Adventures in Non-supersymmetric String Theory

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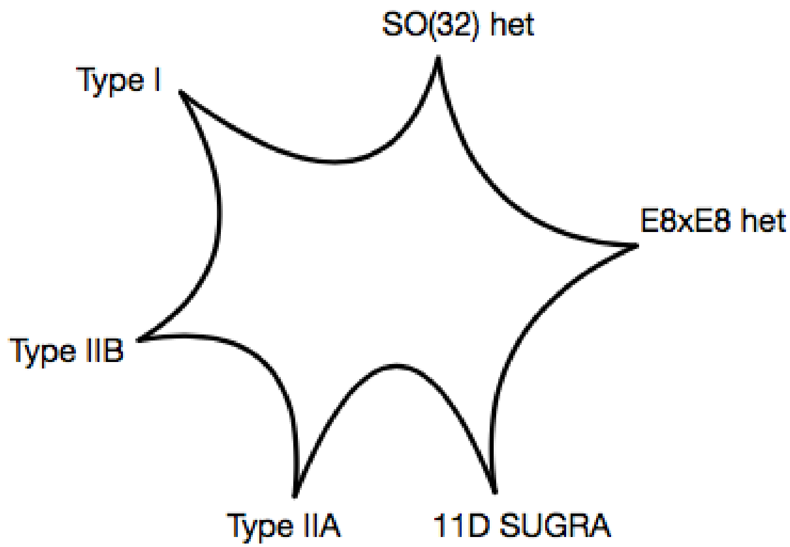
[2010.10521] **JK**
[1908.04805] **JK**, Parra-Martinez, Tachikawa
[1911.11780] **JK**, Parra-Martinez, Tachikawa

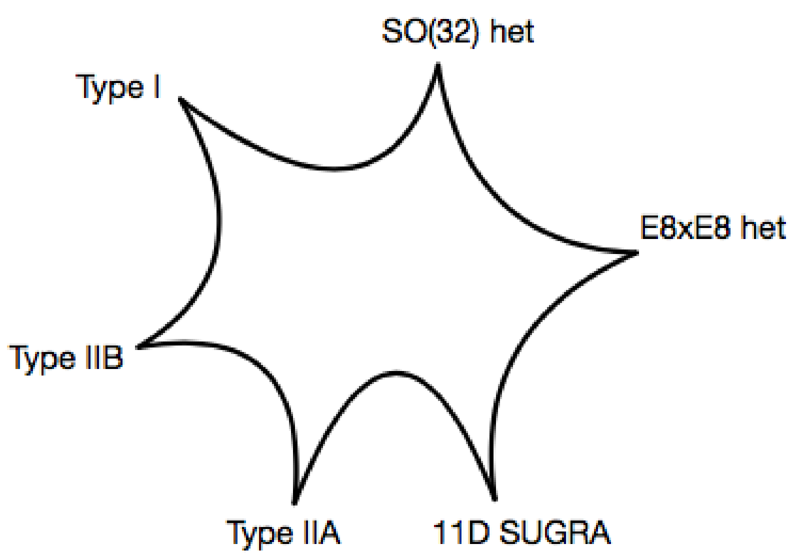
Spectrum of String Theories

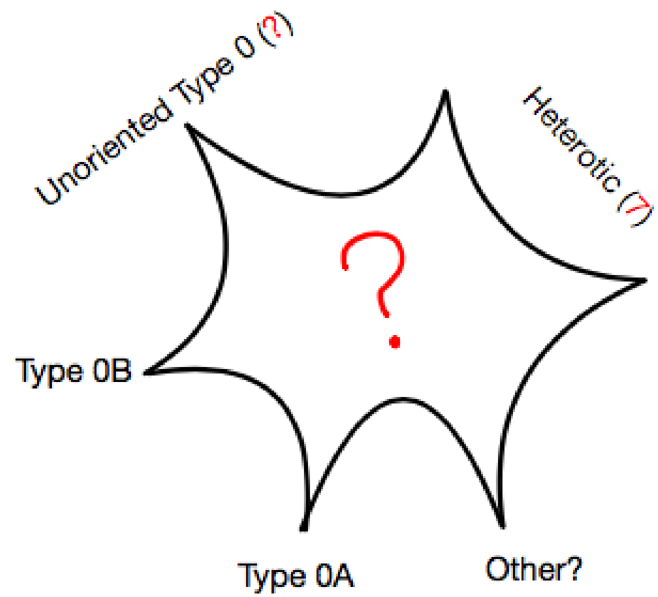
- **In the 80s, it was discovered that SUSY string theories are different limits of a single underlying theory:**



Spectrum of String Theories

- **In the 80s, it was discovered that SUSY string theories are different limits of a single underlying theory.**
- **But there also exist *non-SUSY* string theories**
  - Type 0A/B, unoriented Type 0 strings, and 7 heterotic strings





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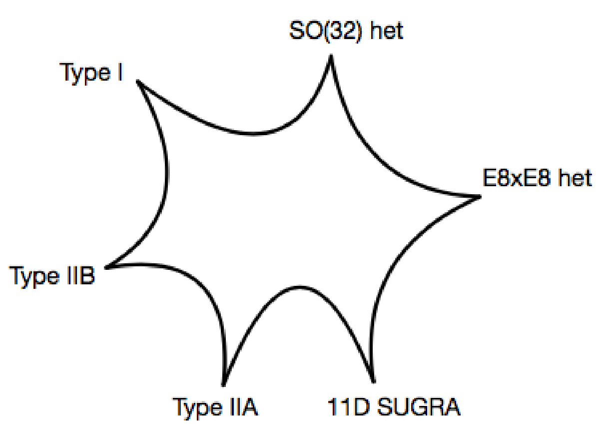
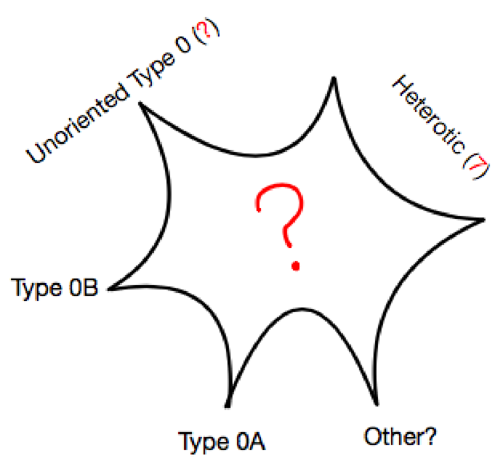
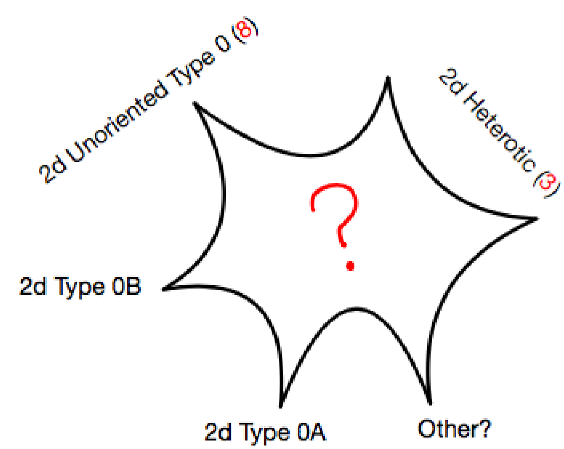
- **But there also exist *non-SUSY* string theories**
  - Type 0A/B, unoriented Type 0 strings, and 7 heterotic strings
- **There are also string theories in dimensions lower than 10d. In particular, much study on 2d strings** [Takayanagi, Toumbas '03; Douglas, Klebanov, Kutasov, Maldacena, Martinec, Seiberg '03]
  - 2d Type 0A/B, unoriented 2d Type 0 strings, and 3 heterotic strings

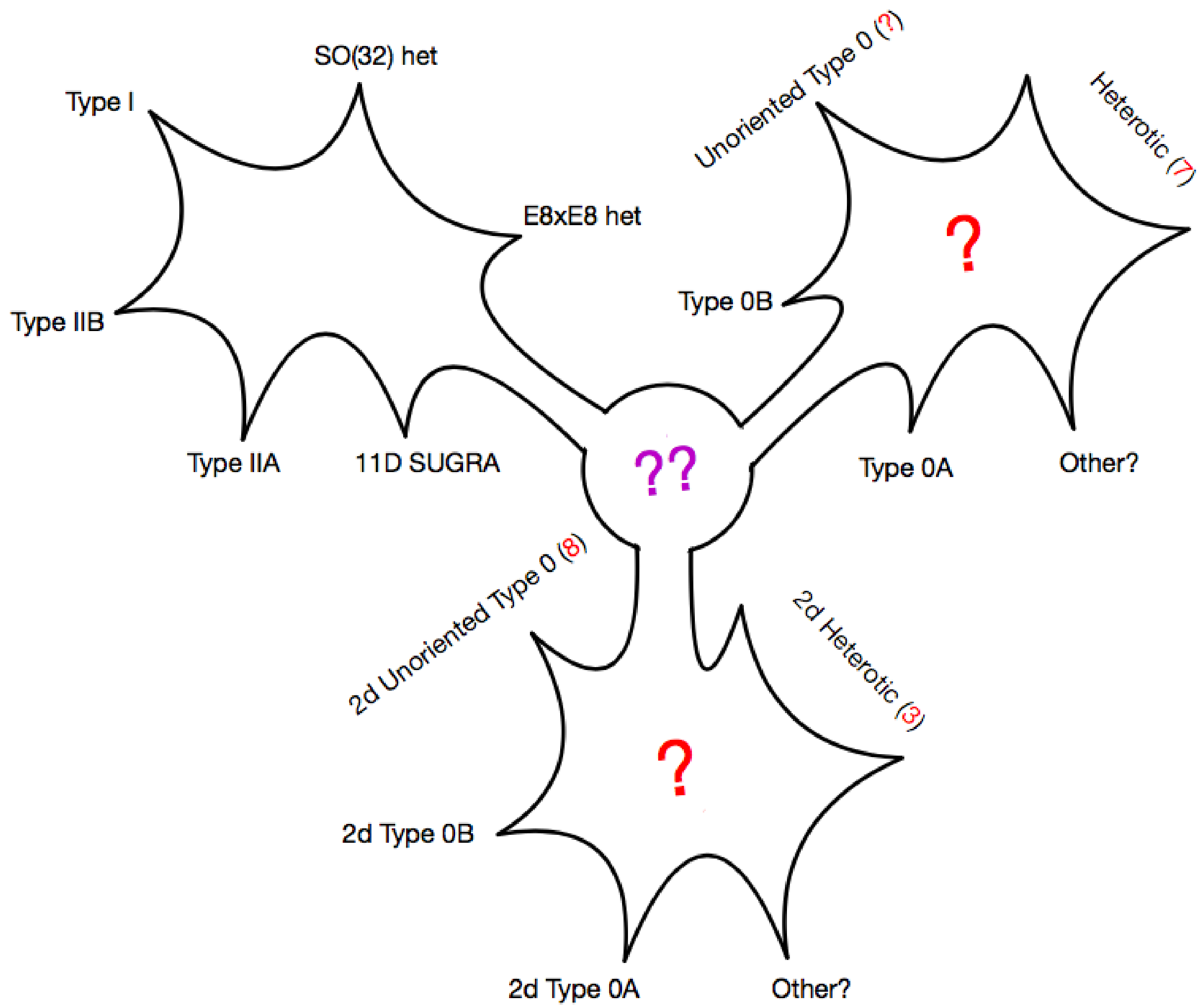






Guiding Principle

- **String theory is a unique theory of Quantum Gravity**



Tachyons

- **non-SUSY string theories generically have closed string tachyons**
  - Exceptions: Pin$^+$ Type 0, $O(16) \times O(16)$ heterotic, Type $\tilde{I}$
- **But the presence of a tachyon doesn't mean the theory is inconsistent. Rather, it means we're expanding about the wrong vacuum.**
- **As long as there exists one stable vacuum, the theories are consistent**
- **The stable vacua may be lower-dimensional**
  - In many cases, they will be known two-dimensional string theories
- **Goal: Classify the non-SUSY 10d strings and find stable vacua for them**

  Part 1: Classify unoriented Type 0 strings

  Part 2: Find stable vacua for non-SUSY heterotic strings

Part I

Part I: Classifying unoriented Type 0 strings

GSO Projection

- **Work in NS-R formalism in lightcone gauge;** $(X^i_{L,R}, \psi^i_{L,R})$ **with** $i = 1, \dots, 8$
- **The spectrum of the physical string is obtained by doing a GSO projection** [Gliozzi, Scherk, Olive '77]
  - Allowed GSO projections must project onto subsectors satisfying e.g. closure of OPE, mutual locality, and modular invariance of torus amplitudes.
- **GSO projection can also be thought of as a sum over spin structures** [Seiberg, Witten '86]
  - Different GSO projections correspond to possible phases assigned to spin structures in a way compatible with cutting/gluing of worldsheet.
  - For example for Type IIA/B, can have

$$Z[T^2;\alpha] = \left(\sum_{gh=hg} \alpha_L(g,h) \, \square \right) \times \overline{\left(\sum_{gh=hg} \alpha_R(g,h) \, \square \right)}$$

with $\alpha(\mathrm{NS},\mathrm{NS}) = \alpha(\mathrm{NS},\mathrm{R}) = \alpha(\mathrm{R},\mathrm{NS}) = 1$ and $\alpha(\mathrm{R},\mathrm{R}) = \pm 1$.

SPT Phases

- For our purposes, an SPT phase is a gapped theory with unique ground state on a manifold without boundary,

$$Z[X] \in U(1) \qquad \partial X = \emptyset$$

- The partition function will only depend on the bordism class of X,
  [Kapustin '14; Kapustin, Thorngren, Turzillo, Wang '15]

$$Z : \Omega_d^s(BG) \to U(1)$$

$s =$ structure of tangent bundle, $G =$ structure of principal bundle.

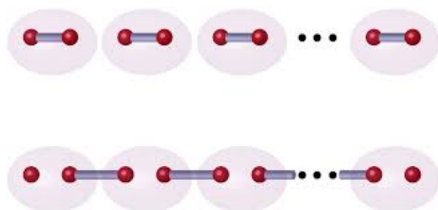
- Hence d-dimensional SPT phases are classified by

$$\mho_s^d(BG) := \mathrm{Hom}\left(\Omega_d^s(BG), U(1)\right)$$

- In the presence of boundary, we can have gapless edge modes.

Kitaev Chain and Arf Invariant

- **We'll mainly be concerned with fermionic SPTs. The simplest case is the Kitaev chain/Majorana wire** [Kitaev '01]

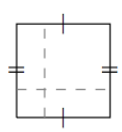
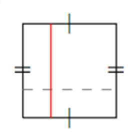


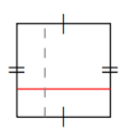
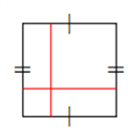
$$\mho^2_{\text{Spin}}(pt) = \mathbb{Z}_2$$

- **Has been realized experimentally** [Mourik et al '12]
- **The partition function of this phase on a manifold with spin structure σ is**

$$Z[\Sigma,\, \sigma] = e^{\pi i \text{Arf}(\Sigma,\, \sigma)}$$

- **One can calculate**

$\exp\left\{ i\pi\, \text{Arf} \left(\right.\right.$  $\left.\left. \right) \right\} = 1$ $\qquad$ $\exp\left\{ i\pi\, \text{Arf} \left(\right.\right.$  $\left.\left. \right) \right\} = 1$

$\exp\left\{ i\pi\, \text{Arf} \left(\right.\right.$  $\left.\left. \right) \right\} = 1$ $\qquad$ $\exp\left\{ i\pi\, \text{Arf} \left(\right.\right.$  $\left.\left. \right) \right\} = -1$

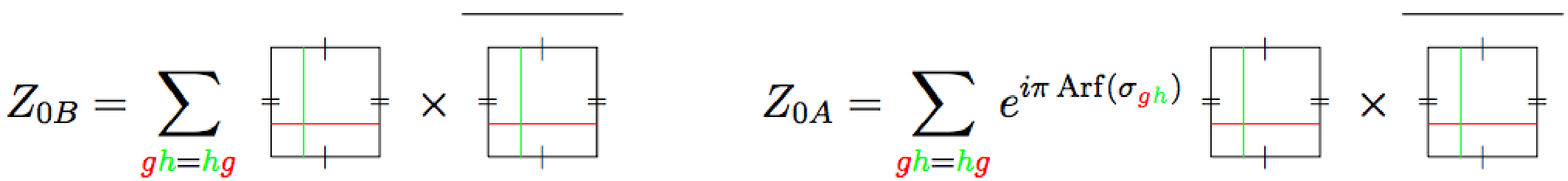
Kitaev Chain

Type 0 Strings

- **Indeed, consider possible SPT phases we can add to the worldsheet,**

$$\mho^2_{\mathrm{Spin}}(pt) = \mathbb{Z}_2 = \left\langle (-1)^{\mathrm{Arf}(\Sigma,\sigma)} \right\rangle$$

- **Two different worldsheet theories with following torus partition functions**



- **So two different points of view**
  (1) No SPT phase, but use different projectors, i.e. $P_{0\mathrm{B}}$ vs. $P_{0\mathrm{A}}$.
  (2) Sum over spin structure with non-trivial SPT phase.

Unoriented Type 0 Strings

- **Let's now allow worldsheets to be unoriented.**
  - Two options: $\mathrm{Pin}^- : \; w_2 + w_1^2 = 0 \qquad \mathrm{Pin}^+ : \; w_2 = 0$
- **Possible SPT phases:**

$$\mho^2_{\mathrm{Pin}^-}(pt) \;=\; \mathbb{Z}_8 \;=\; \left\langle e^{\pi i \mathrm{ABK}(\sigma)/4} \right\rangle$$
$$\mho^2_{\mathrm{Pin}^+}(pt) \;=\; \mathbb{Z}_2 \;=\; \left\langle (-)^{\mathrm{Arf}(\hat{\sigma})} \right\rangle$$

with $\hat{\sigma}$ the spin structure on the oriented double cover.

- **Prediction: an $8 + 2(?) = 10$-fold classification of unoriented Type 0** strings, matching with the Altland-Zirnbauer classification of topological superconductors.
- **Some scattered results in the literature** [Bergman, Gaberdiel '99; Blumenhagen, Font, Lüst '99], **but nothing complete.**

D-branes and K-theory

- **This leads to a rich spectrum of stable D-branes:**

	-1	0	1	2	3	4	5	6	7	8	9
$\widetilde{K}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\widetilde{K}^1$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
DKO	$2\mathbb{Z}_2$	$2\mathbb{Z}_2$	$2\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$2\mathbb{Z}_2$	$2\mathbb{Z}_2$	$2\mathbb{Z}$
DKO^1	$\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$
DKO^2	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
DKO^3	0	$\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\mathbb{Z}$	0
DKO^4	0	0	$2\mathbb{Z}$	0	$2\mathbb{Z}_2$	$2\mathbb{Z}_2$	$2\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
DKO^5	0	$\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\mathbb{Z}$	0
DKO^6	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
DKO^7	$\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}_2$

- **Additional information:**
  - None of the unoriented Type 0 strings has spacetime SUSY
  - All RR tadpoles can be cancelled. NSNS tadpoles can be cancelled by adding branes, or via Fischler-Susskind mechanism
  - The Pin^- strings have closed string tachyons, but Pin^+ strings are tachyon-free

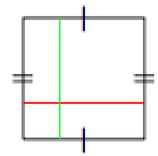
Part II

Part II: Stable vacua for heterotic strings

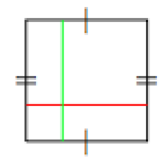
Tachyonic Heterotic Strings

- **Tachyonic heterotic strings are constructed as follows:**
- **Start with** $(X^i_{L,R}, \psi^i_L)$ **with** $i = 1, \dots, 8$ **and** λ^a_R **with** $a = 1, \dots, 32$
- **Simplest partition function is**

$$Z = \frac{1}{2|\eta|^{16}} \sum_{gh=hg} \left(\vphantom{\frac{1}{1}} \right.$$



$$\left. \vphantom{\frac{1}{1}} \right)^8 \times \left(\vphantom{\frac{1}{1}} \right.$$



$$\left. \vphantom{\frac{1}{1}} \right)^{32}$$

$$= 32(q\bar{q})^{-\frac{1}{2}} + 4032 + \dots$$

- **This theory has** 32 **tachyons,** 4032 **massless bosons**
  - graviton (35), B-field (28), dilaton (1), and 496 gauge bosons of $SO(32)$

Tachyonic Heterotic Strings

- **The worldsheet theory studied before has a $(\mathbb{Z}_2)^5$ symmetry,**

$$
\begin{aligned}
g_1 &= \sigma_3 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \, , & g_2 &= \mathbb{1}_2 \otimes \sigma_3 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \, , \\
g_3 &= \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \sigma_3 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \, , & g_4 &= \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \sigma_3 \otimes \mathbb{1}_2 \, , \\
g_5 &= \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \sigma_3 \, .
\end{aligned}
$$

- **Each $\mathbb{Z}_2$ acts as -1 on 16 of the λ^a and as $+1$ on others**

- **We can now gauge $(\mathbb{Z}_2)^n$ for $0 \leq n \leq 5$.**
  - This breaks $SO(32) \not\to SO(2^{5-n}) \times SO(32-2^{5-n})$

n	tachyons	massless fermions	gauge bosons	gauge group
0	32	0	496	$SO(32)$
1	16	256	368	$O(16) \times E_8$
2	8	384	304	$O(8) \times O(24)$
3	4	448	272	$(E_7 \times SU(2))^2$
4	2	480	256	$U(16)$
5	1	496	248	E_8

Tachyon Condensation

- **All of the heterotic strings above have tachyons. We now try to condense them** [Hellerman, Swanson '06; '07]

- **Say $\tilde{\lambda}^a$ are subset of λ^a invariant under $SO(2^{5-n})$. Condensation produces superpotential**

$$W = \sum_{a=1}^{2^{5-n}} \tilde{\lambda}^a \mathcal{T}^a(X)$$

- **Linearized equation of motion for $\mathcal{T}^a(X)$,**

$$\partial^\mu \partial_\mu \mathcal{T}^a - 2\partial^\mu \phi \partial_\mu \mathcal{T}^a + \frac{2}{\alpha'} \mathcal{T}^a = 0$$

- **One solution**

$$\phi = -\frac{2^{\frac{3-n}{2}}}{\sqrt{\alpha'}} X^- \ , \qquad \mathcal{T}^a = m\sqrt{\frac{2}{\alpha'}} e^{\beta X^+} X^{a+1}$$

Condensation to $d > 2$

- **Previous solution gives the following scalar potential**

$$V = Ae^{2\beta X^+}\sum_{a=1}^{2^{5-n}}(X^{a+1})^2 - Be^{\beta X^+}\sum_{a=1}^{2^{5-n}}\tilde{\lambda}^a\psi^{a+1} + \dots$$

- **As $X^+ \to \infty$, fluctuations along $X^1, \dots, X^{2^{5-n}}$ are suppressed, and we get a theory in $d = 10 - 2^{5-n}$ localized at $X^1 = \dots = X^{2^{5-n}} = 0$.**

n	d	massless fermions	gauge bosons	gauge group
3	6	112	266	$E_7 \times E_7$
4	8	240	255	$SU(16)$
5	9	248	248	E_8

- **Low-energy gravity+gauge theories can be checked to be anomaly-free!**

Condensation to $d = 2$

- **For $n < 2$, then $d = 10 - 2^{5-n} < 0$ so this doesn't work.**
- **In these cases we simply condense to $d = 2$, where dilaton background lifts remaining tachyons:**

n	d	massless bosons	massless fermions	gauge group
0	2	24	0	$O(24)$
1	2	8	8	$O(8) \times E_8$
2	2	0	12	$O(24)$

- **The three theories obtained in this way are precisely the three 2d heterotic strings known in the literature!** [Davis, Larsen, Seiberg '05]
- **We have thus connected the known 2d theories with non-SUSY 10d theories via dynamical transitions.**

Conclusion

I) The worldsheets of different string theories can differ by subtle topological terms. These terms explain the different GSO projections, D-brane spectra, and orientifoldings allowed in the theories.

2) Tachyonic strings admit lower-dimensional stable vacua. Many of these are known 2d strings.

- Possible future extensions:
  1) SPT phases for heterotic worldsheets, e.g. $|\mho^2_{\text{Spin}}(B\mathbb{Z}_2^5)| = 65,536$!
  2) Worldsheet domain walls?
  3) Orbifolds: beyond discrete torsion
- There is still much to explore in perturbative string theory!

The End (for now)

Thank you!