Scattering in chiral strong backgrounds

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Work in progress with L. Mason & A. Sharma
see also work with E. Casali, A. Ilderton & S. Nekovar

Amplitudes: what's it all about?

To compute S-matrix, usually follow recipe:

- Perturbation theory around trivial background
- Space-time Lagrangian $\rightarrow$ Feynman rules
- Draw diagrams & compute

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- Lots of progress, especially for massless QFTs
- *All-multiplicity* formulae at tree-level and beyond
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New formulation(s) of perturbative QFT?

Examples

Consider Yang-Mills theory. At tree-level, we know *everything*:

$$A_{n,0}^{(0)} = \delta^4\left(\sum_{i=1}^{n} k_i\right) \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle \langle 2\, 3\rangle \cdots \langle n\, 1\rangle}$$

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$$A_{n,k}^{(0)} = \int \frac{\prod_{r=0}^{k+1} \mathrm{d}^{4|4} U_r}{\mathrm{vol}\, \mathrm{GL}(2,\mathbb{C})} \prod_{i=1}^{n} \frac{\mathrm{d}\sigma_i\, \mathcal{A}_i(Z(\sigma_i))}{\sigma_i - \sigma_{i+1}}$$ [Roiban-Spradlin-Volovich-Witten]

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[Roiban-Spradlin-Volovich-Witten]

$$A_n^{(0)} = \delta^d\left(\sum_{i=1}^{n} k_i\right) \int \mathrm{d}\mu_n \prod_{j=1}^{n} \delta(\mathcal{S}_j) \prod_{i=1}^{n} \frac{1}{\sigma_i - \sigma_{i+1}} \mathrm{Pf}'(M)$$

[Cachazo-He-Yuan]

Strong backgrounds

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MANY reasons to be interested:

- Strong field QED/laser physics – (electromagnetic plane waves)
- Strong field QCD/colour glass condensates – (Yang-Mills plane waves & shockwaves)
- Gravitational waves – (gravitational plane waves & shockwaves)
- Cosmology – (de Sitter or FLRW space-times)
- Strongly-coupled CFTs/holography – (anti-de Sitter)
- Non-perturbative physics – (shockwaves)

Knowledge gap

What do we know about amplitudes in strong backgrounds?

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What do we know about amplitudes in strong backgrounds?

Best results at tree-level: *4-points*

- QED in plane wave [Ilderton]
- YM/GR in AdS [Raju]
- YM in plane wave [TA-Casali-Mason-Nekovar]

But a novel formulation of pQFT should work on *any* perturbative background...

Today

Question:

Can we make all-multiplicity statements about scattering in strong backgrounds?

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Strategy:

Consider 4d gluon scattering on simplest non-trivial background – chiral plane waves

Plane Waves

Solution to vacuum equations (in d dim.) with:

- covariantly constant null symmetry n,
- $(2d - 4)$ additional symmetries,
- commuting to form Heisenberg algebra w/ center n

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For Yang-Mills theory, PWs valued in Cartan of gauge group

[Trautman, Basler-Hadicke, TA-Casali-Mason-Nekovar]

$$\mathrm{d}s^2 = 2\mathrm{d}x^+\,\mathrm{d}x^- - (\mathrm{d}x^\perp)^2\,, \qquad A = x^\perp\,\dot{a}_\perp(x^-)\,\mathrm{d}x^-$$

$a_\perp(x^-)$ are $d-2$ Cartan-valued free functions

Null symmetry: $n = \partial_+$

Scattering on plane waves

Sandwich waves: $\dot{a}_\perp(x^-)$ compactly supported:

- Asymptotically flat regions
- Unitary evolution
- No particle creation (in quadratic theory)

[Gibbons, Garriga-Verdaguer,

TA-Casali-Mason-Nekovar]

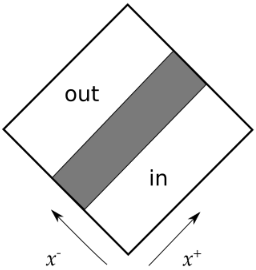

Figure: Sandwich wave

Lots to like about PWs:

- Symmetries
- Physical interpretation $\leftrightarrow$ coherent superposition of gluons
- Universality (Penrose limits)
- Well-defined S-matrix
- Explicit Feynman rules [Volkov, Gibbons, Ward, Mason, TA-Casali-Mason-Nekovar]

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But many new subtleties:

- No d-dimensional momentum conservation – integrals always left over due to wave profile
- Memory effect [Zhang-Duval-Gibbons-Horvathy, TA-Casali-Mason-Nekovar]
- Tails [Günther-Wünsch, Mason, Harte]

4d Plane Waves

Even more structure for $d = 4$:

$$\mathrm{d}s^2 = 2\left(\mathrm{d}x^+\,\mathrm{d}x^- - \mathrm{d}z\,\mathrm{d}\bar{z}\right) = \mathrm{d}x_{\alpha\dot{\alpha}}\,\mathrm{d}x^{\alpha\dot{\alpha}}\,,$$

for

$$x^{\alpha\dot{\alpha}} = \begin{pmatrix} x^+ & \bar{z} \\ z & x^- \end{pmatrix}$$

Propagation direction of wave: $n = \partial_+$

$$\text{Since } n^2 = 0\,, \qquad n^{\alpha\dot{\alpha}} = \iota^\alpha\,\tilde{\iota}^{\dot{\alpha}}\,, \qquad \iota^\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \tilde{\iota}^{\dot{\alpha}}$$

Result:

$$A = \left(z\,\dot{a}(x^-) + \bar{z}\,\dot{\bar{a}}(x^-)\right)\,\iota_\alpha\tilde{\iota}_{\dot{\alpha}}\,\mathrm{d}x^{\alpha\dot{\alpha}}$$

Spinor-helicity on PWs [TA-Ilderton]

On-shell gluon perturbations proportional to $\mathrm{e}^{\mathrm{i}\phi_k}$

$$\phi_k = k_+ \, x^+ + k \, z + \bar{k} \, \bar{z} + e \, (z \, a + \bar{z} \, \bar{a}) + \frac{1}{k_+} \int^{x^-} \mathrm{d}s \, |k + e \, a(s)|^2$$

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Momentum $K_{\alpha\dot{\alpha}}(x^-) = -\mathrm{i}\mathrm{e}^{-\mathrm{i}\phi_k}\, D_{\alpha\dot{\alpha}}\mathrm{e}^{\mathrm{i}\phi_k}$ is on-shell:

$$K^2(x^-) = 0 \Rightarrow K_{\alpha\dot{\alpha}} = \Lambda_\alpha\, \tilde{\Lambda}_{\dot{\alpha}}\,, \quad \Lambda_\alpha = \lambda_\alpha + \frac{e\, a(x^-)}{\sqrt{k_+}}\, \iota_\alpha$$

On-shell polarizations: $\mathcal{E}^-_{\alpha\dot{\alpha}} = \dfrac{\Lambda_\alpha\, \tilde{\iota}_{\dot{\alpha}}}{[\tilde{\iota}\, \tilde{\lambda}]}\,, \qquad \mathcal{E}^+_{\alpha\dot{\alpha}} = \dfrac{\iota_\alpha\, \tilde{\Lambda}_{\dot{\alpha}}}{\langle \iota\, \lambda\rangle}$

So far...

In a 4d PW background, we can:

- use explicit Feynman rules
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- exploit 3-momentum conservation (in $x^+, z, \bar{z}$ directions)

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Can we simplify any further?

Self-dual plane waves

Complexify background and require:

$$*F = \mathrm{i}\, F$$

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Result: only 1 functional d.o.f.

$$A = \bar{z}\,\dot{f}(x^-)\,\mathrm{d}x^- = \bar{z}\,\dot{f}(x^-)\,\iota_\alpha\tilde{\iota}_{\dot{\alpha}}\,\mathrm{d}x^{\alpha\dot{\alpha}}$$

$$F = \dot{f}(x^-)\,\mathrm{d}\bar{z}\wedge\mathrm{d}x^- = \dot{f}\,\tilde{\iota}_{\dot{\alpha}}\tilde{\iota}_{\dot{\beta}}\,\mathrm{d}x_\alpha{}^{\dot{\alpha}}\wedge\mathrm{d}x^{\alpha\dot{\beta}}$$

Coherent superposition of positive-helicity gluons

SDPW kinematics

SDPW have *chiral* on-shell kinematics

Gluon with incoming momentum $k_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$: $\mathsf{T}^{\mathsf{a}}\, \mathcal{E}^{\pm}_{\alpha\dot{\alpha}}(x^-)\, \mathrm{e}^{\mathrm{i}\phi_k}$

$$\phi_k = k \cdot x + e\bar{z}\, f(x^-) + \frac{k}{k_+} \int^{x^-} \mathrm{d}t\, e\, f(t)$$

On-shell kinematics:

$$K_{\alpha\dot{\alpha}}(x^-) = \lambda_\alpha\, \tilde{\Lambda}_{\dot{\alpha}}\,, \qquad \tilde{\Lambda}_{\dot{\alpha}} := \tilde{\lambda}_{\dot{\alpha}} + \frac{e}{\sqrt{k_+}}\, \tilde{\iota}_{\dot{\alpha}}\, f(x^-)$$

$$\mathcal{E}^{-}_{\alpha\dot{\alpha}} = \frac{\lambda_\alpha\, \tilde{\iota}_{\dot{\alpha}}}{[\tilde{\iota}\, \tilde{\lambda}]}\,, \qquad \mathcal{E}^{+}_{\alpha\dot{\alpha}} = \frac{\iota_\alpha\, \tilde{\Lambda}_{\dot{\alpha}}}{\langle \iota\, \lambda \rangle}$$

So what?

MHV scattering = helicity flip on a SD background [Mason-Skinner]

Shift YM action by topological term:

$$-\frac{1}{2\,\mathrm{g}^2}\int \mathrm{tr}\, F \wedge *F + \frac{1}{8\,\mathrm{g}^2}\int \mathrm{tr}\, F \wedge F = -\frac{1}{2\,\mathrm{g}^2}\int \mathrm{tr}\, F^- \wedge F^-$$

for F^- the ASD part of F.

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for F^- the ASD part of F.

Introduce Lagrange multiplier $B \in \Omega^2_-(\mathfrak{g})$.

$-\frac{1}{2\,\mathrm{g}^2}\int \mathrm{tr}\, F^-\wedge F^-$ equivalent to

$$S[A,B] = \int \mathrm{tr}\, F^-\wedge B + \frac{\mathrm{g}^2}{2}\int \mathrm{tr}\, B\wedge B$$

Field equations:

$$\boxed{F^- = -\mathrm{g}^2\, B\,, \qquad \mathrm{D}B = 0}$$

Upshot

Yang-Mills admits a pert. expansion around SD sector

[Chalmers-Siegel]

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Need: something that manifests the integrability/triviality of the SD background...

Twistor theory

Twistor space: $Z^A = (\mu^{\dot\alpha}, \lambda_\alpha)$ homog. coords. on $\mathbb{CP}^3$

$$\mathbb{PT} = \mathbb{CP}^3 \setminus \{\lambda_\alpha = 0\}$$

$x \in \mathbb{C}^4$ given by $X \cong \mathbb{CP}^1 \subset \mathbb{PT}$ via $\mu^{\dot\alpha} = x^{\alpha\dot\alpha}\lambda_\alpha$

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On a flat background:

- Massless free fields $\leftrightarrow$ cohomology on $\mathbb{PT}$ [Penrose, Sparling, Eastwood-Penrose-Wells]
- Representation for on-shell scattering kinematics [Hodges]
- Full tree-level S-matrix of $\mathcal{N} = 4$ SYM [Witten, Berkovits, Roiban-Spradlin-Volovich]
- Full tree-level S-matrix of $\mathcal{N} = 8$ SUGRA [Cachazo-Skinner]

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Theorem [Ward, 1977]

There is a 1:1 correspondence between:

- SD SU(N) Yang-Mills fields on $\mathbb{C}^4$, and
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Upshot: twistor theory *trivializes* the SD sector

SDPWs in Twistor Space

Can construct $E \to \mathbb{PT}$ explicitly; holomorphicity encoded by partial connection on E:

$$\bar{D} = \bar{\partial} + \mathsf{A}\,, \qquad \mathsf{A} = \frac{\langle a\,\lambda\rangle}{\langle a\,\iota\rangle}\,\bar{\partial}\left(\frac{1}{\langle\lambda\,\iota\rangle}\right)\int^{\frac{\langle a\,\iota\rangle}{\langle a\,\lambda\rangle}[\tilde{\iota}\,\mu]}\mathrm{d}t\,f(t)$$

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Penrose transform: gluons encoded by E-twisted cohomology on $\mathbb{PT}$

$$\text{- helicity } \leftrightarrow H^{0,1}_{\bar{D}}(\mathbb{PT},\, \mathcal{O}(-4)\otimes E)$$

$$+ \text{ helicity } \leftrightarrow H^{0,1}_{\bar{D}}(\mathbb{PT},\, \mathcal{O}\otimes E)$$

MHV generating functional

MHV amplitudes on the SDPW background generated by:

$$\int \mathrm{d}^4 x \int_{X_1 \times X_2} \mathrm{D}\lambda_1\, \mathrm{D}\lambda_2\, \langle \lambda_1\, \lambda_2 \rangle^2\, \mathrm{tr}\left[b_1\, \gamma_1 \gamma_2^{-1}\, b_2\, \gamma_2 \gamma_1^{-1} \right]$$

- $b_{1,2} \in H^{0,1}_{\bar{D}}(\mathbb{PT}, \mathcal{O}(-4) \otimes E)$
- $\gamma \leftrightarrow$ holomorphic trivialization of $E|_X$:

$$\gamma\, \bar{D}|_X \gamma^{-1} = \bar{\partial}|_X\,, \qquad \gamma(x, \lambda) := \exp\left(\mathrm{i} \frac{\langle o\, \lambda \rangle}{\langle \iota\, \lambda \rangle} \int^{x^-} \mathrm{d}t\, f(t) \right)$$

Perturbative expansion

Take $\bar{D} \to \bar{D} + a$, for $a \in H^{0,1}_{\bar{D}}(\mathbb{PT}, \mathcal{O} \otimes E)$

$\gamma_1 \gamma_2^{-1}$ must solve $(\bar{D} + a)|_{X_1} \gamma_1 \gamma_2^{-1} = 0 \Rightarrow$ Born series:

$$\gamma_1 \gamma_2^{-1} = \frac{1}{1 - \bar{D}^{-1}|_X a}$$
$$= 1 + \sum_{k=3}^{\infty} \int_{X^{k-2}} \frac{\gamma_1}{\langle \lambda_1 \, \lambda_3 \rangle} \left(\prod_{i=3}^{k} \frac{\mathrm{D}\lambda_i \, \gamma_i^{-1} \, a_i \, \gamma_i}{\langle \lambda_i \, \lambda_{i+1} \rangle} \right) \frac{\gamma_2}{\langle \lambda_k \, \lambda_2 \rangle}$$

Re-labeling the field insertions, n^{th}-order term is

$$\int \mathrm{d}^4 x \int_{X^n} \langle\, \lambda_i\, \lambda_j \rangle^4 \prod_{k=1}^{n} \frac{\mathrm{D}\lambda_k}{\langle \lambda_k\, \lambda_{k+1} \rangle}\, \mathrm{tr}\left[a_1 \cdots b_i\, a_{i+1} \cdots b_j\, a_{j+1} \cdots a_n\right]$$

$$\times \exp\left[\mathrm{i} \sum_{k=1}^{n} e_k\, \frac{\langle o\, \lambda_k \rangle}{\langle \iota\, \lambda_k \rangle} \int^{x^-} \mathrm{d}t\, f(t)\right]$$

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$$\times \exp\left[\mathrm{i} \sum_{k=1}^{n} e_k \, \frac{\langle o \, \lambda_k \rangle}{\langle \iota \, \lambda_k \rangle} \int^{x^-} \mathrm{d}t \, f(t) \right]$$

Evaluate on twistor representatives for $\mathcal{E}^{\pm}_{\alpha\dot{\alpha}} \, \mathrm{e}^{\mathrm{i}\phi_k}$:

$$a_i = \frac{\langle a \, \lambda \rangle}{\langle a \, i \rangle} \, \bar{\partial} \left(\frac{1}{\langle \lambda \, i \rangle} \right) \, \mathrm{e}^{\mathrm{i} \frac{\langle a \, i \rangle}{\langle a \, \lambda \rangle} [\mu \, i]} \, , \quad b_i = \frac{\langle a \, i \rangle^3}{\langle a \, \lambda \rangle^3} \, \bar{\partial} \left(\frac{1}{\langle \lambda \, i \rangle} \right) \, \mathrm{e}^{\mathrm{i} \frac{\langle a \, i \rangle}{\langle a \, \lambda \rangle} [\mu \, i]}$$

MHV amplitude

Evaluating the $\mathbb{CP}^1$ integrals gives:

$$\boxed{\delta^3_{+,\perp}\left(\sum_{i=1}^{n} k_i\right) \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\, \langle 2\, 3\rangle \cdots \langle n\, 1\rangle} \int\limits_{-\infty}^{+\infty} \mathrm{d}x^- \, \mathrm{e}^{\mathrm{i}\mathcal{F}_n(x^-)}}$$

for *Volkov exponent*

$$\mathbb{K}^{\alpha\dot{\alpha}}(x^-) := \sum_{i=1}^{n-1} K_i^{\alpha\dot{\alpha}}(x^-)\,, \qquad \boxed{\mathcal{F}_n(x^-) := \frac{1}{\mathbb{K}_+} \int^{x^-} \mathrm{d}t\, \mathbb{K}^2(t)}$$

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Resolution: field redefinition recasts Yang-Mills action such that all MHV vertices have *single* lightfront integral [Mansfield]

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Other sanity checks & features:

- Explicit checks at 3- and 4-points
- Perturbative limit (MHV$_n$+ background $\rightarrow$ MHV$_{n+1}$)
- Flat background limit
- Generalization to $\mathcal{N}=4$ SYM

Full tree-level S-matrix?

Easy guess for N^{k}MHV, based on holomorphic maps $Z : \mathbb{CP}^1 \to \mathbb{PT}$

$$\int \frac{\prod_{r=0}^{k+1} \mathrm{d}^{4|4} U_r}{\mathrm{vol}\,\mathrm{GL}(2,\mathbb{C})} \, \mathrm{tr}\left(\prod_{i=1}^{n} \frac{\mathrm{d}\sigma_i \, \gamma_i^{-1} \, \mathcal{A}_i(Z(\sigma_i)) \, \gamma_i}{\sigma_i - \sigma_{i+1}} \right)$$

where:

- $Z(\sigma) = \sum_{r=0}^{k+1} U_r \, \sigma^r$ is a degree $k+1$ holomorphic map
- $\{\sigma_i\} \subset \mathbb{CP}^1$ punctures on $\mathbb{CP}^1$
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Currently just a conjecture...

Summary

Upshot: it *is* possible to make all-multiplicity statements in strong backgrounds!

Also a (more complicated) version of this story for **gravity**!

Many exciting things to do:

- Prove/correct N^{k}MHV conjecture
- Double copy for full tree-level SDPW S-matrix
- Generalize to generic PW backgrounds
- Other SD backgrounds?