Seminar Ecole Polytechnique

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1. Introduction to the A^h-model
2. LKFT's
   - Motivation
   - non-Abelian Theories
   - Computations
3. Resummation
   - Introduction
   - Numerical
   - Analytical
4. Conclusion and Future Projects

QCD in linear covariant gauges

$$\partial_\mu A^a_\mu = \alpha b^a \tag{1}$$

represented by the following gauge fixed action

$$S_{FP} = \int \mathrm{d}^4x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) , \tag{2}$$

Has the known properties of

- BRST symmetry,
- renormalizability,
- physical observables should be gauge invariant.

Working in linear covariant gauges $\partial_\mu A^a_\mu = \alpha b^a$, an overcounting problem remains. Consider the gauge transformation

$$A \to A' = A - D\omega \tag{3}$$

which preserves $\partial_\mu {A'_\mu}^a = \alpha b^a$ if

$$\partial D\omega = 0 \tag{4}$$

Meaning that there is an overcounting problem if the FP operator ∂D has zeromodes.

- Originally introduced [1] by minimisation of the A^2-functional, this yields a positive FP operator.
- It can be represented by adding an extra term to the action

$$S_h = \int \mathrm{d}^4x \left(\tau^a \partial_\mu A_\mu^{h,a} + \frac{m^2}{2} A_\mu^{h,a} A_\mu^{h,a} + \bar{\eta}^a \partial_\mu D_\mu^{ab}(A^h) \eta^b \right) , \qquad (5)$$

- Where A_μ^h is defined as, with $h = e^{ig\phi^a T^a}$

$$A_\mu^h = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h \, . \qquad (6)$$

- The minimisation $\partial A^h = 0$ is evident from the presence of the Lagrangian multiplier τ

- The minimum can be soved iteratively, yielding

$$
\begin{aligned}
A^h_\mu &= A_\mu - \frac{\partial_\mu}{\partial^2}\partial A + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\mu \frac{1}{\partial^2}\partial A\right] \\
&+ ig\frac{\partial_\mu}{\partial^2}\left[\frac{\partial_\nu}{\partial^2}\partial A, A_\nu\right] + i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + \mathcal{O}(A^3)\,.
\end{aligned} \tag{7}
$$

- A^h is transversal and gauge invariant by construction
- A^h can be written as a (non-local) power series in g
- This formalism is BRST invariant, and proven renormalizable [1]

One calculates for instance the gluon propagator in some gauge.

- This is (often) computated perturbatively, meaning truncation at some order.
- This introduces an error, which might be gauge dependent.

Secondly, how can you relate a propagator calculated in a (linear covariant) gauge α to the same propagator in gauge α'?
Landau-Khalatnikov-Fradkin transformations can help us shed some light on these questions.

The gauge invariance of A^h can be exploited to calculate gauge invariant propagators, $\left\langle A^h A^h \right\rangle_\alpha = \left\langle A^h A^h \right\rangle_{\alpha'}$.
The power series reduces significantly for Abelian theories, to

$$A^h_\mu = A_\mu - \frac{\partial_\mu}{\partial^2}\partial A \tag{8}$$

and the LKFT for the photon can be found [2]

$$\langle A_\mu A_\nu \rangle_\alpha = \langle A_\mu A_\nu \rangle_{\alpha'=0} + \alpha \frac{p_\mu p_\nu}{p^2} \tag{9}$$

which is indeed the known result.

For general non-Abelian theories, the gauge invariance of A^h still holds. We can now find the relation of the gluon propagator through different gauges

$$\langle A_\mu A_\nu \rangle_\alpha = \langle A_\mu A_\nu \rangle_{\alpha'=0} + \text{'corrections'} \tag{10}$$

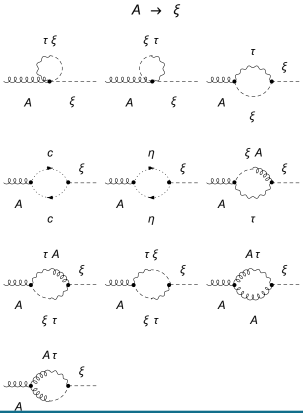
Where the correction term is again a power series in g (or A).
The zeroth order in g retrieves the Abelian result, a more satifying result would be to compute the second order corrections in g. Due to the high number of diagrams we resort to a computational method in Mathematica.

1. Introduce the Lagrangian
2. Model file containing all fields, vertices, and propagators

```
M$ClassesDescription = {
F[1] == {
     SelfConjugate -> False,
     PropagatorType -> Straight,
     PropagatorArrow -> Forward,
     Mass -> mq,
     Indices -> {Index[Colour]},
     PropagatorLabel -> "q" },

V[1] == {
     SelfConjugate -> True,
     Indices -> {Index[Gluon]},
     PropagatorLabel -> A,
     PropagatorType -> Cycles,
     PropagatorArrow -> None,
     Mass -> MA },

(* external sources coupled to composed operators *)

V[2] == {
     SelfConjugate -> True,
     Indices -> {Index[Gluon]},
     PropagatorLabel -> J1,
     PropagatorType -> Straight,
     PropagatorArrow -> None,
     InsertOnly -> External,
     Mass -> 0 },
```

```
(*      Couplings (calculated by FeynRules)      *)

M$CouplingMatrices = {

C[ -U[1, {e1x1}] , U[1, {e2x1}] , V[1, {e3x2}] ] == {{gc1
*FASUNF[e3x2, e1x1, e2x1], 0}, {0, 0}},

C[ -U[2, {e1x1}] , U[2, {e2x1}] , V[1, {e3x2}] ] == {{gc2
*FASUNF[e3x2, e1x1, e2x1], 0}, {0, 0}},

C[ S[2, {e3x1}] , -U[2, {e1x1}] , U[2, {e2x1}] ] == {{I*gc3
*FASUNF[e3x1, e1x1, e2x1], 0}, {0, 0}, {0, 0}},

C[ V[1, {e1x2}] , V[1, {e2x2}] , V[1, {e3x2}] ] == {{-(gc4
*FASUNF[e1x2, e2x2, e3x2]), 0}, {gc4*FASUNF[e1x2, e2x2, e3x2],
0}, {gc4*FASUNF[e1x2, e2x2, e3x2], 0}, {-(gc4*FASUNF[e1x2,
e2x2, e3x2]), 0}, {-(gc4*FASUNF[e1x2, e2x2, e3x2]), 0}, {gc4
*FASUNF[e1x2, e2x2, e3x2], 0}},

C[ V[1, {e1x2}] , V[1, {e2x2}] , V[1, {e3x2}] , V[1, {e4x2}] ]
== {{(-I)*gc5*(FASUNF[e1x2, e2x2, e3x2, e4x2] + FASUNF[e1x2,
e3x2, e2x2, e4x2]), 0}, {I*gc5*(FASUNF[e1x2, e2x2, e3x2,
e4x2] - FASUNF[e1x2, e4x2, e2x2, e3x2]), 0}, {I*gc5
*(FASUNF[e1x2, e3x2, e2x2, e4x2] + FASUNF[e1x2, e4x2, e2x2,
e3x2]), 0}},

C[ -F[1, {e1x2}] , F[1, {e2x2}] , V[1, {e3x2}] ] == {{I*gc6
*FASUNT[e3x2, e1x2, e2x2], 0}, {I*gc6*FASUNT[e3x2, e1x2,
e2x2], 0}},

C[ S[3, {e2x1}] , S[2, {e3x1}] , V[1, {e1x2}] ] == {{gc7
*FASUNF[e1x2, e2x1, e3x1], 0}, {0, 0}},
```

1. Draw all topologies, eg. $1 \to 1$ up to specified order
2. Load FeynRules model and introduce fields
3. Create amplitudes and evaluate one-loop integral
4. Repeat for all external fields



Using Mathematica packages FeynCalc and FeynRules to draw and calculate all possible diagrams, and be combining these results, the 'correction' term in (10) is calculated up to g^2 and found to be

$$'corrections' = -\frac{\alpha g^2 C_A \left(\epsilon(a+2\gamma+2) + 4\epsilon \log(p) - 4\right)}{4\epsilon p^2} P_{\mu\nu} + 0 \times L_{\mu\nu} \quad (11)$$

With $m_{A^h} = 0$ in (5) but we introduced a mass $M \to 0$ in $\langle \tau\tau \rangle$.
In future research, in collaboration with Pietro Dall'Olio, we would like to reintroduce the mass m_{A^h} in the action $\frac{m^2}{2} A_\mu^{h,a} A_\mu^{h,a}$.
With the addition of the mass this can again be linked with the Curci-Ferrari model (see work of Urko et al.), while again solving the Gribov problem.

Next, we will include fermions by addition of

$$S_f = \int \mathrm{d}^4x \Big(\bar{\psi}(i\not{D} + m_f)\psi \Big) \,, \tag{12}$$

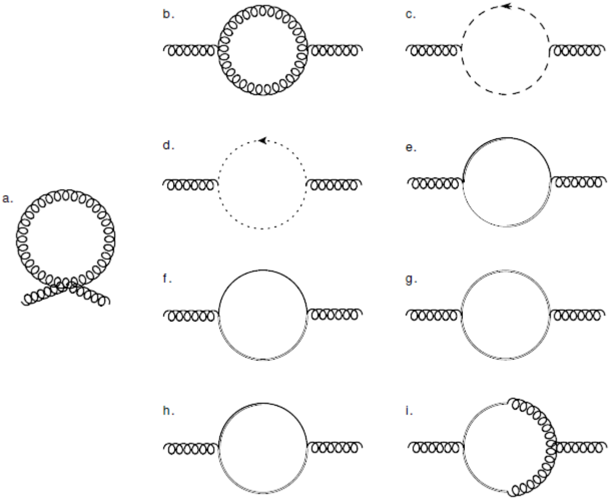
and by defining the corresponding gauge invariant fermion fields

$$\psi^h = h^\dagger \psi \quad \bar{\psi}^h = \bar{\psi} h \tag{13}$$

one can study arbitrary transformations

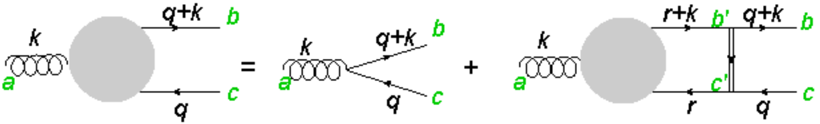
$$\begin{aligned} &\langle A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n)\bar{\psi}(y_1)\psi(z_1)\dots\bar{\psi}(y_m)\psi(z_m)\rangle_\alpha = \\ &\langle A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n)\bar{\psi}(y_1)\psi(z_1)\dots\bar{\psi}(y_m)\psi(z_m)\rangle_{\alpha=0} - \mathcal{R}_\alpha(x_1, y_1, z_1\dots), \end{aligned} \tag{14}$$

Try to evaluate the gluon self-energy, in collaboration with Urko Reinosa.



Take a look at diagram g. Include extra $\phi\phi\tau$-vertices, to introduce τ-ladders.

All these diagrams are IR-divergent. We try to resum these represented by the following diagram



This diagram can be translated in the following rule

$$Y^{\mu}_{abc}(k,q) = Y^{(0)\mu}_{abc}(k,q) + \alpha^2 \int_r Y^{\mu}_{ab'c'}(k,r) X_{b'bcc'}(r,q,q+k) \frac{1}{r^4} \frac{1}{(r+k)^4} \quad (15)$$

Which can be cleaned up a bit, to

$$q^4 Z(q^2) = 1 - \frac{\alpha^2 g^2 m^2}{128\pi^2} \left[\int_0^{q^2} \mathrm{d}r^2 \, \frac{\left(r^2 - q^2\right)^2}{q^4} Z\left(r^2\right) + \int_{q^2}^{\infty} \mathrm{d}r^2 \, \frac{\left(r^2 - q^2\right)^2}{r^4} Z\left(r^2\right) \right] \quad (16)$$

The self energy (for this diagram) is then found by closing this diagram with the $\phi\phi A$-vertex on the right, and is given by

$$\Pi(0) = -\frac{g^2 m^4 \alpha^2}{64\pi^2} \int_0^{\infty} \mathrm{d}q^2 \, Z\left(q^2\right) \quad (17)$$

Trying to solve (16) we combined Gauss-Legendre and Gauss-Laguerre quadrature.

1. Split the interal $0 \to 1$ and $1 \to \infty$
2. Combine Legendre and Laguerre quadratures, eg. Legendre quadrature

$$\int_0^1 f(x,y)Z(x)\,\mathrm{d}x = \frac{1}{2}\sum_{i=1}^{n_{Leg}} w_i f\left(\frac{x_i+1}{2}, y\right) Z\left(\frac{x_i+1}{2}\right) \tag{18}$$

3. Z is known on several node points $Z(x_i')$
4. Solve this group of equations to find all $Z(x_i')$'s
5. With these solutions, calculate $\Pi(0)$

We found inconsistent results ($\Pi(0)$ non-convergent), but the solutions to Z tell a story.

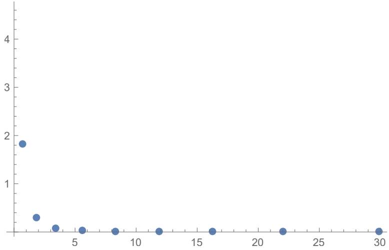


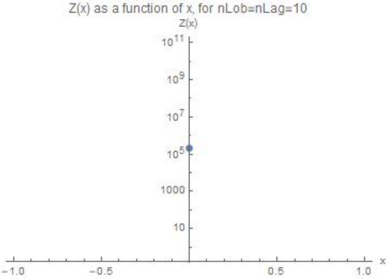
Figure: $Z(x)$ on the node points x_i, for $n_{Lag} = 10$

$Z(x)$ increases as $x \to 0$. But the exact zero point $x_i = 0$ is not included in Legendre quadrature. Switch the first integral to Lobatto quadrature, which has zero included.

Reran the computation with the new nodes x_i and weights w_i (with the constants α, g, and m fixed). We found a consistent result over all orders of n_{Lob} and n_{Lag}, namely

$$\Pi(0) = -2 \tag{19}$$

But even more interesting



We propose the following Anzatz

$$Z(x) = \frac{1}{cc}\delta(x). \tag{20}$$

which solves (16) for $cc = \frac{128\pi^2}{\alpha^2 g^2 m^2}$, and

$$\Pi(0) = -2m^2 \tag{21}$$

Exactly the numerical result, proving furthermore that the resummation (of this subsection of diagrams) yields a finite result.

We met two classes of divergent diagrams.

- Diagram (g) proportional to $\frac{\alpha^2 m^2}{k^2}$ could be resolved by this ladder resummation.
- Diagrams $\propto \log k^2$

The second class could not be solved by adding diagrams as you cannot add extra powers of m^2, meaning extra $A\xi\xi$ vertices for instance.
To solve this class we turned to Renormalisation Group.

Effective action takes the form of

$$\Pi \propto \alpha g^2 m^2 \ln \frac{\mu}{k} \tag{22}$$

but we propose the following resummation

$$\Pi_{resum} = \sum_n m^2 \left(\alpha g^2 \ln\left(\frac{\mu}{k}\right)\right)^n \tag{23}$$

which, after using the RG equation, becomes

$$\Pi_{resum} = \left[-1 + \frac{3+\alpha}{16\pi^2} N\alpha g^2 \ln\left(\frac{\mu}{k}\right)\right]^{-\frac{3+2\alpha}{6+2\alpha}} \tag{24}$$

As $\alpha \geq 0$ this result is now protected from infrared divergences.
This solves the log-divergent class of diagrams, similar reasoning can solve the other class. And it explains why resummation worked in the first class but not in the second.

- (dynamic) CF model and introduce the operator $A^h A^h$ through source term $J A^h A^h$.
- transformation of schemes $\overline{\text{MS}} \leftrightarrow$ IS.
- fix mass m at one scale via the effective potential and use this to predict the running of the propagators.

- Finish the work concerning LKFT's, in collaboration with Pietro Dall'Olio
  - Include m_{A^h} to the Lagrangian and rerun computations
  - Include fermions and study LKFT's in this sector, which can be linked with chiral symmetry
  - Investigate the connection with the Nielson identities
- Finish the work in resummation, in collaboration with Urko Reinosa and others
  - Further interpreting the CF and RG results and the comparison to lattice data
  - Writing the paper for the gluon self-energy, both numeric and analytic, and the relation to RG

[1] M. A. L. Capri, D. Fiorentini, M. S. Guimaraes, B. W. Mintz, L. F. Palhares and S. P. Sorella, Phys. Rev. D **94** (2016) no.6, 065009 [arXiv:1606.06601 [hep-th]].

[2] T. De Meerleer, D. Dudal, S. P. Sorella, P. Dall'Olio and A. Bashir, Phys. Rev. D **97** (2018) no.7, 074017 doi:10.1103/PhysRevD.97.074017 [arXiv:1801.01703 [hep-th]].

[3] J. A. Gracey, M. Peláez, U. Reinosa and M. Tissier, Phys. Rev. D **100** (2019) no.3, 034023 doi:10.1103/PhysRevD.100.034023 [arXiv:1905.07262 [hep-th]].