includes
  - "Auxiliary field anomaly" [Gates, Grisaru, Siegel (1982)]
  - "Compensator fields and anomalies" [de Wit, Grisaru (1985)]
- Sometimes one may arrange to cancel the R-anomaly (and thus the susy anomaly) via a Green-Schwarz mechanism [Cardoso, Ovrut (1992)].

Outline

Phenomenology

These results may have implication to

- Phenomenological models that use SCFTs with anomalous R-symmetry.
- Cosmological models that use matter with anomalous R-symmetries.

- In the context of supersymmetric model building, one does not usually work with theories with an R-symmetry, anomalous or non-anomalous.
- In general, one does not expect a theory with continuous symmetry to emerge from a consistent quantum theory of gravity, such as string theory.
- However, such models may have been considered in bottom-up approaches.

Outline

Supersymmetric theories on curved backgrounds

- There has been renewed interest in SUSY QFTs on curved backgrounds in recent years, following [Pestun (2007)].
- If the curved background admits a notion of rigid supersymmetry, one can use Q as a BRST operator to compute (some) observables exactly for any value of the coupling (localisation).
- Typical backgrounds of interest: spheres, often with SUSY preserving background gauge fields.

Localisation

- Consider an action $S[\Phi]$ with fermionic symmetry Q such that $QS = 0$.
- Q either squares to zero, or to a bosonic symmetry δ_B.
- Now consider the path integral

$$Z(t) = \int [D\Phi] e^{-S[\Phi] - tQV[\Phi]}$$

Localisation

- Then

$$\frac{dZ}{dt} = -\int [D\Phi] QV e^{-S-tQV} = -\int [D\Phi] Q \left(V e^{-S-tQV} \right)$$

- If the fermionic symmetry is non-anomalous, and the measure is Q invariant,

$$\frac{dZ}{dt} = 0$$

so Z is independent of t.
- Setting $t = 0$ we recover the original theory of interest.
- Alternatively, we may calculate Z at $t \to \infty$, where for suitable V the path integral localises, i.e. it becomes a finite dimensional integral.

Anomalies and localisation

- For CFTs with R-anomaly, the localisation argument needs to be reconsidered.
- These considerations resolved puzzles regarding the matching of SUGRA results and localisation results reported in [Genolini, Cassani, Martelli, Sparks (2016)], as discussed in [Papadimitriou (2017)].
- The localisation arguments were recently examined in view of the susy anomaly associated with flavour anomaly in [Closset, Di Pietro, Kim (2019)]

Outline

Conclusions/Outlook

- Supersymmetry is anomalous when the R-symmetry is anomalous.
- This conclusion follows from holographic computations, free field computations and the WZ condition.
- This observation may have implications in:
  1. phenomenology
  2. cosmology
  3. localisation computations
- The results we obtained were for SCFTs but it is very likely that supersymmetry is anomalous for any SQFT with anomalous R-symmetry.
- It would be interesting to extend the results to different spacetime dimensions and different amounts of supersymmetry.