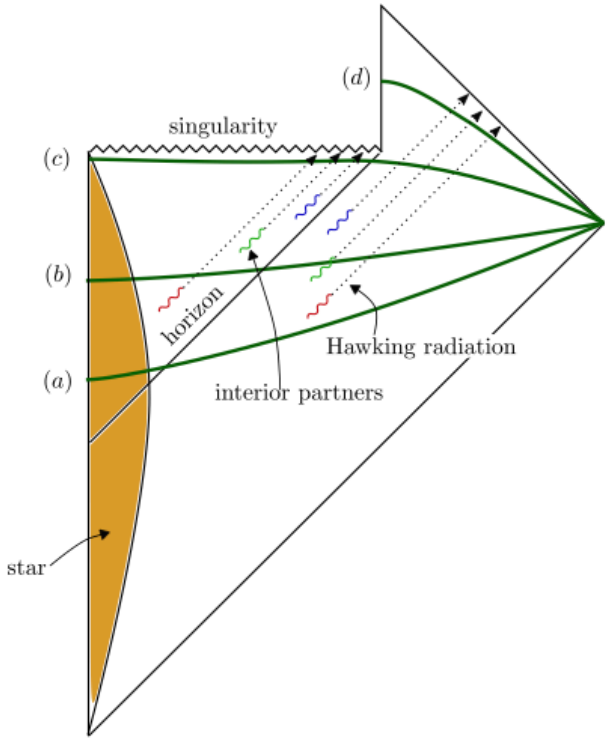
Slow scrambling in extremal BTZ and microstate geometries

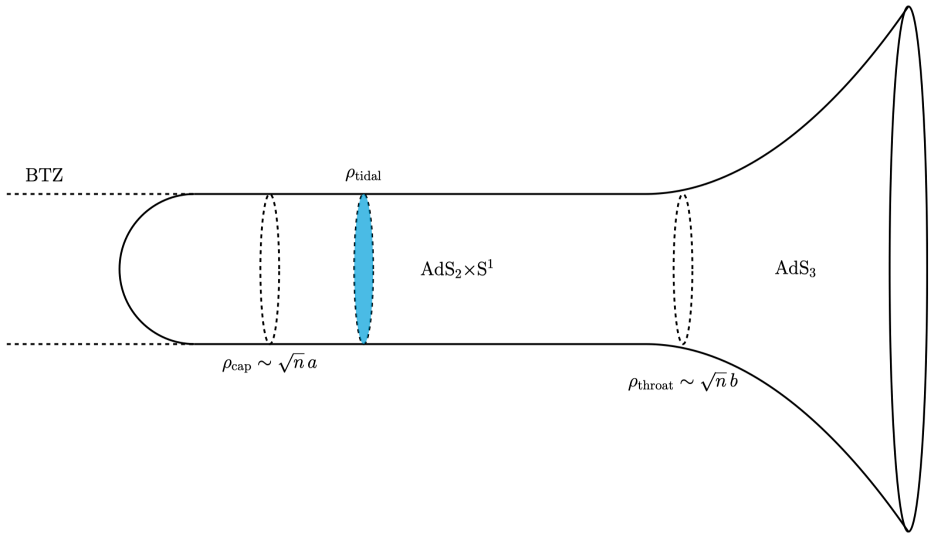
Ben Craps

Based on arXiv:2009.08518 w/ Marine De Clerck, Philip Hacker, Kévin Nguyen and Charles Rabideau

Rencontres théoriciennes, October 22, 2020, Paris, online

Do black hole microstates have horizons?





Left figure from [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini]

Observables in microstate geometries

- Approximate thermality of probes in gravitational microstates

  [Balasubramanian et al, 2005-2019],…

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- Thermalization and revivals of retarded 2pt functions in microstates of extremal BH

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- Will study observable related to quantum chaos

Black holes are chaotic

Black holes are thermal and chaos underlies thermal behavior:

1) Relaxation to thermal equilibrium

2) Sensitivity to initial conditions: $$\frac{dq(t)}{dq(0)} = \{q(t), p(0)\} \sim e^{\lambda t}$$

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Semiclassical approximation: replace Poisson bracket by commutator and consider growth of

$$-\langle [V(0), W(t)]^2 \rangle_\beta$$

as $\hat{W}(t) \equiv e^{-itH} \hat{W}(0) e^{itH}$ “grows” (spreads over the system).

[Larkin, Ovchinnikov 1969]

Lyapunov growth probes black hole horizon

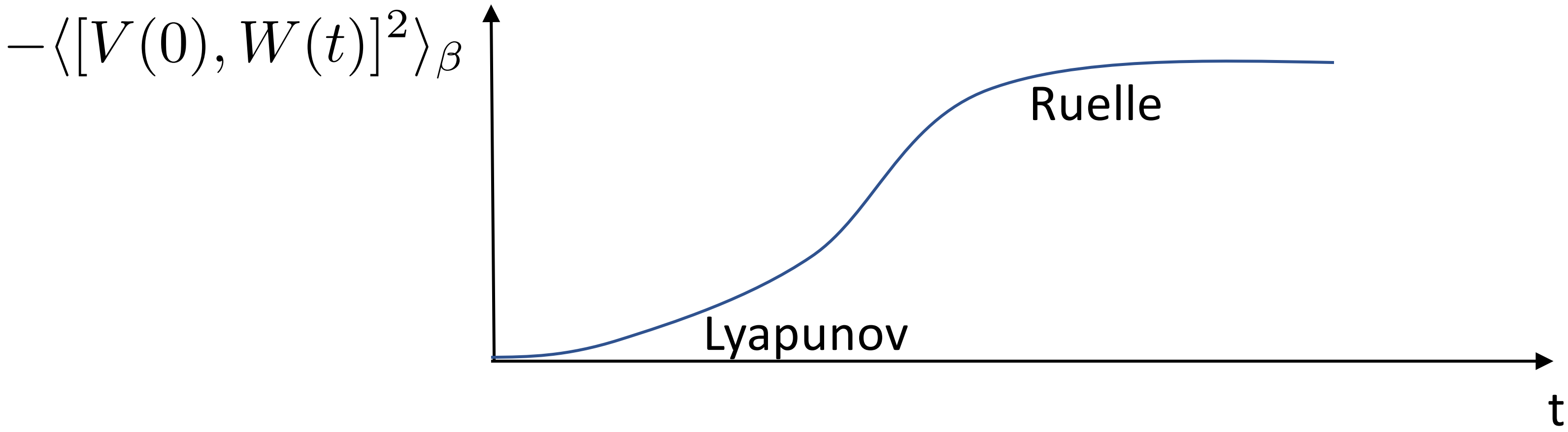

Figure based on [Polchinski]

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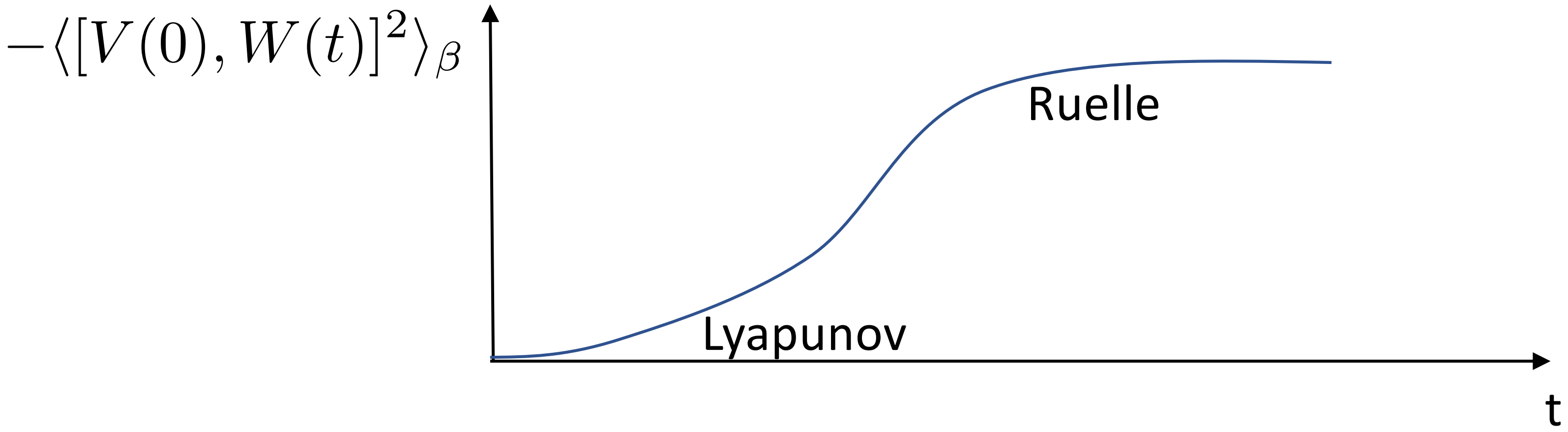

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Two exponential behaviors:

1) Exponential saturation (Ruelle) ⟺ QNM (cf. 2pt function)

AdS/CFT

[Horowitz, Hubeny 1999]

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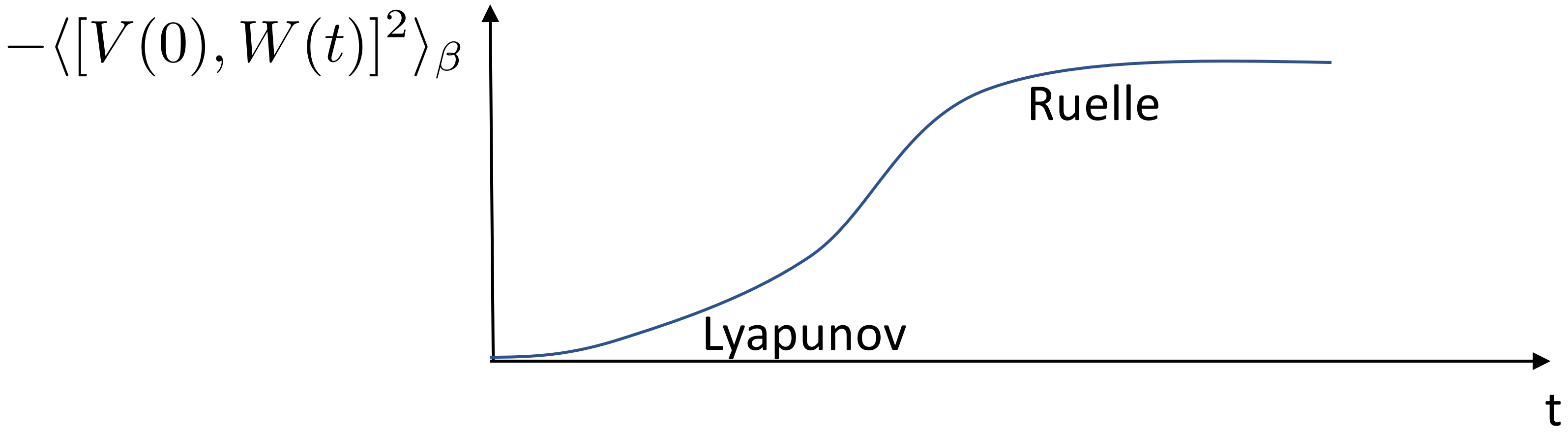


Figure based on [Polchinski]

<u>Two exponential behaviors</u>:

1) Exponential saturation (Ruelle) ⟺ QNM (cf. 2pt function)

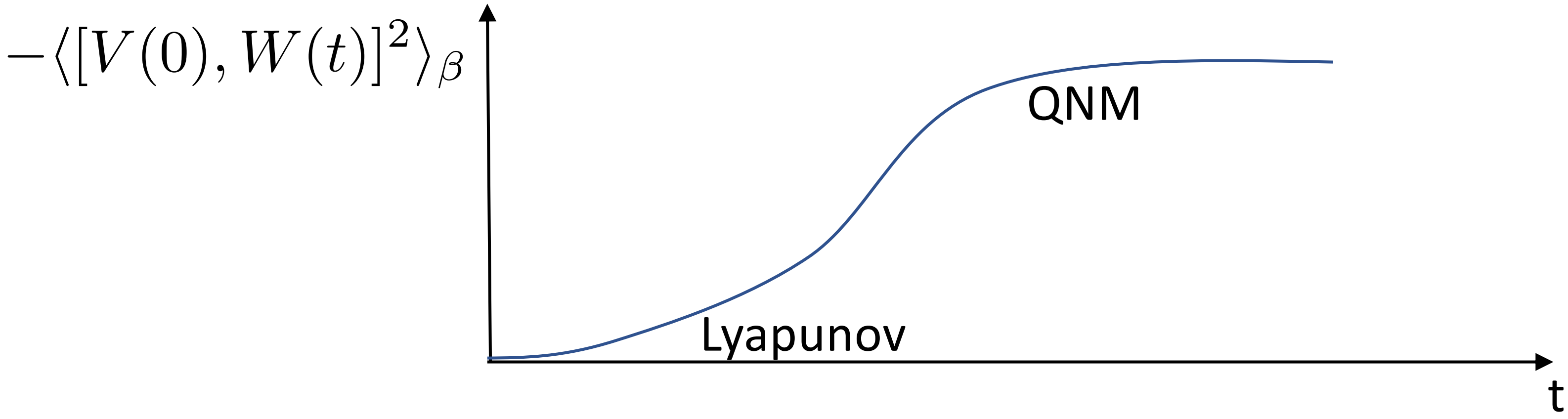
[Horowitz, Hubeny 1999]

AdS/CFT

2) Transient Lyapunov growth ⟺ redshift $\frac{dt}{d\tau} \sim e^{2\pi t/\beta}$

[Kitaev] [Shenker, Stanford]

OTOC



$$C(t) \equiv \frac{-\langle [V(0), W(t)]^2 \rangle_\beta}{2\langle VV \rangle_\beta \langle WW \rangle_\beta} \approx 1 - \mathrm{Re}\,\mathrm{OTOC}(t) \qquad (t \gg \beta)$$

$$\mathrm{OTOC}(t) \equiv \frac{\langle V(0)W(t)V(0)W(t) \rangle_\beta}{\langle VV \rangle_\beta \langle WW \rangle_\beta}$$

Figure based on [Polchinski]

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If Lyapunov growth reflects what happens at the horizon, then what happens in horizonless microstate geometries?

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→ Compute OTOCs for extremal black holes and microstate geometries.

Outline

1. Introduction
2. Geodesic approximation to the OTOC
3. OTOC in extremal BTZ
4. Microstate geometries
5. Discussion

Chaos in holographic 2d CFT

$$h_W \gg h_V \gg 1$$

$$\frac{\langle V(i\epsilon_1, x) W(t+i\epsilon_3, 0) V(i\epsilon_2, x) W(t+i\epsilon_4, 0)\rangle_\beta}{\langle V(i\epsilon_1, 0) V(i\epsilon_2, 0)\rangle_\beta \langle W(i\epsilon_3, 0) W(i\epsilon_4, 0)\rangle_\beta} \approx \left(\frac{1}{1 - \frac{24\pi i h_W}{\epsilon_{12}^* \epsilon_{34} c} e^{\frac{2\pi}{\beta}(t-|x|)}} \right)^{2h_V}$$

$$\epsilon_{ij} = i \left(e^{\frac{2\pi}{\beta} i\epsilon_i} - e^{\frac{2\pi}{\beta} i\epsilon_j} \right)$$

[Roberts, Stanford]

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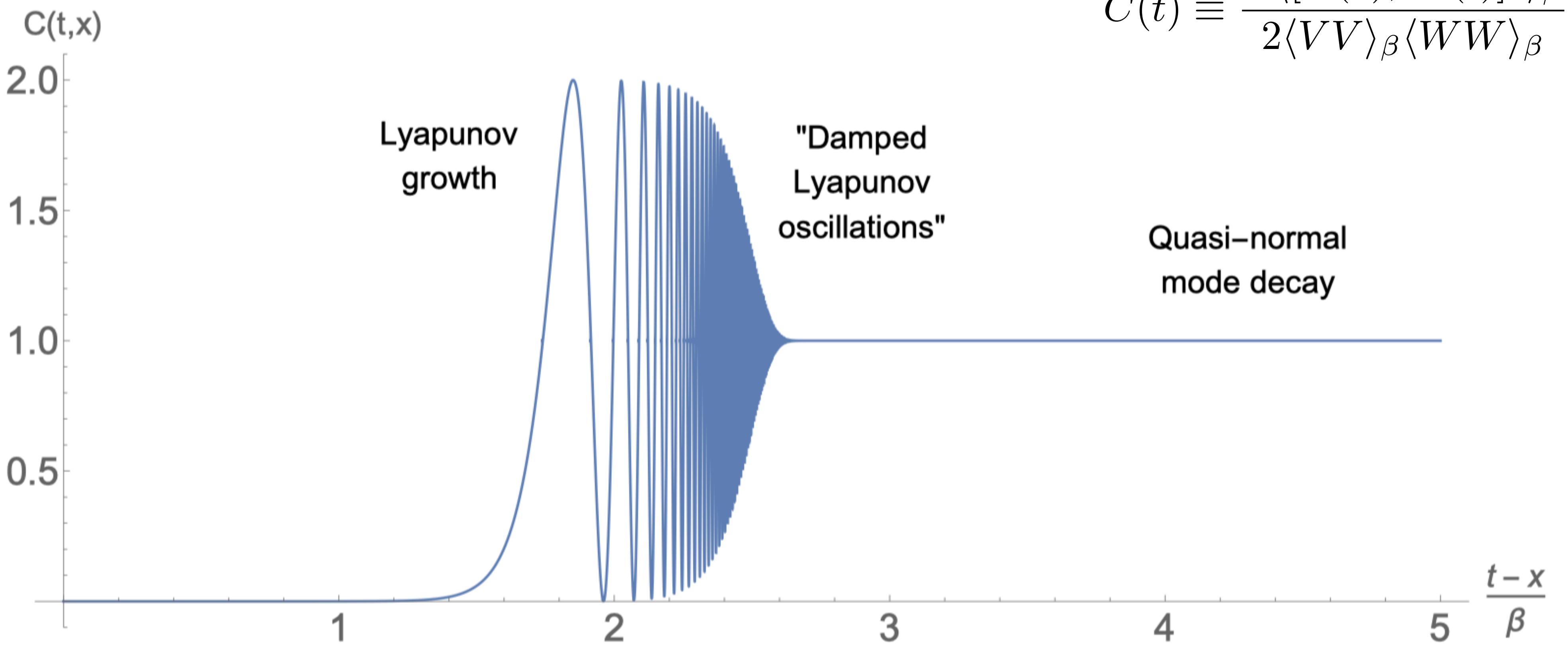
Scrambling time: $$t_s = |x| + \frac{\beta}{2\pi}\log\frac{|\epsilon_{12}^*\epsilon_{34}|c}{48\pi h_W h_V}$$

[Roberts, Stanford]

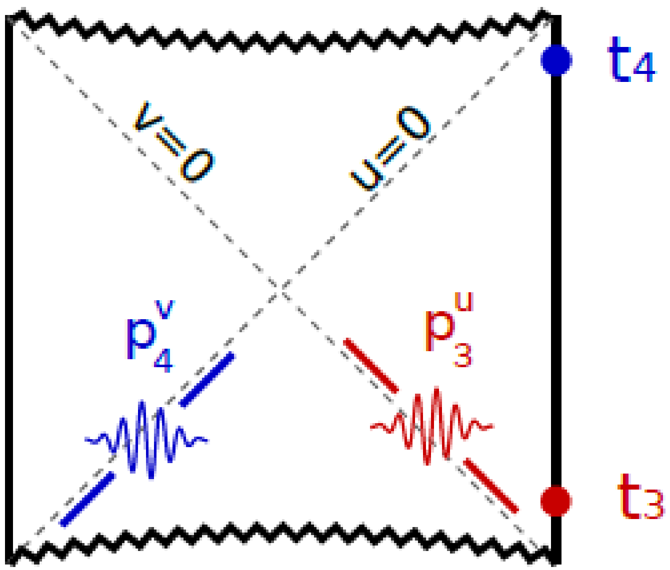
Aside: damped Lyapunov oscillations

$$C(t) \equiv \frac{-\langle [V(0), W(t)]^2 \rangle_\beta}{2\langle VV \rangle_\beta \langle WW \rangle_\beta}$$

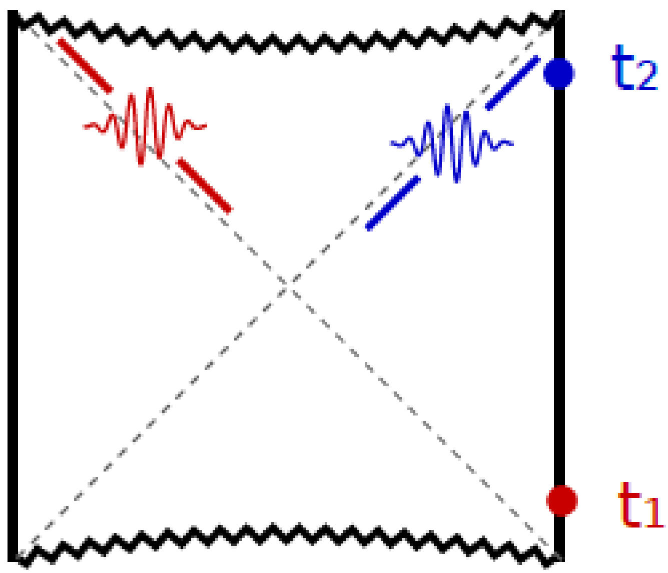


OTOC from gravitational scattering

$\langle V(t_1)W(t_2)V(t_3)W(t_4)\rangle_\beta$ = overlap of two states:



$V(t_3)W(t_4)|TFD\rangle$



$W(t_2)V(t_1)|TFD\rangle$

[Shenker, Stanford]

Geodesic approximation

$$\frac{\langle\phi_V(t_{in},x_-)\phi_W(t_{out},x_+)\phi_V(t_{in},x_-)\phi_W(t_{out},x_+)\rangle}{\langle\phi_V\phi_V\rangle\langle\phi_W\phi_W\rangle}\approx e^{i\delta}\Big|_{\text{saddle}}$$

[Shenker, Stanford] [Balasubramanian, BC, De Clerck, Nguyen]

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$$\delta = \frac{1}{4}\int \left(h^V_{\mu\nu}T^{\mu\nu}_W + h^W_{\mu\nu}T^{\mu\nu}_V\right) + O(G_N^2)$$

eikonal phase

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$$\delta \sim G_N s \sim G_N m_W m_V e^{\kappa(t_{out}-t_{in})}$$

$$\lambda_L = \kappa = \frac{2\pi}{\beta}$$

[Shenker, Stanford] [Balasubramanian, BC, De Clerck, Nguyen]

Slow scrambling in vacuum

Warm-up example of zero-temperature: OTOC in vacuum of 2d holographic CFT

$\beta = \infty$ $\qquad$ $t \gg |x|$ $\qquad$ $h_W \gg h_V \gg 1$

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$$= 1 + \frac{48\pi i h_V h_W}{c(\epsilon_1 - \epsilon_2)(\epsilon_3 - \epsilon_4)}(t - |x|)^2 + \ldots$$

Power law rather than exponential: “slow scrambling”

[Roberts, Stanford]

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Extremal BTZ

Rotating BTZ:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \left(d\varphi - \frac{r_+ r_-}{r^2} dt \right)^2 \ , \quad f(r) = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}$$

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Extremal BTZ: $r_- = r_+$

$$ds^2 = \ell_{AdS}^2\left[-\left(r^2 - 2r_+^2\right)dt^2 + \frac{r^2\ dr^2}{\left(r^2 - r_+^2\right)^2} - 2r_+^2\ dtd\varphi + r^2\ d\varphi^2\right]$$

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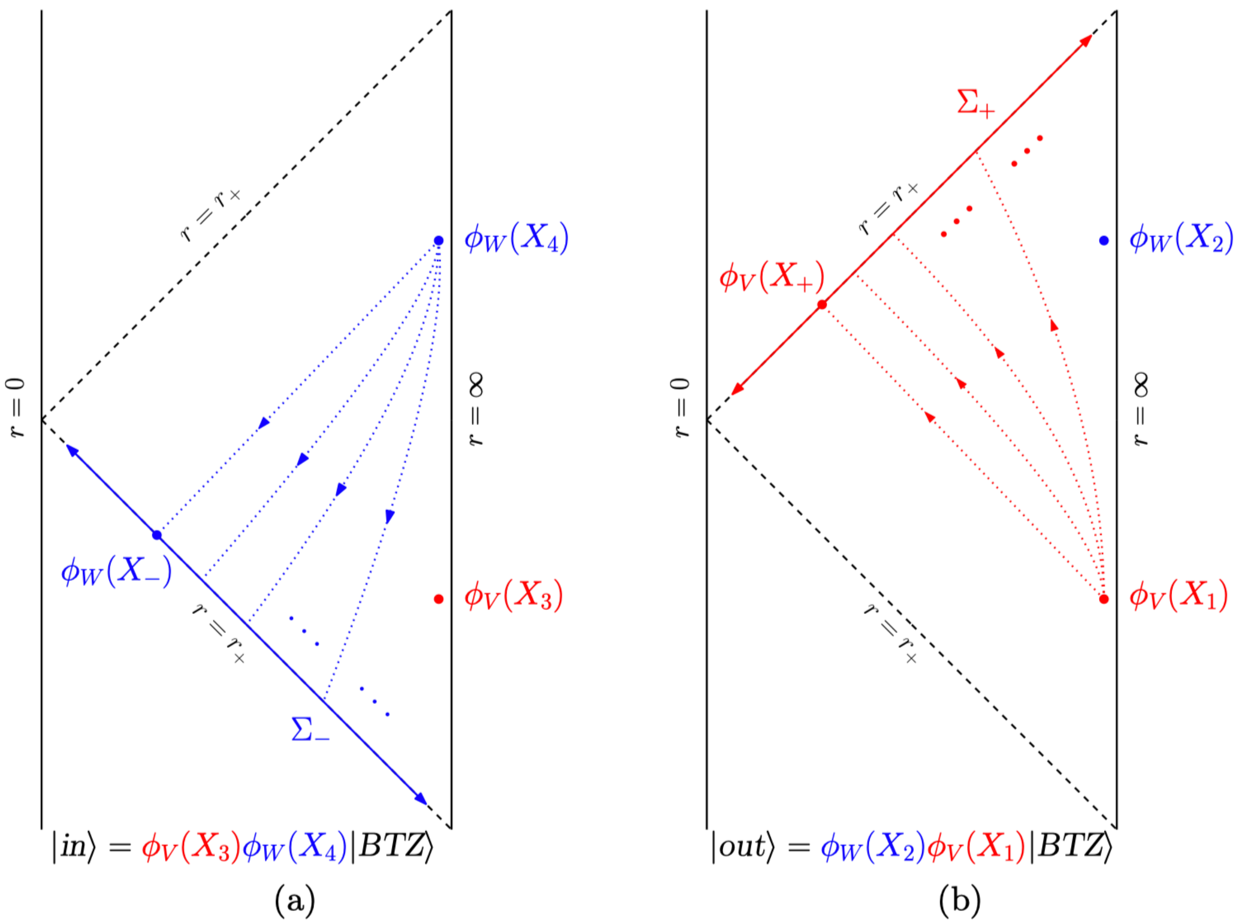
$$T_L = 0, \qquad T_R = \frac{r_+}{\pi}$$

OTOC in extremal BTZ



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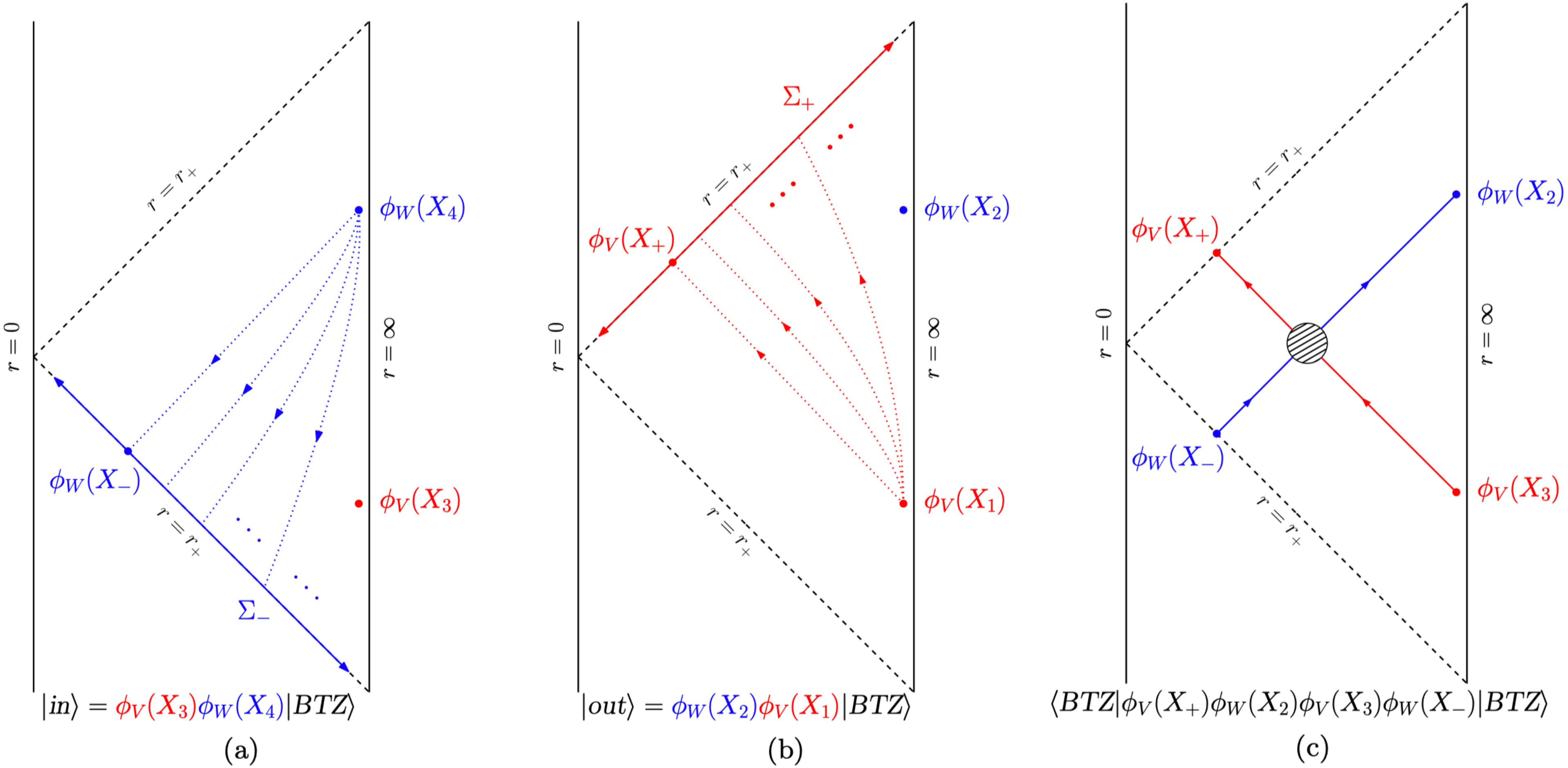

Center-of-mass energy in rotating BTZ

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\varphi - \frac{r_+ r_-}{r^2}dt\right)^2 , \quad f(r) = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}$$

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Infalling and outgoing (almost) null geodesic that cross at radius r_*

Each has conserved energy E and angular momentum $L = 0$

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$$t_\infty - t_* = \pm \int_{r_*}^{\infty} \frac{dr}{f(r)}$$

Center-of-mass energy in rotating BTZ

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$$\int_{r^*}^{\infty} \frac{dr}{f(r)} \sim \frac{r_+}{2(r_+^2 - r_-^2)} \int_{r_*} \frac{dr}{r - r_+} = -\frac{r_+ \log(r_* - r_+)}{2(r_+^2 - r_-^2)} \qquad (r_*^2 - r_+^2 \ll r_+^2 - r_-^2)$$

Center-of-mass energy in rotating BTZ

$$s = \frac{4 m_V m_W E^2}{f(r_*)} \qquad t_\infty - t_* = \pm \int_{r_*}^{\infty} \frac{dr}{f(r)} \qquad f(r) = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}$$

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$$\sim \frac{1}{4} \int_{r_*} \frac{dr}{(r - r_+)^2} = \frac{1}{4(r_* - r_+)} \qquad (r_+^2 - r_-^2 \ll r_*^2 - r_+^2 \ll r_+^2)$$

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$$s \sim \frac{2m_V m_W r_+ E^2}{(r_+^2 - r_-^2)} e^{\frac{r_+^2 - r_-^2}{r_+} \Delta t} \qquad (r_*^2 - r_+^2 \ll r_+^2 - r_-^2)$$

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$$\sim 4m_V m_W E^2 \Delta t^2 \qquad (r_+^2 - r_-^2 \ll r_*^2 - r_+^2 \ll r_+^2)$$

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Infalling and outgoing (almost) null geodesic that cross at radius r_*

Each has conserved energy E and angular momentum $L = 0$

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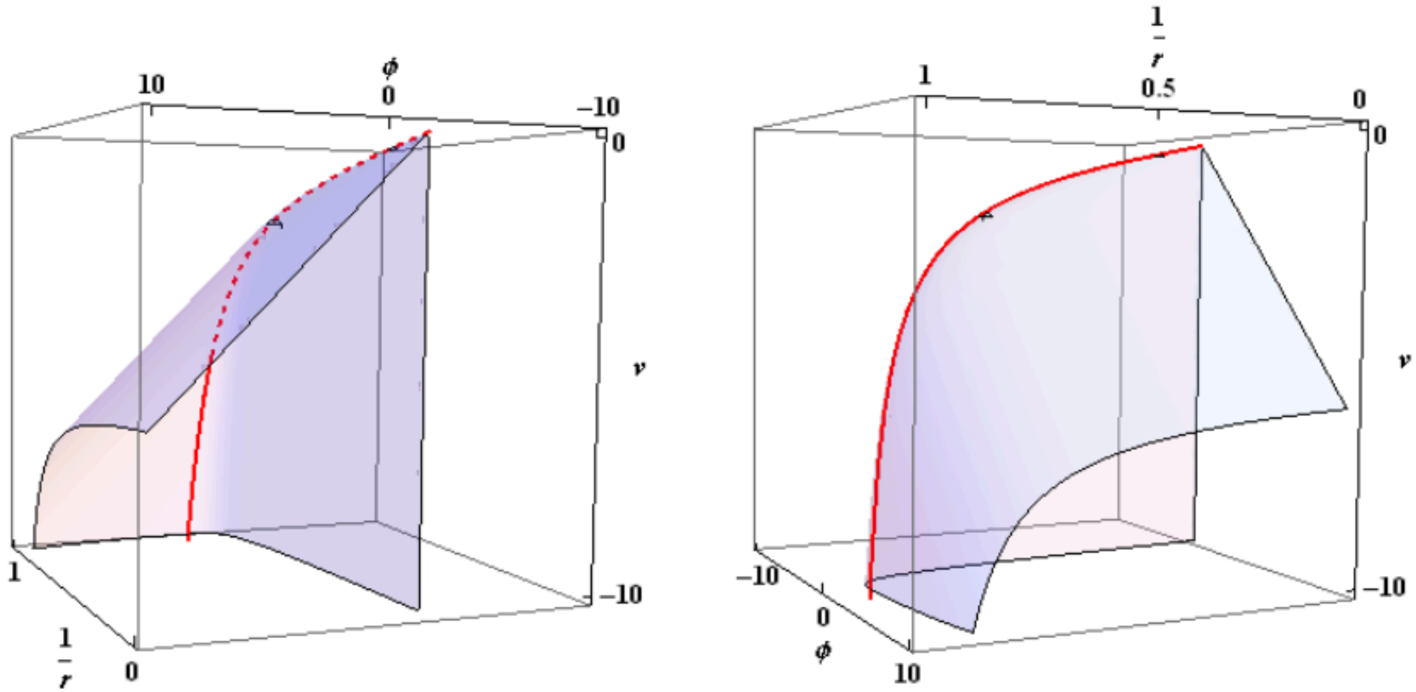
In exponential (Lyapunov) regime of non-extremal BH, we used to have $\delta \sim G_N s$.

In slow scrambling regime of extremal BH, we will find additional time-dependence.

Computing OTOC using shockwaves

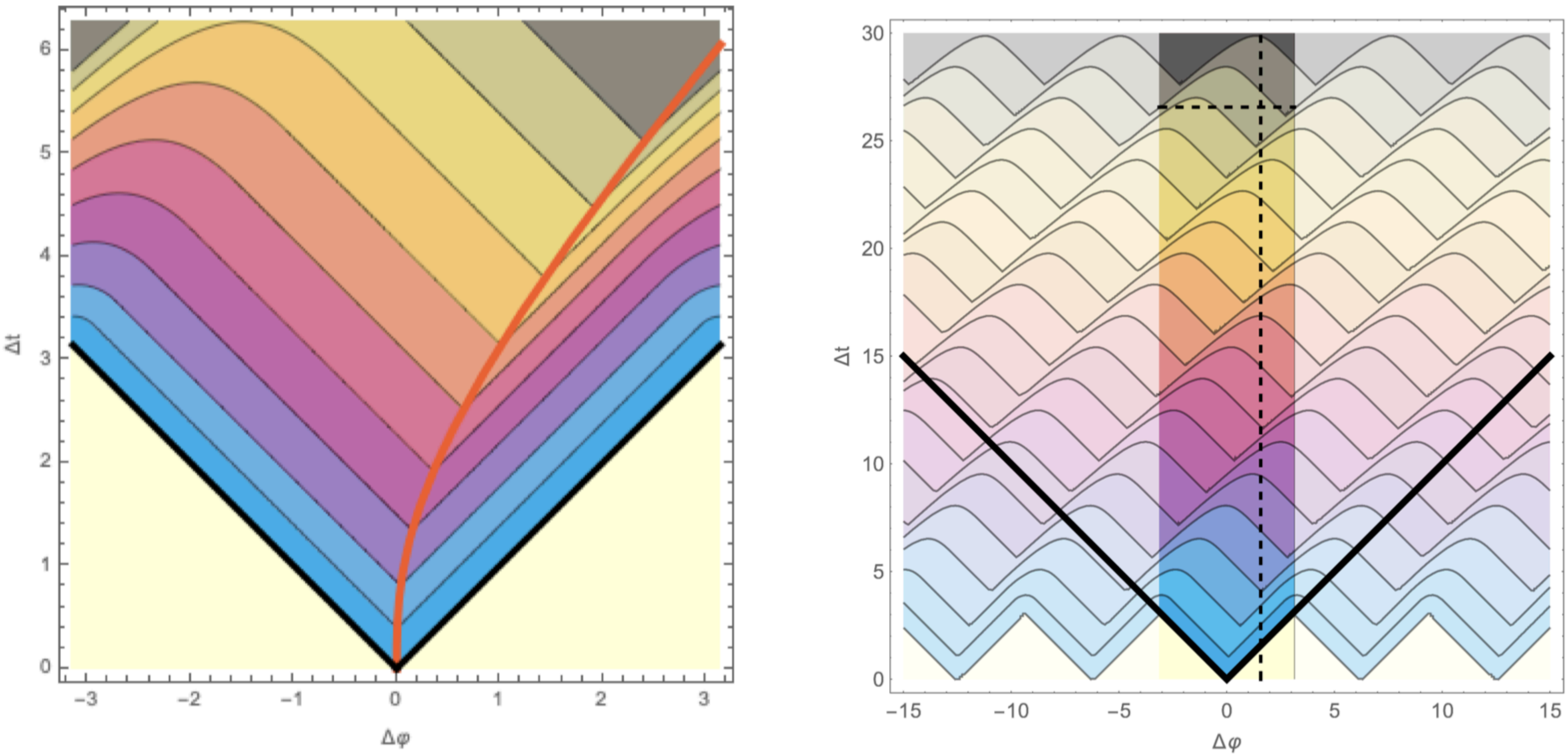
Support of shockwave emitted by outgoing null geodesic in retarded coordinates for non-compact φ:



Shockwave is emitted from geodesic towards conformal boundary.

Computing OTOC using shockwaves: compact φ

Contour plots of eikonal phase (cubic function would have equidistant contour lines):



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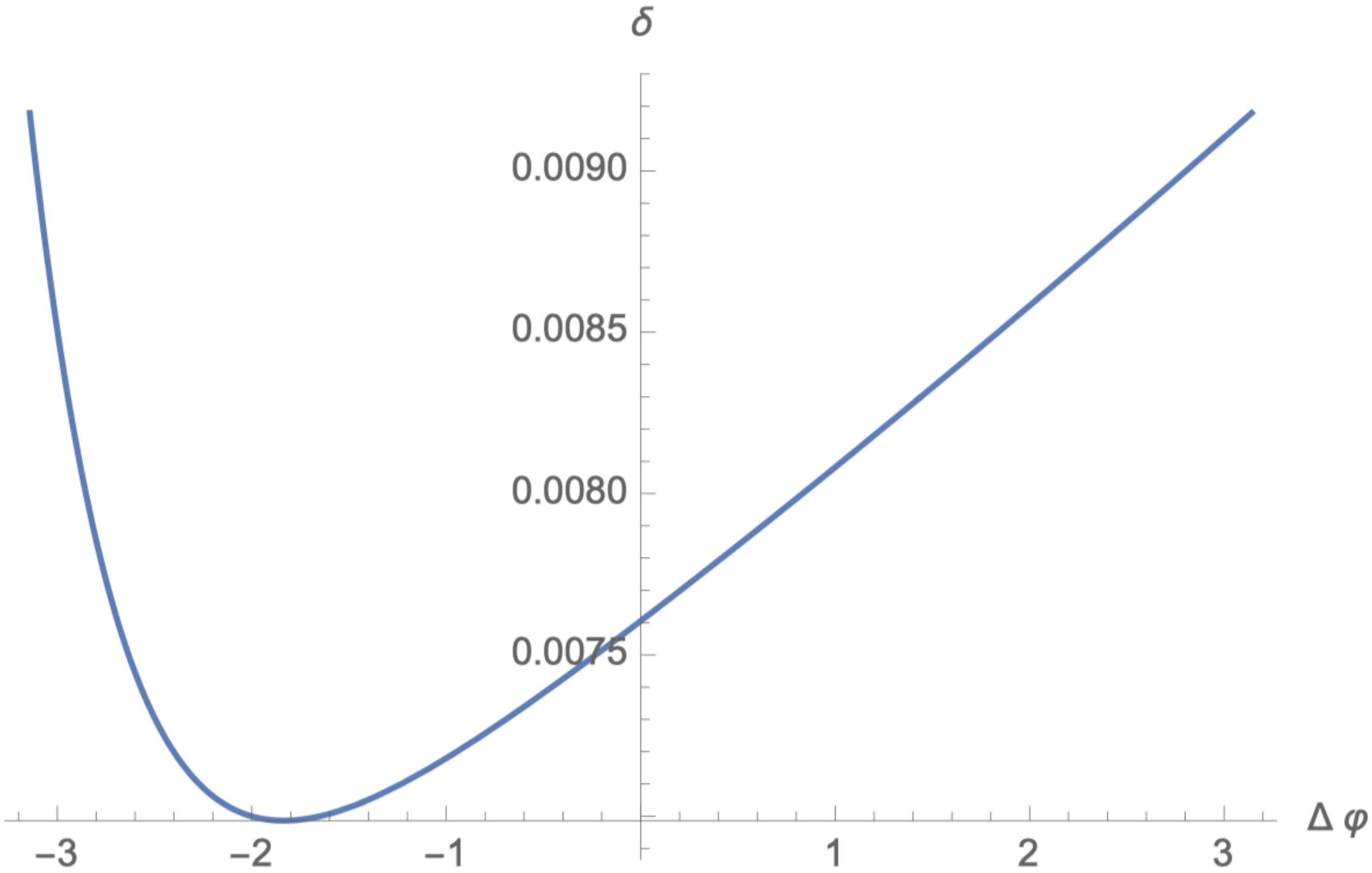
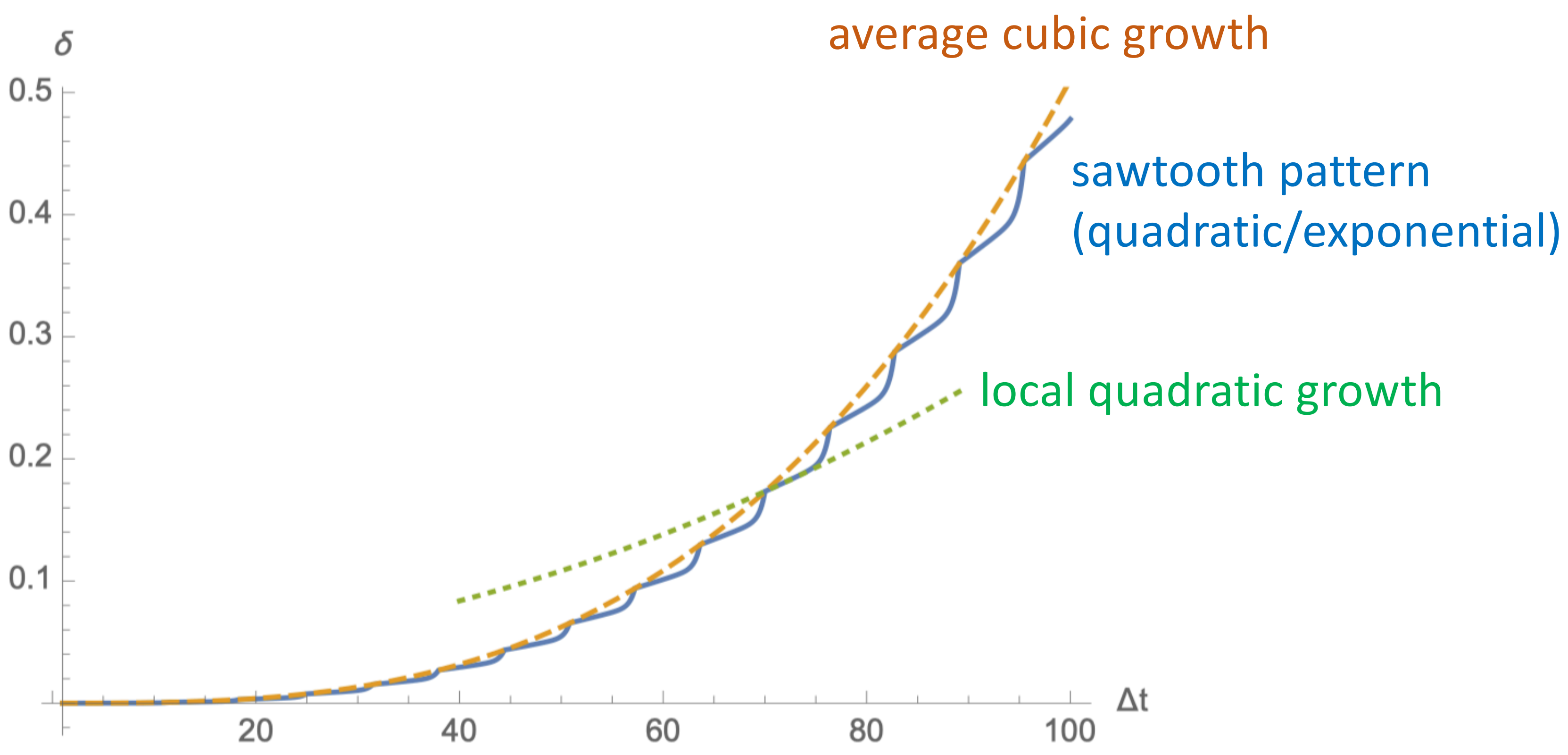


Power-law growth and sawtooth modulation



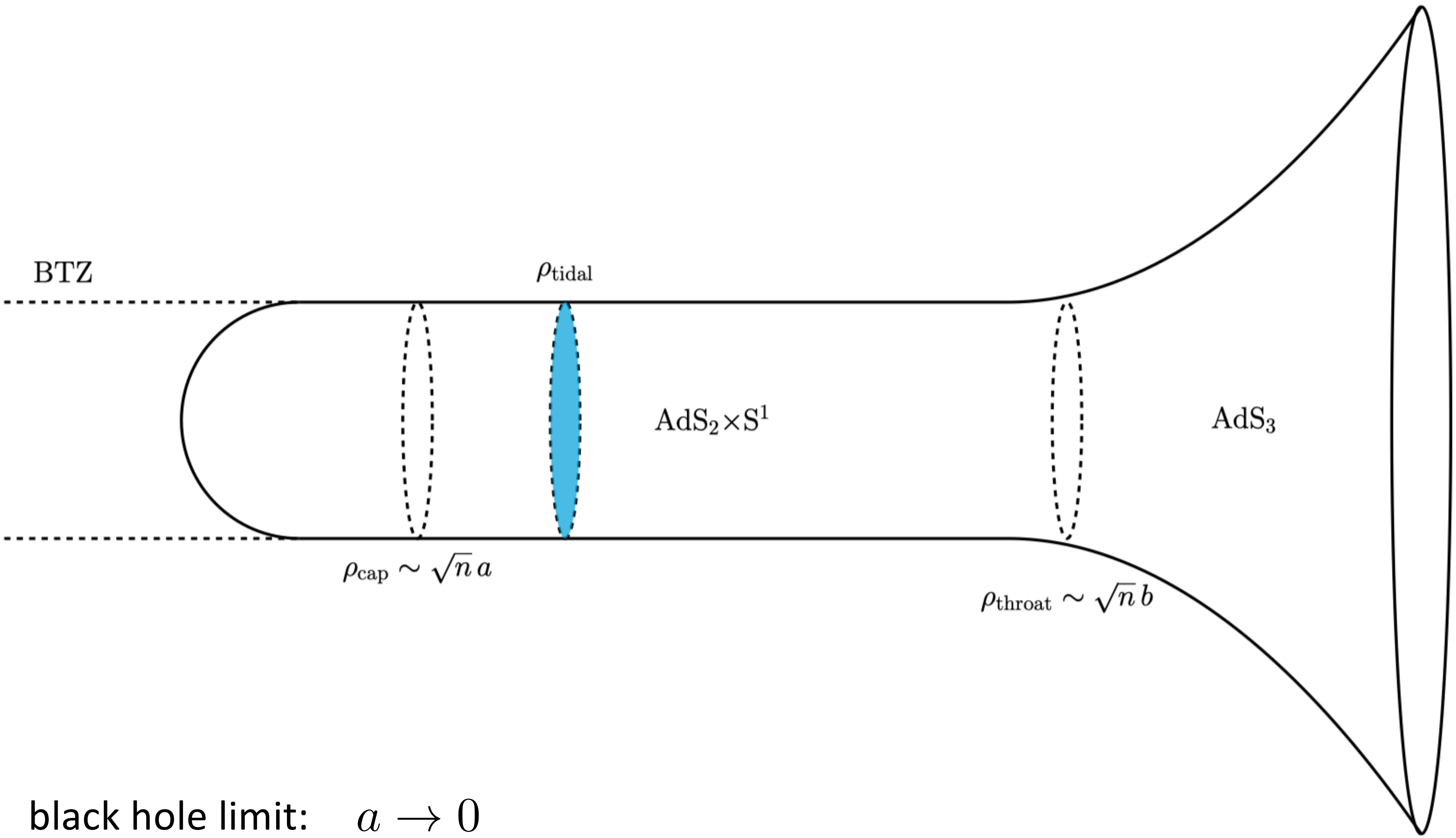
Comments

- Periodic φ : shockwave obtained via method of images.
- Number of images that contribute $\sim \Delta t$.
- This combines with $s \sim (\Delta t)^2$ into overall cubic growth of eikonal phase.
- Local quadratic and exponential growth reflects $T_L = 0, \; T_R = \frac{r_+}{\pi}.$
- For $\Delta t \ll \beta$, scattering happens away from the horizon. If we had replaced shockwave by expression at the horizon, an infinite number of images would have contributed, yielding a divergent result.
- Sawtooth pattern had been observed in [Mezei, Sárosi] for non-extremal rotating BTZ, in the regime $\Delta t \gg \beta$.

Outline

1. Introduction
2. Geodesic approximation to the OTOC
3. OTOC in extremal BTZ
4. Microstate geometries
5. Discussion

Microstate geometries: (1,0,n) superstrata



black hole limit: $a \to 0$

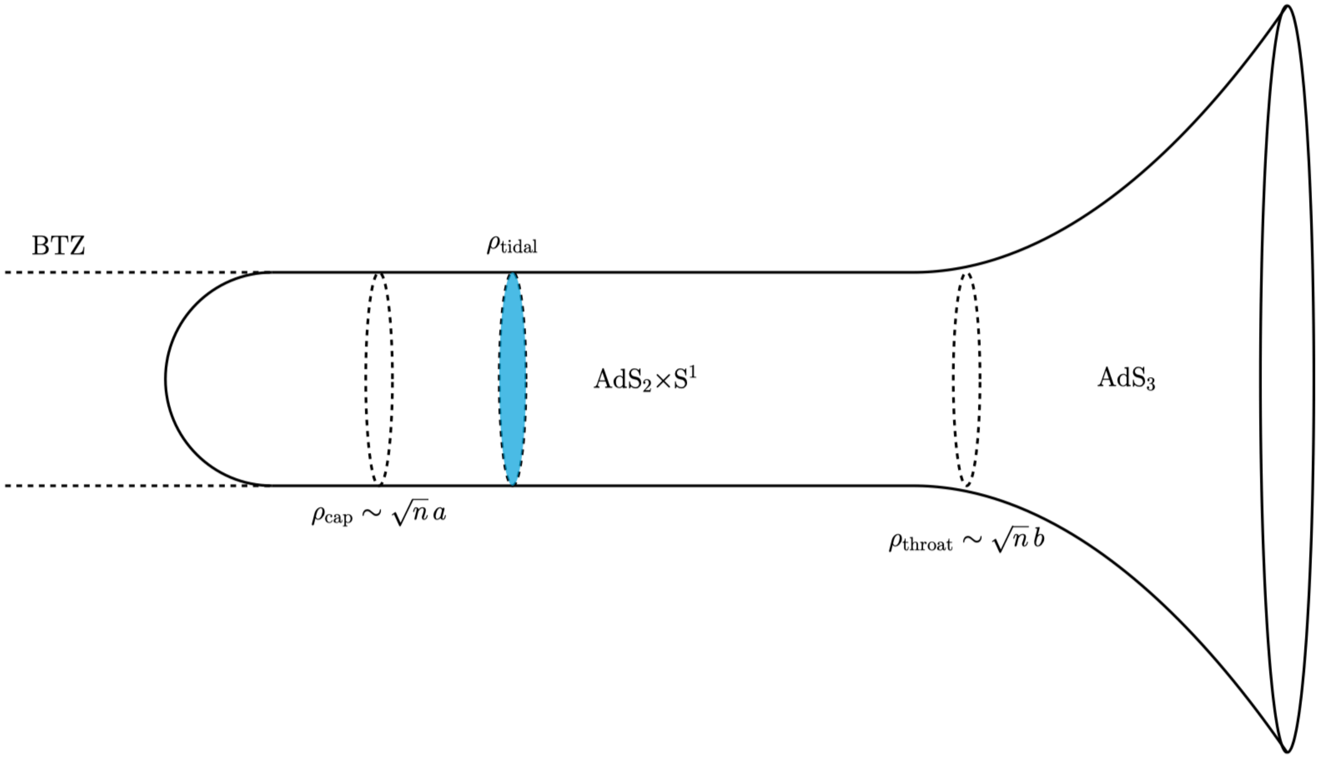
[Bena, Giusto, Martinec, Russo, Shigemori, Turton, Warner]

Dual CFT states

- (1,0,n) superstrata correspond to (coherent superpositions of) explicitly known states in the dual D1-D5 CFT, which has $c = 6N_1N_5$.
- Solutions are supersymmetric → BPS states, which can be constructed at symmetric orbifold point in moduli space.
- Constructed by adding momentum to 2-charge solutions/states, while decreasing angular momentum.
- Parameters a and b are related to angular momentum and momentum, respectively.

[Bena, Giusto, Martinec, Russo, Shigemori, Turton, Warner]

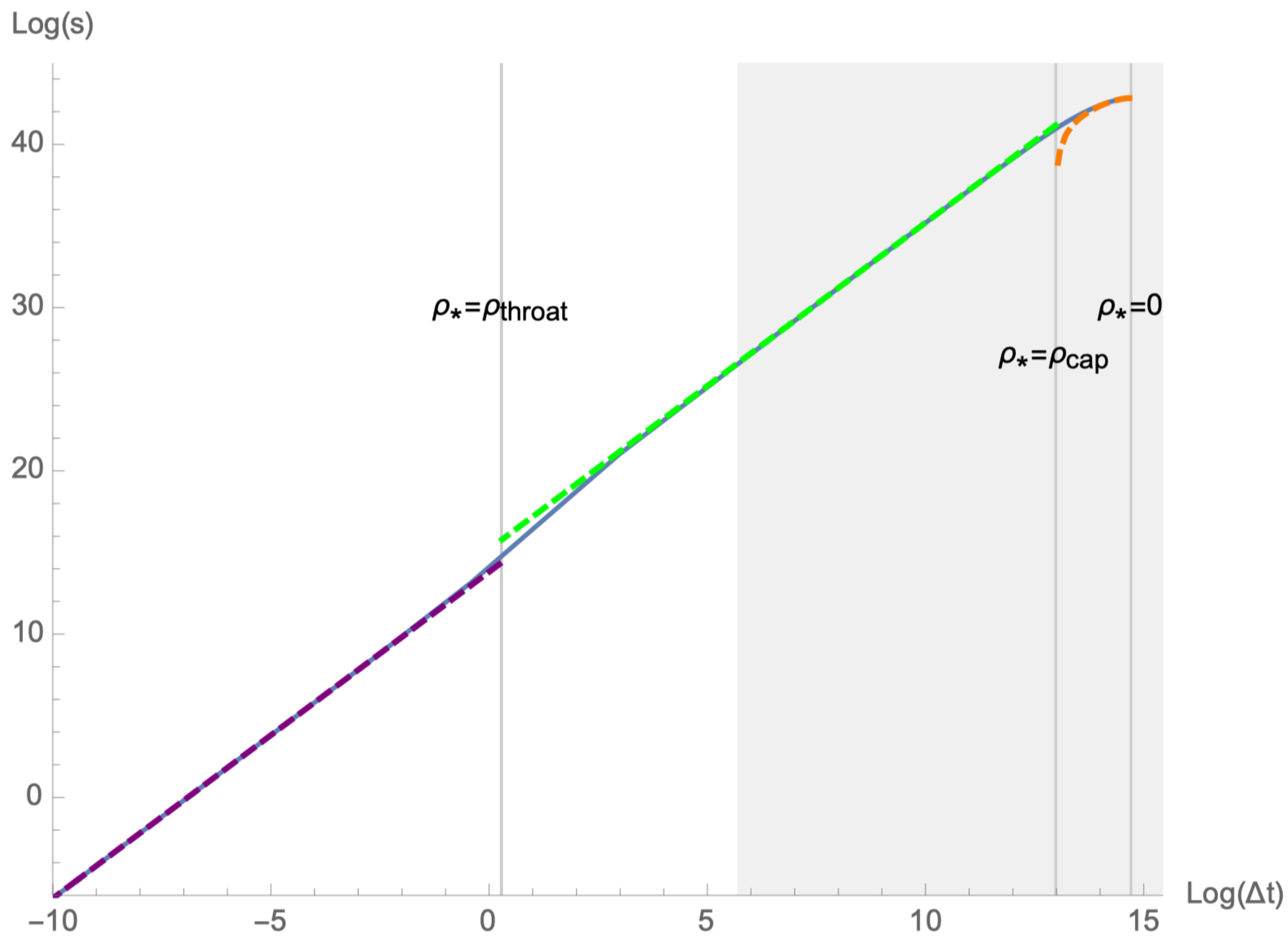
Interaction locus of geodesics



$$\Delta t_{\rm throat} \sim T_R^{-1}$$

$$\Delta t_{\rm cap} \sim \frac{1}{\sqrt{n}} \frac{b^2}{a^2} \sim \frac{N_1 N_5}{T_R}$$

Center-of-mass energy is bounded



Shockwave modifications

- If interactions happen outside cap region, BTZ shockwave can be used.
- When interactions reach cap region, detailed form of OTOC depends on shockwave in 6d superstrata geometry. Not yet computed.

Tidal forces

Geodesic approximation: $\langle\psi|\phi(X)\phi(Y)|\psi\rangle = A(X,Y)e^{imS(X,Y)}$

with $S(X,Y) = \int_Y^X dx \cdot k$

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Raychaudhuri eq: $\dot{\theta} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu$

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In superstratum: $-R_{\mu\nu}k^\mu k^\nu \sim -\frac{2\sqrt{2}}{b} - \frac{n(n+1)a^2b^3}{\sqrt{2}\varepsilon^2\rho^6}$

Tidal forces

Validity of geodesic approximation requires $|\theta| \ll m$

Raychaudhuri eq: $\dot{\theta} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu$

In superstratum: $-R_{\mu\nu}k^\mu k^\nu \sim -\frac{2\sqrt{2}}{b} - \frac{n(n+1)a^2 b^3}{\sqrt{2}\varepsilon^2\rho^6}$

Second term grows as geodesic falls into throat, invalidating geodesic approximation at

$$\Delta t_{\mathrm{tidal}} \sim \sqrt{\frac{\pi T_R}{\pi^2 T_R^2 + 1}\min(h_V, h_W)\varepsilon N_1 N_5}$$

Cf. [Tyukov, Walker, Warner] and [Bena, Martinec, Walker, Warner]

Results

$$\frac{\Delta t_{\text{cap}}}{\Delta t_s} = \left(\frac{8 h_V h_W (N_1 N_5)^2}{3 \varepsilon T_R^3} \right)^{\frac{1}{3}}$$

→ Scrambling happens before the cap region is reached unless $T_R \gtrsim \left(\frac{h_V h_W (N_1 N_5)^2}{\varepsilon} \right)^{\frac{1}{3}}$

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$$\frac{\Delta t_{\text{tidal}}}{\Delta t_s} = \frac{2}{3^{\frac{1}{3}}}\sqrt{\frac{\pi T_R}{(\pi^2 T_R^2 + 1)}}\sqrt{\min(h_V, h_W)}\left(\frac{h_V^2 h_W^2 N_1 N_5}{\varepsilon}\right)^{\frac{1}{6}}$$

→ Scrambling happens before tidal forces invalidate the geodesic approximation unless

$$T_R \gtrsim \min(h_V, h_W)\left(\frac{h_V^2 h_W^2 N_1 N_5}{\varepsilon}\right)^{\frac{1}{3}}$$

Outline

Comments on dual CFT

- Reproduce sawtooth pattern of OTOCs corresponding to rotating BTZ from CFT.

  [work in progress with Surbhi Khetrapal]

- BTZ does not dominate canonical ensemble for low enough temperature, in particular for extremal BTZ.

- So our gravity computation of an OTOC in extremal BTZ should not correspond to a thermal OTOC in CFT.

- However, it does provide a good approximation to an OTOC in a microstate.

Summary

- Lyapunov behavior of OTOC: gravitational scattering near BH horizon.
- OTOC can be computed using geodesic approximation.
- Extremal BTZ: gravitational scattering away from horizon, slow scrambling,
  OTOC displays sawtooth pattern.
- Microstate geometries: reproduce slow scrambling properties of extremal BTZ,
  deviations kick in after the scrambling time except for extremely high temperatures.