# *Symmetries of Celestial Amplitudes*

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Joint String Theory Meeting

June 18, 2020

based on:

A. Fotopoulos, St.St., T.R. Taylor, Bin Zhu:

- **BMS Algebra from Soft and Collinear Limits**

arXiv:1912.10973

JHEP 03 (2020) 130

Wei Fan, A. Fotopoulos, St.St., T.R. Taylor:

- **On Sugawara construction on Celestial Sphere**

arXiv:2005.10666

see also:

St.St., T.R. Taylor:

- **Strings on Celestial Sphere**

arXiv:1806.05688

Nucl. Phys. B935 (2018) 388-411

- **Symmetries of Celestial Amplitudes**

arXiv:1812.01080

Phys. Lett. B793 (2019) 141-143

Traditional momentum space

$$p_k^\mu, \quad k = 1, \ldots, n$$
$$p_k^2 = -m_k^2$$

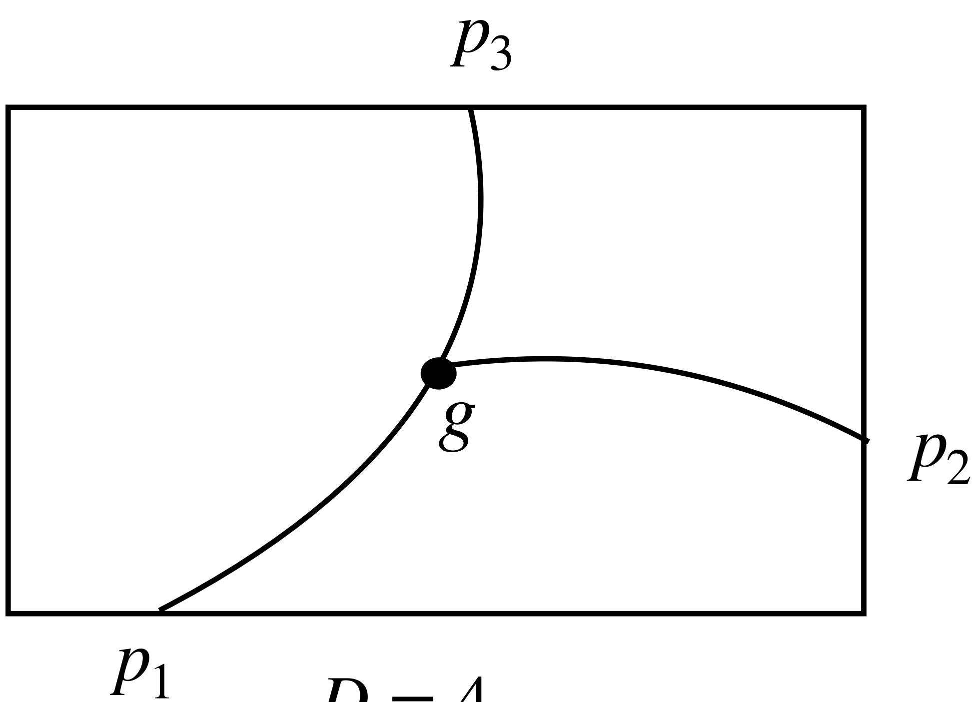
- amplitudes specified by asymptotic wave functions, which transform simply under space-time translations
- with manifest translation symmetry
- traditional amplitudes describe transitions between momentum eigenstates

D=4 Minkowski space probably not the right space
to see **all** symmetries
of scattering amplitudes

Scattering amplitudes in D=4
have interpretation
as Euklidian **D=2** conformal correlators

# Basic Idea

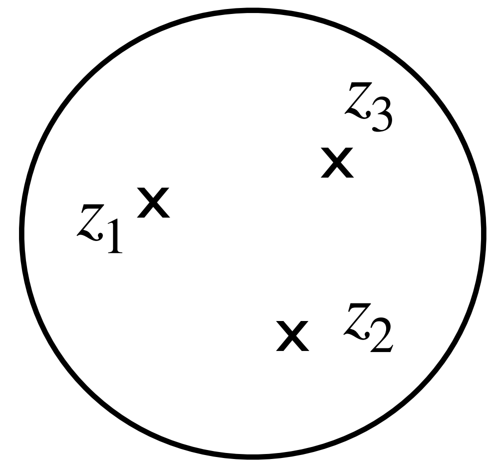
Amplitudes = conformal correlators of primary fields on celestial sphere




$D = 4$

$$z_k = \frac{p_k^1 + ip_k^2}{p_k^0 + p_k^3}$$

$$=$$




celestial sphere

$D = 2$

$$\sim \frac{g}{|z_1 - z_2|^{h_1+h_2-h_3}\ |z_2 - z_3|^{h_2+h_3-h_1}\ |z_1 - z_3|^{h_1+h_3-h_2}}$$

D=4 space-time QFT correlators

D=2 Euklidian CFT correlators

Lorentz symmetry

$SO(1,3) \simeq SL(2,\mathbf{C})$

$$z_i \to \frac{az_i + b}{cz_i + d}$$

global conformal symmetry on $CS^2$

# Why ?

- Constrain S-matrix and understand amplitude relations

*From studying scattering amplitudes:*
***deep connections between***
***gravity and gauge interactions***
*e.g.: KLT, BCJ, EYM (double-copy-construction)*

- scattering amplitudes in both gauge and gravity theories suggest a deeper connection

- indication for the existence of some gauge structure in quantum gravity

- New way of looking at quantum field theory and quantum gravity

- flat space-time holography

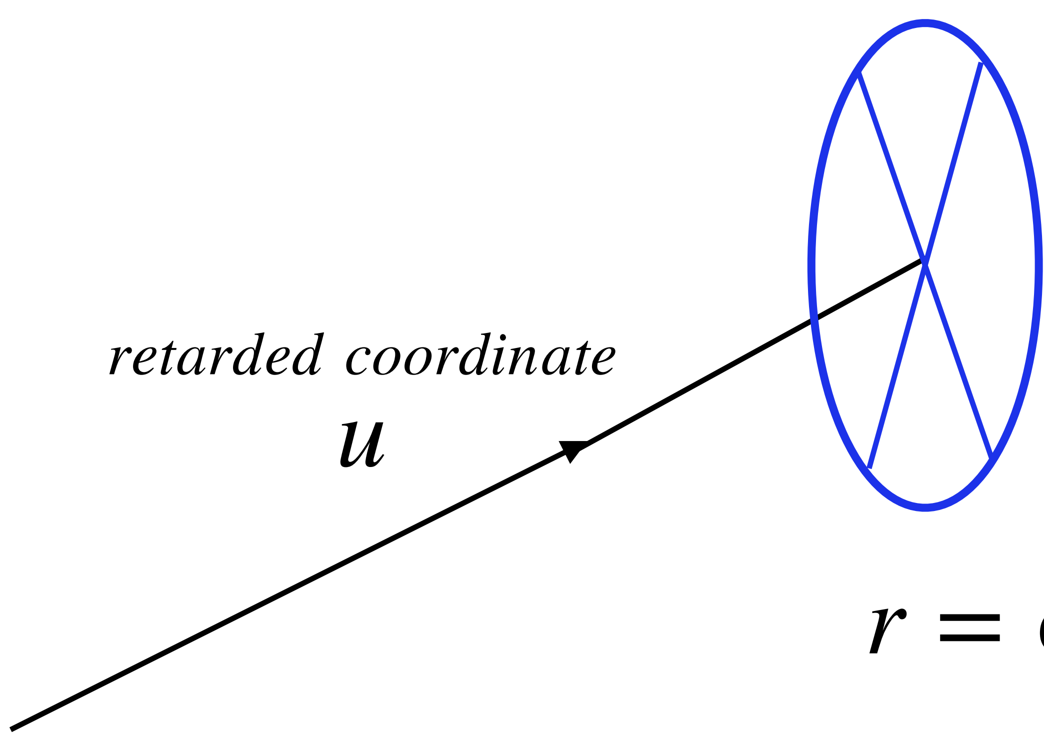
$$ds^2 = -dt^2 + d\vec{x}^2$$

Flat Minkowski metric in <u>retarded (or Bondi) coordinates</u> $(u, r, z, \bar{z})$

$$\begin{cases} x^0 = u + r \\ x^1 = \dfrac{r\,(z+\bar{z})}{1+|z|^2} \\ x^2 = -i\dfrac{r\,(z-\bar{z})}{1+|z|^2} \\ x^3 = \dfrac{r(1-|z|^2)}{1+|z|^2} \end{cases}$$

$$r^2 = \vec{x}^2$$

$$ds^2 = -du^2 - 2\,dudr + \underbrace{\frac{4r^2}{(1+|z|^2)^2}}_{S^2}\,dzd\bar{z}$$




*Null infinity* $\mathscr{I}^+$

$S^2 \times \mathbf{R}$

$(u, z, \bar{z})$

*natural arena*
*for in and out states*

# Massless particle on celestial sphere

described by
- the point $z \in CS^2$ at which it enters or exits the celestial sphere
- SL(2,C) Lorentz quantum numbers $(h, \bar{h})$

$$z \in CS^2 \quad \Longrightarrow \quad p^\mu = \frac{\omega}{1+|z|^2}\, q^\mu(z,\bar{z})$$

Null vector $q$

$$q_\mu q^\mu = 0$$

with: $$q^\mu = (1+|z|^2, z+\bar{z}, -i(z-\bar{z}), 1-|z|^2)$$

$$\omega = E$$

invert: $$z = \frac{p^1 + ip^2}{p^0 + p^3}$$

$$(\vec{p})^2 = (p^0)^2 \quad , \quad E = p^0$$

$$p^\mu \longrightarrow (\omega, z, \bar{z})$$

plane waves in Minkowski: $\exp\{\pm i p_\mu x^\mu\}$

boost eigenstates: $\exp\{\pm iEu\}$

# Particles <-> Operators

in momentum basis: plane waves with momentum $p = \omega q(z)$

in conformal basis: conformal primary wave functions $\Phi$

"state operator correspondence"

$$\Phi_{h,\bar{h}}\left(\frac{az+b}{cz+d}, \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}\right) = (cz+d)^{2h}\,(\bar{c}\bar{z}+\bar{d})^{2\bar{h}}\,\Phi_{h,\bar{h}}(z,\bar{z})$$

with:

$h + \bar{h} = \Delta$ dimension

$h - \bar{h} = J$ spin

$$(h,\bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

In the massless case, with or without spin,
transition from momentum space to conformal primary wavefunctions
with conformal dimension $\Delta$
is implemented by Mellin transform:

$$\tilde{\phi}(\Delta) = \int_0^\infty d\omega \; \omega^{\Delta-1} \phi(\omega)$$

E.g.: plane wave $\exp\{\pm i p_\mu x^\mu\}$

$$\phi^{\pm}_{\Delta}(x, z, \bar{z}) = \int_0^\infty d\omega \; \omega^{\Delta-1} \; \exp\left\{ \pm i\omega q_\mu x^\mu - \epsilon\omega \right\}$$

$$= \left\{ x^\mu q_\mu(z, \bar{z}) \mp i\epsilon \right\}^{-\Delta}$$

solves D=4
Klein-Gordon equation

scalar: J=0 $h = \bar{h} = \frac{\Delta}{2}$ $\Delta = 1 + i\lambda, \; \lambda \in \mathbf{R}$

*Pasterski, Shao (2017)*

# n-point amplitude on celestial sphere

$$\mathscr{A}(\{p_i, \epsilon_j\}) = i(2\pi)^4\ \delta^{(4)}\left(p_1 + p_2 - \sum_{k=3}^{n} p_k\right) A(\{p_i, \epsilon_j\})$$

with:
$$\langle ij \rangle = 2\ (\omega_i\omega_j)^{1/2}\ (z_i - z_j)$$
$$[ij] = 2\ (\omega_i\omega_j)^{1/2}\ (\bar{z}_i - \bar{z}_j)$$

$$\epsilon^\mu(q)_\pm = \frac{1}{\sqrt{2}} \begin{cases} \partial_z q^\mu = (\bar{z}, 1, -i, -\bar{z}) \\ \partial_{\bar{z}} q^\mu = (z, 1, \ i, -z) \end{cases}$$

Celestial amplitudes $\tilde{\mathscr{A}}$ of massless particles are obtained from momentum-space amplitudes $\mathscr{A}$ by Mellin transforms w.r.t. particle energies $\Delta_j = 1 + i\lambda_j$

$$\tilde{\mathscr{A}}_{\{\Delta_l\}}(z_l, \bar{z}_l) = \left(\prod_{l=1}^{n}\int_0^\infty \omega_l^{\Delta_l - 1}\ d\omega_l\right)\ \delta^{(4)}(\omega_1 q_1 + \omega_2 q_2 - \sum_{k=3}^{N} \omega_k q_k)$$
$$\times A(\omega_n, z_n, \bar{z}_n)$$

D=2 CFT correlators involve conformal wave packets

# Gauge Amplitudes

four-gluon amplitude:

$$\tilde{\mathscr{A}}_4(-,-,+,+) = 8\pi\, \delta(r-\bar{r})\, \theta(r-1) \left( \prod_{i<j}^{4} z_{ij}^{\frac{h}{3}-h_i-h_j}\, \bar{z}_{ij}^{\frac{\bar{h}}{3}-\bar{h}_i-\bar{h}_j} \right)$$

$$\times\, r^{\frac{5}{3}}\, (r-1)^{\frac{2}{3}}\, \delta\left( -4 + \sum_{i=1}^{4} \Delta_i \right)$$

$$r = \frac{z_{12}\, z_{34}}{z_{23}\, z_{41}}$$

conformal invariant cross-ratio on $CS^2$

$$r^{-1} = \sin^2\left(\frac{\theta}{2}\right)$$

*Pasterski, Shao, Strominger (2017)*

$$h_1 = \tfrac{i}{2}\lambda_1,\quad h_2 = \tfrac{i}{2}\lambda_2,\quad h_3 = 1+\tfrac{i}{2}\lambda_3\,,\quad h_4 = 1+\tfrac{i}{2}\lambda_4$$

$$\bar{h}_1 = 1+\tfrac{i}{2}\lambda_1,\quad \bar{h}_2 = 1+\tfrac{i}{2}\lambda_2,\quad \bar{h}_3 = \tfrac{i}{2}\lambda_3\,,\quad \bar{h}_4 = \tfrac{i}{2}\lambda_4$$

higher-point: involve Gaussian hypergeometric functions like string amplitudes

*Schreiber, Volovich, Zlotnikov (2017)*

# Graviton Amplitudes

four-graviton amplitude:

$$\tilde{\mathscr{A}}_4(--,--,++,++) = 2\pi\ \delta(r-\bar{r})\ \theta(r-1)\ \left(\prod_{i<j}^{4} z_{ij}^{\frac{h}{3}-h_i-h_j}\ \bar{z}_{ij}^{\frac{\bar{h}}{3}-\bar{h}_i-\bar{h}_j}\right)$$

$$\times\ r^{\frac{11}{3}-\frac{\beta}{3}}\ (r-1)^{-\frac{1}{3}-\frac{\beta}{3}}\ \delta\left(-2+\sum_{i=1}^{4}\Delta_i\right)$$

*St.St., Taylor (2018)*

$$h_1 = -\frac{1}{2}+\frac{i}{2}\lambda_1,\ \ h_2 = -\frac{1}{2}+\frac{i}{2}\lambda_2,\ \ h_3 = \frac{3}{2}+\frac{i}{2}\lambda_3\ , h_4 = \frac{3}{2}+\frac{i}{2}\lambda_4$$

$$\bar{h}_1 = \frac{3}{2}+\frac{i}{2}\lambda_1,\ \ \bar{h}_2 = \frac{3}{2}+\frac{i}{2}\lambda_2,\ \ \bar{h}_3 = -\frac{1}{2}+\frac{i}{2}\lambda_3\ , \bar{h}_4 = -\frac{1}{2}+\frac{i}{2}\lambda_4$$

- *first calculation of graviton amplitude in the conformal basis*
- *important for the soft graviton theorems* $\Delta \to 1, 0, \ldots$ *in celestial basis*

*no holomorphic factorization (due to supertranslation operator P)*

# Operator product expansion

## Celestial conformal field theory (CCFT)

$$\mathcal{O}^a_{\Delta_1,-1}(z,\bar{z})\mathcal{O}^b_{\Delta_2,+1}(w,\bar{w}) = \frac{C_{(-,+)-}(\Delta_1,\Delta_2)}{z-w}\sum_c f^{abc}\ \mathcal{O}^c_{(\Delta_1+\Delta_2-1),-1}(w,\bar{w})$$
$$+\frac{C_{(-+)+}(\Delta_1,\Delta_2)}{\bar{z}-\bar{w}}\sum_c f^{abc}\ \mathcal{O}^c_{(\Delta_1+\Delta_2-1),+1}(w,\bar{w})$$
$$+\ C_{(-+)--}(\Delta_1,\Delta_2)\ \frac{\bar{z}-\bar{w}}{z-w}\ \delta^{ab}\,\mathcal{O}_{(\Delta_1+\Delta_2),-2}(w,\bar{w})$$
$$+\ C_{(-+)++}(\Delta_1,\Delta_2)\ \frac{z-w}{\bar{z}-\bar{w}}\ \delta^{ab}\,\mathcal{O}_{(\Delta_1+\Delta_2),+2}(w,\bar{w})\ + reg\,.$$

Derive from collinear limits of D=4 EYM amplitudes

*Fan, Fotopoulos, St.St., Taylor, Zhu (2019)*

D=4 S-matrix constrains OPE
or vice versa

Derive from first principles and consistency conditions

*Pate, Raclariu, Strominger, Yuan (2019)*

extended BMS symmetry

# Symmetries

At null infinity $\mathscr{I}^{\pm}$ more (hidden) symmetries present
to constrain S-matrix

→ non-trivial consistency on amplitudes

$$z_i \to \frac{az_i + b}{cz_i + d}$$

$$SL(2,Z)_{z_i}: \quad \tilde{\mathscr{A}}_n(\{\Delta_i, J_i\}) \longrightarrow (cz_i + d)^{\Delta_i + J_i} (\bar{c}\bar{z}_i + \bar{d})^{\Delta_i - J_i} \tilde{\mathscr{A}}_n(\{\Delta_i, J_i\})$$

$$P_{-1/2,-1/2} = e^{(\partial_h + \partial_{\bar{h}})/2} = P^0 + P^3$$

*St.St., Taylor (2018)*

$$P^{(j)}_{-1/2,-1/2}: \quad \tilde{\mathscr{A}}_n(\{\Delta_i, J_i\}) \longrightarrow \tilde{\mathscr{A}}_n(\{\Delta_j + 1, J_i\})$$

*comprises into translation operator* $P^{\mu}$ *shifts conformal dimension* $\Delta_j$

celestial gravitational amplitudes appear
as gauge amplitudes translated in space-time

In usual QFT soft theorems $E_s \to 0$ play an important role
in consistency and structure of amplitudes
(in fact, soft theorems completely constrain almost all amplitudes)

in D=4: $p_s \to 0$

$$M_{n+1} \longrightarrow \left( \frac{1}{\epsilon^3} S_{\rm G}^{(0)} + \frac{1}{\epsilon^2} S_{\rm G}^{(1)} + \frac{1}{\epsilon} S_{\rm G}^{(2)} + \dots \right) M_n$$

*Weinberg (1965)* — *Cachazo, Strominger (2014)*

$$A_{n+1} \longrightarrow \left( \frac{1}{\epsilon^2} S_{\rm YM}^{(0)} + \frac{1}{\epsilon} S_{\rm YM}^{(1)} + \dots \right) A_n$$

soft theorems imply
Ward identities for asymptotic symmetries

*Strominger (2013)*

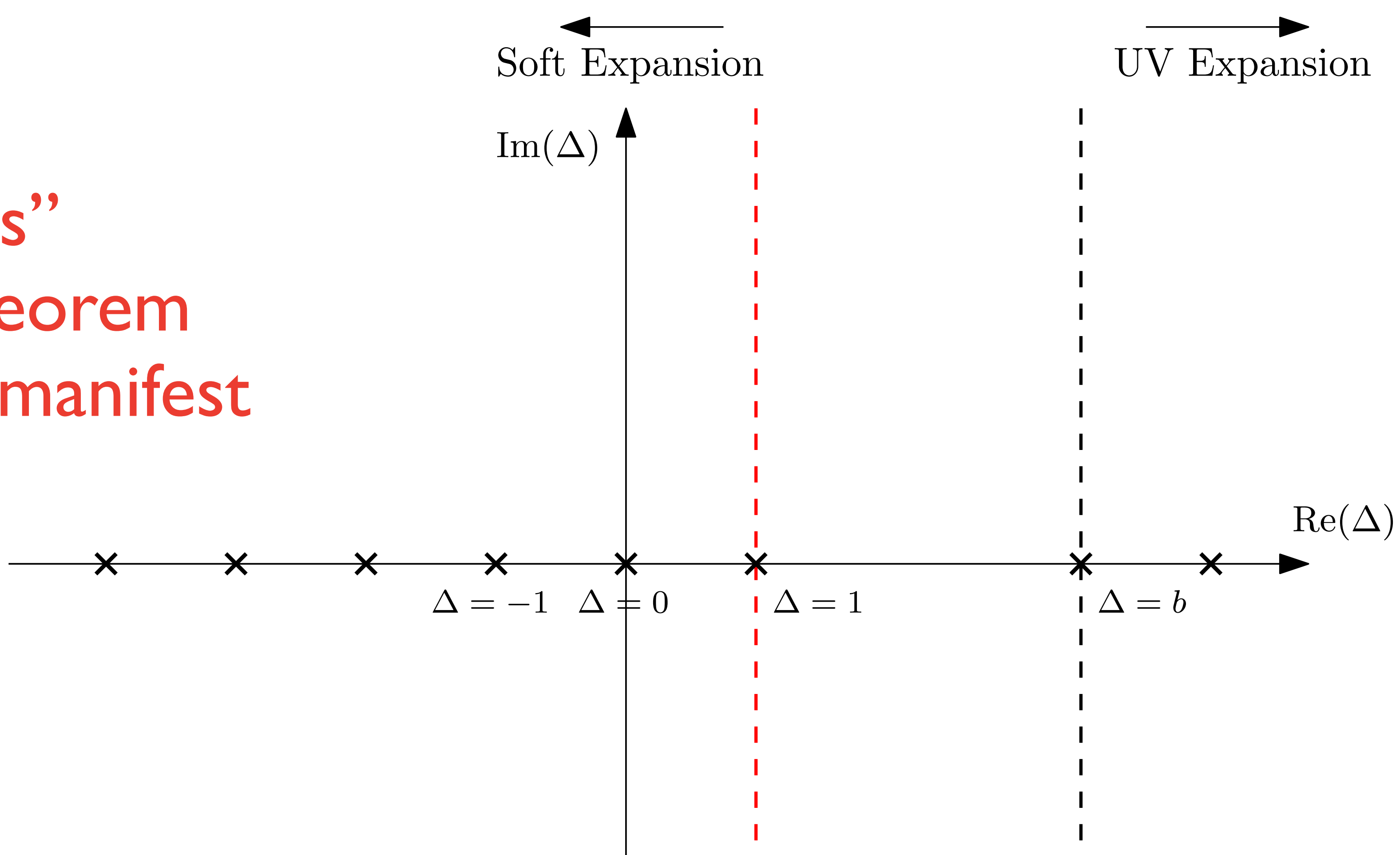
## on $CS^2$: $\omega_s \to 0$

$$\mathcal{M}_{n+1} \longrightarrow \Big( \underbrace{\omega_s^{-1}\, S_G^{(0)}}_{\Delta \to 1} + \underbrace{\omega_s^{0}\, S_G^{(1)}}_{\Delta \to 0} + \underbrace{\omega_s\, S_G^{(2)}}_{\Delta \to -1} + \dots \Big)\, \mathcal{M}_n$$

$$\mathcal{A}_{n+1} \longrightarrow \Big( \underbrace{\omega_s^{-1}\, S_{YM}^{(0)}}_{\Delta \to 1} + \underbrace{\omega_s^{0}\, S_{YM}^{(1)}}_{\Delta \to 0} + \dots \Big)\, \mathcal{A}_n$$

$$\Delta \to 0, 1, \dots$$

in Mellin space "soft-limits"
reproduce Weinberg's soft theorem
and more symmetries become manifest
...




*Kapec, Mitra, Raclariu, Strominger (2016)*

*Donnay, Puhm, Strominger (2018)*

## also relate Ward identities and BMS symmetries:

### explicit field realization

(i) energy-momentum tensor T(z):

soft-graviton $\Delta \to 0$

$$T(z) := \tilde{\mathcal{O}}_{\Delta=2,J=+2}(z,\bar{z}) = \frac{3}{\pi} \int d^2w \; (z-w)^{-4} \; \mathcal{O}_{\Delta=0,J=-2}(w,\bar{w})$$

$(h,\bar{h}) = (2,0)$

Fotopoulos, Taylor (2019)

### shadow transformation:

$$\tilde{\mathcal{O}}^a_{\tilde{\Delta},\tilde{J}}(z,\bar{z}) = \tilde{\mathcal{O}}^a_{2-\Delta,-J}(z,\bar{z}) = \frac{(\Delta+J-1)}{\pi} \int_{\mathbf{C}} \frac{d^2w}{(z-w)^{2-\Delta-J}(\bar{z}-\bar{w})^{2-\Delta+J}} \; \mathcal{O}^a_{\Delta,J}(w,\bar{w})$$

Osborn (2012)

then:

$$\langle T(z) \prod_{i=1}^{n} O_{\Delta_i}(z_i, \bar{z}_i)\rangle = \sum_{i=1}^{n} \left( \frac{h_{O_i}}{(z-z_i)^2} + \frac{\partial_{z_i}}{z-z_i} \right) \langle \prod_{i=1}^{n} O_{\Delta_i}(z_i, \bar{z}_i)\rangle$$

OPE:

$$T(w)T(z) = \frac{2T(z)}{(w-z)^2} + \frac{\partial_z T(z)}{w-z} + \ldots$$

$$T(w)\overline{T}(z) = reg\,.$$

$\rightarrow$ $L_n, \bar{L}_m$ $\qquad c = 0$

*Fotopoulos, St.St., Taylor, Zhu (2019)*

## (ii) supertranslation operator P(z):

soft-graviton $\Delta \to 1$

$$P(z,\bar{z}) := \partial_{\bar{z}} \mathcal{O}_{\Delta=1,J=+2}(z,\bar{z}) \qquad (h,\bar{h}) = (\tfrac{3}{2},\tfrac{1}{2})$$

then:

$$\boxed{\left\langle P(z_0)\prod_{j=1}^{n}\mathcal{O}_{\Delta_j,l_j}(z_j,\bar{z}_j)\right\rangle = \frac{1}{4}\sum_{i=1}^{n}\frac{c_i(\Delta_i)}{c_i(\Delta_i+1)}\frac{1}{z_0-z_i}\left\langle\prod_{n=1}^{n}\mathcal{O}_{\Delta_j,l_j}(z_j,\bar{z}_j)\right\rangle\Bigg|_{\Delta_i\to\Delta_i+1}}$$

OPE:

$$T(w)P(z) = \frac{3}{2(w-z)^2}P(z) + \frac{1}{w-z}\partial_z P(z) + reg\,.$$

In addition to Virasoro symmetry, we construct
all supertranslation generators acting on primary fields

*Fotopoulos, St.St., Taylor, Zhu (2019)*

construct:

$$P_{n-\frac{1}{2},-\frac{1}{2}} = \frac{1}{i\pi(n+1)}\oint dw\, w^{n+1}\left[T(w), P_{-\frac{1}{2},-\frac{1}{2}}\right]$$

$$P_{n-\frac{1}{2},m-\frac{1}{2}} = \frac{1}{i\pi(m+1)}\oint d\bar{w}\, \bar{w}^{m+1}\left[\bar{T}(\bar{w}), P_{n-\frac{1}{2},-\frac{1}{2}}\right]$$

$$P_{-1/2,-1/2} = e^{(\partial_h+\partial_{\bar{h}})/2} = P^0 + P^3$$

we find:

$$\left[P_{n-\frac{1}{2},m-\frac{1}{2}}, \phi^{h,\bar{h}}(z,\bar{z})\right] = z^n \bar{z}^m \phi^{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(z,\bar{z})$$

→ $P_{k,l}, \bar{P}_{k,l}$

→ local (or extended) BMS algebra:

$$[P_{ij}, P_{k,l}] = 0 \ ,$$

$$[L_n, P_{k,l}] = \left(\frac{1}{2}n - k\right) P_{n+k,l} + n(n^2-1)\ C_{n,k} \ ,$$

$$[\bar{L}_n, P_{k,l}] = \left(\frac{1}{2}n - l\right) P_{k,n+l} + n(n^2-1)\ \bar{C}_{n,l} \ .$$

$m, n \in \mathbf{Z},\ i, j, k, l \in \mathbf{Z}+\frac{1}{2}$

*Barnich (2017)*

Conformal soft-theorems ⟷ Ward identities ⟷ BMS algebra

BMS$^{\pm}$ group = symmetry of asymptotically flat D=4 space-time at null infinity $\mathscr{I}^{\pm}$

| global BMS symmetry on celestial sphere | local BMS symmetry on celestial sphere |
|---|---|
| <u>Lorentz group:</u><br>global conformal transformations on celestial sphere SL(2,C)<br>$z \to \frac{az+b}{cz+d}$<br>$L_{-1} = \partial$<br>$L_0 = z\partial + h$<br>$L_1 = z^2\partial + 2hz$ | local conformal transformations = superrotations T(z)<br>$[L_m, L_n] = (m-n)\ L_{m+n}$<br>$[\bar{L}_m, \bar{L}_n] = (m-n)\ \bar{L}_{m+n}$ |
| <u>global space-time translation:</u><br>Abelian subgroup of supertranslations<br>$P_{-1/2,-1/2} = e^{(\partial_h+\partial_{\bar{h}})/2}$ $\quad P_{1/2,1/2} = z\ e^{(\partial_h+\partial_{\bar{h}})/2}$<br>$P_{-1/2,1/2} = \bar{z}\ e^{(\partial_h+\partial_{\bar{h}})/2}$ $\quad P_{-1/2,1/2} = \|z\|^2\ e^{(\partial_h+\partial_{\bar{h}})/2}$ | local space-time translations =supertranslations P(z)<br>$P_{n-\frac{1}{2},m-\frac{1}{2}} \quad n,m \in \mathbf{Z}$ |

→ Symmetries of the celestial OPEs and correlators
S-matrix (non-trivial consistency)

Can 2D CFT on celestial sphere offer some new insights into gauge-gravity connections ?

*related questions:*

- celestial double-copy structure
- celestial KLT structure
- … ?

Sugawara construction:

$$T(w) = \frac{1}{2k + C_2} \lim_{z \to w} \left\{ \sum_a J^a(w) J^a(z) - \frac{k\ \dim(g)}{(w_1 - w_2)^2} \right\}$$

*Sugawara (1968)*

*assumes Kac-Moody current algebra*

## (holomorphic) Kac-Moody current algebra:

$$j^a(z) = \mathcal{O}^a_{\Delta=1,J=+1}(z,\bar{z})$$
$$\bar{j}^a(\bar{z}) = \mathcal{O}^a_{\Delta=1,J=-1}(z,\bar{z})$$

$$j^a(z)j^b(w) \sim \frac{f^{abc}\ \ j^c(w)}{z-w} + reg\,.$$

*furthermore:*

$$j^a(z)\bar{j}^b(\bar{w}) \sim \frac{f^{abc}\ \ \bar{j}^c(\bar{w})}{z-w}$$

anti-holomorphic currents
transform in adjoint representation
of holomorphic Kac-Moody symmetry

$$\bar{j}^a(\bar{z})j^b(w) \sim \frac{f^{abc}\ \ j^c(w)}{\bar{z}-\bar{w}}$$

*follows from CCFT OPE*

*first look:*

*Fan, Fotopoulos, St. St., Taylor (2020)*

CCFT:

$$T^S(z) := \gamma \sum_a j^a(z)j^a(z) = \gamma \lim_{\Delta,\Delta'\to 1} \lim_{z'\to z} \sum_a \mathcal{O}^a_{\Delta,+1}(z,\bar{z})\mathcal{O}^a_{\Delta',+1}(z',\bar{z}')$$

consider n-gluon MHV amplitude $A_n(-,-,+\ldots,+)$
with insertion of pair of gauge currents

$$\lim_{z_j\to z_{n+1}} \langle \mathcal{O}^{a_1}_{\Delta_1 J_1}\ldots j^{a_j}(z_j)\ldots \mathcal{O}^{a_n}_{\Delta_n J_n} j^a(z_{n+1}) j^a(z_{n+1})\rangle$$

$$= \begin{cases} \tilde{C}_2(G)\left(\frac{1}{(z_j - z_{n+1})^2} + \frac{\partial_j}{(z_{n+1}-z_j)}\right)\langle \mathcal{O}^{a_1}_{\Delta_1 J_1}\ldots j^{a_j}(z_j)\ldots \mathcal{O}^{a_n}_{\Delta_n J_n}\rangle\ , & j = 3,\ldots,n \\ 0\ , & j = 1,2 \end{cases}$$

follows from:

$$\lim_{z_{n+1}\to z_j} A_{n+2}(\{g^+_{n+2}, g_1, \ldots, g_n, g^+_{n+1}\}) = -\frac{\tilde{C}_2(G)}{\omega_{n+1}\omega_{n+2}}\left(\frac{1}{(z_{n+1}-z_j)^2} + \frac{\tilde{\partial}_{z_j}}{z_{n+1}-z_j}\right) A_n(\{g_1,\ldots,g_n\})$$

this Sugawara energy-momentum tensor

- only describes <u>soft sector</u> of the theory
- decouples <u>negative helicity states</u>
- only treats <u>holomorphic sector</u>

# A double copy construction of the energy momentum tensor

consider OPE of two gluon operators of opposite helicity
and perform a shadow transformation:

$$\mathcal{O}^a_{\Delta_2,+1}(u,\bar{u})$$

$$\tilde{\mathcal{O}}^a_{2-\Delta_1,+1}(w,\bar{w}) \sim \int d^2z\ (z-w)^{-3}(\bar{z}-\bar{w})^{-1}\ \mathcal{O}^a_{\Delta_1,-1}(z,\bar{z})$$

$$T(w) = \frac{1}{2\dim(g)} \lim_{\Delta_1,\Delta_2\to 0} [\Delta_2(\Delta_1+\Delta_2)] \lim_{u\to w} \sum_a \mathcal{O}^a_{\Delta_2,+1}(u,\bar{u})\tilde{\mathcal{O}}^a_{2-\Delta_1,+1}(w,\bar{w})$$

$$\frac{1}{2k+C_2} \simeq \frac{1}{2\dim(g)},$$

$$k=0$$

- puts both soft and hard modes on equal footing:

$$T(z)\mathcal{O}_{\Delta,J}(w,\bar{w}) = \frac{h}{(z-w)^2}\mathcal{O}_{\Delta,J}(w,\bar{w}) + \frac{1}{z-w}\partial_w\mathcal{O}_{\Delta,J}(w,\bar{w}) + reg\,.$$

*Fan, Fotopoulos, St. St., Taylor, (2020)*

# Further Directions

- understand Virasoro central charge (-one-loop ?)

- establish double-copy structure

  (elaborate on gauge/gravity connections)

- high-energy (large $\lambda$) limit: string world-sheet = celestial sphere

  celestial $CFT_2 \simeq$ string world-sheet $CFT_2$

- understanding the nature of 2D CFT on celestial sphere would enable a holographic description of flat spacetime

- uplift $AdS_3/CFT_2$ holography to $\mathscr{M}_4$

  towards flat space-time holography