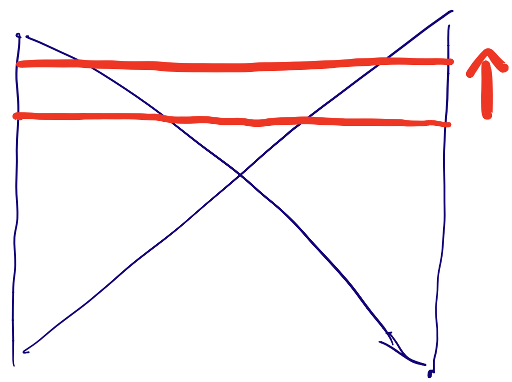
Spacetime as a quantum circuit

(based on arXiv: 2101.01185 with
Ramesh Chandra, Mario Flory, Michal Heller,
Sergio Hörtner, Andrew Rolph)

Complexity



State complexity

$$C = \min_{k} \left| \Psi_{fin} - U_k U_{k-1} \cdots U_1 \Psi_0 \right| < \varepsilon$$

$U_i \in$ gate set

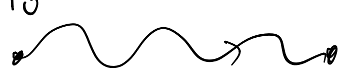
Operator complexity

$$C = \min_{K} \left| U_{fin} - U_K \cdots U_1 \right| < \varepsilon$$

(other things exist like K-complexity)

Continuum Limit
(Nielsen)

Ψ_0 Ψ_{fin}



$U(t) \in U(N)/U(1) \times U(N-1)$

$\mathbb{1}$ U_{fin}

$U(t) \in U(N)$

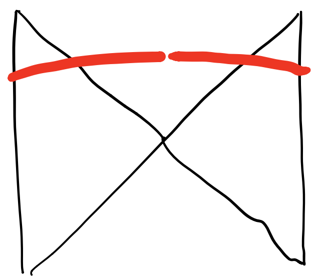
Choice of gate set $\Rightarrow$ choice of metric / cost function on space of unitaries

Complexity = shortest path

Metric could be strange, like $ds = |dx| + |dy|$?

AdS/CFT :

CV



CA

· don't know gate set / Nielsen geometry

don't know Ψ_0 (completely unentangled state)?

but CV, CA pass some consistency checks that any reasonable notion of complexity should obey

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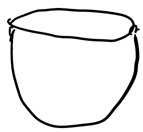
(C)FT — free fields, unitaries which rotate $a_\omega \circlearrowright a^\dagger_\omega$

— use conformal group, Virasoro ...

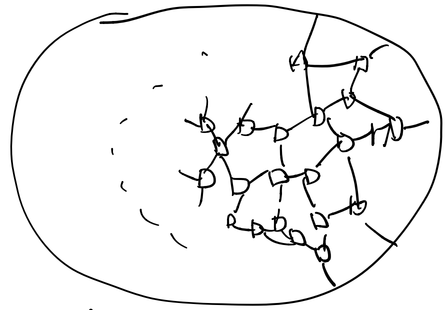
(eg Chagnet, Chapman, JdB, Zukowski, to appear)

Ground state preparation in 2d CFT



g_{ij}

discretize



tensor network = circuit of operations

What would be a reasonable cost ($\sim$ length of path) to associate to this method of preparation?

<u>proposal</u> ("path integral optimization") (Takayanagi, Caputa, Kundu, Miyaji, Watanabe; Czech)

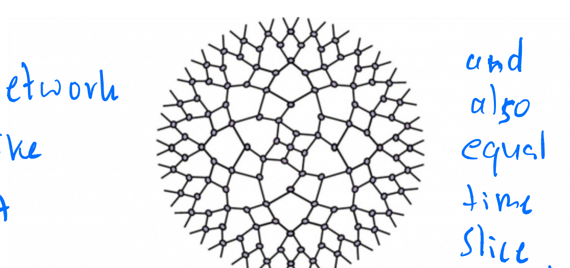
$$C(g_{ij}) \sim \log Z[g_{ij}]$$

$\hookrightarrow$ unormalized path integral

$$\sim S_L(g_{ij}) \quad \text{Liouville action}$$

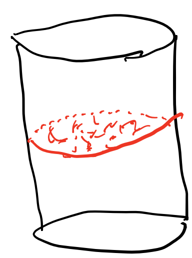
Optimized when $ds^2 \sim dr^2 + e^{2r} dx^2$ Euclidean AdS_2

tensor network looks like MERA



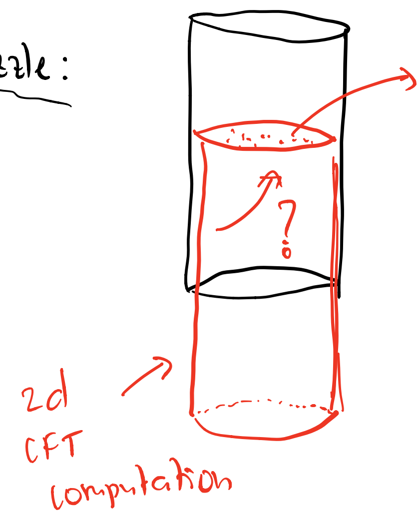
(Swingle)

and also equal time slice of AdS_3



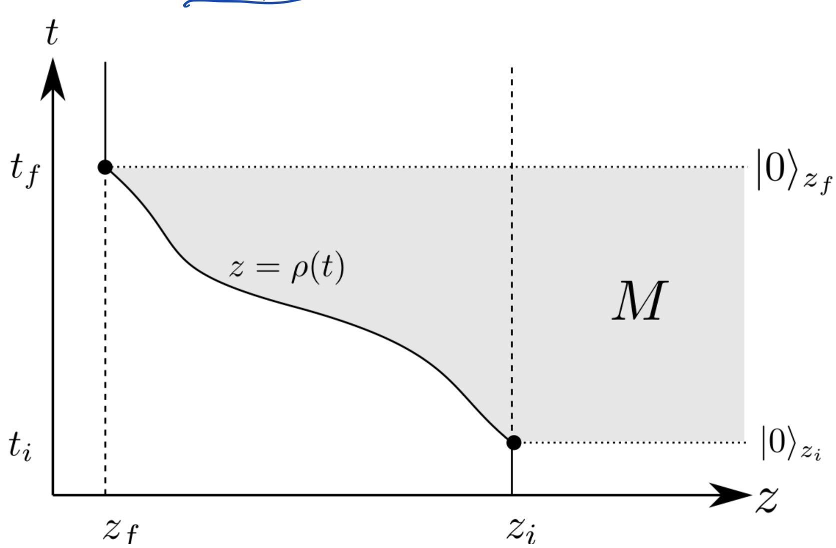
complexity $\sim$ # tensors $\sim$ volume $\sim$ CV ...

Puzzle:



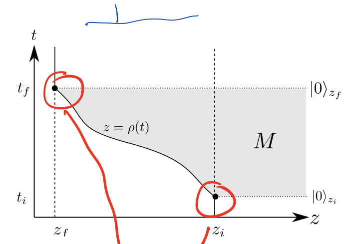
& the role of the UV cutoff is rather confusing....

proposal:



On-shell action of M $\equiv$ complexity of path $|0\rangle_{z_i} \longrightarrow |0\rangle_{z_f}$ in a CFT (WITH VARYING CUTOFF)

Computation in vacuum AdS_3:



$$ds^2 = \frac{dz^2 + dt^2 + dx^2}{z^2},$$

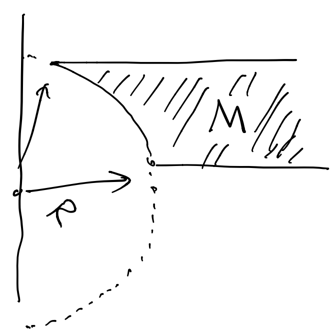
$$I = \frac{1}{\kappa}\int_M d^3x\,\sqrt{G}\,(R+2) + \frac{2}{\kappa}\int_{\partial M} d^2x\,\sqrt{g}K + I_c.$$

corner terms! (Hayward)

$$\Downarrow$$

$$I = \frac{2V_x}{\kappa}\int_{t_i}^{t_f} dt\,\left(\frac{1}{\rho^2} + \frac{\dot\rho\arctan\dot\rho}{\rho^2}\right) + \frac{\pi V_x}{\kappa}\left(\frac{1}{z_f} + \frac{1}{z_i}\right).$$

optimal solution:



$$\rho(t) = \sqrt{\mathcal{R}^2 - (t-t_0)^2}$$

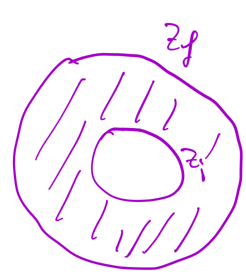
optimize over t_i:

$t_i \to t_f$

M collapses to disc

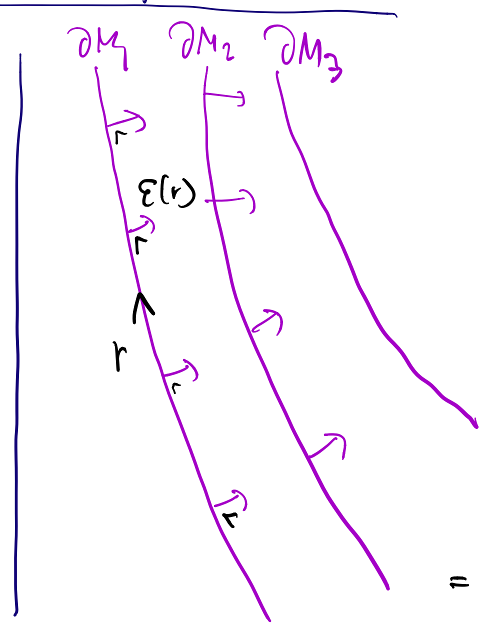
$$I_{min} = \frac{c\pi V_x}{24}\left(\frac{1}{z_f} - \frac{1}{z_i}\right),$$

$\sim$ volume (annulus)



* agrees with complexity = volume = MERA

General Optimalization:



$$\Pi_{\mu\nu} = \frac{1}{\sqrt{g}} \frac{\partial S}{\partial g^{\mu\nu}} \sim \text{"}T_{\mu\nu}\text{"}$$

$$= -(K_{\mu\nu} - K g_{\mu\nu})$$

$$ds^2 = dr^2 + g_{\mu\nu}(x,r)\,dx^\mu dx^\nu$$

$$\Pi_{\mu\nu} = -\frac{1}{2}\left(\partial_r g_{\mu\nu} - g_{\mu\nu} g^{\rho\sigma} \partial_r g_{\rho\sigma}\right)$$

$$\delta_\epsilon S = \int_{\partial M} \epsilon(x)\, \partial_r g^{\mu\nu} \frac{\partial S}{\partial g^{\mu\nu}}$$

$$= 2\int_{\partial M} \sqrt{g}\, \varepsilon(x) \underbrace{\left(\Pi^{\mu\nu}\Pi_{\mu\nu} - \left(\frac{1}{d-2}\right)\left(\Pi^\rho_\rho\right)^2\right)}_{T\bar{T}\text{ deformation}}$$

* For a foliation, get an interpretation as a sequence of $T\bar{T}$ deformations with non-constant parameter

* Nontrivial to express as $T\bar{T}$ deformation of boundary theory

* S = action w/o counterterms. Can also do w/ counterterms.

* We do not vary the background as this would change Ψ_{in} or Ψ_{fin}

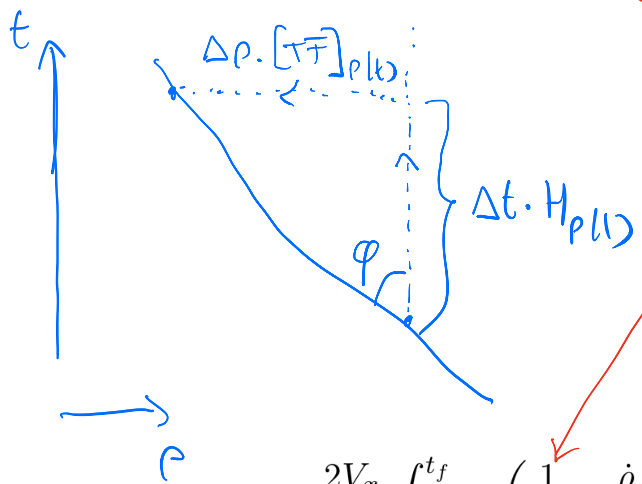
Solution $\delta_\epsilon S = 0 \Rightarrow \Pi^{\mu\nu}\Pi_{\mu\nu} - \frac{1}{(d-2)}(\Pi^\rho_\rho)^2 = 0$ Hamiltonian constraint

$$\Rightarrow R^{(d-1)} + (d-1)(d-2) = 0$$

$$\boxed{\partial M \text{ has constant scalar curvature}}$$

Gate counting interpretation of S_{bulk} ?

$$|0\rangle_{z_f} = P\exp\left[i\int_{t_i}^{t_f} dt\,(H_{\rho(t)} + \dot\rho\,[T\bar{T}]_{\rho(t)})\right]|0\rangle_{z_i}.$$



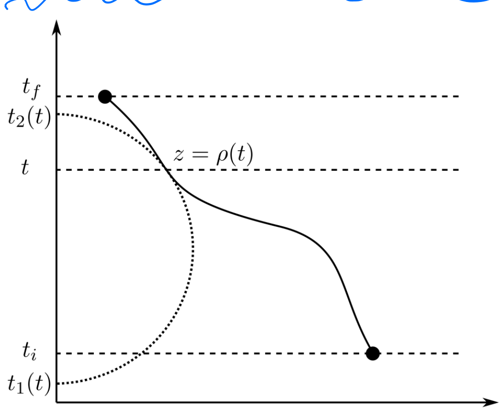
?

$$I = \frac{2V_x}{\kappa}\int_{t_i}^{t_f} dt\left(\frac{1}{\rho^2} + \frac{\dot\rho\arctan\dot\rho}{\rho^2}\right) + \frac{\pi V_x}{\kappa}\left(\frac{1}{z_f} + \frac{1}{z_i}\right).$$

$$= \int d\rho\,\frac{\varphi}{\rho^2}$$

* perhaps there is an interpretation using the algebra of T, $\bar{T}$, $T\bar{T}$?

Kinematic space interpretation (~ effort of combining entanglement wedges)



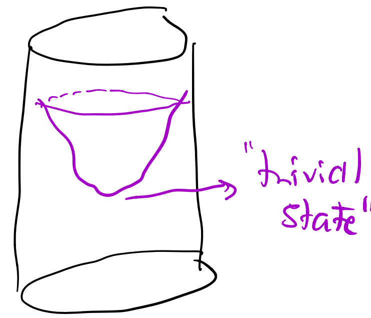
$\rho(t) \to t_1(t),\ t_2(t)$

$$ds^2_{ks} = \frac{-dt_1\,dt_2}{(t_1-t_2)^2}$$

$$S \approx \int \frac{dx}{\rho}\, ds_{ks}(t)$$

Remarks:

* Global AdS, trivial initial state:

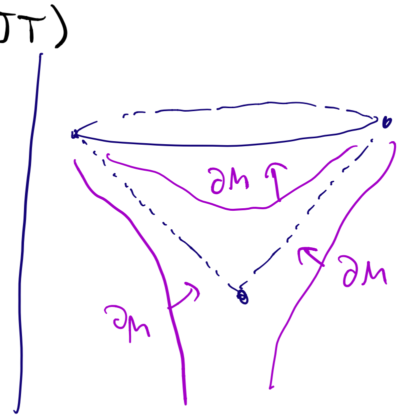


* Choice of time slice?

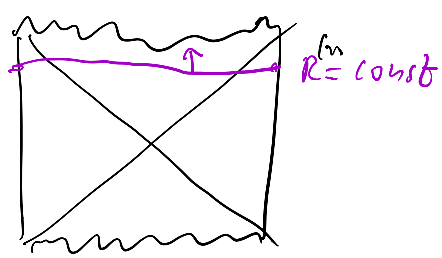
* Finite deformation of Liouville?

* AdS_2 circuit $H \to f(H)$, cost function? $S \sim \int dt \frac{1}{\ell}$
(perhaps need JT)

* Lorentzian case
get both CV and CA



* BTZ black hole



* Relation to entanglement wedge reconstruction?

THE END