FRACTONS IN CURVED SPACE

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[2111.03973] AJ, K Jensen

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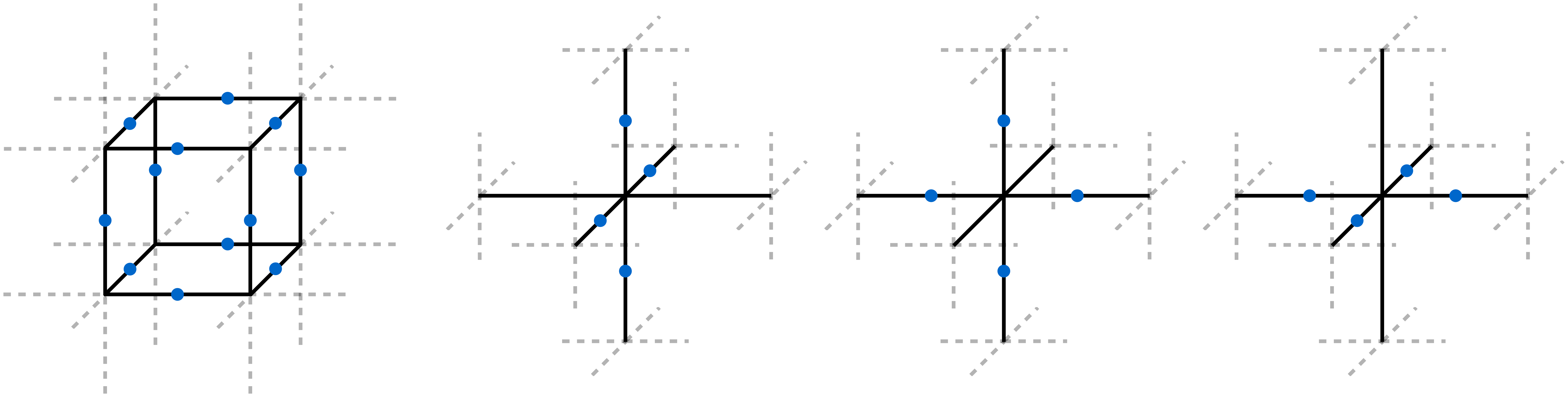
MOTIVATION

- Given a lattice model with local interactions, there typically exists an effective Quantum Field Theoretic description at low energy or long distances.
- However, recently, a large class of lattice spin models have surfaced that do not seem to admit a standard continuum field theoretic description [1].
- The most striking feature of these models are quasi-particle excitations with restricted mobility, e.g. **fractons** that are pinned to a point, **lineons** that are confined to a line, **planons** that are confined to a plane, etc.
- Physicists are still trying to understand how to adapt the field theoretic techniques to describe the continuum limit of fractonic systems [2].

[1] Haah [1101.1962]; Vijay, Haah, Fu [1505.02576, 1603.04442]; Pretko [1604.05329]
[2] Seiberg, Shao [2003.10466, 2004.00015]

X-CUBE MODEL



$$A_C = \prod_{\ell \in C} X_\ell \qquad B_V^x = \prod_{\ell \in V_{yz}} Z_\ell \qquad B_V^y = \prod_{\ell \in V_{zx}} Z_\ell \qquad B_V^z = \prod_{\ell \in V_{xy}} Z_\ell$$

$$X_\ell Z_\ell = - Z_\ell X_\ell$$
$$X_\ell^2 = Z_\ell^2 = 1$$

$$H = - \sum_C A_C - \sum_V \left(B_V^x + B_V^y + B_V^z \right)$$

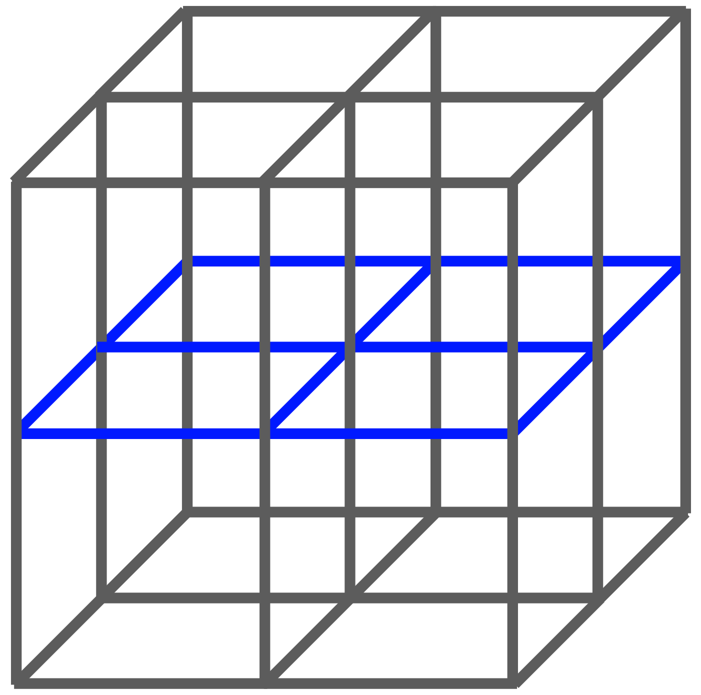
[1] Vijay, Haah, Fu (2016) [1603.04442].

X-CUBE MODEL

- **Large number of ground states.**
  On a 3-torus with periodic boundary conditions, the ground state degeneracy scales as

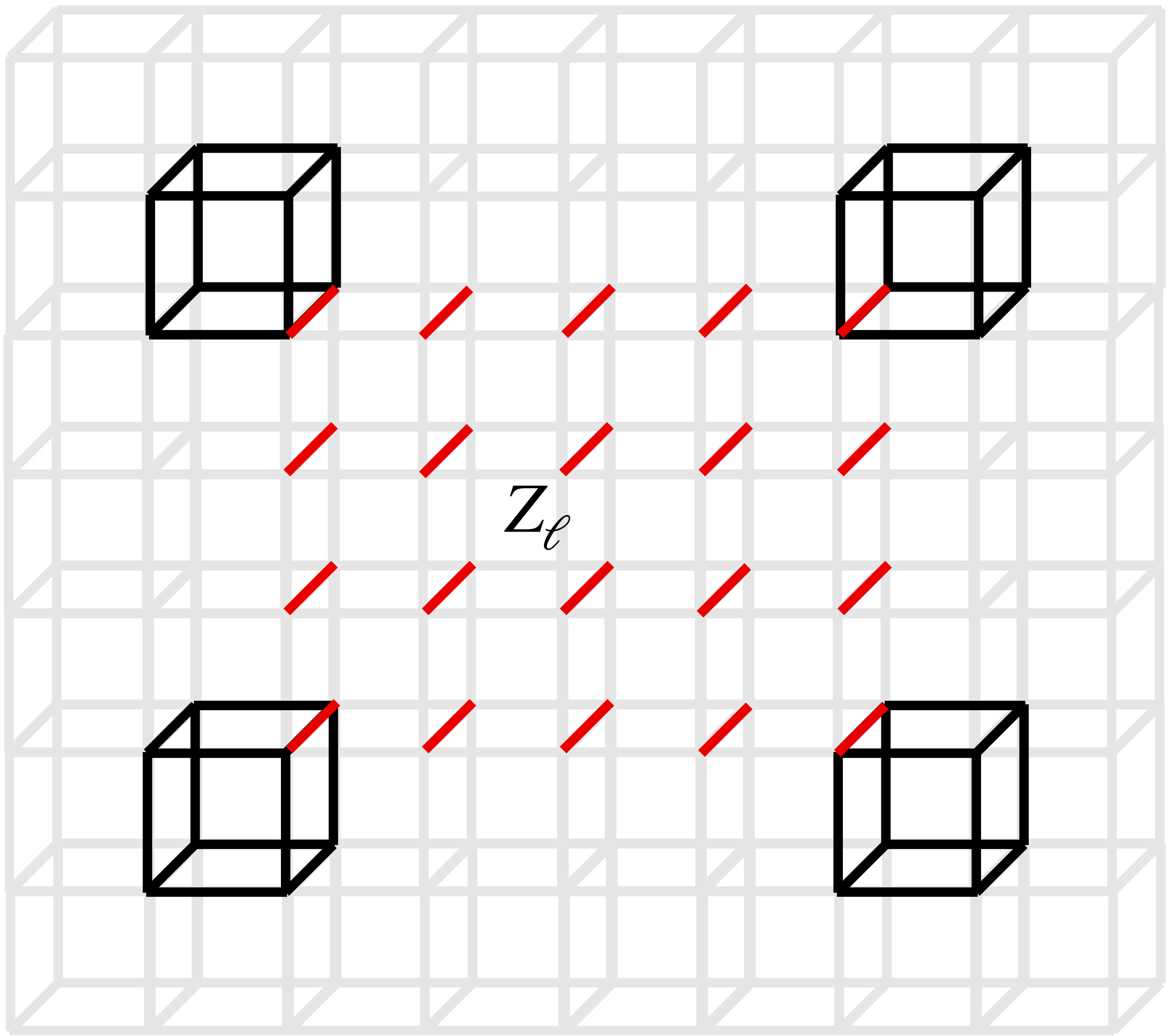
$$\#\,\textit{ground states} = 2^{2L_x+2L_y+2L_z-3}$$

- **Subsystem Symmetry.**
  The Hamiltonian is invariant under spin-flips acting independently on planes.

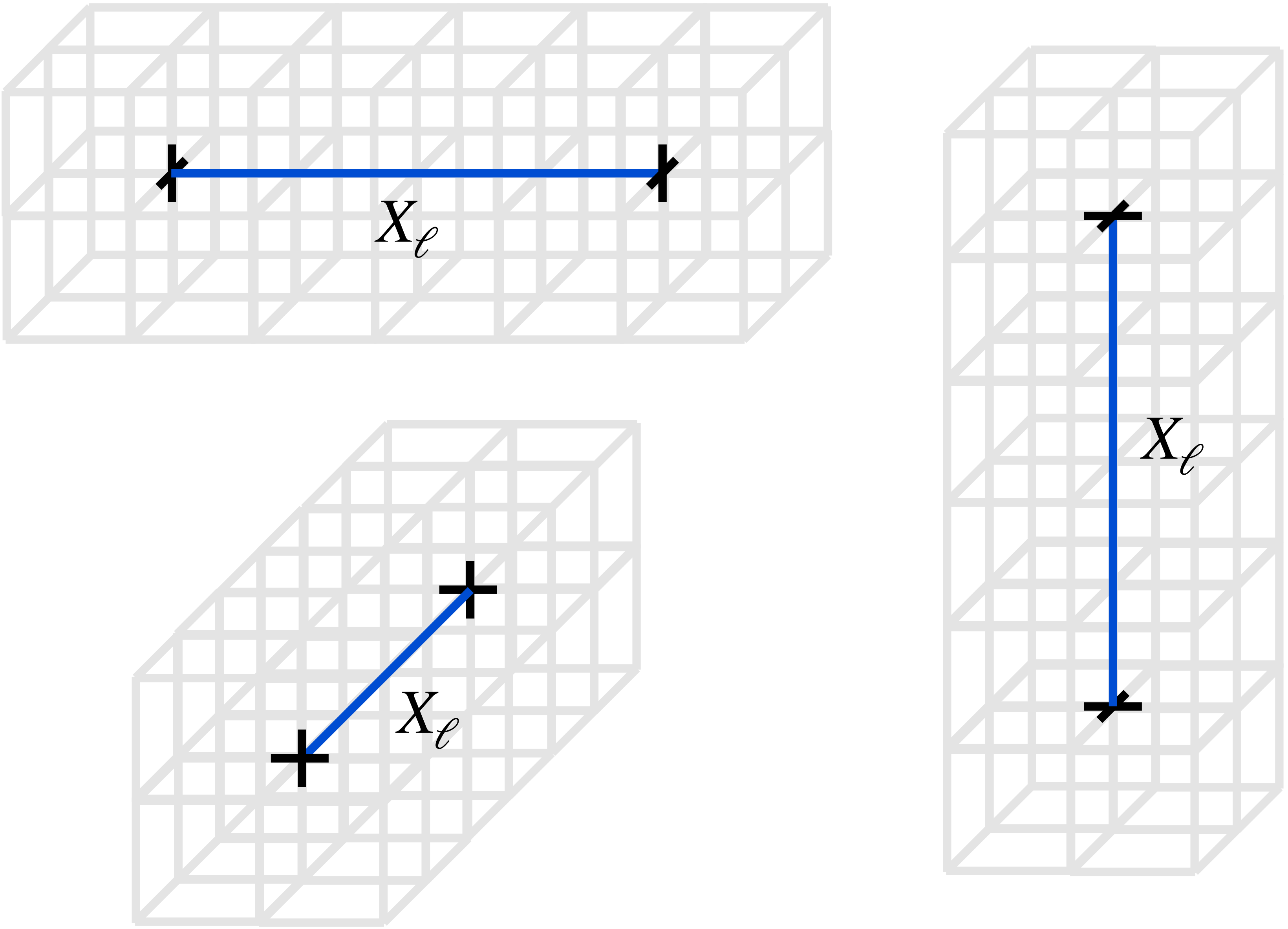


- **Restricted Mobility.**
  The model admits quasiparticle excitations that are unable to move freely in space.

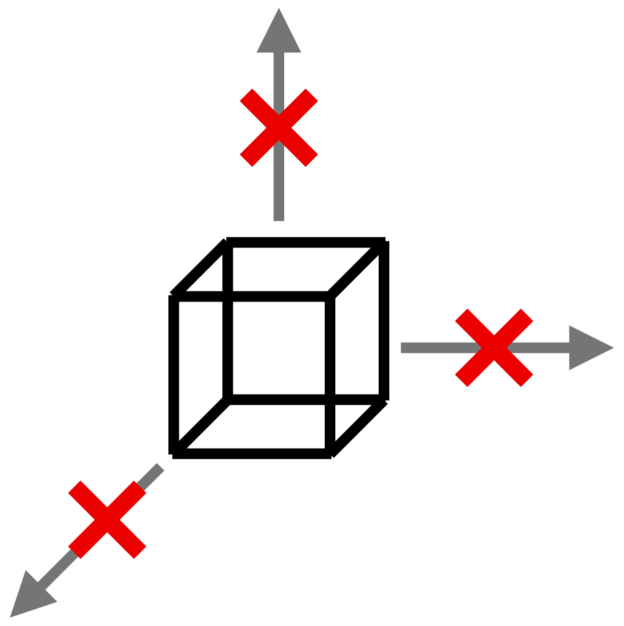
FRACTONS AND LINEONS IN X-CUBE MODEL



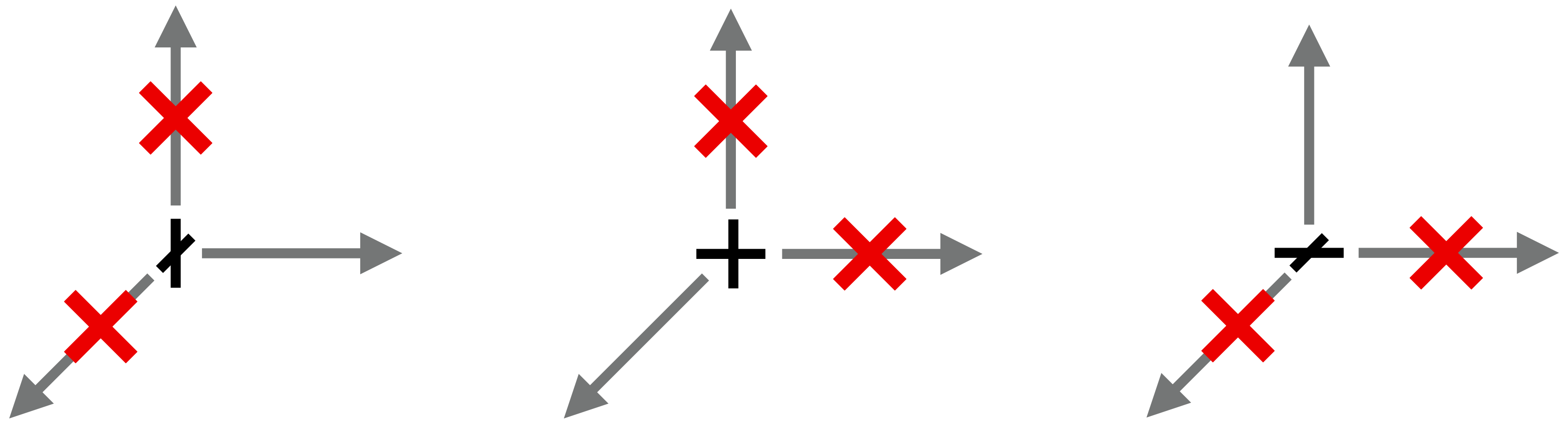
Fractons



Lineons



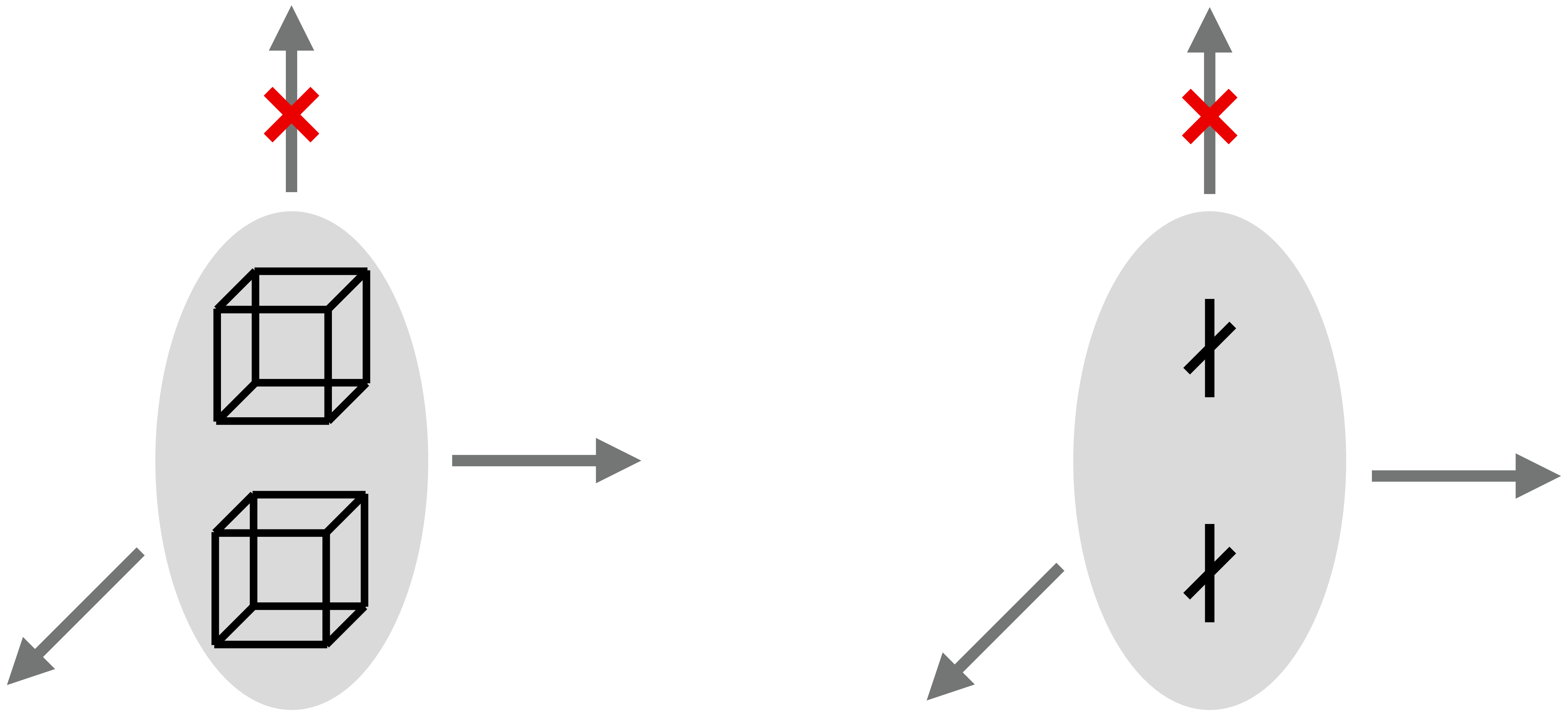
unable to move



can only move in one direction

FRACTONS AND LINEONS IN X-CUBE MODEL

- Dipolar bound states of fractons and lineons can move in a plane.



Planons

DIPOLE SYMMETRY

- From the point of view of continuum description, these exotic features can be understood as the consequence of **dipole** and **multipole** symmetries.
- If dipole moment is conserved in a field theory, charged excitations can only be created in quadrupoles. Once created, a charged excitation cannot move on its own without violating dipole moment conservation.
- In continuum, fractonic lattice models are described by a phase where the dipole symmetry is spontaneously broken.

DIPOLE SYMMETRY

- Consider a field theory with a conserved U(1) charge

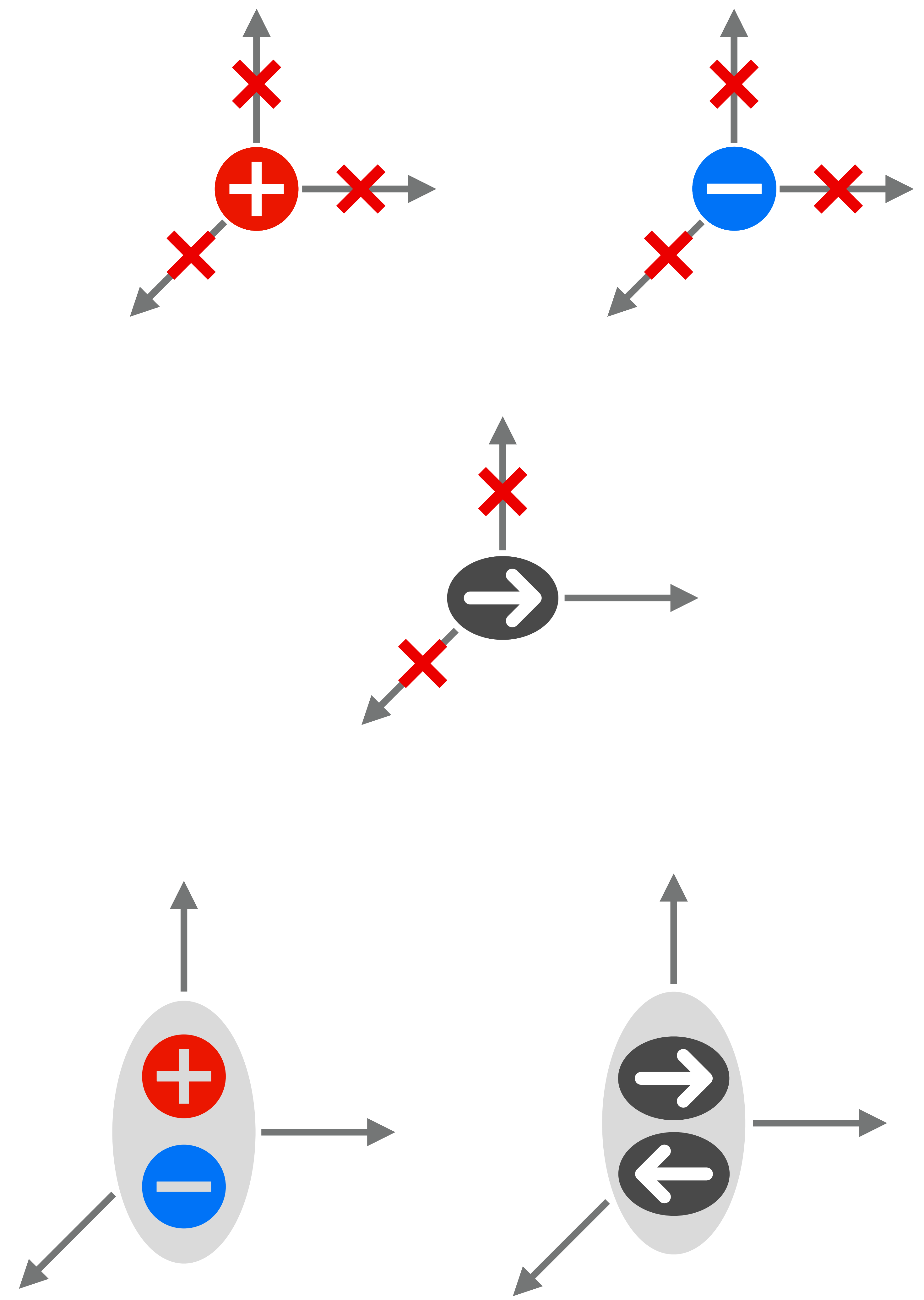
$$\partial_t J^t + \partial_i J^i = 0 \implies \frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, J^t = -\oint \mathrm{d}^2x\, J^\perp$$

such that the flux is $J^i = \partial_j J^{ij}$ with $J^{ij} = J^{ji}$.
It follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, J^t x^i = \oint \mathrm{d}^2x\left(J^{i\perp} - J^\perp x^i\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, J^i = \oint \mathrm{d}^2x\, \partial_t J^{i\perp}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, (J \times x)^i = \oint \mathrm{d}^2x\, \epsilon_{ijk}\partial_t J^{j\perp} x^k$$



DIPOLE SYMMETRY

- We will focus on field theories with conserved dipole moment.
- The dipole symmetry algebra is given as

$$[\mathrm{H}, \ldots] = 0 \qquad [\mathrm{Q}, \ldots] = 0$$

$$[\mathrm{P}_i, \mathrm{P}_j] = 0 \qquad [\mathrm{J}_i, \mathrm{P}_j] = i\epsilon_{ijk}\mathrm{P}_k \qquad [\mathrm{J}_i, \mathrm{J}_j] = i\epsilon_{ijk}\mathrm{J}_k$$

$$[\mathrm{J}_i, \mathrm{D}_j] = i\epsilon_{ijk}\mathrm{D}_k \qquad [\mathrm{P}_i, \mathrm{D}_j] = i\delta_{ij}\mathrm{Q} \qquad [\mathrm{D}_i, \mathrm{D}_j] = 0$$

- The associated Ward identities are

$$\mathrm{H}: \quad \partial_t \epsilon^t + \partial_i \epsilon^i = 0$$

$$\mathrm{P}_i: \quad \partial_t \pi^i + \partial_j \tau^{ij} = 0$$

$$\mathrm{Q}: \quad \partial_t J^t + \partial_i J^i = 0$$

$$\mathrm{J}_i: \quad \epsilon_{ijk}\tau^{jk} = 0 \implies \partial_t\left(\epsilon_{ijk}\pi^j x^k\right) = -\,\partial_l\left(\epsilon_{ijk}\tau^{jl}x^k\right)$$

$$\mathrm{D}_i: \quad J^i = \partial_j J^{ij} \implies \partial_t\left(J^t x^i\right) = \partial_j\left(J^{ij} - J^j x^i\right)$$

SCALAR CHARGE THEORY

- Consider the scalar field theory [1]

$$S = \int \mathrm{d}t\,\mathrm{d}^3x \left(i\Phi^*\partial_t\Phi + \lambda\,\mathrm{D}_{ij}(\Phi^*,\Phi^*)\mathrm{D}^{ij}(\Phi,\Phi) - V(\Phi^*\Phi) \right)$$

$$\mathrm{D}_{ij}(\Phi,\Phi) = \Phi\partial_i\partial_j\Phi - \partial_i\Phi\partial_j\Phi$$

It is invariant under global monopole and dipole transformations

$$\Phi \to \exp(iq\Lambda)\,\Phi \qquad \Phi \to \exp(iq\psi_i x^i)\,\Phi$$

- The monopole and dipole conserved currents are given as

$$J^t = \Phi^*\Phi$$

$$J^i = \partial_j\Big(i\lambda\,\mathrm{D}^{ij}(\Phi^*,\Phi^*)\Phi^2 + c.c. \Big)$$

$$J^{ij} = i\lambda\,\mathrm{D}^{ij}(\Phi^*,\Phi^*)\Phi^2 + c.c.$$

$$\partial_t J^t + \partial_i J^i = 0$$

$$\partial_j J^{ij} = J^i$$

$$\Phi = \bar{\Phi}\,\mathrm{e}^{i\varphi}$$

$$\begin{aligned}&\mathrm{D}_{ij}(\Phi,\Phi)\\ &= \mathrm{e}^{2i\varphi}\mathrm{D}_{ij}(\bar{\Phi},\bar{\Phi})\\ &\quad + i\bar{\Phi}^2\mathrm{e}^{2i\varphi}\partial_i\partial_j\varphi\end{aligned}$$

SCALAR CHARGE THEORY

- We can gauge the monopole and dipole symmetries using a set of gauge fields

$$A_t \to A_t + \partial_t \Lambda, \qquad A_i \to A_i + \partial_i \Lambda + \psi_i, \qquad a_{ij} \to a_{ij} + \partial_i \psi_j + \partial_j \psi_i$$

Note: we can gauge fix the dipole symmetry by setting $A_i = 0$ leading to $\psi_i = -\partial_i \Lambda$.

- The modified scalar field theory is

$$S = \int \mathrm{d}t\, \mathrm{d}^3x \left(i\Phi^* \mathrm{D}_t \Phi + \lambda\, \mathrm{D}_{ij}(\Phi^*, \Phi^*) \mathrm{D}^{ij}(\Phi, \Phi) - V(\Phi^* \Phi) \right) + S_{gauge}$$

$$\mathrm{D}_t \Phi = \partial_t \Phi - iqA_t \Phi, \qquad \mathrm{D}_i \Phi = \partial_i \Phi - iqA_i \Phi$$

$$\mathrm{D}_{ij}(\Phi, \Phi) = \Phi \mathrm{D}_i \mathrm{D}_j \Phi - \mathrm{D}_i \Phi \mathrm{D}_j \Phi - \frac{iq}{2} a_{ij} \Phi^2$$

J^t, J^i, J^{ij} can be obtained by varying the action with respect to A_t, A_i, a_{ij}.

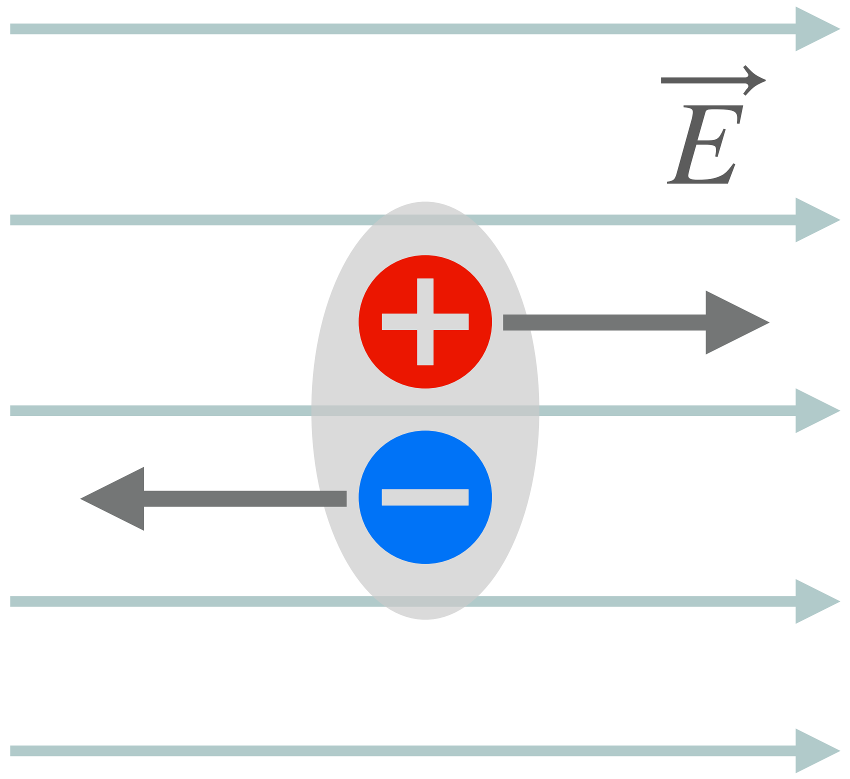
SYMMETRIC TENSOR GAUGE THEORY

- The monopole field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is not dipole-invariant

$$E_i = F_{it} \to E_i - \partial_t \psi_i, \qquad B^i = \frac{1}{2}\epsilon^{ijk} F_{jk} \to B^i + \epsilon^{ijk}\partial_j \psi_k$$

$$S_{gauge} = \int \mathrm{d}t\, \mathrm{d}^3x \left(\frac{\epsilon_0}{2} E_i E^i - \frac{1}{2\mu_0} B_i B^i \right)$$

not allowed



- Define dipole gauge field and field strength

$$A_t^k = F_{tj}\delta^{jk}, \qquad A_i^k = \frac{1}{2}\left(F_{ij} + a_{ij}\right)\delta^{jk} \qquad \Longrightarrow \qquad A_\mu^k \to A_\mu^k + \partial_\mu \psi^k$$

$$F_{\mu\nu}^k = \partial_\mu A_\nu^k - \partial_\nu A_\mu^k$$

Dipole field strength is invariant under both monopole and dipole transformations.

SYMMETRIC TENSOR GAUGE THEORY

- We can use the dipole field strength $F^k_{\mu\nu}$ to obtain a dipole-invariant gauge theory

$$E_{ij} = 2F^k_{it}\,\delta_{kj} = -\,\partial_t a_{ij} - \partial_i E_j - \partial_j E_i$$

$$B^{kl} = \epsilon^{kij} F^l_{ij} = \epsilon^{kij}\partial_i a_j{}^l - \partial^l B^k$$

$$S_{gauge} = \int \mathrm{d}t\,\mathrm{d}^3x \left(\frac{\epsilon_0}{4} E_{ij}E^{ij} - \frac{1}{2\mu_0} B_{ij}B^{ij} \right)$$

- The gauge field equations of motion lead to

$$\partial_i\partial_j E^{ij} = \frac{1}{\epsilon_0} J^t, \qquad \epsilon^{kli}\partial_k B_l{}^j + \epsilon^{klj}\partial_k B_l{}^i = \mu_0 \left(J^{ij} + \epsilon_0 \partial_t E^{ij} \right)$$

- We will see that there is an obstruction is coupling this theory to curved space.

[1] Pretko [1604.05329]

COUPLING TO CURVED BACKGROUND

- We wish to couple quantum field theories with conserved dipole moment to curved spacetime background.
- These theories have no boost symmetry — Galilean or Lorentzian. Therefore the "observer" or "reference frame" makes an integral part of the spacetime geometry. Must couple to **Aristotelian spacetimes** [1].
- We can use variations with respect to the spacetime sources to obtain the spacetime conserved currents: energy density/flux, momentum density, and stress tensor.

[1] de Boer, Hartong, Have, Obers, Sybesma [1710.04708]

ARISTOTELIAN SPACETIMES

- Aristotelian background sources:

Clock-form: n_μ, Frame-vector: v^μ

Spatial (co-)metric: $h_{\mu\nu}$, $h^{\mu\nu}$

Gauge field: A_μ

$$n_\mu h^{\mu\nu} = v^\mu h_{\mu\nu} = 0$$

$$v^\mu n_\mu = 1, \quad h_{\mu\lambda}h^{\lambda\nu} + n_\mu v^\rho = \delta_\mu^\nu$$

$$h_{\mu\nu} = h_{\nu\mu}, \quad h^{\mu\nu} = h^{\nu\mu}$$

Non-covariant notation: n_t, n_i, h_{ij}, v^i, A_t, A_i

Flat limit: $n_t = 1$, $h_{ij} = \delta_{ij}$, $n_i = v^i = 0$, $A_t = 0$, $A_i = 0$

- Connection:

$$\Gamma^\lambda_{\mu\nu} = v^\lambda \partial_\mu n_\nu + \frac{1}{2} h^{\lambda\rho}\left(\partial_\mu h_{\nu\rho} + \partial_\nu h_{\mu\rho} - \partial_\rho h_{\mu\nu}\right)$$

$$\nabla_\mu n_\nu = 0, \quad \nabla_\mu h^{\nu\rho} = 0, \qquad \nabla_\mu v^\nu \neq 0, \quad \nabla_\mu h_{\nu\rho} \neq 0$$

[1] de Boer, Hartong, Have, Obers, Sybesma [1710.04708, 2004.10759]; [2] Novak, Sonner, Withers [1911.02578]; [3] Armas, AJ [2010.15782]

ARISTOTELIAN SPACETIMES

- We demand invariance under background diffeomorphisms and gauge transformations

$$n_\mu \to n_\mu + L_\chi n_\mu, \qquad h_{\mu\nu} \to h_{\mu\nu} + L_\chi h_{\mu\nu} \qquad A_\mu \to A_\mu + L_\chi A_\mu + \partial_\mu \Lambda$$

$$v^\mu \to v^\mu + L_\chi v^\mu, \qquad h^{\mu\nu} \to h^{\mu\nu} + L_\chi h^{\mu\nu}$$

L_χ: Lie derivative along χ^μ

- Leads to Ward identities

$$\delta \ln \mathcal{Z} = \int dt\, d^3x \sqrt{\gamma} \left[J^\mu \delta A_\mu - \epsilon^\mu \delta n_\mu - \pi_\mu \delta v^\mu + \frac{1}{2} \tau^{\mu\nu} \delta h_{\mu\nu} \right]$$

$$\pi_\mu v^\mu = 0, \quad \pi^\mu \equiv h^{\mu\nu} \pi_\nu$$

$$\tau^{\mu\nu} = \tau^{\nu\mu}, \ \tau^{\mu\nu} n_\nu = 0$$

$$\begin{aligned} \nabla'_\mu \epsilon^\mu &= -\, v^\mu f_\mu - \tau^{\mu\nu} h_{\lambda\nu} \nabla_\mu v^\lambda \\ \nabla'_\mu \left(v^\mu \pi^\nu + \tau^{\mu\nu} \right) &= h^{\nu\mu} f_\mu - \pi_\mu h^{\nu\lambda} \nabla_\lambda v^\mu \\ \nabla'_\mu J^\mu &= 0 \end{aligned} \quad \xrightarrow{\text{flat}} \quad \begin{aligned} \partial_t \epsilon^t + \partial_i \epsilon^i &= 0 \\ \partial_t \pi^i + \partial_j \tau^{ij} &= 0 \\ \partial_t J^t + \partial_i J^i &= 0 \end{aligned}$$

$$\gamma = \det(h_{\mu\nu} + n_\mu n_\nu)$$

$$\nabla'_\mu = \nabla_\mu - L_\nu n_\mu$$

f_μ: generalised Lorentz force

[1] de Boer, Hartong, Have, Obers, Sybesma [1710.04708, 2004.10759]; [2] Novak, Sonner, Withers [1911.02578]; [3] Armas, AJ [2010.15782]

ASIDE: BOOST SYMMETRIES

- For Galilean-invariant systems, we can demand invariance under *infinitesimal* Milne boosts

$$n_\mu \to n_\mu, \qquad h_{\mu\nu} \to h_{\mu\nu} - n_\mu \alpha_\nu - n_\nu \alpha_\mu \qquad\qquad A_\mu \to A_\mu + m\,\alpha_\mu$$

$$v^\mu \to v^\mu + \alpha^\mu, \qquad h^{\mu\nu} \to h^{\mu\nu}$$

$$\psi_\mu v^\mu = 0$$

$$\psi^\mu = h^{\mu\nu}\psi_\nu$$

This implies $\pi_\mu = m\, h_{\mu\nu} J^\nu$, or in the flat limit $\pi^i = m\, J^i$.

- For relativistic systems, we can demand invariance under *infinitesimal* Lorentz boosts

$$n_\mu \to n_\mu - \frac{1}{c^2}\alpha_\mu, \qquad h_{\mu\nu} \to h_{\mu\nu} - n_\mu \alpha_\nu - n_\nu \alpha_\mu \qquad\qquad A_\mu \to A_\mu$$

$$v^\mu \to v^\mu + \alpha^\mu, \qquad h^{\mu\nu} \to h^{\mu\nu} + \frac{1}{c^2}\left(v^\mu \alpha^\nu + \alpha^\mu v^\nu\right)$$

This implies $\pi_\mu = h_{\mu\nu}\epsilon^\nu / c^2$, or in the flat limit $\pi^i = \epsilon^i / c^2$.
We can define the relativistic metric: $g_{\mu\nu} = -\, c^2 n_\mu n_\nu + h_{\mu\nu}$.

CONSERVED DIPOLES IN ARISTOTELIAN SPACETIMES

- Introduce dipole source $a_{\mu\nu}$ and dipole shift parameter ψ_μ

$$A_\mu \to A_\mu + L_\chi A_\mu + \partial_\mu \Lambda + \psi_\mu$$

$$a_{\mu\nu} \to a_{\mu\nu} + L_\chi a_{\mu\nu} + h^\rho_\mu h^\sigma_\nu \left(\nabla_\rho \psi_\sigma + \nabla_\sigma \psi_\rho \right)$$

$$a_{\mu\nu} = a_{\nu\mu}, \quad v^\mu a_{\mu\nu} = 0$$

$$\psi_\mu v^\mu = 0$$

$$h^\mu_\nu = h^{\mu\lambda} h_{\lambda\nu}$$

- The Ward identities are now

$$\delta \ln \mathcal{Z} = \int dt\, d^3x \sqrt{\gamma} \left[\ldots + \frac{1}{2} J^{\mu\nu} \delta a_{\mu\nu} \right]$$

$$\nabla'_\mu \epsilon^\mu = -v^\mu f_\mu - \left(\tau^{\mu\nu} + \tau^{\mu\nu}_{\mathrm{d}} \right) h_{\lambda\nu} \nabla_\mu v^\lambda$$

$$\nabla'_\mu \left(v^\mu \pi^\nu + \tau^{\mu\nu} + \tau^{\mu\nu}_{\mathrm{d}} \right) = h^{\nu\mu} f_\mu - \pi_\mu h^{\nu\lambda} \nabla_\lambda v^\mu$$

$$\nabla'_\mu J^\mu = 0$$

$$\nabla'_\mu J^{\mu\nu} = h^\nu_\mu J^\mu$$

$$\nabla'_\mu = \nabla_\mu - L_v n_\mu$$

f_μ: generalised Lorentz force

$\tau^{\mu\nu}_{\mathrm{d}}$: asymmetric dipole stress

CONSERVED DIPOLES ON ARISTOTELIAN SPACETIMES

- All the conserved densities and fluxes are invariant under U(1) monopole transformations and transform appropriately under diffeomorphisms.
- Under dipole shifts, their transformation properties are given as

$$\epsilon^\mu \to \epsilon^\mu + \left(2J^{\mu(\rho}\psi^{\sigma)} - J^{\rho\sigma}\psi^\mu\right)\frac{1}{2}L_v h_{\rho\sigma}$$

$$\pi^\mu \to \pi^\mu - (J^\nu n_\nu)\psi^\mu + J^{\rho\mu}F^n_{\rho\sigma}\psi^\sigma$$

$$\tau^{\mu\nu} \to \tau^{\mu\nu} - 2J^\lambda h^{(\mu}_\lambda \psi^{\nu)} + \nabla'_\lambda(\psi^\lambda J^{\mu\nu})$$

$\xrightarrow{\text{flat}}$

$$\epsilon^t \to \epsilon^t, \qquad \epsilon^i \to \epsilon^i$$

$$\pi^i \to \pi^i - J^t\psi^i$$

$$\tau^{ij} \to \tau^{ij} - 2J^{(i}\psi^{j)} + \partial_k(\psi^k J^{ij})$$

- Note that momentum density π^i and stress-tensor τ^{ij} are non-invariant under dipole shift symmetry, even on flat spacetime.

SCALAR CHARGE THEORY ON CURVED SPACE

- We can write the covariant version of the scalar charge theory

$$S = \int \mathrm{d}t\, \mathrm{d}^3x \sqrt{\gamma} \left(i\Phi^* v^\mu \mathrm{D}_\mu \Phi + \lambda\, h^{\mu\rho} h^{\nu\sigma}\, \mathrm{D}_{\mu\nu}(\Phi^*, \Phi^*) \mathrm{D}_{\rho\sigma}(\Phi, \Phi) - V(\Phi^*\Phi) \right) + S_{gauge}$$

$$\mathrm{D}_\mu \Phi = \partial_\mu \Phi - iqA_\mu \Phi$$

$$\mathrm{D}_{\mu\nu}(\Phi, \Phi) = \frac{1}{2} h^\rho_\mu h^\sigma_\nu \left(\Phi \mathrm{D}_\mu \mathrm{D}_\nu \Phi + \Phi \mathrm{D}_\nu \mathrm{D}_\mu \Phi - 2\mathrm{D}_\nu \Phi \mathrm{D}_\mu \Phi \right) - \frac{iq}{2} a_{\mu\nu} \Phi^2$$

- We can vary with respect to background sources to read out the conserved currents.

SYMMETRIC TENSOR GAUGE THEORY ON CURVED SPACE?

- The covariantised definition of the dipole connection is

$$A_\mu^\lambda = n_\mu v^\rho F_{\rho\sigma} h^{\sigma\lambda} + \frac{1}{2}\left(h_\mu^\rho F_{\rho\sigma} h^{\sigma\lambda} + a_{\mu\sigma} h^{\sigma\lambda}\right) \qquad A_t^k = F_{tj}\delta^{jk}, \qquad A_i^k = \frac{1}{2}\left(F_{ij} + a_{ij}\right)\delta^{jk}$$

$$A_\mu^\lambda \to A_\mu^\lambda + \nabla_\mu \psi^\lambda + n_\mu \psi^\nu \nabla_\nu v^\lambda \qquad A_\mu^k \to A_\mu^k + \partial_\mu \psi^k$$

We note that the dipole connection does not transform "nicely" anymore.

- Consequently, covariant dipole field strength is not invariant under dipole transformations

$$F_{\mu\nu}^\lambda = \nabla_\mu A_\nu^\lambda - \nabla_\nu A_\mu^\lambda + F_{\mu\nu}^n v^\rho A_\rho^\lambda + 2n_{[\mu} A_{\nu]}^\rho \nabla_\rho v^\lambda \qquad F_{\mu\nu}^k = \partial_\mu A_\nu^k - \partial_\nu A_\mu^k$$

$$F_{\mu\nu}^\lambda \to F_{\mu\nu}^\lambda + \left(R^\lambda{}_{\rho\mu\nu} + F_{\mu\nu}^n \nabla_\rho v^\lambda - 2n_{[\mu}\nabla_{\nu]}\nabla_\rho v^\lambda\right)\psi^\rho \qquad F_{\mu\nu}^k \to F_{\mu\nu}^k$$

$$S_{gauge} = \int dt\, d^3x \sqrt{\gamma}\, h_{\lambda\tau} h^{\nu\sigma} F_{\mu\nu}^\lambda F_{\rho\sigma}^\tau \left(\epsilon_0 v^\mu v^\rho - \frac{1}{\mu_0} h^{\mu\rho}\right) \quad \text{not allowed} \qquad \int dt\, d^3x \left(\frac{\epsilon_0}{4} E_{ij}E^{ij} - \frac{1}{2\mu_0} B_{ij}B^{ij}\right)$$

SYMMETRIC TENSOR GAUGE THEORY ON CURVED SPACE?

- We can write down dipole-invariant terms coupled to the charged scalar

$$S_{gauge} = \int \mathrm{d}t\, \mathrm{d}^3x \sqrt{\gamma}\, h_{\lambda\tau}\, h^{\nu\sigma}\, \mathscr{F}^{\lambda}_{\mu\nu}\, \mathscr{F}^{\tau}_{\rho\sigma} \left(\epsilon_0' v^\mu v^\rho - \frac{1}{\mu_0'} h^{\mu\rho} \right)$$

$$\mathscr{F}^{\lambda}_{\mu\nu} = \Phi^*\Phi\, F^{\lambda}_{\mu\nu} - \frac{i}{2q} h^{\rho\sigma} \left(\Phi^* \mathrm{D}_\rho \Phi - \Phi \mathrm{D}_\rho \Phi^* \right) \left(R^{\lambda}{}_{\sigma\mu\nu} + F^{n}_{\mu\nu} \nabla_\sigma v^\lambda - 2 n_{[\mu} \nabla_{\nu]} \nabla_\sigma v^\lambda \right)$$

- In the Higgs phase for the charged scalar, this gives rise to the flat space limit

$$S_{gauge} = \int \mathrm{d}t\, \mathrm{d}^3x \left(\frac{\epsilon_0}{4} E_{ij} E^{ij} - \frac{1}{2\mu_0} B_{ij} B^{ij} \right) + interactions$$

$$\epsilon_0 = |\Phi_0|^2 \epsilon_0'$$

$$\mu_0 = \frac{1}{|\Phi_0|^2} \mu_0'$$

OUTLOOK

- Continuum description of fractonic lattice models feature exotic dipole and multipole symmetries.
- We have learnt how to couple field theories with conserved dipole (and multipole) moment to curved spacetime.
- Fracton field theories have no boost invariance, therefore they can only be coupled to Aristotelian spacetimes.
- Free symmetric tensor gauge theory cannot be coupled to a generic curved spacetime.

OUTLOOK

- Is there a mixed dipole-gravitational anomaly in free symmetric tensor gauge theory? [1]
- Curved spacetime Ward identities and transformation properties of conserved Noether currents will prove pivotal for constructing dissipative hydrodynamic description for fractonic systems. [2]
- Construct field theories for quasiparticle excitations with "internal dipole moment".

[1] Burnell, Devakul, Gorantla, Lam, Shao [2110.09529]; Yamaguchi [2110.12861]
[2] Gromov, Lucas, Nandkishore [2003.09429]; etc.

FRACTON FLUIDS

- The thermal state can be described by the Grand-Canonical Partition Function

$$\exp(-\beta W) = \operatorname{tr}\exp(-\beta \mathscr{H}) \qquad\qquad \mathscr{H} = \int \mathrm{d}^3x \left(\epsilon^t - u^i\pi_i - \mu\, J^t\right)$$

- Dipole-transformation properties of π_i imply that μ is not dipole invariant

$$\pi^i \to \pi^i - J^t\psi^i \qquad \Longrightarrow \qquad \mu \to \mu + u^i\psi_i$$

- Given the grand-canonical free-energy density $F = -\,p(T,\mu,\overrightarrow{u}^2)$, it immediately follows that

$$u^i \neq 0, \quad J^t = \frac{\partial p}{\partial \mu} = 0 \qquad\quad or \qquad\quad u^i = 0, \quad J^t = \frac{\partial p}{\partial \mu} \neq 0.$$

A local fluid particle can move if and only if the local charge density is zero.

FRACTON FLUIDS

- Constitutive relations for a boost-agnostic ideal fluid are given as [1]

$$\epsilon^i = (\epsilon^t + p)\, u^i$$
$$\pi^i = \rho\, u^i$$
$$\tau^{ij} = \rho\, u^i u^j + p\, \delta^{ij}$$
$$J^i = J^t\, u^i$$

- For $J^i = \partial_j J^{ij}$, we must have that either of J^t, u^i is zero.
  If $J^t = 0,\ u^i \neq 0$, the fluid is just neutral.
  If $J^t \neq 0,\ u^i = 0$, all fluxes are zero.

[1] de Boer, Hartong, Have, Obers, Sybesma [1710.04708, 2004.10759]; [2] Novak, Sonner, Withers [1911.02578]; [3] Armas, AJ [2010.15782]

FRACTON FLUIDS

- These properties of fracton fluids are intimately tied with the free tensor gauge theory not being able to couple to curved spacetime.
- We expect that the equilibrium configurations of a fluid should be obtained from an local equilibrium partition function [1]

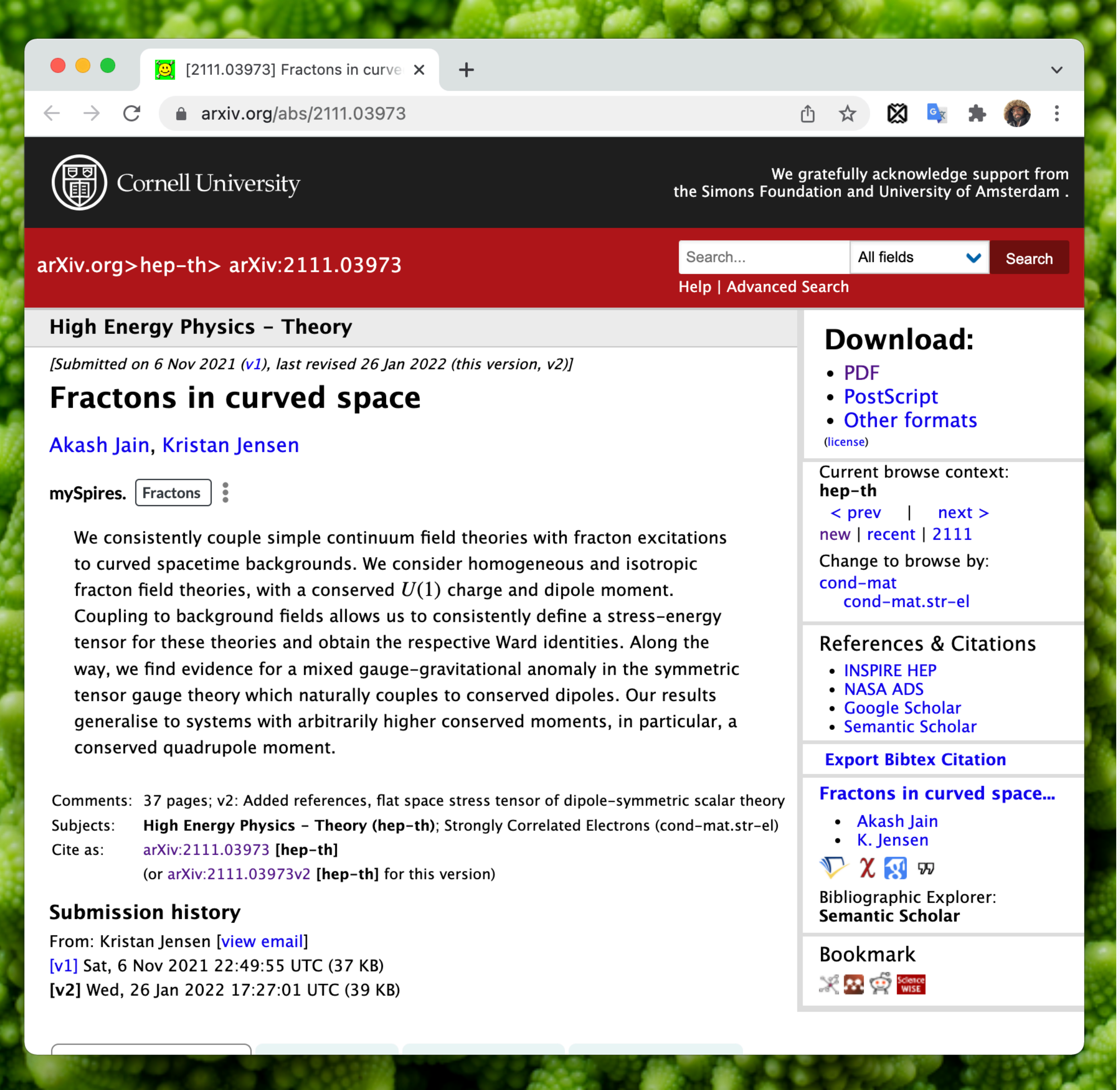
$$W[n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij}] = \int \mathrm{d}^3x \sqrt{\gamma}\, \mathscr{F}(n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij})$$

However, symmetries forbid us to write any such partition function, at least at the leading order in derivatives.

- It might be possible to write a non-local partition function in the presence of some additional low-energy degrees of freedom, such as Goldstones,

$$W[n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij}] = -T \ln \int \mathscr{D}\varphi \exp\left(\frac{1}{T}\int \mathrm{d}^3x \sqrt{\gamma}\, \mathscr{F}(\varphi; n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij})\right)$$

[1] Banerjee, JBhattacharya, Bhattacharyya, Jain, Minwalla, Sharma [1203.3544]; Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom [1203.3556]



THANK YOU

AND STAY HEALTHY

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