Quantum BTZ black hole

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Based on 2007.15999 [hep-th]

Antonia Frassino

Benson Way

Earlier refs:

Quantum black holes as holograms in AdS brane worlds hep-th/0206155

w/ A Fabbri + N Kaloper

Exact description of black holes on branes.2 hep-th/9912135

w/ G Horowitz + R Myers

What's a Quantum Black Hole?

A large-N matrix, with

fastly scrambling entries?

quantum black hole

Simpler goal:

Classical geometry of black hole modified

(possibly a lot) by quantum fields

Much insight gained this way

Quantum backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G\langle T_{\mu\nu}(g_{\alpha\beta})\rangle$$

classical Einstein tensor & metric

quantum matter renorm stress tensor
(many fields)

Coupled system: metric + ⟨QFT⟩

Very hard to solve simultaneously

Perturbative backreaction: limited insight

Quantum backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G\langle T_{\mu\nu}(g_{\alpha\beta})\rangle$$

Exact backreaction:

2D models: CGHS, JT+CFT

Holographic reformulation

RE+Fabbri+Kaloper 2002
Tanaka 2002

Quantum backreaction — What for?

Radiating/Evaporating Black hole

Quantum (generalized) entropy $S_{gen} = \frac{A}{4G} + S_{out}$

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Quantum (generalized) entropy $S_{gen} = \frac{A}{4G} + S_{out}$

Page curve turns when $S_{out} \sim \frac{A}{4G}$

Penington 2019
Almheiri+al 2019

Quantum Extremal Surfaces

Engelhardt+Wall 2014

Quantum backreaction — What for?

Radiating/Evaporating Black hole

Quantum (generalized) entropy $S_{gen} = \frac{A}{4G} + S_{out}$

Page curve turns when $S_{out} \sim \frac{A}{4G}$

Penington 2019
Almheiri+al 2019

Quantum Extremal Surfaces

Engelhardt+Wall 2014

Other phenomena: Quantum dressing of singularities

RE+Fabbri+Kaloper 2002

Holographic approach

$\langle T_{\mu\nu} \rangle$: CFT on boundary geometry of bulk dual

classical bulk $\Leftrightarrow$ planar CFT ($N \to \infty$)

Conventional AdS/CFT has *fixed boundary* geometry

Make boundary geometry dynamical

Holographic CFT+gravity

1. Modify boundary conditions: “Setting the boundary free”

Compère+Marolf 2008
Andrade+Marolf 2011

2. Move the bdry inwards: Braneworld holography

H Verlinde, Gubser 1999

both combined: “RS+DGP” braneworld

Braneworld gravity

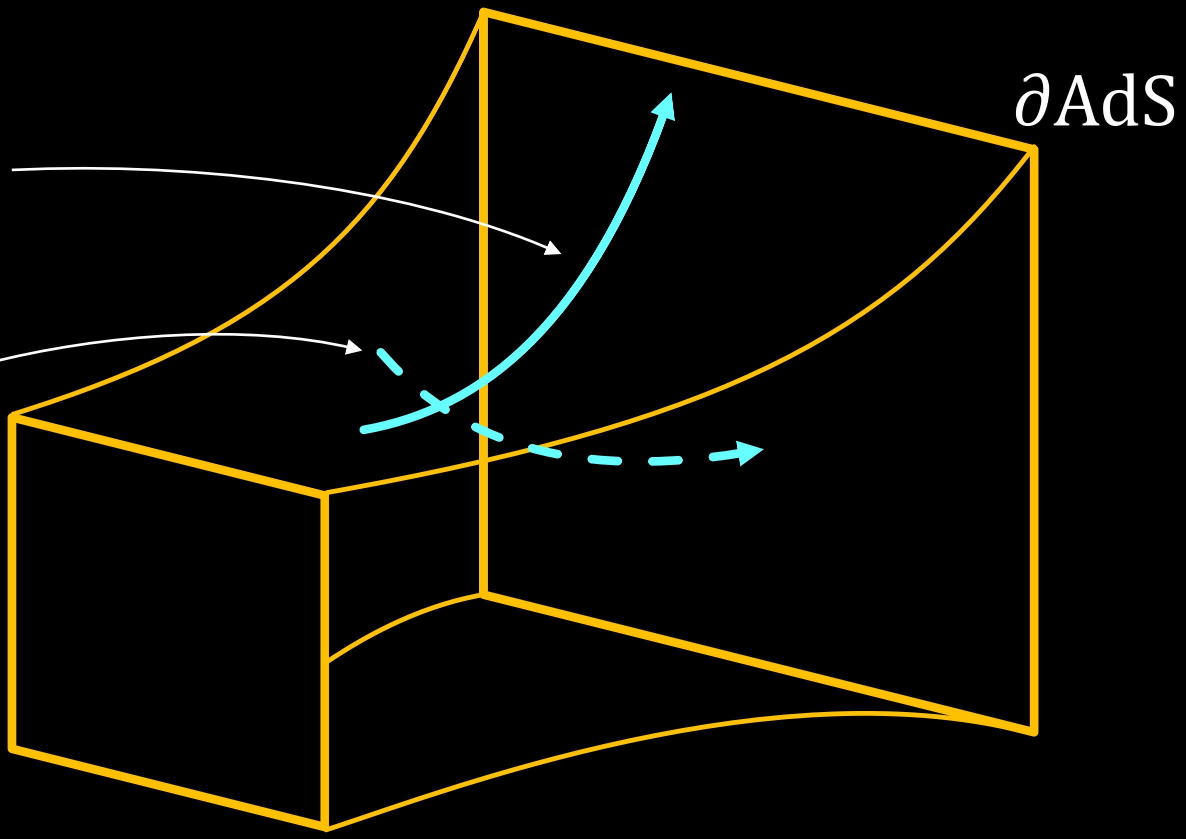
Randall+Sundrum 1999

AdS bulk

Gravitational fluctuations

non-normalizable

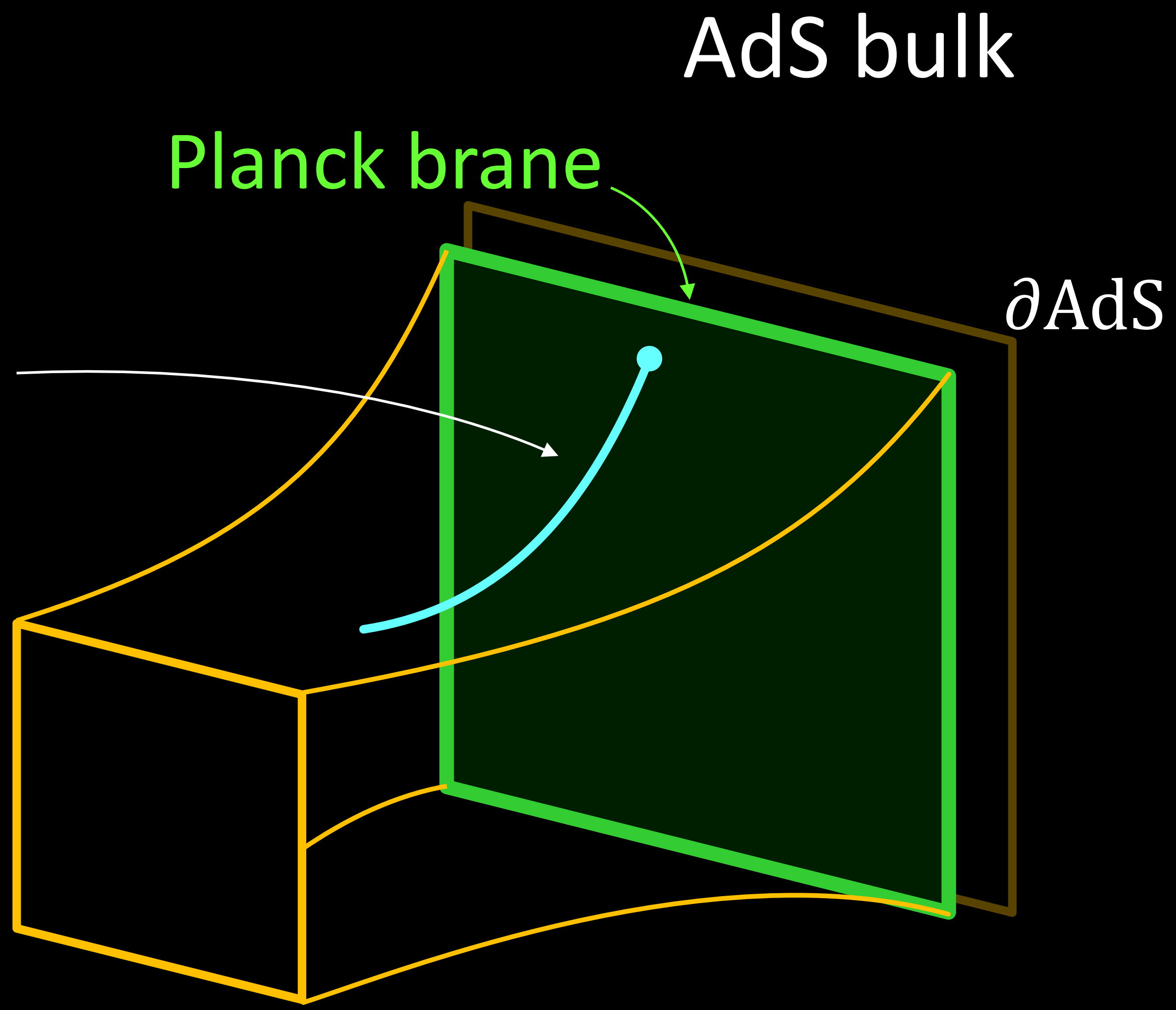
normalizable



Braneworld gravity

Randall+Sundrum 1999

Gravitational fluctuations



non-normalizable

Braneworld dynamics

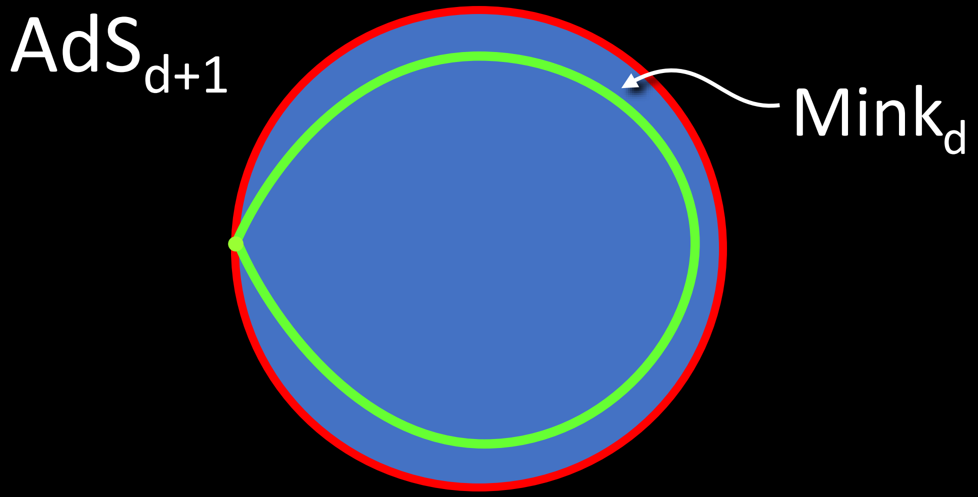
Graviton on the brane

Remaining grav bulk dynamics

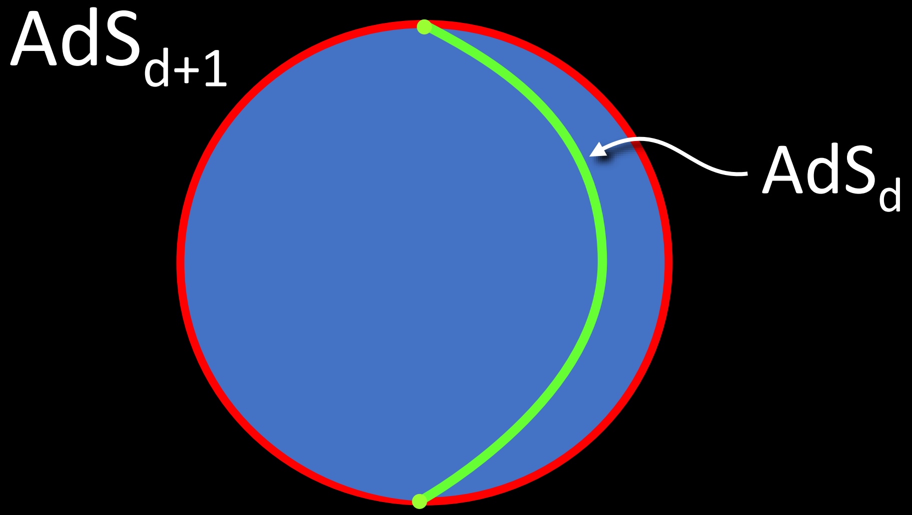
is dual to CFT (cutoff & planar)

Slicing AdS

Poincaré slicing



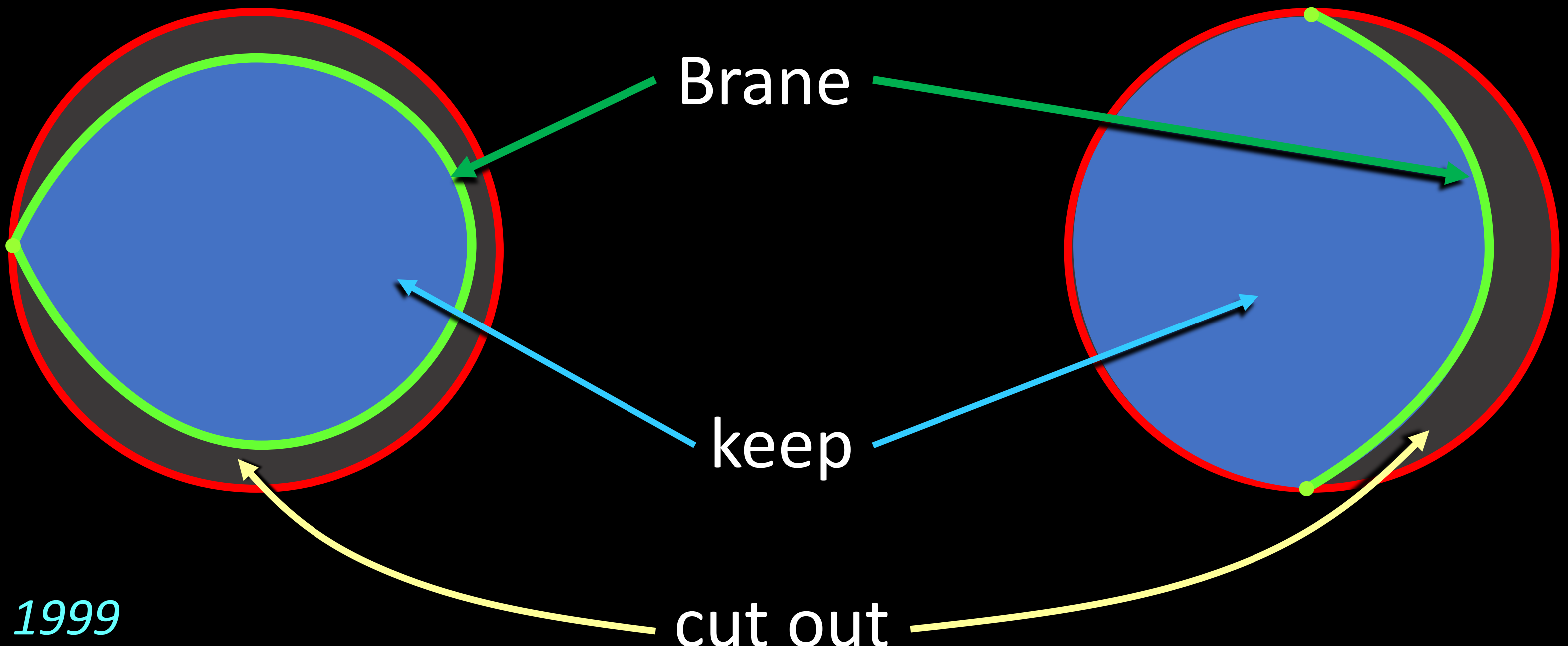
AdS slicing



Braneworlds

Randall-Sundrum: Minkowski branes

Karch-Randall: AdS branes



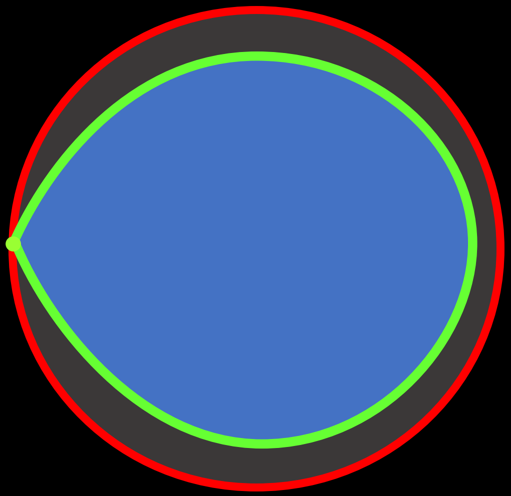
Randall+Sundrum 1999
Karch+Randall 2000

& paste to copy: 2-sided brane, $\mathbb{Z}_2$ orbifold

Braneworld holographies

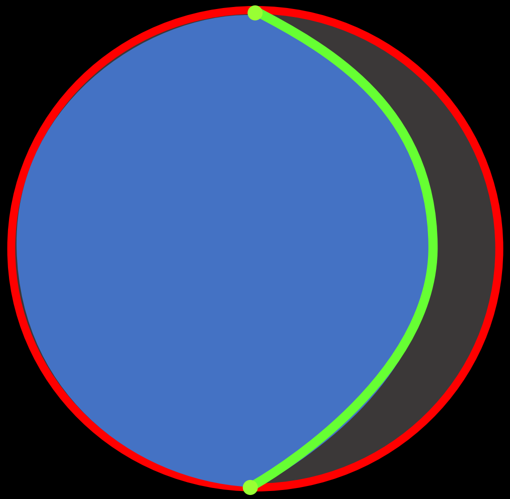
Geng+Karch 2020
Chen+Myers+Neuenfeld
+Reyes+Sandor 2020

RS Minkowski branes



Finite bulk volume
Normalizable massless graviton @brane

KR AdS branes



Infinite bulk volume
Nonnormalizable graviton @brane
Massive graviton bound state

Braneworld holography

$$\mathcal{G}_{\mu\nu}(g_{\alpha\beta}) = 8\pi G\langle T_{\mu\nu}(g_{\alpha\beta})\rangle_{planar}$$

Einstein+higher curvature

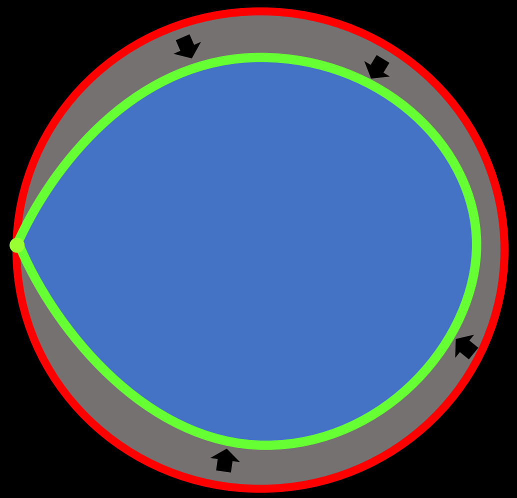
large-N CFT stress tensor

Effective theory w/ cutoff (brane position)

Higher-curvature terms: induced by *CFT above cutoff*

$\langle T_{\mu\nu}\rangle$: holographic *CFT below cutoff*

Effective gravity + CFT

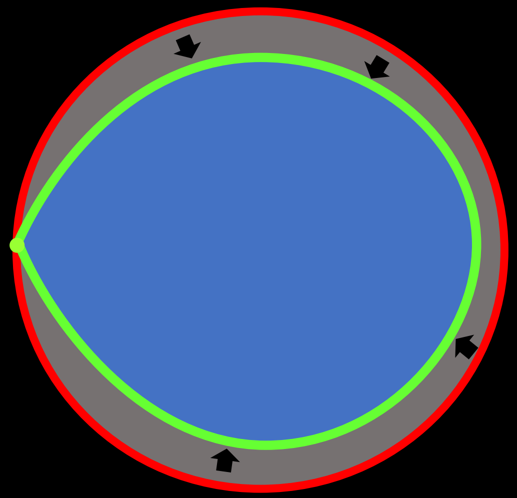


deHaro+Skenderis+Solodukhin 2000

Integrate from boundary to brane
à la Fefferman-Graham
(holographic renormalization)

i.e. solve bulk Einstein eqs with prescribed boundary metric, perturbatively away from the bdry

Effective gravity + CFT



deHaro+Skenderis+Solodukhin 2000

Do not introduce counterterms!
Keep finite cutoff: brane position

This integrates the CFT UV degrees of freedom and generates the effective action

Effective action gravity + CFT

RE+Johnson+Myers 1999

Integrate 4D bulk action

→ 3D eff action

3D h_{ab}, R_{ab}

$\frac{1}{L_3^2}$: 4D $\frac{1}{\ell_4^2}$ and brane tension $\frac{1}{\ell}$

ℓ: brane position ~ tension^{-1} ~ cutoff $^{-1}$

$$I = \frac{1}{16\pi G_3} \int d^3x \sqrt{-h} \left[\frac{2}{L_3^2} + R + \ell^2 \left(\frac{3}{8} R^2 - R_{ab} R^{ab} \right) + \cdots \right]$$

$$+ I_{CFT}$$

$G_3 = \frac{G_4}{2\ell_4}$

holographic CFT

same as "New 3D massive gravity"

Bergshoeff+Hohm+Townsend 2009

Effective action gravity + CFT

$$I = \frac{1}{16\pi G_3}\int d^3x\,\sqrt{-h}\,\left[\frac{2}{L_3^2} + R + \ell^2\left(\frac{3}{8}R^2 - R_{ab}R^{ab}\right) + \cdots\right]$$

$$+ I_{CFT}$$

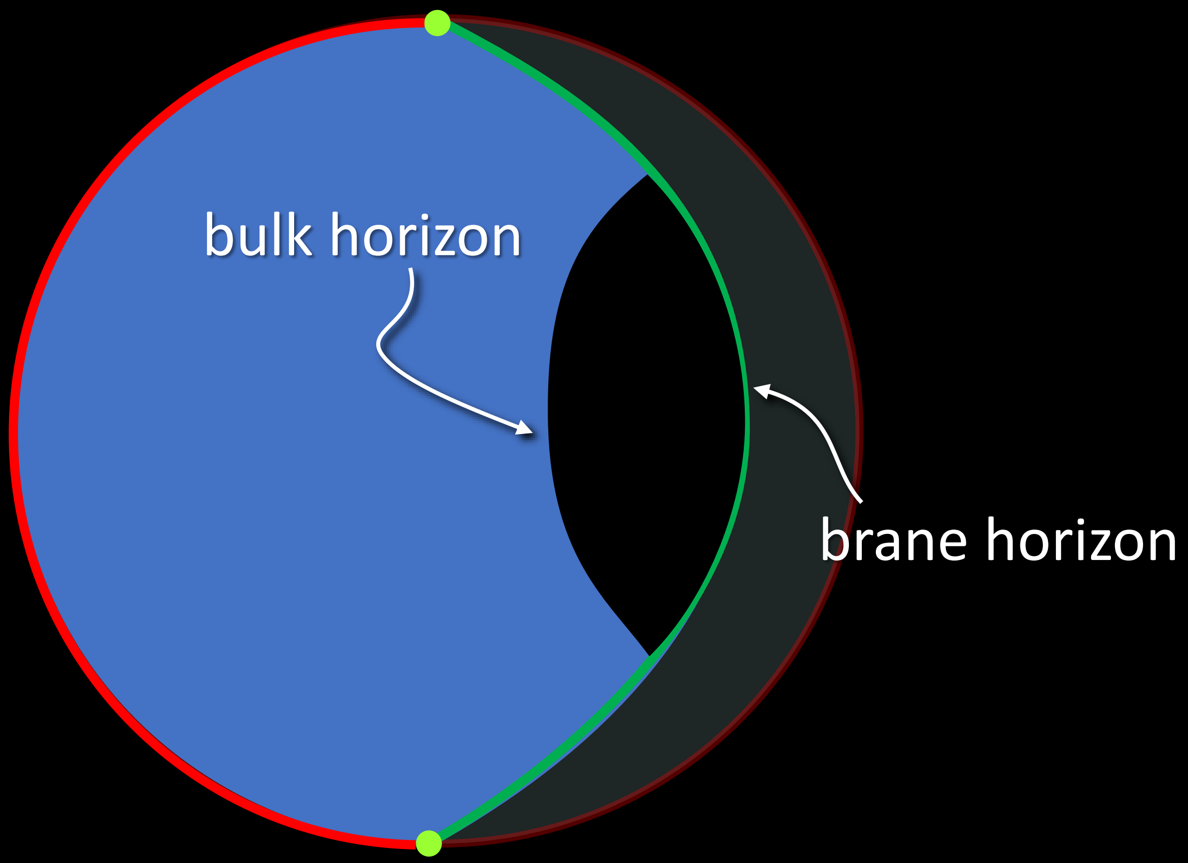
An exact 4D bulk solution is a 3D effective theory solution

exact CFT stress tensor

exact backreaction of the CFT (planar)

exact in *all* higher curvature corrections

Black hole on brane

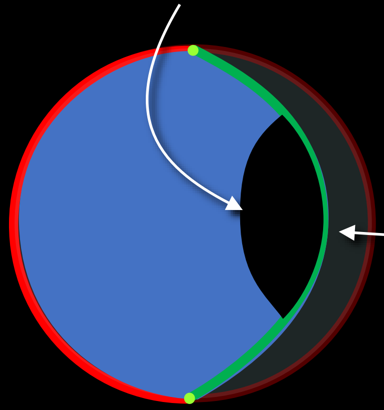


Quantum BH Entropy in bw holography

$$S_{gen} = S_{Wald} + S_{out}$$

Bulk bh entropy

$$S_{gen} = \frac{A_{d+1}}{4G_{d+1}}$$



Brane bh entropy

$$S_{Wald} = \frac{A_d}{4G_d} + \cdots$$

induced by entanglement of CFT above cutoff

CFT entanglement entropy

$$S_{out} = S_{gen} - S_{Wald}$$

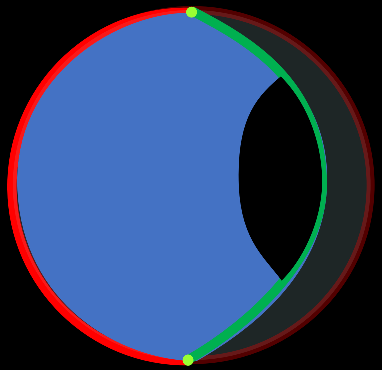
(CFT below cutoff)

RE 2006

Quantum BH Entropy

If the holographic interpretation of braneworlds is consistent, then

$$TdS_{gen} = dM - \Omega\, dJ$$

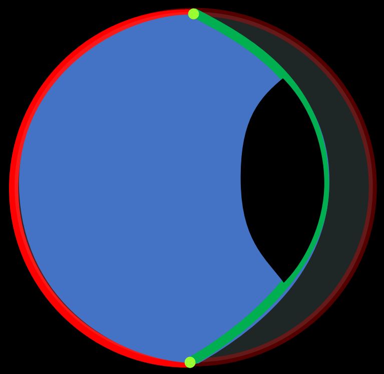


$$\Delta S_{gen} \geq 0$$

S_{Wald} should not satisfy these

Quantum BH Entropy: Second law

$\Delta S_{gen} \geq 0$ bulk 2nd law



S_{Wald} can decrease

for perturbative backreaction (non-holo): *Wall 2011*

Quantum BH Entropy: First law

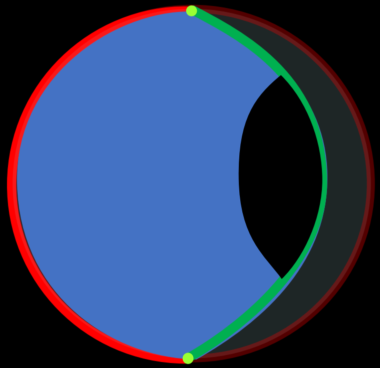
$$TdS_{gen} = dM - \Omega\, dJ$$

Not trivial!

In $d + 1$ dim bulk

On d dim brane
w/ higher curvature gravity

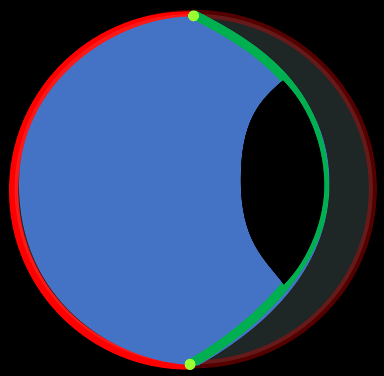
Bulk 1st law?



S_{Wald}: no 1st law

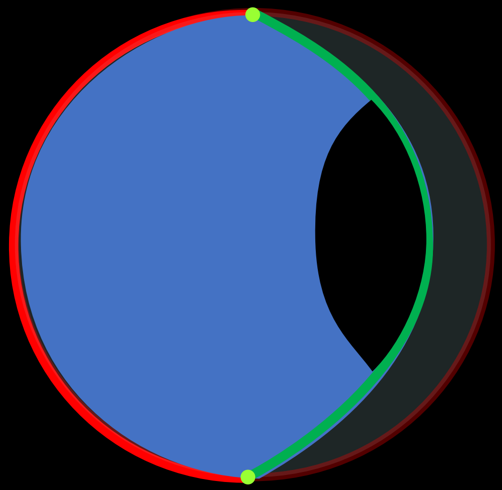
Quantum BH Entropy: First law

$$T dS_{gen} = dM - \Omega \, dJ$$



We will test this in explicit exact solutions

Black hole on the brane: exact solutions?



Bulk BH is accelerating (towards "missing bdry")

Exact solutions known in AdS_4 *RE+Horowitz+Myers 1999*

No solutions known in $AdS_{>4}$ for localized bhs

Black hole on the brane: AdS C-metric

Plebański + Demiański 1976

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left(-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right)$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}$$

$$G(x) = 1 - \kappa x^2 - \mu x^3$$

ℓ_3: brane curvature radius

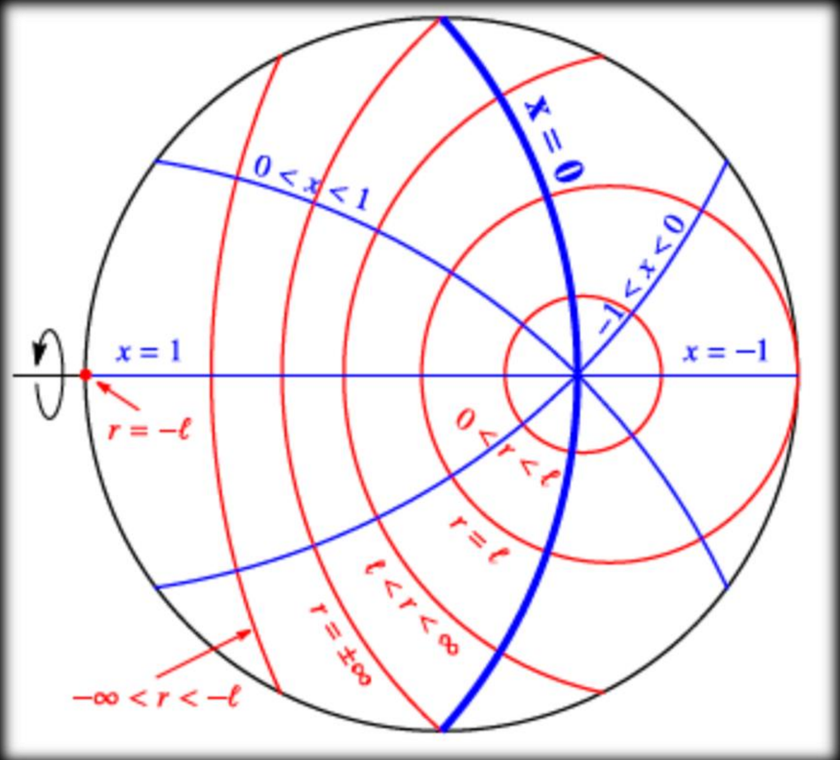
ℓ: brane position, tension^{-1}

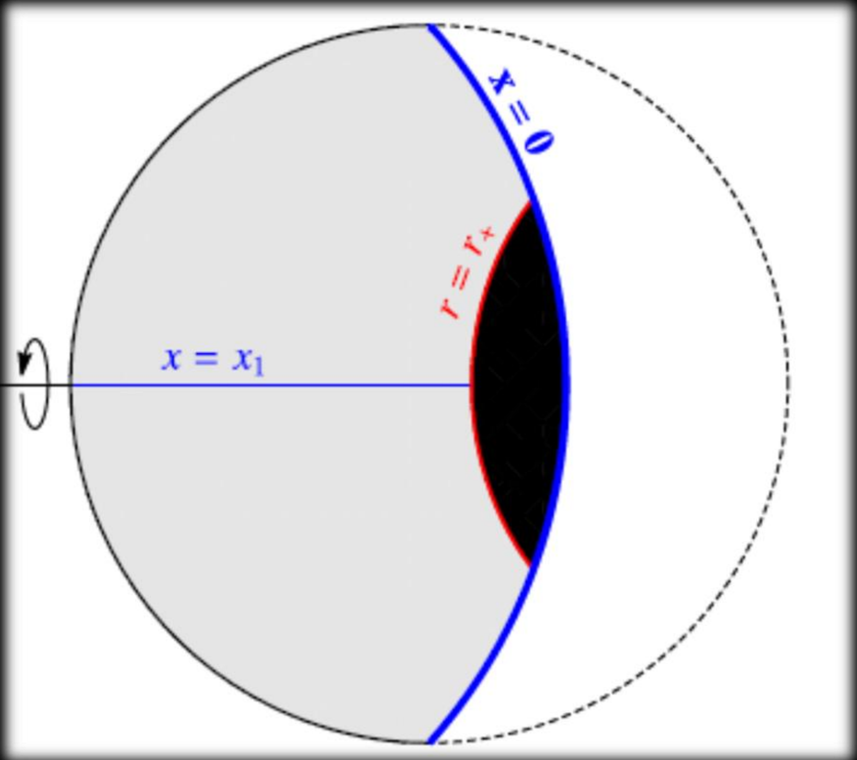
μ: quantum corrections

$\kappa = \pm 1$: small/large 3D bh

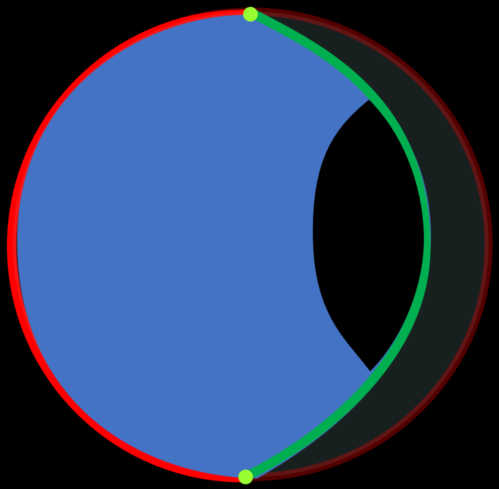
Adapted coordinates

Brane at $\mathbf{x = 0}$: $K_{ab} = -\frac{1}{\ell} h_{ab}$





Strength of backreaction: ℓ



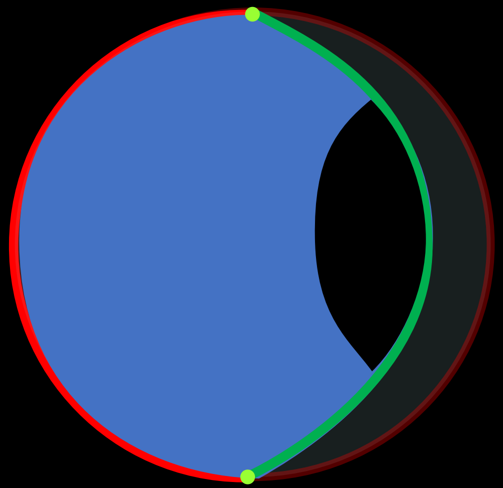
$$0 < \ell < \infty$$

brane tension $\propto 1/\ell$

backreaction $\propto \ell$

cutoff energy $\propto 1/\ell$

Strength of backreaction: ℓ



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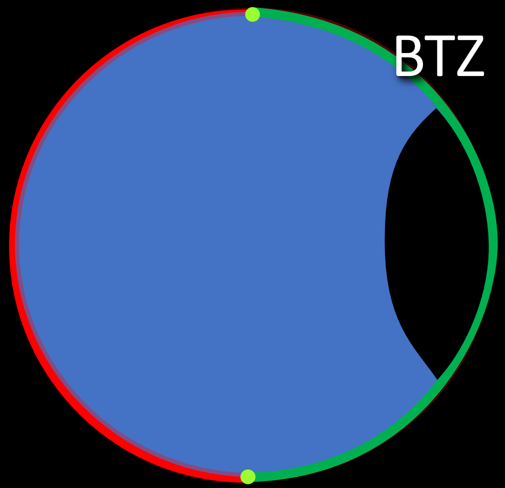


BTZ @bdry

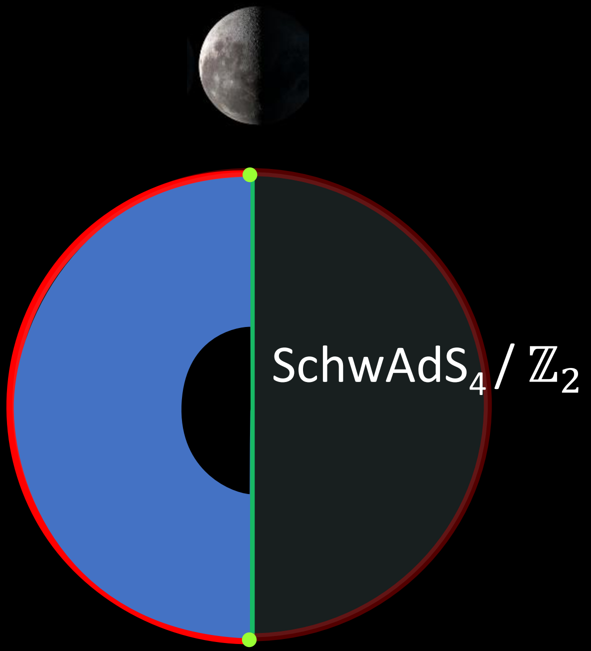
$\ell \to 0$

brane tension → ∞

backreaction → 0

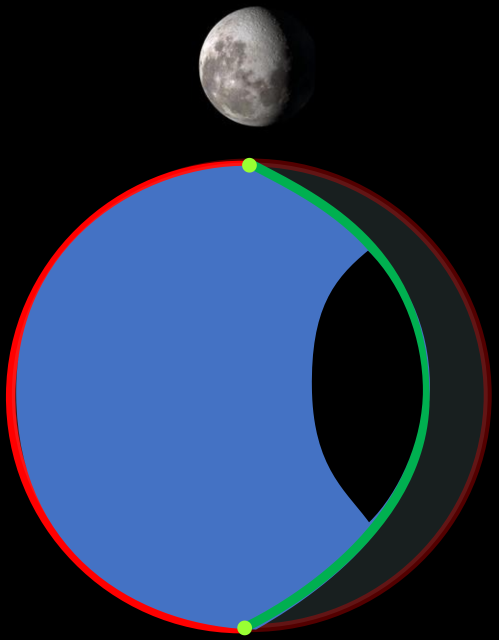
Hubeny+Marolf+Rangamani 2009

Strength of backreaction: ℓ



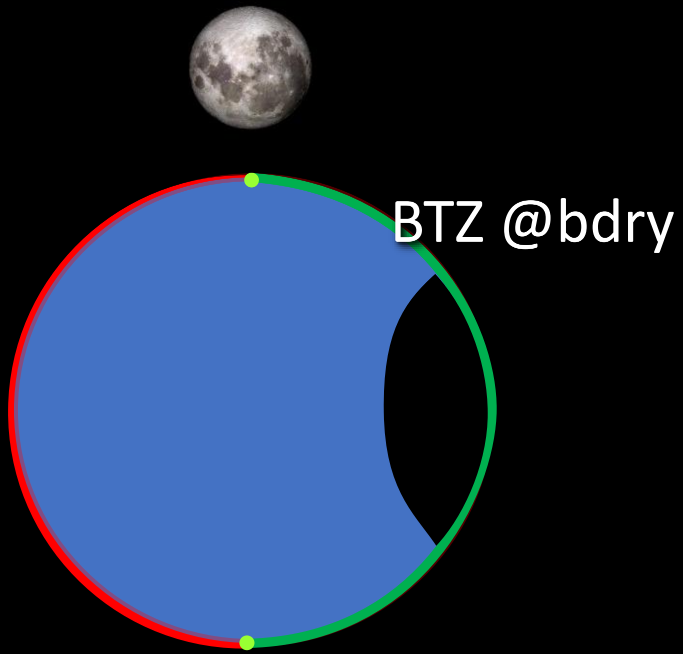
$\ell \to \infty$

zero tension brane
“maximal backreaction”
no 3D gravity



$0 < \ell < \infty$

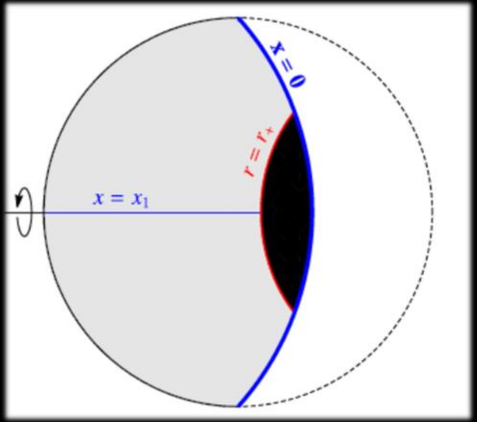
brane tension $\propto 1/\ell$
backreaction $\propto \ell$
cutoff energy $\propto 1/\ell$



$\ell \to 0$

brane tension → ∞
backreaction → 0

quBTZ metric

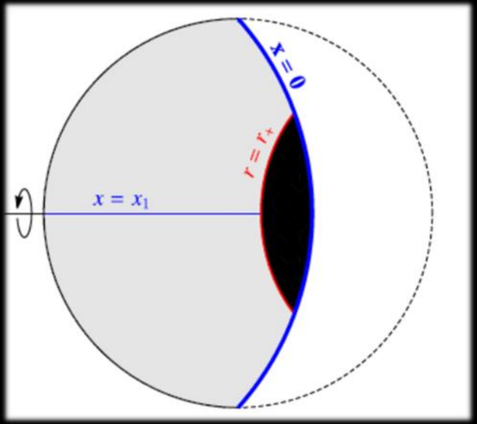


3D metric induced on brane at $x = 0$

$$ds^2 = \frac{\ell^2}{(\ell + \cancel{x}r)^2}\left(-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\cancel{\frac{dx^2}{G(x)}} + G(\cancel{x})d\phi^2\right)\right)$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} \qquad G(x) = 1 - \cancel{\kappa x}^2 - \cancel{\mu x}^3$$

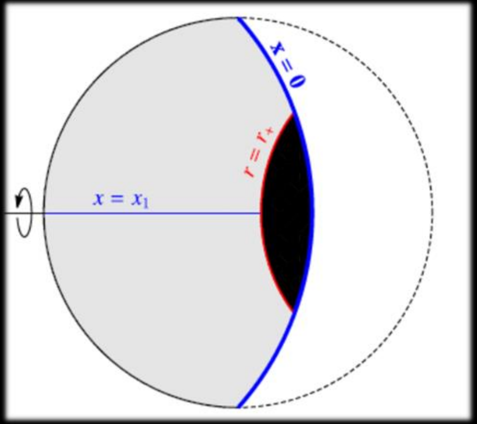
quBTZ metric



3D metric induced on brane at $x = 0$

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}} + r^2 d\phi^2$$

quBTZ metric

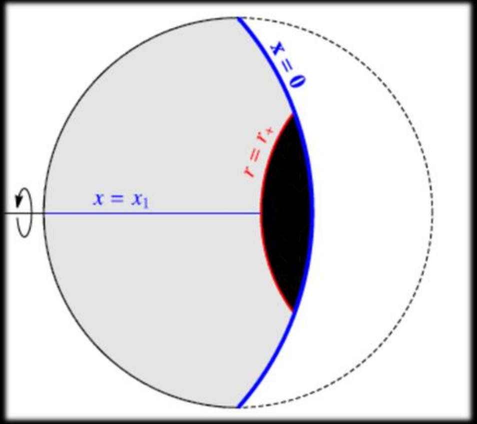


3D metric induced on brane at $x = 0$

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M \qquad\qquad\qquad\right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M \qquad\qquad\qquad} + r^2 d\phi^2$$

$\ell = 0$: BTZ black hole

quBTZ metric



3D metric induced on brane at $x = 0$

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}} + r^2 d\phi^2$$

$\ell = 0$: BTZ black hole

$\ell > 0$: quantum-corrected BTZ $\neq$ AdS$_3/\Gamma$

G_3 “renormalized” by higher curvatures

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\,\frac{F(M)}{r}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\,\frac{F(M)}{r}} + r^2 d\phi^2$$

quantum correction: $\ell = 2\,\hbar\,c\,G_3 + \cdots$

CFT central charge

$F(M)$ determined by bulk regularity outside horizon

$$\langle T^a{}_b\rangle = \frac{\ell}{16\pi G_3}\frac{F(M)}{r^3}\,\mathrm{diag}\{1,1,-2\} + \cdots$$

not thermal plasma $\mathrm{diag}\{-2,1,1\}$

$\langle T^a{}_b \rangle$ for free fields

Steif, Shiraishi+Maki, Lifschytz+Ortiz 1993

Casals+Fabbri+Martínez+Zanelli 2016,2019

Free conformal scalar in BTZ

Method of images: BTZ = AdS_3/Γ

Transparent bc's at $\partial\mathrm{AdS}_3$

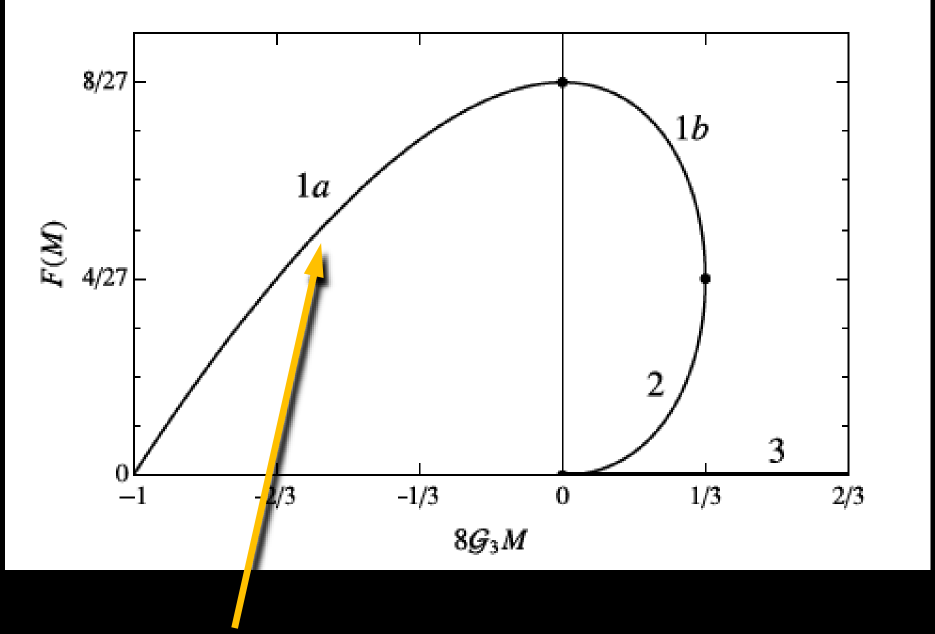
Satisfy KMS, Hartle-Hawking at horizon

Same structure: $\langle T^a{}_b \rangle = \frac{c}{8\pi}\frac{F(M)}{r^3}\mathrm{diag}\{1,1,-2\}$

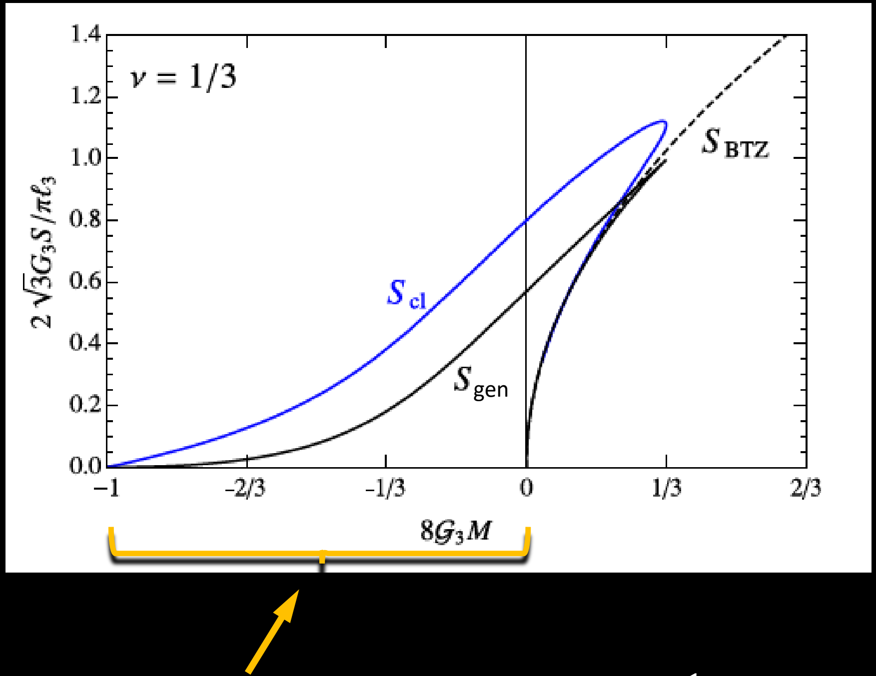
different $F(M) = \sum_{n=1}^{\infty} F_n(M)$

Holographic quantum black holes: **small AdS$_3$ bhs**

Stress tensor



Entropy



Casimir energy on a cone

Conical singularities dressed by quantum horizon

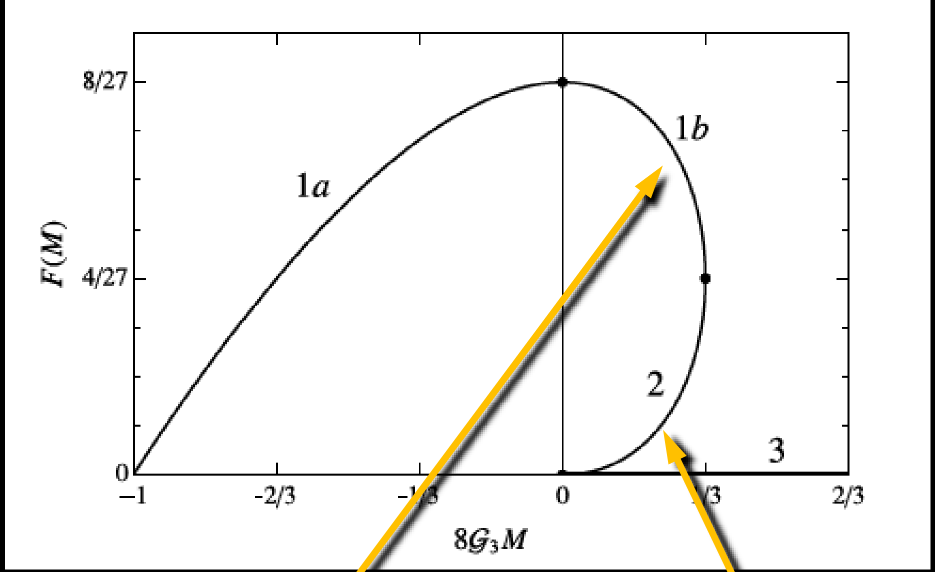
Quantum Cosmic Censorship

Small AdS$_3$ black holes $-\frac{1}{8G_3} < M < 0$

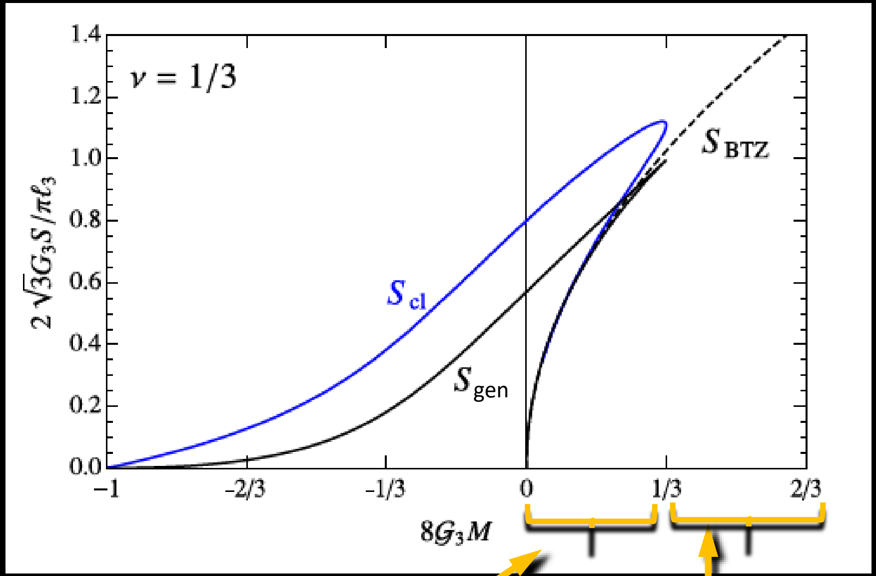
RE+Fabbri+Kaloper 2002

Holographic quantum black holes: **quBTZ**

Stress tensor



Entropy



CFT in BTZ

Casimir-dominated

Thermal-dominated

Quantum-corrected BTZ: $0 < M < \frac{1}{24G_3}$

Quantum-*uncorrected* BTZ: $M > \frac{1}{24G_3}$

Quantum BH Entropy: First law

$$M = \frac{1}{2\mathcal{G}_3} \frac{z^2(1 - \nu z^3)(1 + \nu z)}{(1 + 3z^2 + 2\nu z^3)^2}$$

(measured in 3D eff theory)

ν: backreaction parameter ℓ/ℓ_3

$$T = \frac{1}{2\pi\ell_3} \frac{z(2 + 3\nu z + \nu z^3)}{1 + 3z^2 + 2\nu z^3}$$

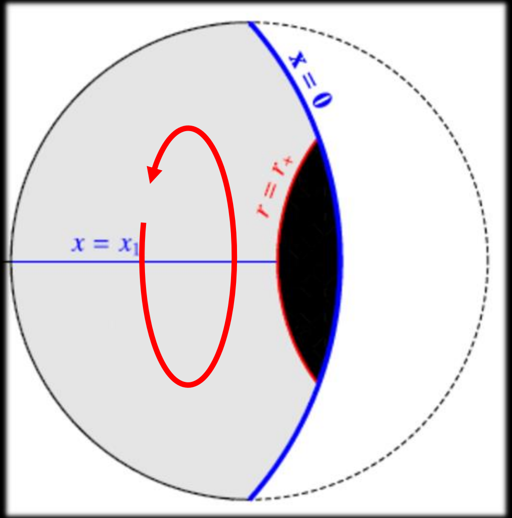
z: mass parameter

$$S_{gen} = \frac{A_{bulk}}{4G_4} = \frac{\pi\ell_3}{\mathcal{G}_3} \frac{z}{1 + 3z^2 + 2\nu z^3}$$

$$T\partial_z S_{gen} = \partial_z M$$

$$\Rightarrow TdS_{gen} = dM$$

Rotating quBTZ



Rotating AdS C-metric

Bulk structure similar to Kerr(-AdS$_4$)

inner&outer horizons, ring singularity

$\ell = 0$: rot BTZ black hole

$\ell > 0$: quantum-corrected rot BTZ $\neq$ AdS$_3$/Γ

Rotating quBTZ

Quantum-censored rotating cones

Two branches of rot quBTZ

Can have $M < J/\ell_3$

Rotating quBTZ

$\langle T_{ab} \rangle_{CFT}$ much simpler than for free conformal scalar

Smooth at inner Cauchy horizon

see *RE+Tomašević 2020 “Strong Cosmic Censorship in BTZ”*

Quantum BH Entropy: First law

$M \quad J$ measured in 3D eff theory

$T \quad \Omega$

$$S_{gen} = \frac{A_{bulk}}{4G_4}$$

Parametrized by

- ν: backreaction parameter ℓ/ℓ_3
- z: mass parameter
- α: rotation parameter

$$T\partial_z S_{gen} = \partial_z M - \Omega\partial_z J, \quad T\partial_\alpha S_{gen} = \partial_\alpha M - \Omega\partial_\alpha J$$

$$\Rightarrow TdS_{gen} = dM - \Omega dJ$$

So

Fully quantum-backreacted BTZ and CFT stress tensor can be described *exactly* and *in detail*

Quantum entropy from braneworld holography: *consistent*

Efficient use of holography to *solve a hard quantum problem*

Going further

General holographic proof of $TdS_{gen} = dM - \Omega dJ$

More classical proofs of quantum theorems?

Holographic duals of massive gravities – 3D and higher d

Extensions: charge, higher-d?

Exact entanglement islands

Dual Hawking evaporation? *RE+Kaloper+Tanaka+al 2002-20??*

Thank you

Antonia Frassino

Benson Way

ERC AdG 692951-GravBHs

Rotating quBTZ metric

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell \frac{F_1(M,J)}{r}\right) dt^2 + \left(r^2 + \ell \frac{F_2(M,J)}{r}\right) d\phi^2$$

$$- 8\mathcal{G}_3 J \left(1 + \ell \frac{F_3(M,J)}{r}\right) dt\, d\phi + \left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M + \frac{(4\mathcal{G}_3 J)^2}{r^2} + \ell \frac{F_4(M,J)}{r}\right)^{-1} dr^2$$

$\ell = 0$: rot BTZ black hole

$\ell > 0$: quantum-corrected rot BTZ $\neq \mathrm{AdS}_3/\Gamma$