

Recent developments in the S-matrix bootstrap program

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Plan

1. A numerical S-matrix procedure. Possible directions.
2. Direction 1: general mass and spin in 4d
[Spinning S-matrix bootstrap in 4d]
3. Direction 2: S-matrix bootstrap + form factor bootstrap
4. Conclusions

Part A: a numerical procedure

A numerical procedure

One-particle asymptotic state: $|m, \vec{p}\rangle_{in}, |m, \vec{p}\rangle_{out}$

Scattering process: $p_1 p_2 \rightarrow p_3 p_4, p_i^2 = -m^2$

Scattering amplitude: $\mathcal{S}(s, t, u) \times (2\pi)^d \delta^{(d)}(p_1 + p_2 - p_3 - p_4) \equiv$
 $\langle m, \vec{p}_3; m, \vec{p}_4 | S | m, \vec{p}_1; m, \vec{p}_2 \rangle.$

$$S = \mathbb{I} + iT \Rightarrow \mathcal{T}(s, t, u)$$

Mandelstam variables:

$$\begin{aligned} s &\equiv -(p_1 + p_2)^2 & t &= -\frac{s - 4m^2}{2}(1 - \cos \theta), \\ t &\equiv -(p_1 - p_3)^2 & s + t + u &= 4m^2 \\ u &\equiv -(p_1 - p_4)^2 & u &= -\frac{s - 4m^2}{2}(1 + \cos \theta) \end{aligned}$$

Partial amplitude: $\mathcal{S}_j(s) = 1 + i \kappa_j s^{-1/2} (s - 4m^2)^{(d-3)/2}$

$$\times \int_{-1}^{+1} dx (1 - x^2)^{\frac{d-4}{2}} C_j^{(d-3)/2}(x) \mathcal{T}(s, t(s, x), u(s, x))$$

$x \equiv \cos \theta$

A numerical procedure

[Miguel Paulos, Joao Penedones, Jonathan Toledo, Balt van Rees, Pedro Vieira; 2016, 2017]

Ansatz: $\mathcal{T}(s, t, u) = \text{poles} + \sum_{i=1}^{\infty} \alpha_i K_i(s, t, u)$

Analiticity: $K_i(s, t, y) = \rho_s^{m_i} \rho_t^{n_i} \rho_u^{p_i}$

$$\rho_z \equiv \frac{\sqrt{4m^2 - z_0} - \sqrt{4m^2 - z}}{\sqrt{4m^2 - z_0} + \sqrt{4m^2 - z}}$$

Crossing equations: $\mathcal{T}(s, t, u) = \mathcal{T}(t, s, u) = \mathcal{T}(u, t, s)$

Unitarity: $|\mathcal{S}_j(s)|^2 \leq 1, \quad j = 0, 2, 4, \dots, \quad s \geq 4m^2$

$$\begin{pmatrix} 1 & \mathcal{S}_j^*(s) \\ \mathcal{S}_j(s) & 1 \end{pmatrix} \succeq 0$$

Directions

1. Various applications in 2d: non-perturbative bounds, integrable models
[Lucia Cordova, Yifei He, Martin Kruczenski, Pedro Vieira; 2019], ...
2. Scattering in 4d with O(3) symmetry. Study of massless and massive pions.
[Andrea Guerrieri, Joao Penedones, Pedro Vieira; 2018, 2020]
3. Improvements of the numerical method. Exploration of alternative methods.
[Andrea Guerrieri, Alexandre Homrich, Pedro Vieira; 2020],
[Martin Kruczenski, Harish Murali; 2020], ...
4. Scattering of particles with generic masses in spins
[Aditya Hebbar, DK, Joao Penedones; 2020]
5. Form factor bootstrap
[DK, Simon Kuhn, Joao Penedones; 2019], [DK; 2020]

Part B:

Spinning S-matrix bootstrap in 4d

[Aditya Hebbar, DK, Joao Penedones; 2020]

Crossing equations

Crossing equations:

$$\mathcal{T}_{12 \rightarrow 34}^{\lambda_3, \lambda_4}_{\lambda_1, \lambda_2}(p_1, p_2, p_3, p_4) = \xi_1 \mathcal{T}_{\bar{4}2 \rightarrow 3\bar{1}}^{+\lambda_3, -\lambda_1}_{-\lambda_4, +\lambda_2}(-p_4, p_2, p_3, -p_1)$$

$$\mathcal{T}_{12 \rightarrow 34}^{\lambda_3, \lambda_4}_{\lambda_1, \lambda_2}(p_1, p_2, p_3, p_4) = \xi_2 \mathcal{T}_{1\bar{3} \rightarrow \bar{2}4}^{-\lambda_2, +\lambda_4}_{+\lambda_1, -\lambda_3}(p_1, -p_3, -p_2, p_4)$$

$$\mathcal{T}_{12 \rightarrow 34}^{\lambda_3, \lambda_4}_{\lambda_1, \lambda_2}(p_1, p_2, p_3, p_4) = \xi_3 \mathcal{T}_{\bar{3}2 \rightarrow \bar{1}4}^{-\lambda_1, +\lambda_4}_{-\lambda_3, +\lambda_2}(-p_3, p_2, -p_1, p_4)$$

$$\mathcal{T}_{12 \rightarrow 34}^{\lambda_3, \lambda_4}_{\lambda_1, \lambda_2}(p_1, p_2, p_3, p_4) = \xi_4 \mathcal{T}_{1\bar{4} \rightarrow 3\bar{2}}^{+\lambda_3, -\lambda_2}_{+\lambda_1, -\lambda_4}(p_1, -p_4, p_3, -p_2)$$

Analytic continuation: $p^\mu \rightarrow$ complex values $\rightarrow -p^\mu$

Phase: $\xi_i = \pm 1$.

Derivation: (A) [\[Trueman, Wick; 1964\]](#) and (B) LSZ formula

Tensor structures

Decomposition in tensor structures:

$$\mathcal{T}_{12 \rightarrow 34}^{\lambda_3, \lambda_4}_{\lambda_1, \lambda_2}(p_1, p_2, p_3, p_4) = \sum_{I=1}^{N_4} \mathcal{T}_{12 \rightarrow 34}^I(s, t, u) \mathbb{T}_{12 \rightarrow 34}^{\lambda_3, \lambda_4}_{\lambda_1, \lambda_2}(p_1, p_2, p_3, p_4)$$

Building blocks for tensor structures: $SO(1, 3) \leftrightarrow SO(3)$

$$p^\mu = \{p^0, \vec{p}\}, \quad \vec{p} = \{\mathbf{p} \cos \phi \sin \theta, \mathbf{p} \sin \phi \sin \theta, \mathbf{p} \cos \theta\}, \quad \mathbf{p} \equiv |\vec{p}|.$$

Example (polarization, massless spin 1):

$$\epsilon_\alpha^\lambda(p) = \frac{e^{i\lambda\phi}}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos \theta \cos \phi - i\lambda \sin \phi \\ \cos \theta \sin \phi + i\lambda \cos \phi \\ -\sin \theta \end{pmatrix}$$

COM amplitudes

Center of mass amplitudes:

$$\mathcal{T}_{12 \rightarrow 34}^{\lambda_3, \lambda_4}_{\lambda_1, \lambda_2}(s, t, u) \equiv \mathcal{T}_{12 \rightarrow 34}^{\lambda_3, \lambda_4}_{\lambda_1, \lambda_2}(p_1^{\text{com}}, p_2^{\text{com}}, p_3^{\text{com}}, p_4^{\text{com}})$$

Center of mass:

$$\begin{aligned} p_1^{\text{com}} &= (E_1, 0, 0, +\mathbf{p}), \\ p_2^{\text{com}} &= (E_2, 0, 0, -\mathbf{p}), \\ p_3^{\text{com}} &= (E_3, +\mathbf{p}' \sin \theta, 0, +\mathbf{p}' \cos \theta), \\ p_4^{\text{com}} &= (E_4, -\mathbf{p}' \sin \theta, 0, -\mathbf{p}' \cos \theta). \end{aligned}$$

Equal mass case:

$$E_i = \frac{\sqrt{s}}{2}, \quad \mathbf{p} = \mathbf{p}' = \sqrt{\frac{s}{4} - m^2}, \quad \sin \theta = \frac{2\sqrt{tu}}{s - 4m^2}, \quad \cos \theta = \frac{t - u}{s - 4m^2}.$$

COM crossing

Wigner d-matrix:

$$d_{\lambda' \lambda}^{(j)}(\beta) = \sqrt{(+\lambda)!(-\lambda)!(+\lambda')!(\ell - \lambda')!} \\ \times \sum_{\nu=0}^{2j} (-1)^\nu \frac{(\cos(\beta/2))^{2j+\lambda-\lambda'-2\nu} (-\sin(\beta/2))^{\lambda'-\lambda+2\nu}}{\nu!(\ell - \lambda' - \nu)!(+\lambda - \nu)!(\nu + \lambda' - \lambda)!}$$

COM crossing:

$$T_{12 \rightarrow 34} \underset{\lambda_1, \lambda_2}{\overset{\lambda_3, \lambda_4}{\lambda'_1, \lambda'_2}}(s, t, u) = \eta \sum_{\lambda'_i} e^{i\pi(\lambda'_1 + \lambda'_4)} \quad \text{[Trueman, Wick; 1964]} \\ \times d_{\lambda'_1 \lambda_1}^{(j_1)}(\alpha_1) d_{\lambda'_2 \lambda_2}^{(j_2)}(\alpha_2) d_{\lambda'_3 \lambda_3}^{(j_3)}(\alpha_3) d_{\lambda'_4 \lambda_4}^{(j_4)}(\alpha_4) T_{1\bar{3} \rightarrow \bar{2}4} \underset{\lambda'_1, \lambda'_3}{\overset{\lambda'_2, \lambda'_4}{\lambda'_1, \lambda'_3}}(t, s, u),$$

$$+ \cos \alpha_1 = - \cos \alpha_2 = - \cos \alpha_3 = + \cos \alpha_4 = + \frac{st}{\sqrt{s(s-4m^2)} \sqrt{t(t-4m^2)}}, \\ + \sin \alpha_1 = - \sin \alpha_2 = + \sin \alpha_3 = - \sin \alpha_4 = - \frac{2m \sqrt{stu}}{\sqrt{s(s-4m^2)} \sqrt{t(t-4m^2)}}.$$

Partial amplitudes in 4d

$$|p^\mu,\ell,\lambda\rangle^{\lambda_1\lambda_2}\equiv\Pi_\ell\left(|m_1,\vec{p}_1,j_1,\lambda_1\rangle\otimes|m_2,\vec{p}_2,j_2,\lambda_2\rangle\right)$$

$$p^\mu \equiv p_1^\mu + p_2^\mu, \quad s \equiv -p^2$$

$$\mathcal{S}_{\ell\lambda_1,\lambda_2}^{\lambda_3,\lambda_4}(s)\times\delta_{\ell'\ell}\delta_{\lambda'\lambda}=\mathop{\langle}^{_{out}}_{_{in}}\!\!p',\ell',\lambda'|p,\ell,\lambda\rangle^{\lambda_1\lambda_2}$$

$$\mathcal{S}_{\ell\lambda_1,\lambda_2}^{\lambda_3,\lambda_4}(s)=\delta_{m_1m_3}\delta_{m_2m_4}\delta_{j_1j_3}\delta_{j_2j_4}\delta_{\lambda_1\lambda_3}\delta_{\lambda_2\lambda_4}+i\mathcal{T}_{\ell\lambda_1,\lambda_2}^{\lambda_3,\lambda_4}(s)$$

$$\mathcal{T}_{\ell\lambda_1,\lambda_2}^{\lambda_3,\lambda_4}(s)=\frac{\sqrt{\mathbf{p}\mathbf{p}'}}{8\pi\sqrt{s}}\times\int_0^\pi d\theta\sin\theta d^{(\ell)}_{\lambda_{12}\lambda_{34}}(\theta)\mathcal{T}_{\lambda_1,\lambda_2}^{\lambda_3,\lambda_4}\left(s,t(s,\theta),u(s,\theta)\right)$$

$$\lambda_{12}\equiv\lambda_1-\lambda_2,\quad \lambda_{34}\equiv\lambda_3-\lambda_4$$

Unitarity

States: $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, \dots$

$$\forall i : \quad \left| |\psi_i\rangle \right| \geq 0 \quad \text{also} \quad \begin{pmatrix} \langle \psi_1 | \psi_1 \rangle & \langle \psi_1 | \psi_2 \rangle & \langle \psi_1 | \psi_3 \rangle \\ \langle \psi_2 | \psi_1 \rangle & \langle \psi_2 | \psi_2 \rangle & \langle \psi_2 | \psi_3 \rangle \\ \langle \psi_3 | \psi_1 \rangle & \langle \psi_3 | \psi_2 \rangle & \langle \psi_3 | \psi_3 \rangle \end{pmatrix} \succeq 0$$

$$|\psi_1\rangle_{\text{in}} \equiv |p, \ell, \lambda\rangle_{\text{in}}^{j_1, j_2},$$

$$|\psi_2\rangle_{\text{in}} \equiv |p, \ell, \lambda\rangle_{\text{in}}^{j_1, j_2 - 1},$$

$$\vdots$$

$$|\psi_{N\text{in}}\rangle_{\text{in}} \equiv |p, \ell, \lambda\rangle_{\text{in}}^{-j_1, -j_2},$$

$$|\psi_1\rangle_{\text{out}} \equiv |p, \ell, \lambda;\rangle_{\text{out}}^{j_3, j_4},$$

$$|\psi_2\rangle_{\text{out}} \equiv |p, \ell, \lambda\rangle_{\text{out}}^{j_3, j_4 - 1},$$

$$\vdots$$

$$|\psi_{N\text{out}}\rangle_{\text{out}} \equiv |p, \ell, \lambda\rangle_{\text{out}}^{-j_3, -j_4}$$

Example: Majorana scattering (1)

Identical particles:

$$\begin{aligned} T_{--}^{--} &= T_{++}^{++}, & T_{--}^{+-} &= T_{--}^{-+}, & T_{-+}^{--} &= T_{++}^{+-}, & T_{-+}^{-+} &= T_{+-}^{+-}, & T_{-+}^{+-} &= T_{+-}^{-+}, \\ T_{++}^{++} &= T_{--}^{-+}, & T_{+-}^{--} &= T_{++}^{+-}, & T_{+-}^{++} &= T_{--}^{-+}, & T_{++}^{-+} &= T_{++}^{+-} \end{aligned}$$

Parity invariance:

$$T_{--}^{-+} = -T_{++}^{+-}, \quad T_{--}^{++} = +T_{++}^{--}$$

Independent amplitudes

$$\begin{array}{lll} \Phi_1(s, t, u) \equiv T_{++}^{++}(s, t, u) & & H_1(s, t, u) \\ \Phi_2(s, t, u) \equiv T_{++}^{--}(s, t, u) & & H_2(s, t, u) \\ \Phi_3(s, t, u) \equiv T_{+-}^{+-}(s, t, u) & \Rightarrow & H_3(s, t, u) \\ \Phi_4(s, t, u) \equiv T_{+-}^{-+}(s, t, u) & & H_4(s, t, u) \\ \Phi_5(s, t, u) \equiv T_{++}^{+-}(s, t, u) & & H_5(s, t, u) \end{array}$$

Example: Majorana scattering (2)

$$H_I(s, t, u) = \sum_{J=1}^5 \tilde{C}_{st}^{IJ}(s, t, u) H_J(t, s, u),$$

$$H_I(s, t, u) = \sum_{J=1}^5 \tilde{C}_{su}^{IJ}(s, t, u) H_J(u, t, s),$$

$$\tilde{C}_{st} = \begin{pmatrix} -\frac{1}{4} & -1 & \frac{3}{2} & 1 & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & 1 & \frac{3}{2} & -1 & -\frac{1}{4} \end{pmatrix}, \quad \tilde{C}_{su} = \begin{pmatrix} -\frac{1}{4} & 1 & -\frac{3}{2} & 1 & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & -1 & -\frac{3}{2} & -1 & -\frac{1}{4} \end{pmatrix}$$

Example: Majorana scattering (3)

$$\Phi_1^\ell(s) \equiv T_{\ell++}^{++}(s)$$

$$\Phi_2^\ell(s) \equiv T_{\ell++}^{--}(s)$$

$$\Phi_3^\ell(s) \equiv T_{\ell+-}^{+-}(s)$$

$$\Phi_4^\ell(s) \equiv T_{\ell+-}^{-+}(s)$$

$$\Phi_5^\ell(s) \equiv T_{\ell++}^{+-}(s)$$

$$\ell \geq 0 \text{ (even)} : \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + i \begin{pmatrix} 0 & -\Phi_1^{\ell*}(s) - \Phi_2^{\ell*}(s) \\ \Phi_1^\ell(s) + \Phi_2^\ell(s) & 0 \end{pmatrix} \succeq 0$$

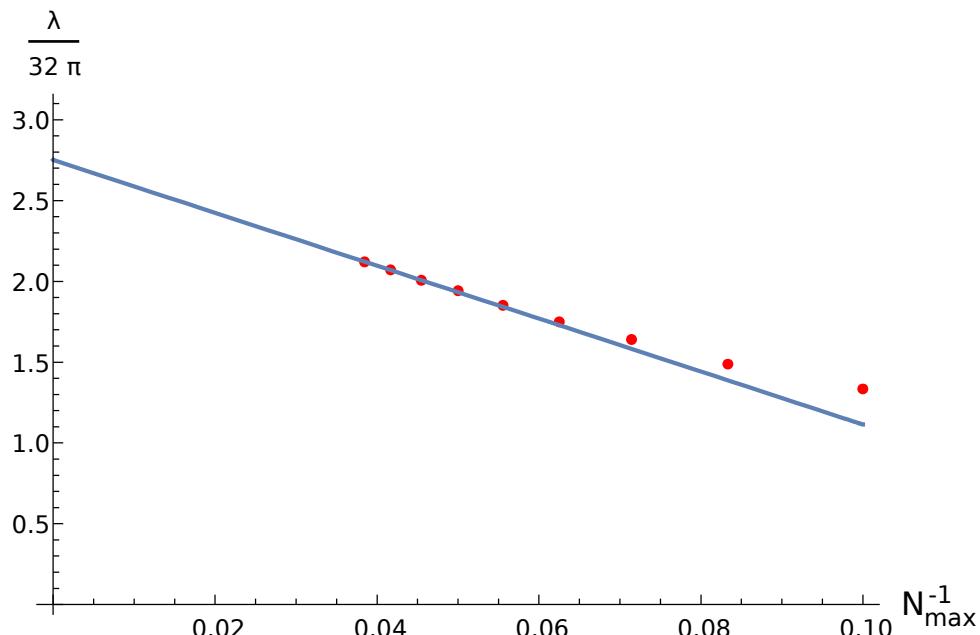
$$\ell = 0 : \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + i \begin{pmatrix} 0 & -\Phi_1^{\ell*}(s) + \Phi_2^{\ell*}(s) \\ \Phi_1^\ell(s) - \Phi_2^\ell(s) & 0 \end{pmatrix} \succeq 0$$

$$\ell \geq 1 \text{ (odd)} : \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 2i \begin{pmatrix} 0 & -\Phi_3^{\ell*}(s) \\ \Phi_3^\ell(s) & 0 \end{pmatrix} \succeq 0$$

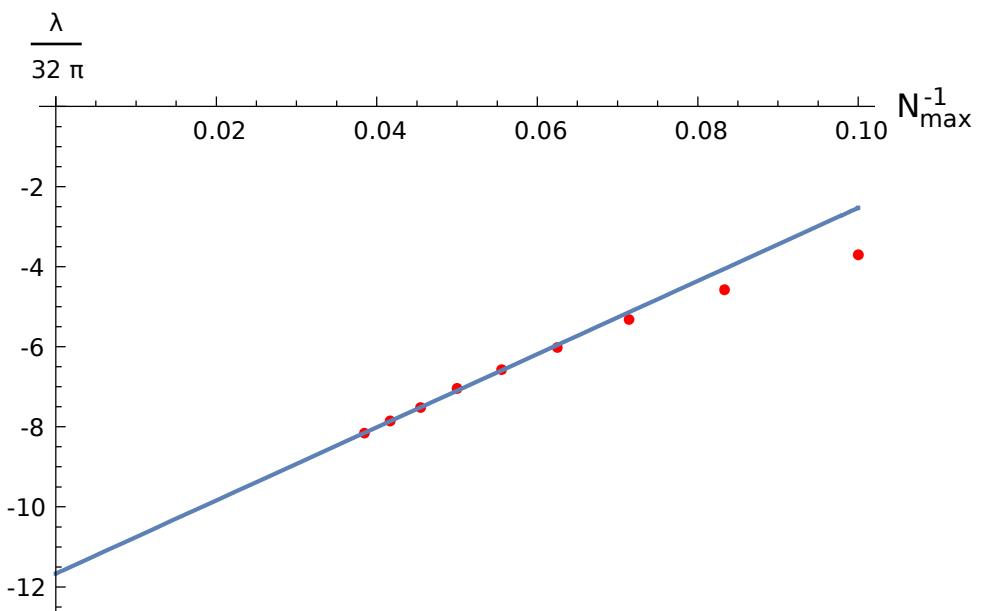
$$\ell \geq 2 \text{ (even)} : \quad \begin{pmatrix} \mathbb{I}_{2 \times 2} & \mathbb{S}_{2 \times 2}^{\ell\dagger}(s) \\ \mathbb{S}_{2 \times 2}^\ell(s) & \mathbb{I}_{2 \times 2} \end{pmatrix} \succeq 0,$$

$$\mathbb{I}_{2 \times 2} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbb{S}_{2 \times 2}^\ell(s) \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + i \begin{pmatrix} \Phi_1^\ell(s) - \Phi_2^\ell(s) & 2\Phi_5^{\ell*}(s) \\ 2\Phi_5^\ell(s) & 2\Phi_3^\ell(s) \end{pmatrix}$$

Numerical results: quartic coupling

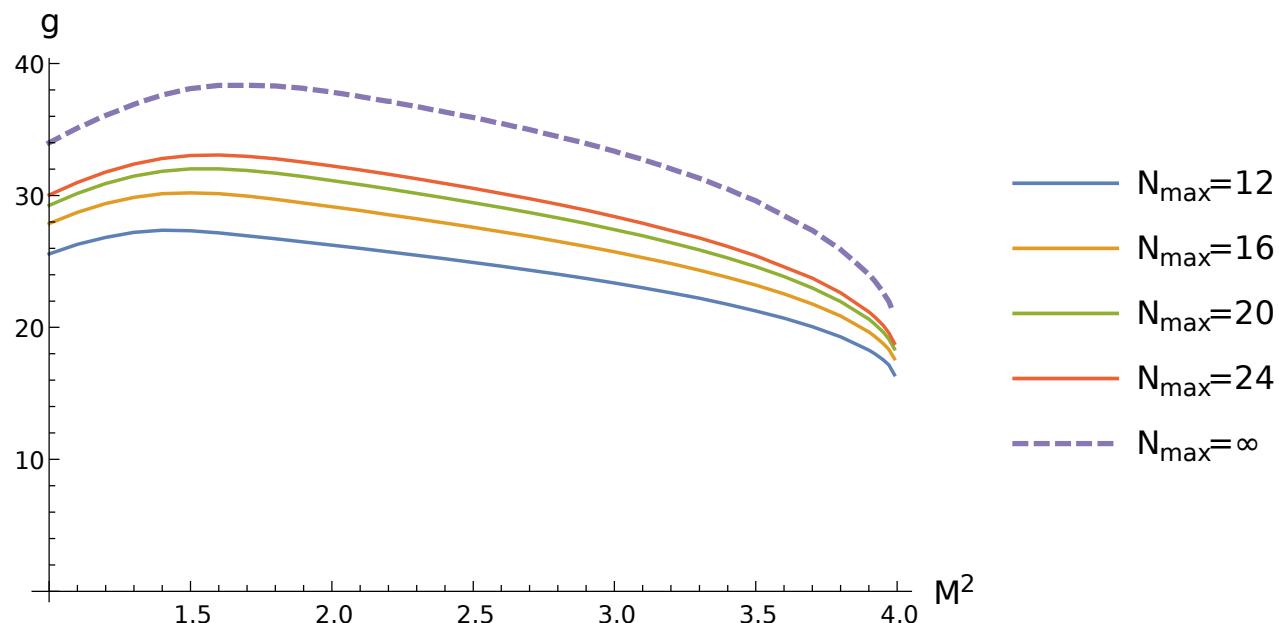


$$\vec{H}(4m^2/3, 4m^2/3, 4m^2/3) = \frac{\lambda}{m^2} \times \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$



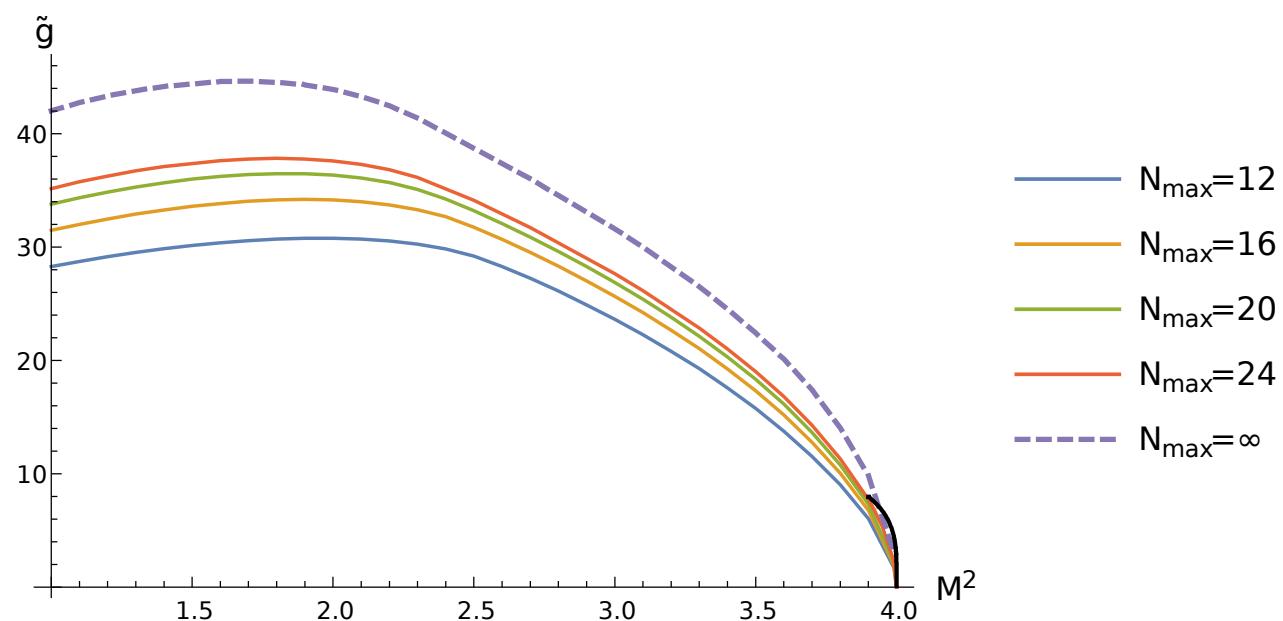
Numerical results: cubic coupling (1)

$$\vec{H}(s, t, u) = \frac{1}{2} g^2 \times \begin{pmatrix} -\frac{4}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2} \\ \frac{1}{t-M^2} - \frac{1}{u-M^2} \\ -\frac{1}{t-M^2} + \frac{1}{u-M^2} \\ -\frac{1}{t-M^2} - \frac{1}{u-M^2} \\ \frac{1}{t-M^2} + \frac{1}{u-M^2} \end{pmatrix} + \dots$$



Numerical results: cubic coupling (2)

$$\vec{H}(s, t, u) = \frac{1}{2} \tilde{g}^2 \times \begin{pmatrix} \frac{1}{t-M^2} + \frac{1}{u-M^2} \\ -\frac{1}{t-M^2} + \frac{1}{u-M^2} \\ -\frac{1}{t-M^2} + \frac{1}{u-M^2} \\ \frac{1}{t-M^2} + \frac{1}{u-M^2} \\ -\frac{4}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2} \end{pmatrix} + \dots$$



Part C: form factor bootstrap

[DK, Simon Kuhn, Joao Penedones; 2019], [DK; 2020]

Form factor bootstrap

Stress-tensor and its trace: $\Theta(x) = \eta_{\mu\nu} T^{\mu\nu}$

$$\mathcal{F}_T^{\mu\nu}(p_1, p_2) \equiv {}_{out}\langle m, \vec{p}_1; m, \vec{p}_2 | T^{\mu\nu}(0) | 0 \rangle$$

Two-particle form factor:

$$\mathcal{F}_\Theta(s) \equiv {}_{out}\langle m, \vec{p}_1; m, \vec{p}_2 | \Theta(0) | 0 \rangle$$

$$\mathcal{F}_\Theta(s), \quad \mathcal{F}_{(2)}(s), \quad s \equiv -(p_1 + p_2)^2$$

Spectral density: $2\pi\theta(p^0)\rho_T^{\mu\nu;\rho\sigma}(p) \equiv \int d^d x e^{-ip \cdot x} \langle 0 | T^{\mu\nu}(x) T^{\rho\sigma}(0) | 0 \rangle_W$

$$2\pi\theta(p^0)\rho_T^{\mu\nu;\rho\sigma}(p) = 2\text{Re} \int d^d x e^{-ip \cdot x} \langle 0 | T^{\mu\nu}(x) T^{\rho\sigma}(0) | 0 \rangle_T$$

$$\rho_\Theta(s), \quad \rho_{(2)}(s), \quad s \equiv -p^2$$

Form factor bootstrap

$$\begin{pmatrix} 1 & \mathcal{S}_0^*(s) & \omega\,\mathcal{F}_{\Theta}^*(s) \\ \mathcal{S}_0(s) & 1 & \omega\,\mathcal{F}_{\Theta}(s) \\ \omega\,\mathcal{F}_{\Theta}(s) & \omega\,\mathcal{F}_{\Theta}^*(s) & 2\pi\,\rho_{\Theta}(s) \end{pmatrix}\succeq 0$$

$$\omega^2 = \frac{1}{\mathcal{N}_d} \frac{\Omega_{d-1}}{2(2\pi)^{d-2}}$$

$$\mathcal{N}_d \equiv 2^{d-1} \sqrt{s} \, \left(s - 4m^2\right)^{(3-d)/2}$$

$$\begin{pmatrix} 1 & \mathcal{S}_2^*(s) & \varepsilon\,\mathcal{F}_{(2)}^*(s) \\ \mathcal{S}_2(s) & 1 & \varepsilon\,\mathcal{F}_{(2)}(s) \\ \varepsilon\,\mathcal{F}_{(2)}(s) & \varepsilon\,\mathcal{F}_{(2)}^*(s) & 2\pi\,s^2\,\rho_{\hat{T}}^2(s) \end{pmatrix}\succeq 0$$

$$\varepsilon \equiv \omega \, \sqrt{\frac{2}{d^2-1}}$$

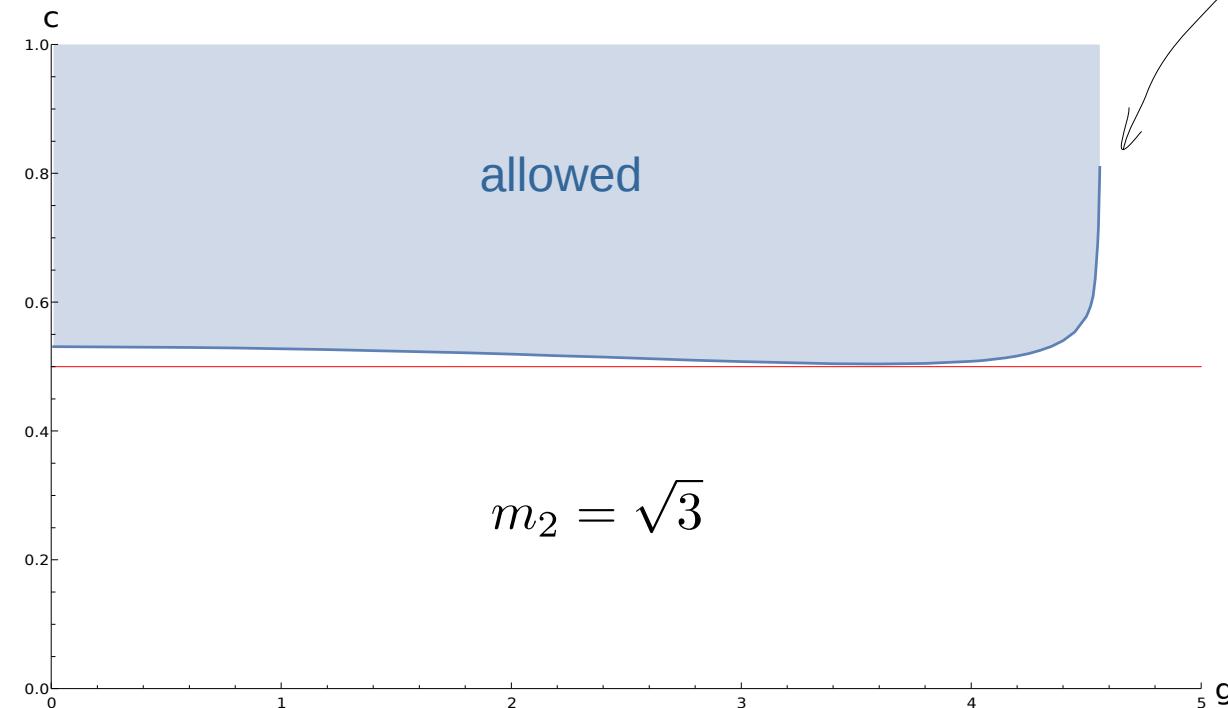
$$d=2:\qquad c_{UV}=12\pi\int_0^\infty\frac{ds}{s^2}\,\rho_\Theta(s)$$

$$d\geq 3:\qquad \lim_{s\rightarrow\infty}s^{2-d/2}\rho_{(2)}(s)=\text{const}\times C_T^{UV}$$

Numerical results

Assumption: two asymptotic states $m_1 = 1, m_2$

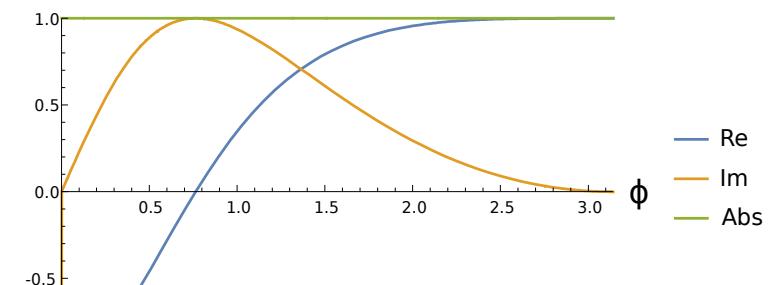
$$\mathcal{S}_{m_1 m_1 \rightarrow m_1 m_1}(s) = -\frac{g^2}{s - 4m_2^2} + \dots$$



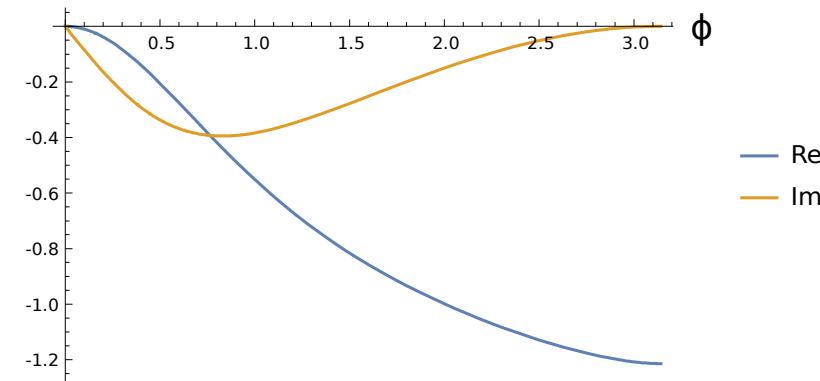
$$g = 4.55901$$

$$c = c_{b_2} + c_{b_1 b_1} + \dots = 0.80921 + \dots, \quad c_{b_2} = 0.72126, \quad c_{b_1 b_1} = 0.08795,$$

sine-Gordon



$$s = \frac{8}{1 + \cos \phi}$$



Conclusions

1. There is at least one concrete numerical method for studying QFTs non-perturbatively:
2. Starting from this method there are many interesting directions to explore:
 - a) applications of the method to concrete problems
 - b) extensions of the theoretical part
 - c) extensions/development of the numerical part
3. I discussed 1: general spin bootstrap setup in 4d & application to Majorana fermions.
 - a) bounds on photon scattering
 - b) pion – proton scattering
4. I discussed 2: form factor bootstrap