Hall fluid at strong coupling

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virtually in Ecole Polytechnique – 10 February 2022

Key references

Mostly about recent work

- Gravity dual of (spontaneous) SDW+CDW
  [1408.1397]
- Conductivities: sliding stripes
  [1612.07323]
- Pinning them
  [1708.07837]
- Nonzero magnetic field with all above
  [2111.14885]

with

- Matti Järvinen (APCTP, South Korea)
- Matthew Lippert (Old Westbury, Long Island US)

Apologies for not citing all relevant papers along the way...

Outline

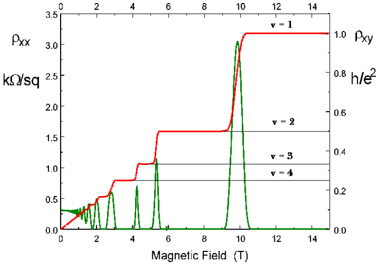
1. Introduction
2. Hall fluid model
3. Conductivities
4. Discussion/outlook

Quantum Hall Effect

- Electrons in 2+1 dimensions, perpendicular B field, low T $\Rightarrow$ A series of plateaux

$$\sigma_{xx} = 0 \;, \qquad \sigma_{yx} = \nu \frac{e^2}{h}$$

$$\nu \equiv \frac{n_e}{2\pi B} = \begin{cases} \text{integers} \\ \text{certain fractions} \end{cases}$$

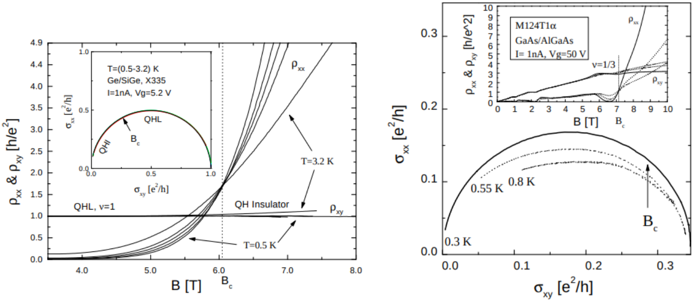


- Integer filling fraction ν: Landau levels plus impurities – pretty well understood
- Fractional ν is a strong coupling effect – less well understood
  - Different descriptions: Laughling wave function, Chern-Simons theory, Jain's composite fermions. . .

Semicircle from Law of Corresponding States

Semicircle: An exact relation in the Integer and Fractional Quantum Hall Effect

M. Hilke, D. Shahar[1], S.H. Song[2], D.C. Tsui, Y.H. Xie[3] and M. Shayegan



$$\sigma_{xx}^2 + (\sigma_{yx} - \sigma_0/2)^2 = (\sigma_0/2)^2 \ , \ \sigma_0 = \nu e^2/h$$

- Indications that similar law holds for striped Hall fluids [MacDonald-Fisher'99,von Oppen-Halperin-Stern'99]
- Duality transformation behind [Burgess-Dolan, also Kivelson-Lee-Zhang]

$$\sigma = \sigma_{yx} + i\sigma_{xx} \rightarrow \frac{a\sigma + b}{c\sigma + d} \ , \ ad - bc = 1, c \text{ even}$$

Striped phases

- Predicted by Peierls in 1930 in quasi-1D metals
- Different kinds of stripes
  - Charge/current density wave (CDW)
  - Spin density wave (SDW) etc.
- Spatially modulated phases often appear in condensed matter systems
  - QH samples w/ high LL [Lilly-Cooper-Störmer-Pfeiffer-West] [Du-Tsui-Störmer-Pfeiffer-West]
  - pseudogap phase of cuprate superconductors [Emery-Kivelson-Tranquada]
  - thin films of superfluid ^{3}He [Sauls-Wiman]
  - Calcium-doped graphene

Holographic striped phases

Translation symmetry-breaking in holographic models

- Mostly **explicit** breaking
- Modulated sources model ionic lattice and impurities
- Linear scalars (massive gravity, Q-lattice, Bianchi VII etc.) homogeneous
- Symmetry breaking $\rightarrow$ Drude-like conductivity

[Horowitz-Santos-Tong 1204.0519]

[Vegh 1301.0537]

[Arean et al. 1308.1920]

[Blake et al. 1308.4970]

[Goutéraux 1401.5436]

[Donos-Gauntlett 1409.6875]

[. . .]

[Amoretti-Areán-Goutéraux-Musso 1812.08118]

[Andrade-Krikun 1812.08132]

[. . .]

Holographic striped phases

Spontaneous breaking of translation symmetry

- Interesting phenomena
  - Phase transition
  - Goldstone mode
  - Interaction with lattice/impurities
- Modulated instabilities in several models
  [typically due CS interactions: Nakamura-Ooguri-Park 0911.0679]
- Inhomogeneous ground state constructed in some examples
  [Donos 1303.7211]
  [Withers 1304.0129]
  [Rozali et al. 1304.3130]
  [...]
- Limited work on transport

Recently, works on both spontaneous and explicit breaking...

Our approach

- Top-down: a concrete string theory set-up
- Control over the dual field theory
- Criterion: system with only fermion matter in the fundamental representation*
- Specific models with 2+1 d fermions interacting with 3+1 d gauge fields: D3-D7' systems

Holographic cousin models

Brane intersections with $\#\mathrm{ND} = 6$ [Rey]

- Fundamental fermions [Davis,Kraus,Shah]
- Probe *Dq* in *Dp* background [Myers,Wapler]
- No SUSY $\rightarrow$ stability?
- Chern-Simons terms [Alanen,Keski-Vakkuri,Kraus,Suur-Uski]

Familiar example: Sakai-Sugimoto D4-D8-$\overline{\mathrm{D8}}$

QH models:

1. The D3-D7' model [Bergman-NJ-Lifschytz-Lippert]
   - 2+1 d defect, filling fraction ν irrational
2. The D2-D8' model [NJ-Järvinen-Lippert]
   - Fully 2+1 d, $\nu = 1$ [Rai-Mukhopadhyay 1909.03458]

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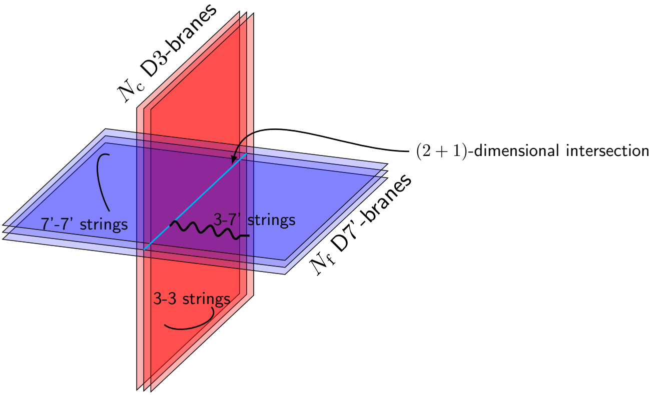
*Unquenched (massive) ABJM: fully 2+1 d, $\nu = M/2 + func(N_f)$, SUSY even w/ $d \neq 0 \neq B$ [Bea-NJ-Lippert-Ramallo-Zoakos]

D3-D7' model



$$S_{\text{defect}} = \int d^3x \sum_{a=1}^{N_f} \left(\bar{\psi}^a(x) i\gamma^\mu(\partial_\mu - iA_\mu)\psi^a(x) + \ldots \right)$$

Defect ψ are doublets of SO(2,1) and singlets of SO(3)$\times$ SO(3)

D3-D7' model

- D3 background, $AdS_5 \times S^5$ (finite temperature)

$$ds_{10}^2 = r^2 \left(-h(r)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{1}{r^2}\left(\frac{dr^2}{h(r)} + r^2 d\Omega_5^2\right)$$

$$h(r) = 1 - \left(\frac{r_T}{r}\right)^4$$

- Five-form flux

$$F_5 = dC_4 = (1 + *)dt \wedge dx \wedge dy \wedge dz \wedge d(r^4)\ ,\ \int_{S^5} F_5 = N$$

- Dilaton is a constant: CFT (w/ D7: dCFT)
- S^5 fibering: $S^2 \times S^2$ over an interval

$$d\Omega_5^2 = d\psi^2 + \cos^2\psi\ d\Omega_{2(1)}^2 + \sin^2\psi\ d\Omega_{2(2)}^2$$

	0	1	2	z	r	ψ	$S^2_{(1)}$	$S^2_{(2)}$
$D3$	X	X	X	X				
$D7$	X	X	X		X		X	X

Probe D7-brane

- Wraps $S^2 \times S^2 \subset S^5$
- Embedding $\psi(r)$ is tachyonic
- Stabilized by wrapped flux on S^2's

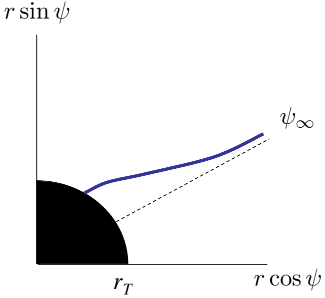
Add magnetic field $F_{12} = B$ and charge density $F_{r0} = A_0'(r)$

D7 probe action

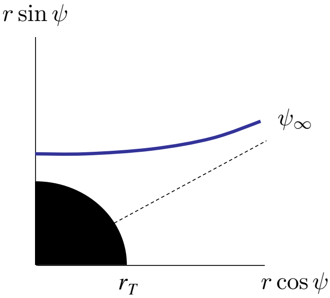
$$S = S_{\text{DBI}} + S_{\text{CS}} \quad = -T_7 \int d^8x\, e^{-\Phi} \sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} - \frac{(2\pi\alpha')^2 T_7}{2} \int \underbrace{P[C_4] \wedge F \wedge F}_{\textit{all physics here!}}$$

Embeddings

Black Hole



Minkowski



- D7 enters horizon
- Metallic ferromagnetic phase
- This talk
- $\psi \sim \psi_\infty + \frac{m_\psi}{r^{\Delta_+}} + \frac{c_\psi}{r^{\Delta_-}}$

- D7 ends where S^2 shrinks
- QH phase, $\nu = \frac{2\pi D}{B} = f(\psi_\infty)$
- $\sigma_{xx} = 0 + e^{-m_{gap}/T}$, $\sigma_{yx} = \frac{\nu}{2\pi}$
- magnetoroton, anyon superfluid

- Choose wrapped S^2 fluxes: $\Delta_+ = 1$ and $\Delta_- = 2$.

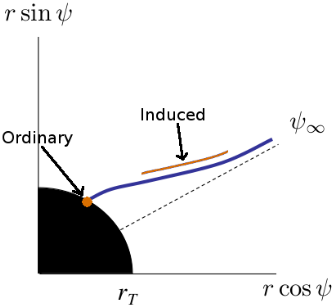
Radial charge distribution

$$\tilde{d}(r) = \frac{\delta S_{\mathrm{DBI}}}{\delta a_0'} = d_\infty - 2B\, c(\psi(r))$$

Total charge $= \tilde{d}(\infty) = d_\infty$

Split between:

- Ordinary (fractionalized) charge $= d_\infty - 2B\, c(\psi_T)$
- Induced (cohesive) charge $= 2B\, c(\psi_T)$



here CS contribution is $\propto$

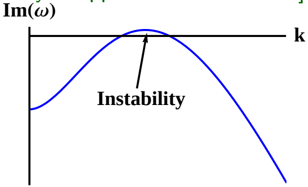
$$c(\psi(r)) = \psi(r) - \frac{1}{4}\sin(4\psi(r)) - \psi_\infty + \frac{1}{4}\sin(4\psi_\infty)\ , \ \ \psi \sim \text{axion}$$

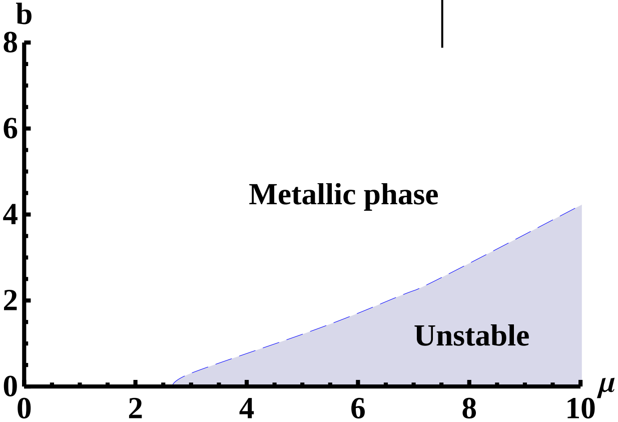
Striped instability

Analysis of quasinormal modes ⇒ instability (Nakamura-Ooguri-Park)

[Bergman-NJ-Lifschytz-Lippert arXiv:1106.3883]

- In the metallic phase at large charge density
- Finite wave number k ⇒ stripes





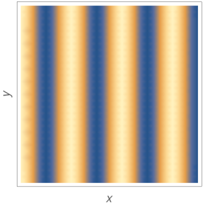
Striped ground state

D7 probe action

$$S = S_{\mathrm{DBI}} + S_{\mathrm{CS}} \quad = -T_7 \int d^8\xi \, e^{-\Phi} \sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} - \frac{(2\pi\alpha')^2 T_7}{2} \int P[C_4] \wedge F \wedge F$$

Look for inhomogeneous ground state

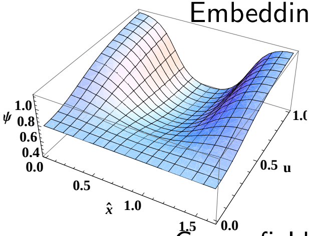
[Järvinen-NJ-Lippert 1408.1397]

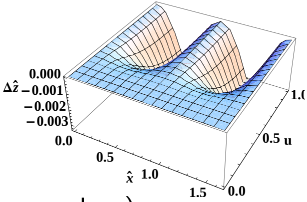


- Periodic with period L in x-direction (wave number $k = 2\pi/L$)
- Translation symmetry intact in y-direction
- Inhomogeneous embedding of the D7-brane $\psi(x,u)$, $z(x,u)$ and gauge fields $a_0(x,u)$, $a_y(x,u)$ with $u = r_T/r$.

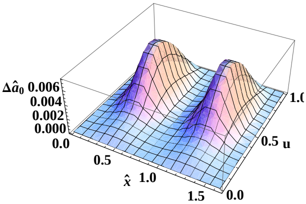
Example solution

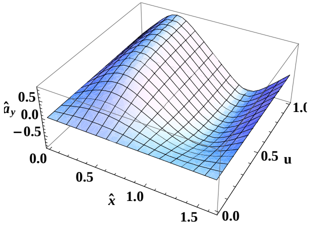
Embedding (ψ and Δz)





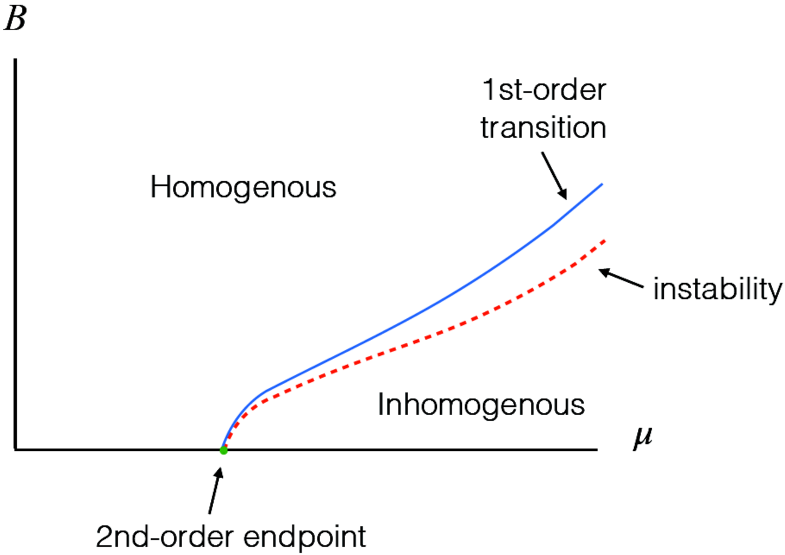
Gauge fields (Δa_0 and a_y)





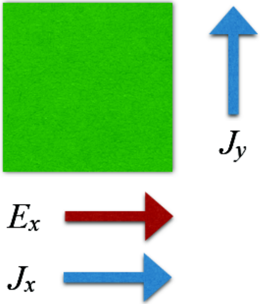
- "SDW" $\gg$ CDW
- Modulated persistent current $J_y(x)$ and magnetization $\frac{\partial \mathcal{L}}{\partial b}(x)$

Phase diagram



Homogeneous: DC Conductivities

- BH phase: metallic behavior
- Use Karch-O'Bannon method
  [Karch-O'Bannon 0705.3870]
- Yields nonlinear DC conductivity at finite (can be large) E_x



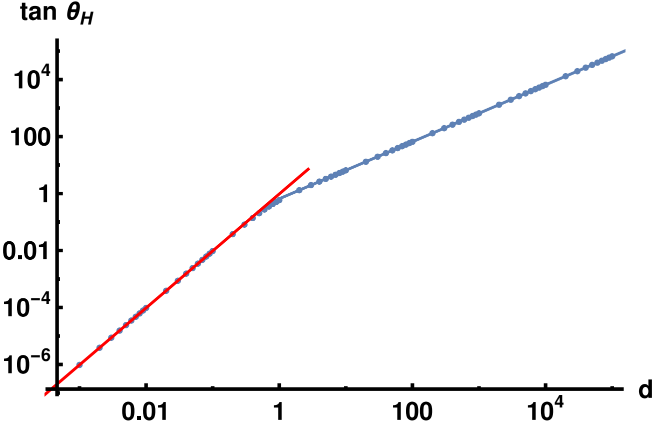
$$\sigma_{yx}|_{E_x=0} = \frac{1}{2\pi^2}\left(\frac{B}{T^4+B^2}\tilde{d}+2c(\psi_T)\right) \quad ; \qquad \tilde{d} = d_\infty - 2B\,c(\psi_T)$$

$$\sigma_{xx}|_{E_x=0} = \frac{1}{2\pi^2}\frac{T^2}{T^4+B^2}\sqrt{\tilde{d}^2+\mathrm{func}(\psi_T)\,(T^4+B^2)}$$

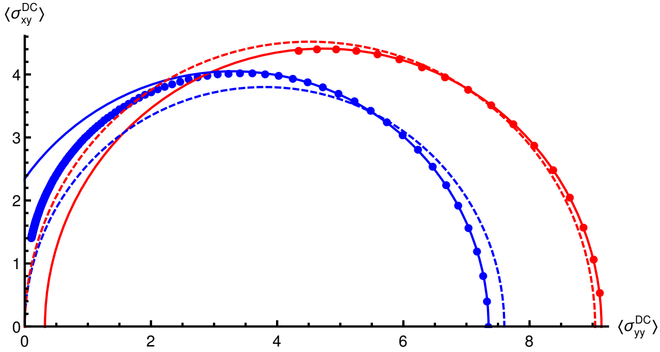
DC Conductivities

Hall angle

$$\tan\theta_H = \frac{\sigma_{yx}}{\sigma_{xx}} \sim \begin{cases} T^{-2} & , \ T \to 0 \\ T^{-4} & , \ T \to \infty \end{cases}$$



DC Conductivities



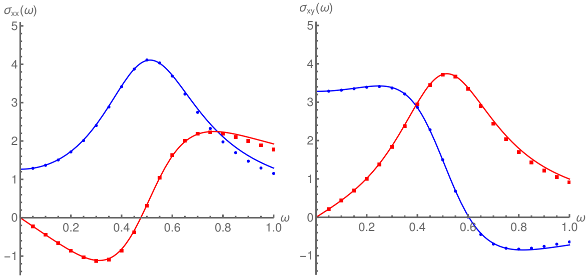
- Find novel semicirle law analytically (homogeneous):

$$\left(\sigma_{yy}^{DC} - \frac{\sqrt{2}\pi D}{\sqrt{\lambda} N_c T^2}\right)^2 + \left(\sigma_{yx}^{DC}\right)^2 = \left(\frac{\sqrt{2}\pi D}{\sqrt{\lambda} N_c T^2}\right)^2$$

  in the limit $D/T^2 \to \infty$ (N.B. $D \gg B$ huge filling fraction)
- Holds also for (unpinned) $\langle$striped$\rangle$ phases

Optical Conductivities

- Turn on oscillating electric field (DC limit $\omega \to 0$):$E_x e^{i\omega t}$



- Good fit to hydro model ($\omega_c \overset{...?}{=} \kappa_\omega B, \gamma \overset{...?}{=} \hat{\gamma} B^2$) also for (unpinned) stripes

[Hartnoll-Kovtun-Müller-Sachdev 0706.3215]
[Delacrétaz-Goutéraux-Hartnoll-Karlsson 1612.04381,1702.05104]

$$\sigma_{xx}|_{E_x=0} = \sigma_Q \left[\frac{(\omega+i/\tau)(\omega+i\gamma+i\omega_c^2/\gamma+i/\tau)}{(\omega+i\gamma+i/\tau)^2-\omega_c^2}\right] \underset{B\to 0}{\overset{...?}{=}} \sigma_Q + \frac{\sigma_D}{1-i\omega\tau}$$

$$\sigma_{yx}|_{E_x=0} = -\frac{\rho}{B}\left[\frac{\omega_c^2+\gamma^2-2i\gamma(\omega+i/\tau)}{(\omega+i\gamma+i/\tau)^2-\omega_c^2}\right] \underset{B\to 0}{\overset{...?}{=}} \frac{2\hat{\gamma}\rho\tau B}{1-i\omega\tau} - \frac{\rho\kappa_\omega^2 B}{(\omega+i/\tau)^2}$$

Striped phases: conductivities

Study time-dependent fluctuations of D7' fields

- $\delta f(t, x, u), \ f = \psi, z, a_t, a_x, a_y$
- All sources vanish, except δE_x or δE_y

Goldstone mode

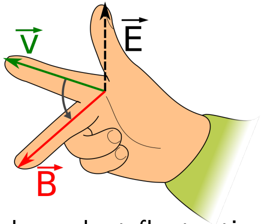
- Translation symmetry spontaneously broken
- For any solution

$$f(t, x, u) \to f(t, x + \kappa, u)$$

Striped phases: conductivities

- Stripes will slide:

$$v_s = v_x \delta E_x + \underbrace{v_y}_{B\neq 0:\text{Hall sliding}} \delta E_y$$



- Determine speeds e.g. from time-independent fluctuation [NJ-Järvinen-Lippert 1612.07323] [Goutéraux-NJ-Pönni 1803.03089]

$$\delta f(t,x,u) = \kappa \partial_x f(x,u)$$

- Even for $v_s \neq 0$ $\exists$ ($\langle$averaged$\rangle$) conserved currents $\mathcal{J}_i \propto \frac{\delta S}{\delta \partial_u \delta a_i}$:

$$\mathcal{J}_x = const.\ , \quad \lim_{u\to 0} \mathcal{J}_x = \delta j_x(x) - v_s d(x)$$
$$\langle \mathcal{J}_y \rangle = const.\ , \quad \lim_{u\to 0} \mathcal{J}_y = \delta j_y(x,\cancel{t}) + v_s t J_y'(x)$$

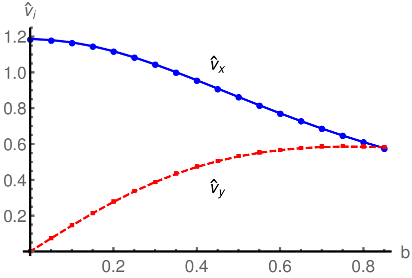
allowing to write ($\langle$averaged$\rangle$) $\sigma^{\mathrm{DC}}(x)$ at the BH horizon

- $\sigma^{AC}(x,\omega)$ numerically

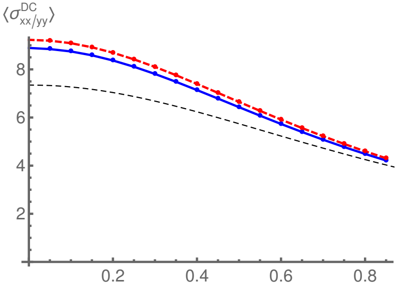
DC conductivities at finite b

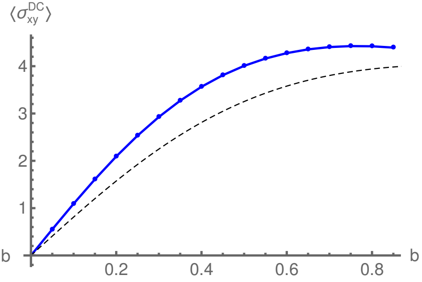
Magnetic field breaks parity
$\Rightarrow$ two sliding velocities
$v_s = v_x \delta E_x + v_y \delta E_y$

$$\langle \sigma^{\rm DC}_{xx} \rangle = \langle \hat{\sigma}^{-1} \rangle^{-1} + (\text{P-odd})\, \hat{v}_x b + (\text{P-even})\, \hat{v}_x$$
$$\langle \sigma^{\rm DC}_{xy} \rangle = (\text{P-odd})\,(1 - \hat{v}_y b) + (\text{P-even})\, \hat{v}_y$$
$$\langle \sigma^{\rm DC}_{yx} \rangle = (\text{P-odd}) + (\text{P-odd})\,(\hat{v}_x b) + (\text{P-even})\, \hat{v}_x$$
$$\langle \sigma^{\rm DC}_{yy} \rangle = (\text{P-even})\,(1 - \hat{v}_y b) + (\text{P-odd})\, \hat{v}_y$$



Again, match with zero frequency limit of optical conductivities:





Pinning

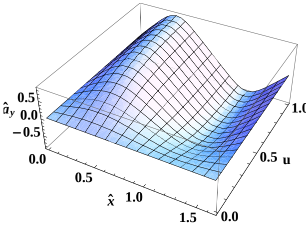
- Mimic impurities via explicit deformed bdry conditions:

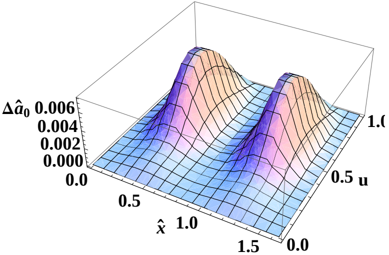
$$\text{Magnetic lattice :} \quad a_y(x, u=0) = bx + \alpha_b \sin(k_0 x)$$

OR

$$\text{Ionic lattice :} \quad a_t(x, u=0) = \mu + \alpha_\mu \cos(2k_0 x)$$

- Fixed lattice wavelength = dynamical wavelength

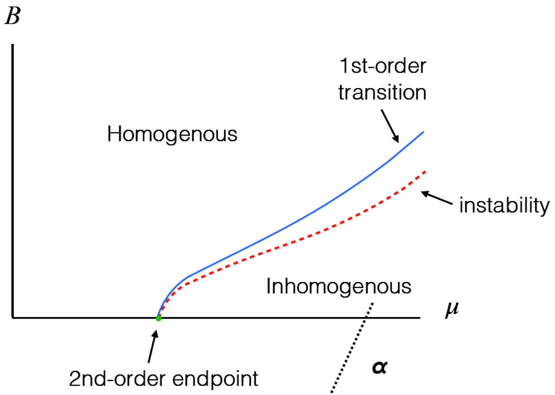




- Commensurability by hand; could take to be different

[Andrade-Krikun 1701.04625,. . .]

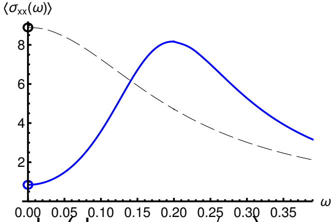
Pinning



- Expect to be in preferred phase for small α's

Glimpse on results

- Lifting of the Goldstone mode: $\alpha = 0 \to \alpha \neq 0$
- Height of the peak shrinks and width broadens



- x-conductivity drops by a decade (decrease w/ α)

$$\langle \sigma_{xx}^{DC,unpinned} \rangle \gg \langle \sigma_{xx}^{DC} \rangle$$
$$\langle \sigma_{xy}^{DC,unpinned} \rangle \gg \langle \sigma_{xy}^{DC} \rangle$$

- y-conductivity $\langle \sigma_{yy}^{DC} \rangle$ increases w/ α
- Pinned stripes also satisfy a ~~semicircle~~ law

$$\langle \sigma_{xx}^{\mathrm{DC}} \rangle \langle \sigma_{yy}^{\mathrm{DC}} \rangle + \langle \sigma_{xy}^{\mathrm{DC}} \rangle^2 = \langle \sigma_{xx}^{\mathrm{DC}} \rangle \sigma_{yy}^{0}$$

- Optical conductivities: tested hydro models for either
  - $\alpha = 0$: as above for homogeneous phase
  - $b = 0$: Drude-Lorentz, e.g. pseudo-Goldstone pole fits well

Discussion/outlook

- Understand the various novel (semi-circle) laws
- Measurables for this strongly coupled Hall fluid
- Probe branes vs. hydro
- Phase diagram for $\alpha \neq 0$: away from locking
- QNM, especially the y-dependent fluctuations: bubble phase?
- Construct stripes w/ finite E_x
- Transport of anyonic stripes
- Stripy quantum Hall phase?
- Backreaction

Back-up slides

Duality symmetry in QH

- Landau Level Addition Transformation (**L**)

$$\sigma_{yx}(\nu+1) \leftrightarrow \sigma_{yx}(\nu)+1\ ,\ \ \sigma_{xx}(\nu+1) \leftrightarrow \sigma_{xx}(\nu)$$

- Flux Attachment Transformation (**F**)

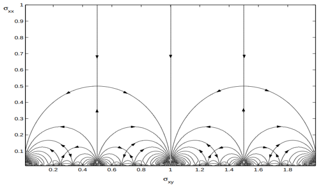
$$\rho_{yx}\left(\frac{\nu}{2\nu+1}\right) \leftrightarrow \rho_{yx}(\nu)+2\ ,\ \ \rho_{xx}\left(\frac{\nu}{2\nu+1}\right) \leftrightarrow \rho_{xx}(\nu)$$

- Particle-Hole Transformation (**P**)

$$\sigma_{yx}(1-\nu) \leftrightarrow 1-\sigma_{yx}(\nu)\ ,\ \ \sigma_{xx}(1-\nu) \leftrightarrow \sigma_{xx}(\nu)$$

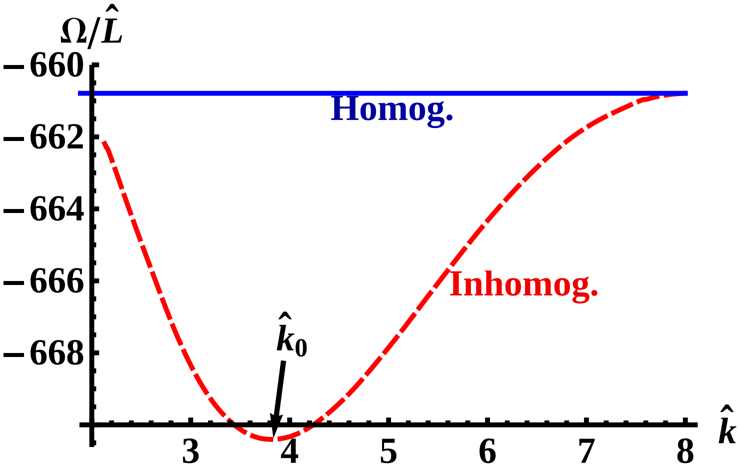
Obtain e.g. series

- Integer $\sigma = 1 \to n$ w/ $\mathbf{L}^{n-1}$
- Laughlin $\sigma = 1 \to \frac{1}{2m+1}$ w/ $\mathbf{F}^{m}$
- Jain $\sigma = 1 \to \frac{p}{2pm+1}$ w/ $\mathbf{F}^{m}\mathbf{L}^{p-1}$



Minimizing the energy

Grand canonical ensemble, $\Omega(\mu, b, L)/L$



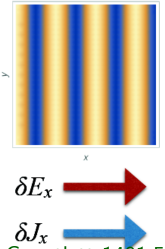
- Inhomogeneous state usually preferred
- Energy minimized at $\hat{k} = \hat{k}_0(\mu, b)$

DC conductivity: $\sigma_{\rm xx}$

Ansatz:

- Turn on constant electric field δE_x
- Must allow stripes to slide

$$\delta f(t,x,u) = \delta f(x,u) - v_s t \partial_x f(x,u)$$



[Donos-Gauntlett 1401.5077]

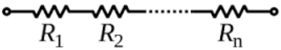
Follow "standard recipe"

- write bdry quantities (e.g. δJ_x) in terms of horizon data
- impose horizon regularity
- speed v_s not fixed

$$\sigma_{xx}(x) = \langle \hat{\sigma}_{xx}^{-1} \rangle^{-1} + \frac{v_s}{\delta E_x}\Big[\underbrace{d(x) - \langle d(x) \rangle}_{\sim CDW} + \underbrace{SDW}_{\gg CDW} + tiny \Big]$$

- "Local" conductivity

$$\langle \hat{\sigma}_{xx}^{-1} \rangle^{-1} \qquad \leftrightarrow$$



$\delta a_x = -(\delta E_x - p'(x))t + \delta a_x(x,u)$, $\delta a_t = p(x) + \delta a_t(x,u) - v_s t \partial_x a_t(x,u)$

DC conductivity: σ_{yx}

Turn on $\delta E_x, \delta J_y \Rightarrow$ Hall conductivity

- Sliding stripes with background current J_y

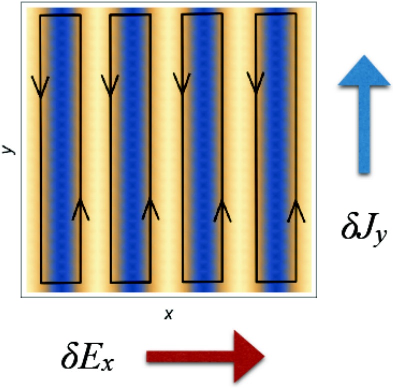
$$J_y(x - v_s t) \approx J_y(x) - v_s t J_y'(x)$$

- Gives modulated divergence

$$\sigma_{yx} = i v_s J_y'(x) \times \infty + \text{finite}$$

- Spatial average vanishes

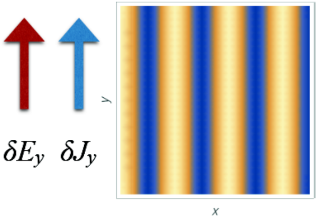
$$\langle \sigma_{yx} \rangle = 0$$



DC conductivity: σ_{yy} and σ_{xy}

Turn on δE_y

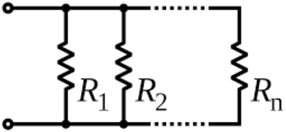
- Stripes don't move
- Parity $\Rightarrow \sigma_{xy}(x) = 0$
- No conserved bulk current for $\delta a_y(x)$



- But, spatially averaged current conserved

$$\langle \sigma_{yy} \rangle = \langle \hat{\sigma}(1 + \mathrm{small}) \rangle + \underbrace{\langle \sigma_{yy}^{SDW} \rangle}_{\text{dominant}}$$

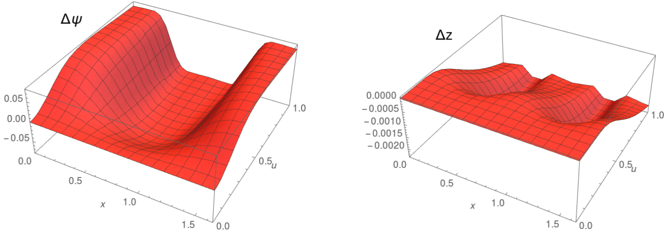
$\langle \hat{\sigma}(x) \rangle \quad \leftrightarrow$



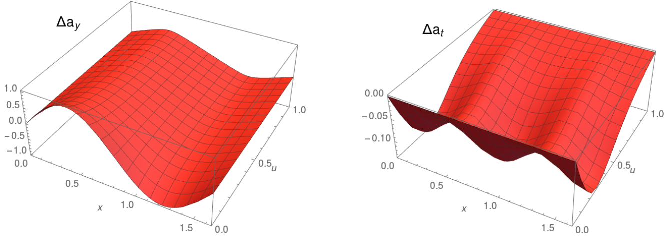
Pinning

Exemplified solutions ($\Delta f = f_{\alpha_b=1} - f_{\alpha_b=0}$):

Embedding ($\Delta\psi$ and Δz)

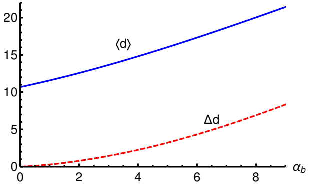


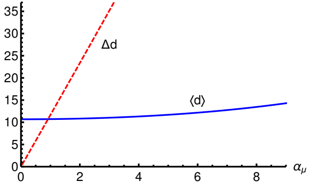
Gauge fields (Δa_y and Δa_t)



Pinning

- Deformed bdry conditions add in more charge





- DC conductivities (from slides 34 & 36):

$$\langle \sigma_{xx} \rangle = \langle \hat{\sigma}_{xx}^{-1} \rangle^{-1} + \delta_{\alpha_b,\alpha_\mu,0} \frac{v_s}{\delta E_x} \left[SDW + tiny \right]$$

$$\langle \sigma_{yy} \rangle = \langle \hat{\sigma}(1 + small) \rangle + \langle \sigma_{yy}^{SDW} \rangle$$

$$\langle \sigma_{xy} \rangle = 0 = \langle \sigma_{yx} \rangle$$

- Parametrically large wrt δE_x: stripes pinned w/ any α's

Optical conductivities

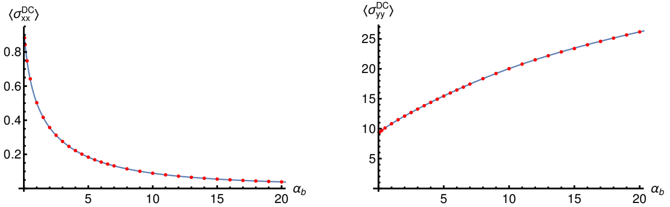
- Turn on electric field

$$\delta E_x e^{-i\omega t}$$
$$OR$$
$$\delta E_y e^{-i\omega t}$$

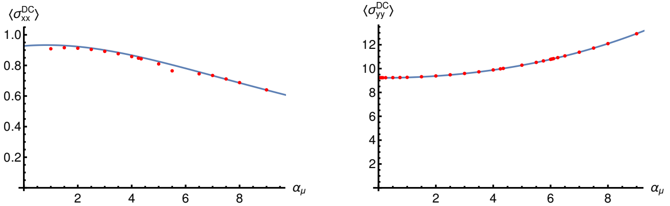
- Solve fluctuation EOM numerically w/ pseudospectral method
- Extract AC conductivities $\sigma_{ij}(\omega, x)$, $i, i = x, y$
- DC conductivities as $\omega \to 0$ limits of AC

DC limits

Magnetic lattice



Ionic lattice

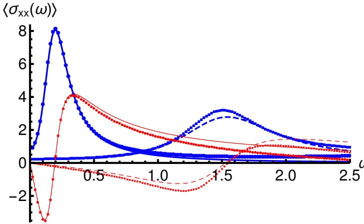


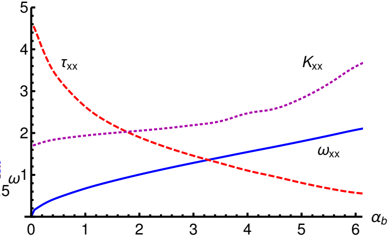
- Recall SDW $\gg$ CDW thus effect w/ $\alpha_b \gg \alpha_\mu$
- $\langle \sigma_{xx} \rangle \ll \langle \sigma_{yy} \rangle$; pinning
- $\langle \sigma_{xx} \rangle$ decreases with α; stripes are more strong
- $\langle \sigma_{yy} \rangle$ increases with α; more charge carriers around

Magnetic lattice: σ_{xx}

Captured by fitting to Drude-Lorentz model

$$\langle \sigma_{xx} \rangle = \frac{\langle \sigma_{xx}^{DC} \rangle}{1 - i\tau_{xx}\omega} + \frac{iK_{xx}\omega}{\omega^2 - \omega_{xx}^2 + i\omega/\tau_{xx}}$$



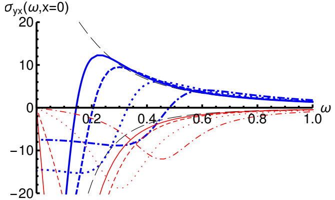


- Three parameter $K_{xx}, \tau_{xx}, \omega_{xx}$ fit; $\langle \sigma_{xx}^{DC} \rangle$ from analytics
- Small α_b, $\omega_{xx} \sim \alpha_b^{1/2}$ as in driven damped harmonic oscillator model

Magnetic lattice: σ_{yx}

- Captured by a modified Lorentzian

$$\sigma_{yx} = \frac{K_{yx}(x)/\tau_{yx}}{\omega^2 - \omega_{yx}^2 + i\omega/\tau_{yx}}$$



- Three parameter $K_{yx}(x) \sim \cos(2\pi x/L), \tau_{yx}, \omega_{yx}$ fit
- Find $\omega_{yx} \approx \omega_{xx}$ and $\tau_{yx} \approx \tau_{xx}$
- Notice that as $\omega_{yx} \to 0$:

$$\sigma_{yx}\Big|_{\alpha_b=0} = \frac{\tau_{yx} K_{yx}(x)}{1 - i\tau_{yx}\omega} - K_{yx}(x)\left(\frac{i}{\omega} + \delta(\omega)\right) \;,\; K_{yx}(x) = v_s J_y'(x)$$

Magnetic lattice: σ_{yy} and σ_{xy}

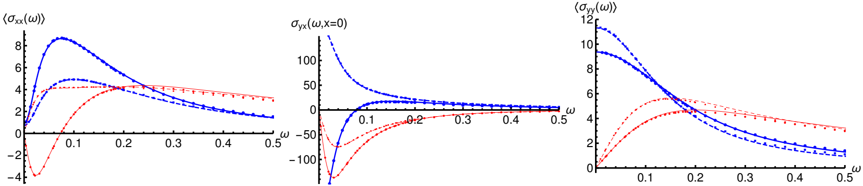
- Electric field in the y-direction: no pinning effects
- Captured by simple Drude

$$\langle \sigma_{yy} \rangle = \frac{\langle \sigma_{yy}^{DC} \rangle}{1 - i\tau_{yy}\omega}$$

- Parity: $\sigma_{xy} = 0$.

Ionic lattice: AC conductivities

- Small α_μ analogous to magnetic lattice; pinning much weaker
- Captured again by σ_{xx}: Drude-Lorentz , σ_{yx}: modified Lorentz, σ_{yy}: Drude
- Find Goldstone mode parameters $\omega_{xx} = \omega_{yx}$, $\tau_{xx} = \tau_{yx}$

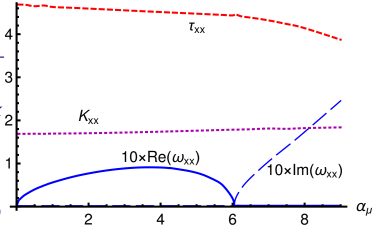


Ionic lattice: instability

$$\langle \sigma_{xx} \rangle = \frac{\langle \sigma_{xx}^{DC} \rangle}{1 - i\tau_{xx}\omega} + \frac{iK_{xx}\omega}{\omega^2 - \omega_{xx}^2 + i\omega/\tau_{xx}}$$

- Poles from the second term

$$\omega = -\frac{i}{2\tau_{xx}} \pm i\sqrt{\frac{1}{4\tau_{xx}^2} - \omega_{xx}^2}$$



- Get an instability for $\alpha_\mu \gtrsim 6$
- Interpretation?

What's up with hydro?

- Both homogeneous and unpinned striped phases fit well w/

$$\sigma_L = \sigma_Q \left[\frac{(\omega+i/\tau)(\omega+i\gamma+i{\omega_c}^2/\gamma+i/\tau)}{(\omega+i\gamma+i/\tau)^2-\omega_c^2}\right]$$

$$\sigma_H = -\frac{\rho}{B}\left[\frac{\omega_c^2+\gamma^2-2i\gamma(\omega+i/\tau)}{(\omega+i\gamma+i/\tau)^2-\omega_c^2}\right]$$

- Instead of 5 parameters, fit 8 parameters instead:

$$\sigma_L = c_L + \frac{\mathcal{R}_L+i\mathcal{I}_L}{\omega-\omega_c+i\Gamma} - \frac{\mathcal{R}_L-i\mathcal{I}_L}{\omega+\omega_c+i\Gamma}$$

$$\sigma_H = c_H + \frac{\mathcal{R}_H+i\mathcal{I}_H}{\omega-\omega_c+i\Gamma} - \frac{\mathcal{R}_H-i\mathcal{I}_H}{\omega+\omega_c+i\Gamma}$$

testing if $c_H \approx 0, \omega_c c_L \approx \mathcal{R}_L, \mathcal{R}_L\mathcal{R}_H \approx -\mathcal{I}_L\mathcal{I}_H$ gives above[1]

- Works for homogeneous but not for (unpinned) stripes

[1] Relations $c_L = \sigma_Q, \Gamma = \gamma + 1/\tau, \mathcal{R}_L = \sigma_Q\omega_c, \mathcal{I}_L = (\sigma_Q\omega_c^2 - \gamma^2\sigma_Q)/(2\gamma), \mathcal{R}_H = (\gamma^2\rho - \rho\omega_c^2)/(2\omega_c), \mathcal{I}_H = -\gamma\rho$