D-instanton Amplitudes in String Theory

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

Paris, February 2021

Plan:

1. Introduction to the problem

2. Solution via string field theory

3. Final remarks

References:

arXiv:2012.11624, 2101.08566

The problem

Amplitudes in string perturbation theory are usually expressed as integrals over the moduli spaces of Riemann surfaces with punctures.

However these integrals often run into divergences.

Example: Veneziano amplitude (in $\alpha' = 1$ unit)

$$\int_0^1 dy\, y^{2p_1.p_2}(1-y)^{2p_2.p_3}$$

– diverges for $2p_1.p_2 \le -1$ or $2p_2.p_3 \le -1$.

Conventional viewpoint: Define the amplitude for $2p_1.p_2 > -1$, $2p_2.p_3 > -1$ and then analytically continue to the other kinematic regions.

String field theory (SFT) provides a different, but equivalent viewpoint.

SFT is a regular quantum field theory, designed so that the Feynman diagrams of this theory ‘formally’ reproduce the amplitudes based on the world-sheet formalism.

However, in SFT we have a clear physical interpretation of the divergences.

Consider Schwinger parametrization of a propagator in QFT:

$$(k^2+m^2)^{-1} = \int_0^\infty ds\, e^{-s\,(k^2+m^2)} = \int_0^1 dq\, q^{(k^2+m^2)-1}, \qquad q = e^{-s}$$

In SFT, the role of k^2+m^2 is played by the eigenvalues of L_0, and we write:

$$(L_0)^{-1} = \int_0^1 dq\, q^{L_0-1}$$

When we translate SFT Feynman diagrams to the world-sheet language, q’s become moduli of Riemann surfaces.

The divergences arise from the $q \to 0$ limit.

$$(L_0)^{-1} = \int_0^1 dq\, q^{L_0-1}$$

– an identity for $L_0 > 0$ states.

For $L_0 < 0$ the l.h.s. is finite but the r.h.s. diverges.

SFT instructs us to use the l.h.s.

– equivalent to analytic continuation when the latter is available.

For $L_0 = 0$ both sides diverge, reflecting the presence of on-shell states in the internal lines of a Feynman diagram.

In such cases we can use SFT to identify the physical origin of these states and use insights from QFT to deal with these divergences.

Today we shall discuss a specific situation where analytic continuation fails and we need to invoke SFT to make sense of the divergences

– D-instanton contribution to string amplitudes.

D-instantons: D-branes with Dirichlet boundary condition on all non-compact directions including (euclidean) time.

They are saddle points of the Euclidean path integral and give non-perturbative contribution to string amplitudes.

Problem: Open strings living on the D-instanton do not carry any continuous momenta

$\Rightarrow$ we cannot move away from the singularities by varying the external momenta.

Polchinski; Green, Gutperle; ...

Even if the divergent parts cancel after suitable choice of regulators, the finite parts that remain after the cancellation become ambiguous.

Fischler, Susskind

We shall study this in the context of bosonic string theory in two dimensions, but the method we describe can be applied to any string theory.

World-sheet theory: A free scalar X describing time coordinate and a Liouville field theory with central charge 25.

Total central charge adds up to 26, cancelling anomalies on the world-sheet.

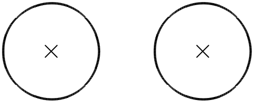
In this case the closed string 'tachyon' is actually a massless state of the theory.

In arXiv:1907.07688 Balthazar, Rodriguez and Yin (BRY) computed the single D-instanton contribution to the two point amplitude of closed string tachyons.

This model is interesting because there is a dual matrix model description that gives the exact results.

Das, Jevicki; Gross, Klebanov; Sengupta, Wadia; Moore, Plesser, Ramgoolam; · · ·

The leading contribution comes from the product of two disk one point functions.



Result:

$$\mathcal{N}\, e^{-1/g_s}\, 2\pi\, \delta(\omega_1 + \omega_2)\, 2\sinh(\pi|\omega_1|)\, 2\sinh(\pi|\omega_2|)$$

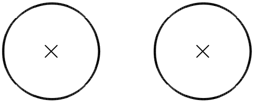
g_s: string coupling constant

$-\omega_1, \omega_2$: energies of incoming / outgoing 'tachyons'

$\mathcal{N}$: An overall normalization constant given (naively) by:

$$\mathcal{N} = i \exp\Big[\text{annulus partition function}\Big] = i \exp\left[\int_0^\infty \frac{dt}{2t}(e^{2\pi t} - 1)\right]$$

– divergent in the world-sheet description of string theory.



Naively one might have expected this to be proportional to $\delta(\omega_1)\delta(\omega_2)$.

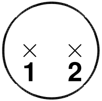
However, for D-instanton boundary conditions, individual disk amplitudes do not conserve energy, since time translation invariance is broken.

The energy conservation is restored at the end after integration over the collective coordinate describing the position of the D-instanton

– will be discussed later.

At the next order, there are three contributions.

1. Two point function on the disk.



Result:

$$8\,\pi\,\mathcal{N}\,e^{-1/g_s}\,\delta(\omega_1+\omega_2)\,\sinh(\pi|\omega_1|)\,\sinh(\pi|\omega_2|)\,g_s\,f(\omega_1,\omega_2)$$

$$f(\omega_1,\omega_2)=\frac{1}{2}\int_0^1 dy\,y^{-2}(1+2\omega_1\omega_2 y)+f_{finite}(\omega_1,\omega_2)$$

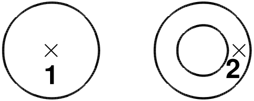
Note the divergences from the $y\to 0$ limit

– cannot be tamed by deforming the ω_i's.

Write

$$f(\omega_1,\omega_2)=A_f+B_f\,\omega_1\,\omega_2+f_{finite}(\omega_1,\omega_2)$$

2. Product of disk one point function and annulus one point function.



$+ (1 \Leftrightarrow 2)$

Result:

$$8\,\pi\,\mathcal{N}\,e^{-1/g_s}\,\delta(\omega_1+\omega_2)\,\sinh(\pi|\omega_1|)\,\sinh(\pi|\omega_2|)\,g_s\{g(\omega_2)+g(\omega_1)\}$$

$$g(\omega) = \int_0^1 dv \int_0^{1/4} dx \left\{ \frac{v^{-2}-v^{-1}}{\sin^2(2\pi x)} + 2\,\omega^2\,v^{-1} \right\} + g_{finite}(\omega)$$

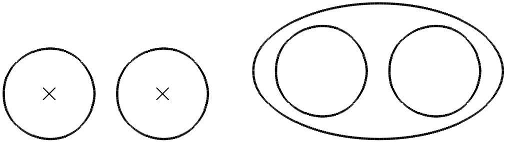
Note the divergences from $x \to 0$ and $v \to 0$ that cannot be tamed by deforming ω.

Write

$$g(\omega) = A_g + B_g\,\omega^2 + g_{finite}(\omega)$$

3. Product of two disk one point functions and zero point function on a surface with Euler number -1

– a disk with two holes and a torus with one hole



$+$1 more

$$8\,\pi\,\mathcal{N}\,e^{-1/g_s}\,\delta(\omega_1+\omega_2)\,\sinh(\pi|\omega_1|)\,\sinh(\pi|\omega_2|)\,g_s\,C$$

Total result to order g_s:

$$\begin{aligned} & 8\,\pi\,\mathcal{N}\,e^{-1/g_s}\,\delta(\omega_1+\omega_2)\,\sinh(\pi|\omega_1|)\,\sinh(\pi|\omega_2|) \\ & \quad\times[1+g_s\{f(\omega_1,\omega_2)+g(\omega_1)+g(\omega_2)+C\}] \\ = \; & 8\,\pi\,\mathcal{N}\,e^{-1/g_s}\,\delta(\omega_1+\omega_2)\,\sinh(\pi|\omega_1|)\,\sinh(\pi|\omega_2|) \end{aligned}$$

$$[1+g_s\{f_{finite}(\omega_1,\omega_2)+g_{finite}(\omega_1)+g_{finite}(\omega_2)+A_f+2A_g+C+(B_f-2B_g)\omega_1\omega_2\}]$$

At this stage A_f, B_f, A_g, B_g, C are unknown constants

– divergent in the world-sheet description.

Using the same functions we can also get the expressions for n-point function of closed string tachyon at order $g_s\, e^{-1/g_s}$.

The leading contribution comes from product of n disk one point functions.

Result:

$$\begin{aligned} & \mathcal{N}\, e^{-1/g_s}\, 2\,\pi\, \delta(\omega_1 + \cdots \omega_n) \prod_{i=1}^{n} \{2 \sinh(\pi|\omega_i|)\} \\ = \quad & 2^{n+1}\, \pi\, \mathcal{N}\, e^{-1/g_s}\, \delta(\omega_1 + \cdots \omega_n) \prod_{i=1}^{n} \sinh(\pi|\omega_i|) \end{aligned}$$

At next order there are 3 types of diagrams:

1. $(n-2)$ disk one point functions and one disk 2-point function

2. $(n-1)$ disk one point functions and one annulus 1-point function

3. n disk one point functions and a zero point function on surfaces of Euler number -1

Result

$$2^{n+1}\,\pi\,\mathcal{N}\,e^{-1/g_s}\,\delta(\omega_1+\cdots\omega_n)\,\left\{\prod_{i=1}^{n}\sinh(\pi|\omega_i|)\right\}\,g_s\,\left[\sum_{j<k}f(\omega_j,\omega_k)+\sum_{j}g(\omega_j)+C\right]$$

Use

$$f(\omega_1,\omega_2)=A_f+B_f\,\omega_1\,\omega_2+f_{finite}(\omega_1,\omega_2),\quad g(\omega)=A_g+B_g\,\omega^2+g_{finite}(\omega)$$

D-instanton contribution to n tachyon amplitude to order g_s:

$$2^{n+1}\,\pi\,\mathcal{N}\,e^{-1/g_s}\,\delta(\omega_1+\cdots\omega_n)\,\left\{\prod_{i=1}^{n}\sinh(\pi|\omega_i|)\right\}\,\left[1+g_s\,\left\{\sum_{j<k} f(\omega_j,\omega_k)+\sum_j g(\omega_j)+C\right\}\right]$$

$$=2^{n+1}\,\pi\,\mathcal{N}\,e^{-1/g_s}\,\delta(\omega_1+\cdots\omega_n)\,\left\{\prod_{i=1}^{n}\sinh(\pi|\omega_i|)\right\}$$

$$\times\left[1+g_s\left\{\sum_{j<k} f_{finite}(\omega_j,\omega_k)+\sum_j g_{finite}(\omega_j)+\frac{n(n-1)}{2}A_f+n\,A_g+C+\left\{B_g-\frac{B_f}{2}\right\}\sum_j\omega_j^2\right\}\right]$$

The result is known from the dual matrix model.

For scattering of a tachyon of energy $-\omega_1$ into tachyons of energy $\omega_2,\cdots,\omega_n$, the result is:

$$-\frac{1}{8\pi^2}\,2^{n+1}\,\pi\,e^{-1/g_s}\,\delta(\omega_1+\omega_2+\cdots\omega_n)\,\left\{\prod_{i=1}^{n}\sinh(\pi|\omega_i|)\right\}$$

$$\left[1-i\,g_s\sum_{j=2}^{n}\omega_j\left(1-\sum_{i=2}^{n}\pi\omega_i\coth(\pi\omega_i)\right)\right],$$

$$
2^{n+1}\,\pi\,\mathcal{N}\,e^{-1/g_s}\,\delta(\omega_1+\cdots\omega_n)\left\{\prod_{i=1}^{n}\sinh(\pi|\omega_i|)\right\}g_s
$$

$$
\times\left[1+g_s\left\{\sum_{j<k}f_{finite}(\omega_j,\omega_k)+\sum_j g_{finite}(\omega_j)+\frac{n(n-1)}{2}A_f+n\,A_g+C+\left(B_g-\frac{B_f}{2}\right)\sum_j\omega_j^2\right\}\right]
$$

$$
=\quad -\frac{1}{8\pi^2}2^{n+1}\,\pi\,e^{-1/g_s}\,\delta(\omega_1+\omega_2+\cdots\omega_n)\left\{\prod_{i=1}^{n}\sinh(\pi|\omega_i|)\right\}
$$

$$
\left[1-i\,g_s\sum_{j=2}^{n}\omega_j\left(1-\sum_{i=2}^{n}\pi\omega_i\coth(\pi\omega_i)\right)\right].
$$

Comparing the two results, one gets (BRY):

$$
\mathcal{N}=-\frac{1}{8\pi^2}\text{(analytical)}
$$

$$
A_f\simeq-0.50,\quad A_g\simeq0.00,\quad C\simeq0.00,\qquad 2B_g-B_f\simeq-1.40\quad\text{(numerical)}
$$

Can we determine them using string field theory?

Solution via string field theory

Strategy

1. Take the world-sheet expression for an amplitude and focus on the divergent part associated with open string degeneration, e.g. $t \to \infty$, $y \to 0$, $v \to 0$, $x \to 0$.

2. Make appropriate change of variables to express the moduli in terms of SFT Schwinger parameters $q_1, q_2, \cdots$.

3. Then we replace divergent integrals of the form

$$\int_0^1 dq\, q^{-\beta-1}, \qquad \beta \neq 0$$

by $-1/\beta$, interpreting this as due to states with $L_0 = -\beta$.

$\beta = 0$ will require special treatment.

4. Divergent integrals of the form:

$$\int_0^1 dq\, q^{-1}$$

can be traced to the zero modes ($L_0 = 0$ states).

These cannot be treated using Feynman diagrams and the first step is to remove these contributions by hand

– corresponds to ‘unintegrating them’ in the path integral.

We then have to find the physical origin of these modes, and treat them accordingly

– involves doing the path integral over them explicitly, instead of treating them via Feynman diagrams.

Results from string field theory (analytical)

$$\mathcal{N} = -\frac{1}{8\pi^2},\ A_f = -\frac{1}{2},\ \boxed{A_g = -\frac{1}{2}},\ C = 0(\text{indirect}),\ 2B_g - B_f \simeq -\ln 4 \simeq -1.386$$

Compare this with the matrix model result of BRY:

$$\mathcal{N} = -\frac{1}{8\pi^2}(\text{analytical})$$

$$A_f \simeq -0.50, \quad A_g \simeq 0.00, \quad C \simeq 0.00, \qquad 2B_g - B_f \simeq -1.40 \quad (\text{numerical})$$

We hope that the discrepancy in the value of A_g will be resolved soon, but at present it remains an enigma.

We shall now illustrate this by describing the calculation of the overall normalization constant $\mathcal{N}$

– technically simpler but conceptually most subtle

The naive world-sheet expression for this is given by:

$$\mathbf{i}\,\exp[\mathbf{A}]$$

A: annulus partition function

$$A = \int_0^\infty \frac{dt}{2t}\, Tr(e^{-2\pi t\, L_0}) = \int_0^\infty \frac{dt}{2t} \left[\sum_i e^{-2\pi t\, h_i^b} - \sum_j e^{-2\pi t\, h_j^f} \right]$$

$\{h_i^b\}$ and $\{h_j^f\}$ are the L_0 eigenvalues of the bosonic and fermionic open string states.

Due to ghost zero modes, there is a projection operator $b_0 c_0$ hidden in the trace and the sum runs over $b_0|s\rangle = 0$ states.

A diverges in string theory due to states with $L_0 \leq 0$.

For the two dimensional string theory:

$$A = \int_0^\infty \frac{dt}{2t}\left[e^{2\pi t} - 1\right] = \int_0^\infty \frac{dt}{2t}\left[e^{2\pi t} + 1 - 2\right]$$

Compare this with the general result:

$$A = \int_0^\infty \frac{dt}{2t} \mathrm{Tr}\left(e^{-2\pi t L_0}\right) = \int_0^\infty \frac{dt}{2t}\left[\sum_i e^{-2\pi t\, h_i^b} - \sum_j e^{-2\pi t\, h_j^f}\right]$$

This shows that in the two dimensional string theory contribution from $L_0 > 0$ states all cancel

$\Rightarrow$ we can focus on the finite number of $L_0 \leq 0$ modes:

$$\text{boson}: \quad c_1|0\rangle,\ c_1\alpha_{-1}|0\rangle, \qquad \text{fermion}: \quad |0\rangle,\ c_{-1}c_1|0\rangle$$

α_{-1}: oscillator of the euclidean time coordinate

$$A = \int_0^\infty \frac{dt}{2t}\left[\sum_i e^{-2\pi t\, h_i^b} - \sum_j e^{-2\pi t\, h_j^f}\right]$$

Typically in all consistent string theories the $[\cdots]$ vanishes as $t \to 0$

$\Rightarrow$ effective number of bosonic and fermionic modes are equal (including ghosts).

In this case we can carry out the integral analytically and write:

$$A = \frac{1}{2} \ln \frac{\prod_j h_j^f}{\prod_i h_i^b}$$

Also fermionic modes come in pairs with equal h_j^f.

$$\Rightarrow \quad e^A = \sqrt{\frac{\prod_j h_j^f}{\prod_i h_i^b}} = \int \prod_i \frac{db_i}{\sqrt{2\pi}} \prod_j df_j\, d\widetilde{f}_j \exp\left[-\frac{1}{2}\sum_i h_i^b b_i^2 + {\sum_j}' h_j^f \widetilde{f}_j\, f_j\right]$$

b_i: bosonic mode, $f_j, \widetilde{f}_j$: fermionic modes with same h_j^f

$$\Rightarrow \quad e^A = \int \prod_i \frac{db_i}{\sqrt{2\pi}} \prod_j df_j\, d\widetilde{f}_j \exp\left[-\frac{1}{2}\sum_i h_i^b b_i^2 + {\sum_j}' h_j^f \widetilde{f}_j f_j\right]$$

Our next task is to

1. Interpret this as the Siegel gauge fixed path integral of a gauge invariant string field theory.

2. Show that the ambiguities associated with the $h_i^b = 0$ and $h_j^f = 0$ states arise due to breakdown of the Siegel gauge.

3. Use the gauge invariant path integral to evaluate e^A.

Since open strings on a D-instanton do not carry any continuous momenta, the string field theory is a zero dimensional field theory.

Define ghost number of open string states by assigning

$c, \bar{c}$: ghost number 1, $b, \bar{b}$: ghost number -1, matter : ghost number 0

$\{|\phi_r^{(n)}\rangle\}$: A basis of states of ghost number n.

For the open string states with $L_0 \leq 0$, this list is finite:

$c_1|0\rangle,\ |0\rangle,\ c_{-1}c_1|0\rangle,\ c_1\alpha_{-1}|0\rangle,\ c_0c_1|0\rangle,\ c_0|0\rangle,\ c_0c_{-1}c_1|0\rangle,\ c_0c_1\alpha_{-1}|0\rangle$

α_{-1}: oscillator of the Euclidean time X

In open string field theory the classical string field $|\psi\rangle$ is an arbitrary state of ghost number 1.

If we expand:

$$|\psi\rangle = \sum_r \psi^r |\phi_r^{(1)}\rangle$$

then $\{\psi^r\}$ are the dynamical degrees of freedom.

The quadratic part of the gauge invariant classical action is:

$$S_{gi} = \frac{1}{2}\langle\psi|Q_B|\psi\rangle$$

Q_B: BRST charge of the world-sheet theory

The gauge transformation parameters of the classical open string field theory are described by an arbitrary state $|\theta\rangle$ of ghost number 0.

If we expand

$$|\theta\rangle = \sum_{\mathbf{u}} \theta_{\mathbf{u}} |\phi_{\mathbf{u}}^{(0)}\rangle$$

then $\{\theta_{\mathbf{u}}\}$ are the gauge transformation parameters.

$$\delta|\psi\rangle = \mathbf{Q_B}|\theta\rangle$$

Partition function in the gauge invariant formalism is defined as:

$$\mathbf{I_{gi}} \equiv \int \prod_{\mathbf{r}} \mathbf{d}\psi^{\mathbf{r}}\, \mathbf{e}^{-\mathbf{S_{gi}}} \Bigg/ \int \prod_{\mathbf{s}} \mathbf{d}\theta_{\mathbf{s}}$$

The normalization has been fixed arbitrarily and will not affect the final result.

$$I_{gi} = \int \prod_r d\psi^r \, e^{-S_{gi}} \Big/ \int \prod_s d\theta_s$$

One can show, using standard Fadeev-Popov formalism, that if we formally gauge fix to Siegel gauge $b_0|\psi\rangle_s = 0$, then this reduces to

$$\int \prod_i db_i \prod_j df_j \, d\tilde{f}_j \, \exp\left[-\frac{1}{2}\sum_i h^b_i b_i^2 + \sum_j{}' h^f_j \tilde{f}_j f_j\right] = (2\pi)^{n_b/2} e^A$$

n_b: number of bosonic modes in the Siegel gauge

In the present case:

$$|\psi\rangle = \psi^0 c_1|0\rangle + \psi^1 c_0|0\rangle + \phi \, c_1\alpha_{-1}|0\rangle, \qquad |\theta\rangle = \theta|0\rangle$$

$$S_{gi} = \frac{1}{2}\langle\psi|Q_B|\psi\rangle = -\frac{1}{2}(\psi^0)^2 - (\psi^1)^2$$

This gives $n_b = 2$ and

$$e^A = \frac{1}{2\pi}\int d\psi^0 d\psi^1 \, d\phi \, e^{\frac{1}{2}(\psi^0)^2 + (\psi^1)^2} \Big/ \int d\theta$$

$$\mathbf{e^A} = \frac{\mathbf{1}}{\mathbf{2}\pi} \int \mathbf{d}\psi^{\mathbf{0}}\mathbf{d}\psi^{\mathbf{1}}\,\mathbf{d}\phi\,\mathbf{e}^{\frac{1}{2}(\psi^0)^2+(\psi^1)^2} \Big/ \int \mathbf{d}\theta$$

The ψ^0 and ψ^1 integration can be carried out by choosing the integration contour along the steepest descent contour (imaginary axis) yielding a factor of $-\pi\sqrt{2}$.

$$\mathbf{e^A} = -\frac{\mathbf{1}}{\sqrt{\mathbf{2}}} \int \mathbf{d}\phi \Big/ \int \mathbf{d}\theta$$

$$\mathbf{e^A} = -\frac{\mathbf{1}}{\sqrt{\mathbf{2}}} \int \mathbf{d}\phi \Big/ \int \mathbf{d}\theta$$

ϕ is related to the D-instanton location $\widetilde{\phi}$ along the time direction.

θ is related to the rigid U(1) symmetry transformation parameter $\widetilde{\theta}$ on the D-instanton

– gives a phase $\mathbf{e}^{\mathbf{i}\widetilde{\theta}}$ to open strings connecting the D-instanton to a second D-brane.

The precise relation between these modes can be found by comparing their amplitudes.

Results: $\pi\sqrt{\mathbf{2}}\,\mathbf{g_o}\,\phi = \widetilde{\phi}, \qquad \theta = \mathbf{i}\widetilde{\theta}/\mathbf{g_o}$

$\mathbf{g_o}$: coupling constant of open string theory.

$$e^A = -\frac{1}{\sqrt{2}} \int d\phi \Big/ \int d\theta$$

$$\pi\sqrt{2}\, g_o\, \phi = \widetilde{\phi}, \qquad \theta = i\widetilde{\theta}/g_o$$

Since $\widetilde{\theta}$ has period 2π we have $\int d\widetilde{\theta} = 2\pi$.

The integration over the D-instanton position $\widetilde{\phi}$ generates the energy conserving δ-function $2\pi\delta(\omega)$ and is not usually included in $\mathcal{N}$.

This gives

$$e^A = -\frac{1}{\sqrt{2}}\, \frac{1}{\pi\sqrt{2}\, g_o}\, \frac{g_o}{2\pi i} = \frac{i}{4\pi^2}$$

But we are not done yet.

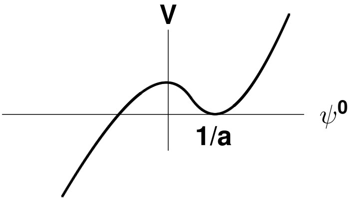
The multiplier factor:

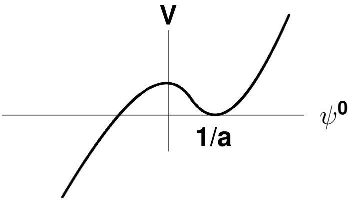
We have computed the leading contribution from the steepest descent contour around the instanton.

How does the steepest descent contour around the instanton fit inside the full contour along which the ‘path integral’ is to be performed?

For this we look at the open string tachyon mode ψ^0.

Tachyon potential has a maximum at 0 representing the D-instanton and a local minimum at some value 1/a representing the vacuum without any D-instanton.

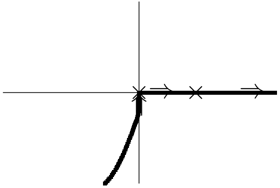




The potential goes to $-\infty$ as $\psi^0 \to -\infty$

This shows that the ψ^0 integration contour cannot be taken to be along the real axis since e^{-V} approaches ∞ as $\psi^0 \to -\infty$.

The correct choice of integration contour:



This covers the full steepest descent contour of the perturbative vacuum at $\psi^0 = 1/a$ but only half of the steepest descent contour of the instanton at $\psi^0 = 0$.

This gives:

$$\mathcal{N} = i\, e^A \, \frac{1}{2} = i\, \frac{i}{4\pi^2}\, \frac{1}{2} = -\frac{1}{8\pi^2}$$

in agreement with the matrix model result.

Final remarks

In this talk we have focussed on D-instantons in two dimensional bosonic string theory.

However D-instantons are present in many string theories, and the problems we discussed are universal problems for all D-instantons.

This includes for example the breakdown of the Siegel gauge, translational zero modes, rigid U(1) transformation etc.

String field theory for superstrings is well established by now and can be used to address these issues.

This gives a fully systematic method for computing D-instanton contribution to string amplitudes

– provides a window into non-perturbative aspects of string theory using perturbative techniques.