Defects and Anomalies in QFTs

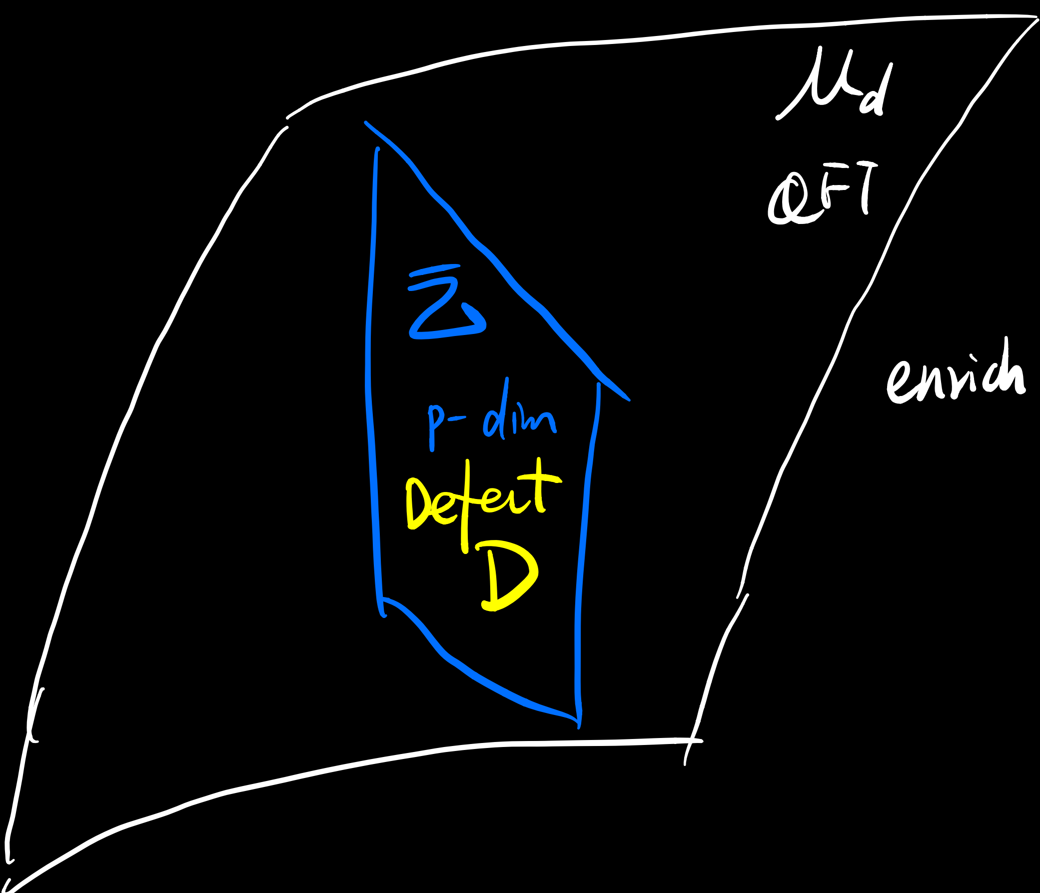
Based on 2012.06574, 2101.12648 Yu

2d defects 4d defects

2012.15861 Thorngren-Yu

boundaries

+ many previous works by others



$$
\begin{tikzcd}
\text{Defects} \arrow[d, bend right=60, "\text{enrich}"'] \\
\text{Anomalies} \arrow[u, bend right=60, "\text{constrain}"']
\end{tikzcd}
$$

Defects

- QFT ~ Local operator algebra

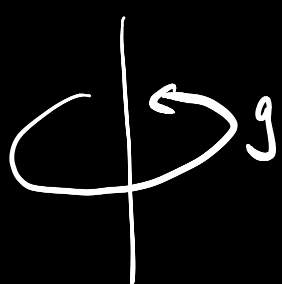
  $\langle \phi_1(x_1)\, \phi_2(x_2) \cdots \rangle$

- Defects $\to$ extended op. D on $\underset{\substack{\uparrow \\ p\text{-dim}}}{\Sigma} \subset M_d$

  e.g. WL $\quad \operatorname{tr}\left(P e^{\oint_\gamma A}\right)$ $\qquad p=1$

  Boundary $\quad \phi|_{\Sigma} = \phi_0$ $\qquad p=d-1$

  Singularity $\quad \phi(x) \sim \dfrac{1}{x_\perp}$ $\qquad p=d-1$



twist $\quad \phi(e^{2\pi i} z) = g \cdot \phi(z)$ $\qquad p=d-2$

- - - - -

- $\exists$ local operators on the defects

Local & Extended Ops Not Independent

$\boxed{\text{Take CFT to be precise}}$

$e^{\lambda \int_\Sigma \phi}$ (not surjective)

$$\text{local ops} \underset{\text{decompose}}{\overset{}{\rightleftarrows}} \text{defects}$$

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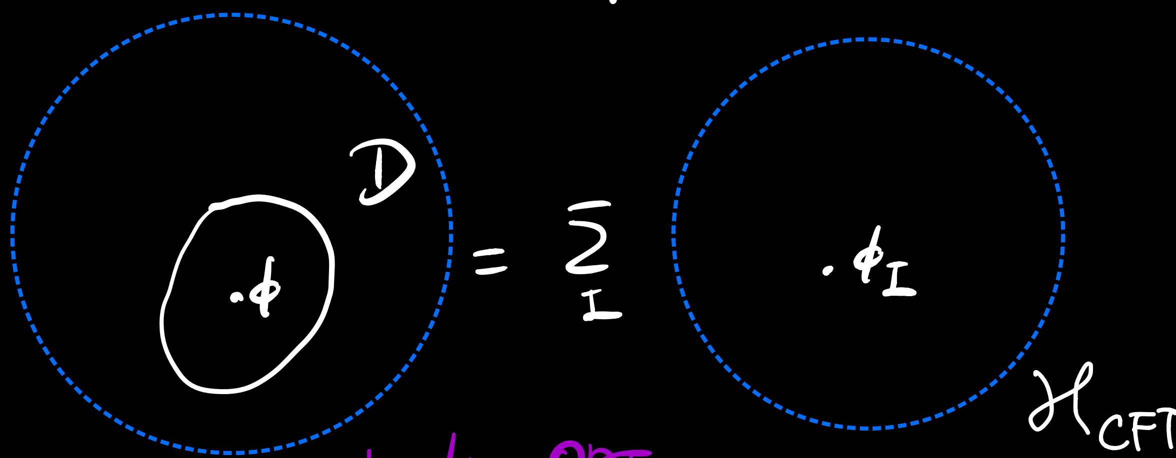

bulk OPE

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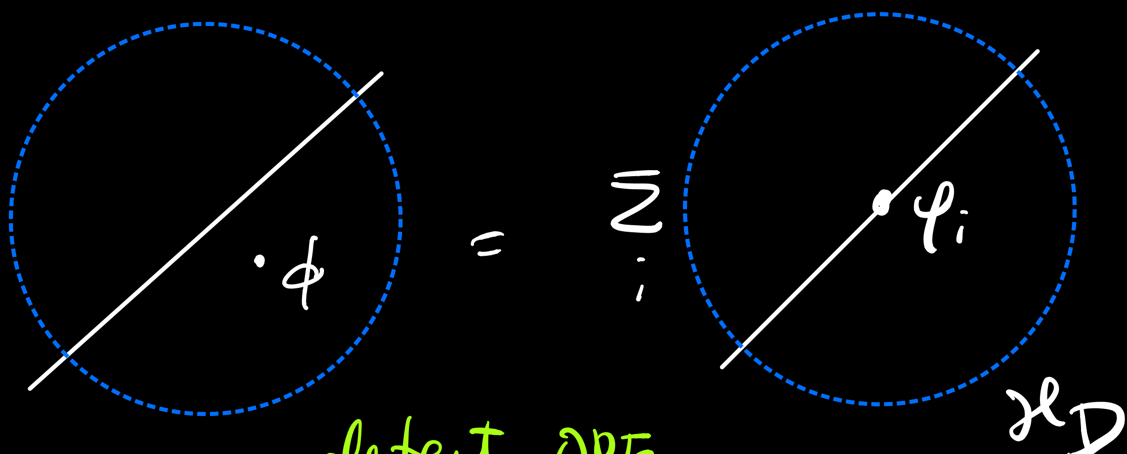


defect OPE

• Defects important to detect global topological structures, higher form symmetries, phase transitions - - -

Anomalies

't Hooft anomaly for G

$\Downarrow$

obstruction to gauging G

Multiple manifestations:

e.g $G = U(1)$ $d=2$

Modified cons. law. $$\partial_\mu J^\mu = \frac{k}{4\pi} \epsilon_{\mu\nu} F^{\mu\nu}$$ 

Contact term $$\langle \partial_\mu J^\mu(x) J^\nu(0) \rangle = \frac{k}{4\pi} \epsilon^{\mu\nu} \partial_\mu \delta^2(x)$$

separated point correlator $$J^\mu(x) J^\nu(0) = C_J \frac{I^{\mu\nu}}{|x|^2} + k \frac{I^\nu{}_\rho \epsilon^{\mu\rho}}{|x|^2}$$

Anomalous var. of PF. $$\delta_\lambda Z[A] = \frac{k}{4\pi} \int \lambda F$$

$$\uparrow$$

$$A \to A + d\lambda$$

Conformal anomalies

$$T^\mu{}_\mu \sim a E_d + \sum_i c_i W_d^i$$

Euler $\quad$ Weyl

$d=2 \qquad T^\mu{}_\mu = \frac{c}{24\pi} R$

Other manifestations (parity even)

Contact term $\qquad \langle T^\rho{}_\rho T_{\mu\nu} \rangle = \frac{c}{24\pi}(\partial_\mu \partial_\nu - \partial^2)\delta^2(x)$

separated point correlator $\qquad \langle T_{\mu\nu} T_{\rho\sigma} \rangle \sim c \frac{I_{\mu\nu\rho\sigma}}{|x|^4}$

Weyl var. of PF $\qquad \delta_\sigma Z[g] = \frac{c}{24\pi} \int d^2x \sqrt{g}\, \sigma R$

$$g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}$$

't Hooft anomalies vs Conformal anomalies

Common :

- obstruction to gauging
- failure of conservation at coincident points
- fine structures of current algebra
- "count" dof
- Constrain RG, IR physics

Diff :

- RG inv vs RG variant
- quantized vs unquantized

Acessibility • Robust, easy vs difficult

Anomaly matching • $k_{UV} = k_{IR}$ vs $c_{UV} > c_{IR}$

$\Uparrow$

c-thm, a-thm ...

Goals (broadly)

1. Structures of Anomalies in the presence of defects?

2. Defect conformal anomalies & RG monotonicity for defects?

3. Given local operator algebra $\Rightarrow$ admissible defect ops? (both top. & general)

relatedly $\rightarrow$ necessary/sufficient data to fully specify a QFT w/o Lagrangian?

methods: bootstrap, anomaly constraints ...

[Watanabe 2016, Jensen-Shaverin-Yarom 2017
Thorngren-Yw 2020, Hellerman-Orlando-Watanabe 2021]

Focus on 1 & 2 here

surface defect in gen. unitary CFTs $p=2$

Defect Anomalies

$(z^a, y^i) = x^\mu$

- Surface defect D on $\Sigma = \mathbb{R}^{1,1} \subset \mathbb{R}^{d-1,1}$

- Defect symmetries:

Interest $\Rightarrow$ ① inherited from bulk
eg bulk currents J_μ

② Locally conserved symmetries
eg defect currents j_a

- Bulk conserved current $\langle \partial_\mu J^\mu \rangle_D = 0$

$\Downarrow$

Charge operator: $U_\alpha = e^{\alpha \int_{M^{d-1}} {}^* J}$ topological

even if $M^{d-1} \cap \Sigma \neq \emptyset$

Defect 't Hooft anomalies

$$\langle \partial_\mu J^\mu \rangle_D = \frac{k}{4\pi} \delta(\bar{z}) \epsilon_{ab} F^{ab}$$

with $U(1)$ bkgd

- Solves WZ consistency condition for defect symmetries
- Present even if $\nexists$ bulk anomalies
- Encode current correlator in the presence of defect

  $\langle J_\mu(x) J_\nu(x') \rangle_D \ldots$
- Robust: easy to obtain after deformations

e.g. anomaly inflow from $d=3$ $\boxed{\text{Dirac Fermion } \psi}$

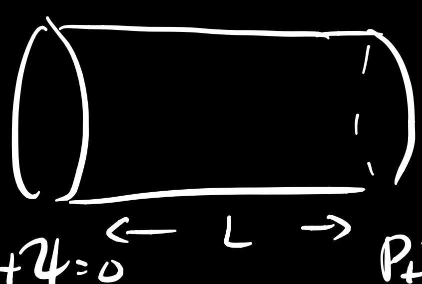
standard U(1) sym b.c. $\quad P_\pm \equiv \dfrac{1\pm\gamma_y}{2}$

$$P_+\psi|_\Sigma = 0 \quad \text{or} \quad P_-\psi|_\Sigma = 0$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

boundary U(1) anomaly $\quad \frac{1}{2} I_4(\chi_-^{2d}) = -\frac{1}{2} I_4(\chi_+^{2d})$

$\uparrow$ 2d chiral fermion

- ψ on 

$\xrightarrow{L\to 0} \chi_-^{2d}$

$$\boxed{\chi_-^{2d} + \int_\Sigma \bar\psi P_+ \chi_-^{2d}}$$

- $P_+\psi|_\Sigma = 0$, $P_-\psi|_\Sigma = 0$ have opposite anomalies by parity flip

Defect conformal anomalies

- conformal surface defect D on $\mathbb{R}^{1,1} \subset \mathbb{R}^{d-1,1}$

$$SO(2,2) \times SO(d-2) \subset SO(2,d)$$

- Bulk stress tensor $T_{\mu\nu}$

w/ defect D

$$\begin{cases} \partial_\mu T^{\mu a} = 0 \\ \partial_\mu T^{\mu i} = \delta(\Sigma)\, D^i \\ T^\mu{}_\mu = 0 \end{cases}$$

displacement op

w/ nontrivial ambient metric

$$\langle T^\mu{}_\mu \rangle_D \supset \frac{\delta(\Sigma)}{24\pi}\left(b R + d_1 \hat{K}^i_{ab} \hat{K}_i^{ab} + d_2 W_{ab}{}^{ab} \right)$$

[Schwimmer-Theisen '08]

Intrinsic

Extrinsic

b-anomaly

- analogous to c_{2d}, count defect dofs (relative)
  defect casimir energy, entanglement entropy
  e.g of defect states $\in \mathcal{H}_D$

- After Weyl transf. $\Rightarrow F_{H^3 \times S^{d-3}} \sim b \log\left(\frac{L}{\epsilon}\right)$

  compute from heat kernel methods

$$b\left(\begin{matrix}\text{real scalar} \\ \text{w/ } \phi|_\Sigma = 0\end{matrix}\right) = -\frac{1}{16}, -\frac{1}{120}, 0, \frac{1}{504} \cdots$$

$d = \quad 3, \quad 4, \quad 5, \quad 6, \cdots$

[Nishioka-Sato 2021]

- Interacting?

- RG monotonicity?

[Jensen-O'Bannon 2015] from generalizing

[Komargodski-Schwimmer 2011]

$$\boxed{\begin{array}{c} UV \\ \\ D_{UV} \end{array}} \xrightarrow{\int_{\Sigma} d^2 z \, \lambda \, O_{UV}} \boxed{\begin{array}{c} IR \\ \\ D_{IR} \end{array}}$$

$$b(D_{UV}) \overset{?}{>} b(D_{IR})$$ on the defect

$\Downarrow$

Idea: . Restore Weyl sym by spurions dilaton τ

. Δb controlled by $S_{dilaton}[\tau]$

. $\Delta b > 0$ by unitarity constraints

Similar for 4d defects . . .

Surface defect anomalies w/ SUSY

- Conformal defects w/ 2 chiral supercharges

  $\mathcal{N}=(0,2)$ superconformal sym.

- Ubiquitous in 3d, 4d, 6d SCFTs

- $U(1)_r$ symmetry

$$J_\mu \xleftrightarrow{\text{SUSY}} T_{\mu\nu}$$

$$\partial_\mu J^\mu \longleftrightarrow T^\mu{}_\mu$$

defect $U(1)_r$ anomaly — conformal anomaly

$$\delta \log Z \sim \int d^2 z \; k \, \lambda F \; + \; \sqrt{h} \, b \, \sigma R$$

h: induced metric

- SUSY completion

$$\frac{k}{4\pi}\int d^2z\, d\theta^+\, \delta\Omega\, R_- + c.c$$

$\sigma + i\lambda$ $\qquad$ $\sim \theta^+ (R\sqrt{h} + 2F)$

$$\Rightarrow \boxed{b = 3k \qquad \left(b = 3k - \frac{k_g}{2} \right)}$$

- How to identity $U(1)_r$?

mixing

$$\boxed{\begin{aligned} r &= \hat{r} + \sum_I t^I q_I \\ J_\mu &= \hat{J}_\mu + \sum_I t^I j^I_a \delta^a_\mu \delta(\Sigma) \end{aligned}}$$

2d local currents

't Hooft

$$b_{trial}(t_I) = 3k_{rr}$$

$\langle \hat{J}_\mu \hat{j}^I_a \rangle = 0$

unmix by:
$$\left.\frac{\partial b_{trial}}{\partial t_I}\right|_{t_I = t^*_I} = 0$$

b-extremization (generalizing c-ext. of [Benini-Bobev 2013])

Application 1

$\mathcal{N}=2$ Super Ising $\qquad W = \Phi^3$

UV description

"Dirichlet boundary" $\quad \phi|_\partial = P_+ \psi|_\partial = 0$

$r_\Phi = \frac{2}{3} \qquad k = -\frac{1}{2}\left(\frac{2}{3}-1\right)^2$ $\Leftarrow$ $U(1)_r$ anomaly

$k_g = -\frac{1}{2}$ $\Leftarrow$ grav. anomaly

$b = 3k - \frac{k_g}{2} = \frac{1}{12}$ $\Leftarrow$ boundary conf. anomaly

$\left(c_L = \frac{1}{3}, \quad c_R = -\frac{1}{6} \right)$

Application 2

$\mathcal{N}=2$ SQED + $q=1$ chiral

$\Updownarrow$ mirror

Free chiral

$B_D[\text{SQED}]$: $U(1)_T \times U(1)_{\hat{R}} \times U(1)_G$

$\Updownarrow$ [Dimofte Gaiotto Paquette 2017]

b-extremization

$$r = \hat{R} + \frac{1}{2}T + \frac{1}{2}G$$

$$\Rightarrow b = \frac{5}{8}$$

$$\left(c_L = \frac{7}{8}, \quad c_R = \frac{3}{8}\right)$$

B_N[free chiral] + Fermi

$$\Rightarrow b = \frac{1}{8}$$

$$\left(c_L = -\frac{1}{8}, \quad c_R = \frac{3}{8}\right)$$

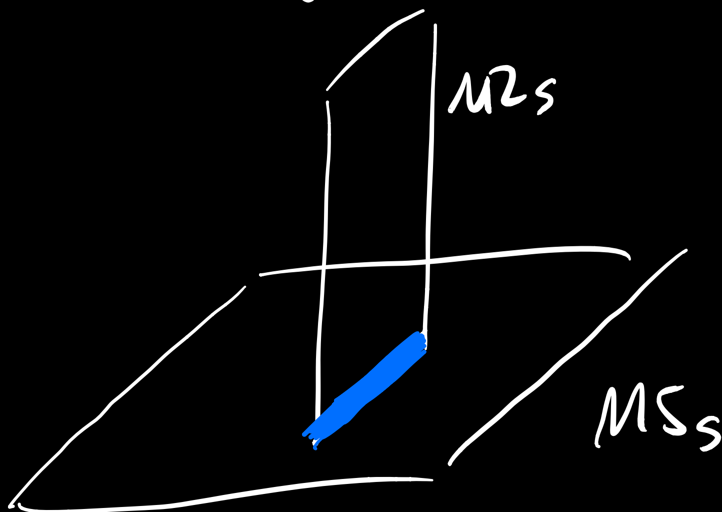
- Further applications

e.g. surface defect

$\mathcal{D}_\lambda[\mathfrak{g}]$ in 6d (2,0) SCFT

$\Lambda(\mathfrak{g})$

ADE Lie algebra



$\to$ tensor branch

Anomaly inflow (after deformation) $\to$ 't Hooft anomaly

[Weyl vector]

$$\Rightarrow \quad b = 3(\lambda,\lambda) + 24(\lambda,\rho)$$

also large N results [Estes et al 2018]

- Similar story for $p=4$ defects

Some open questions

- General boundaries of 3d $\mathcal{N}=2$ SCFTs

- Defects under bulk deformations
  e.g. integrable relevant / irrelevant

- Bounds on defect conf. anomalies?
  b for $p=2$, a & c for $p=4$

- Discrete defect anomalies? ....

- Classification of defect anomalies?
  extended SPT?