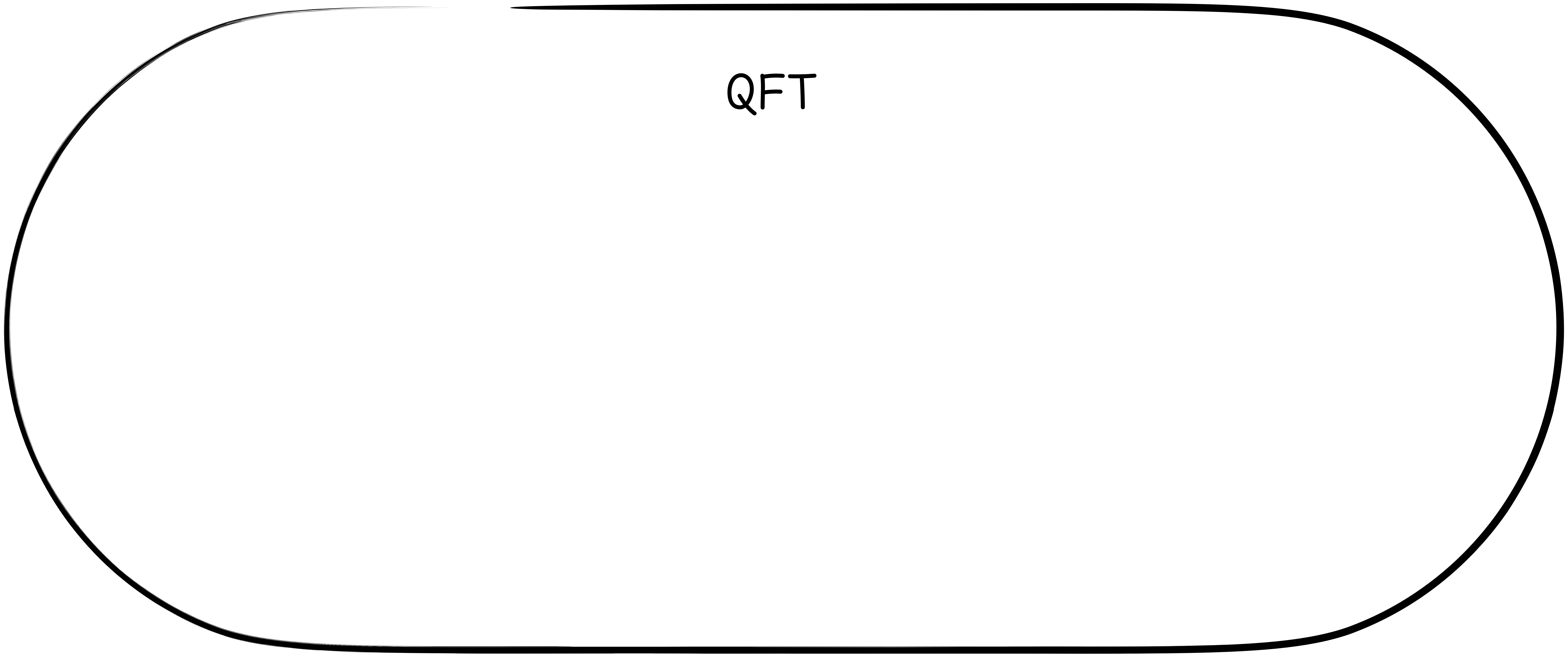
What can we learn from SCFTs with $a = c$?

Craig Lawrie (DESY)

Based on: 2106.12579 w/ M.J. Kang and J. Song
2111.12092 w/ M.J. Kang, K-H. Lee, and J.Song
2206.xxxxx w/ M.J. Kang, K-H. Lee, M. Sacchi, and J. Song
220x.xxxxx w/ M.J. Kang, K-H. Lee, and J. Song

16-06-2022

Motivation



Quantum field theory is a widely applicable framework for answering diverse questions in physics

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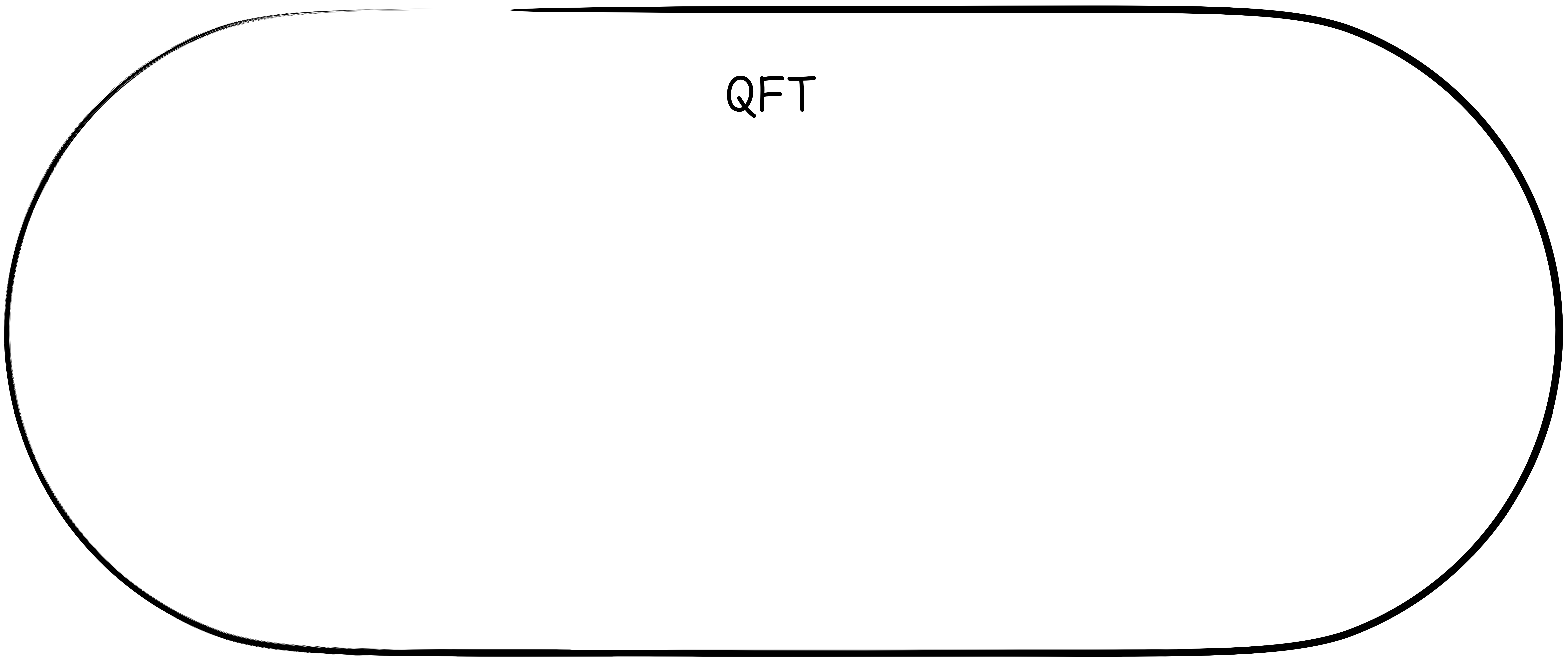
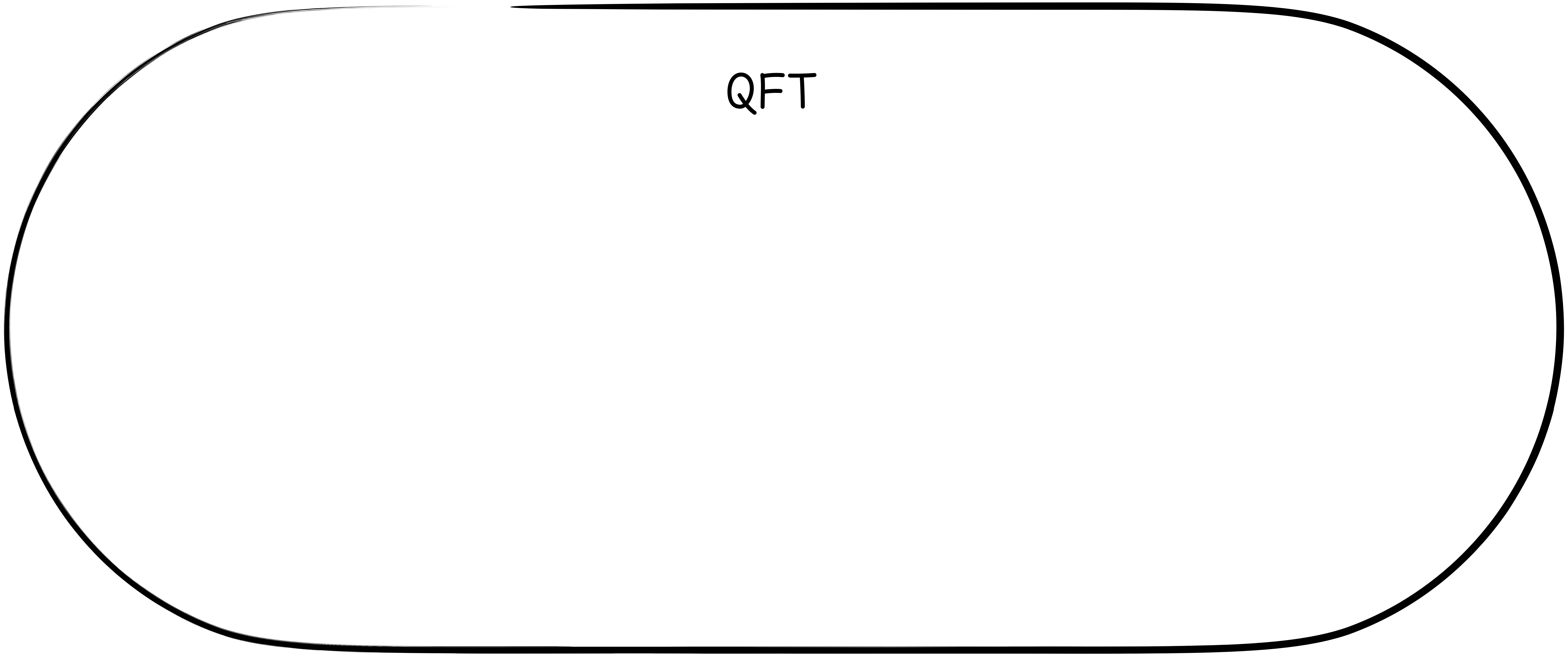


Quantum field theory is a widely applicable framework for answering diverse questions in physics

→ generally hard to study outside of the "perturbative regime"

Motivation



Quantum field theory is a widely applicable framework for answering diverse questions in physics

add new symmetries to make questions tractable

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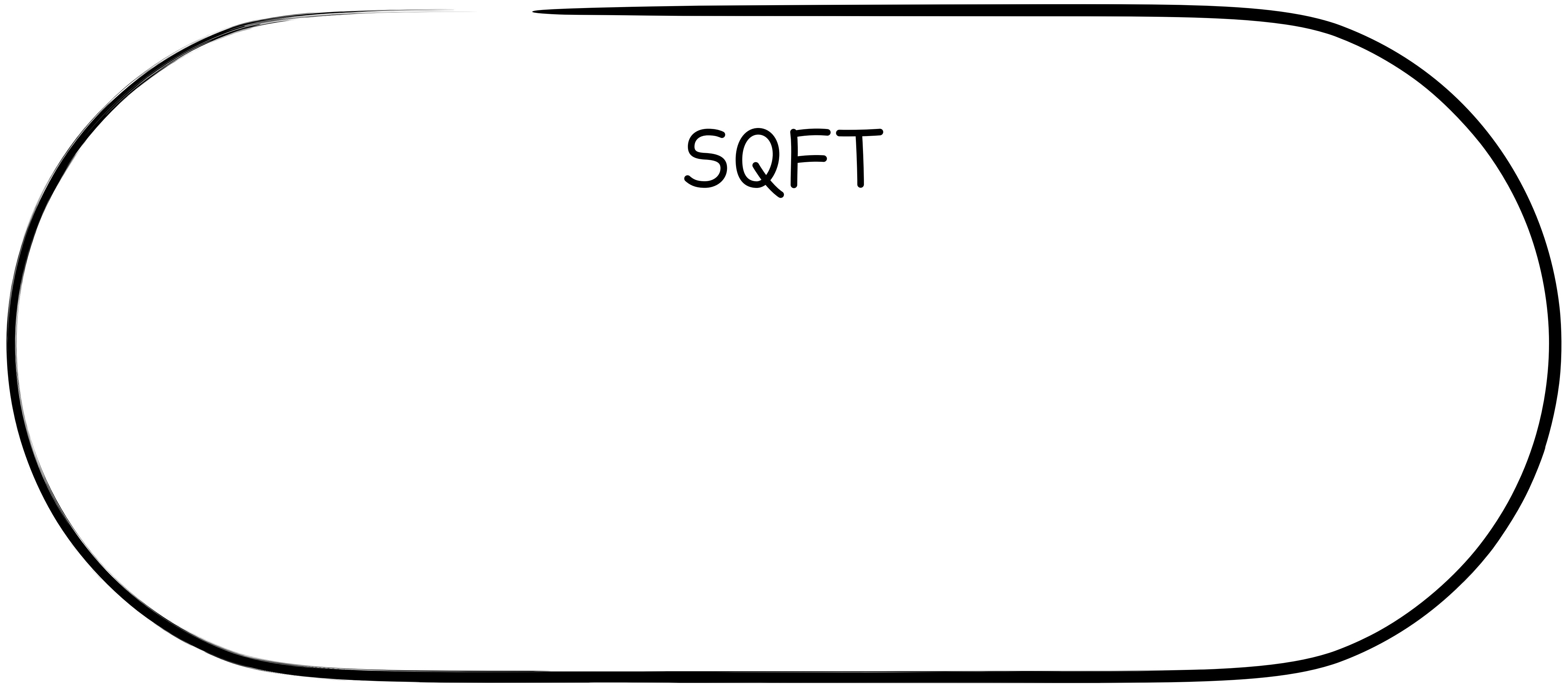

Supersymmetry $\Longrightarrow$ certain quantities are protected from quantum corrections

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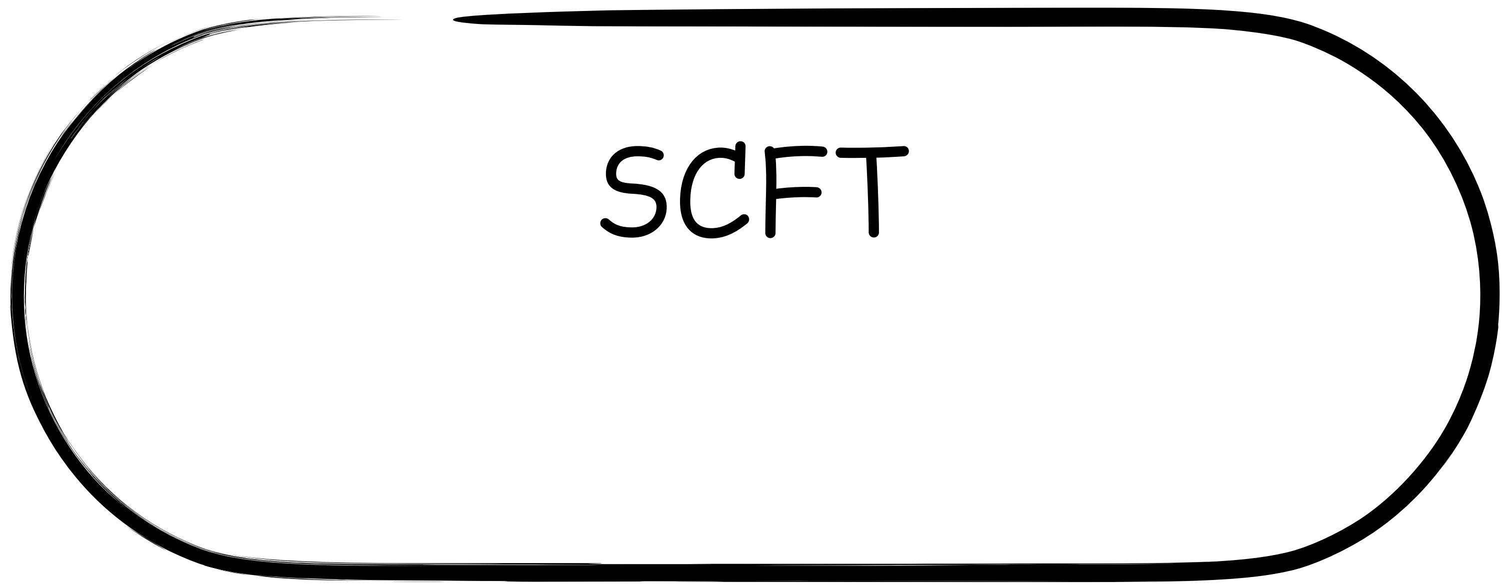

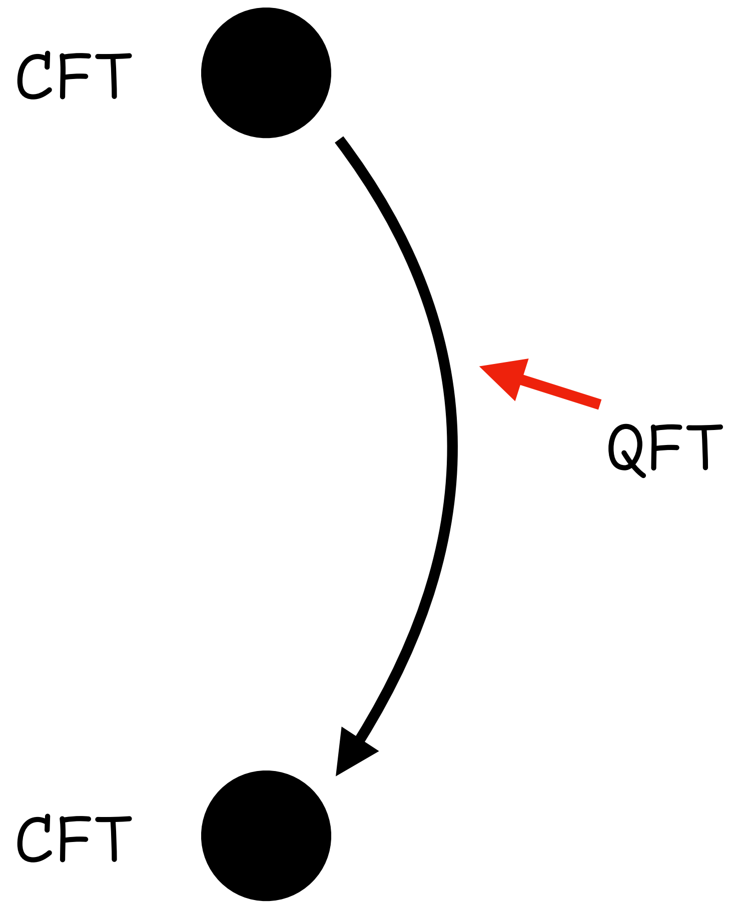

Conformal symmetry $\Longrightarrow$ exists at the fixed points of renormalization group flows between QFTs

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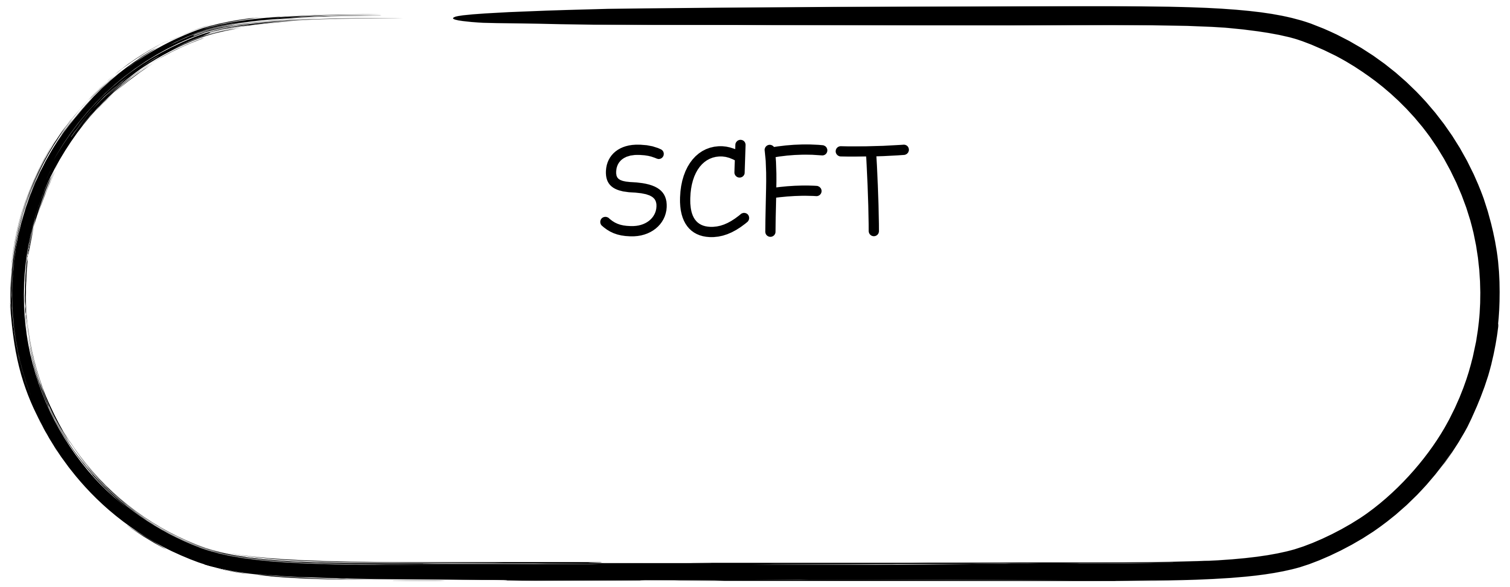

Conformal symmetry $\Longrightarrow$ exists at the fixed points of renormalization group flows between QFTs

Supersymmetry $\Longrightarrow$ certain quantities are protected from quantum corrections

We study this subspace of the space of all QFTs

Central Charges

Conformal symmetry becomes anomalous when the CFT is placed in an arbitrary background

central charges

$$\langle T^{\mu}_{\mu} \rangle = \frac{c}{16\pi^2} W^2 - \frac{a}{16\pi^2} E$$

stress-energy tensor

Weyl tensor

Euler density

The a-theorem

[Komargodski, Schwimmer]

UV Theory

$(a_{\mathrm{UV}}, c_{\mathrm{UV}})$

RG flow

IR Theory

$(a_{\mathrm{IR}}, c_{\mathrm{IR}})$

a monotonically decreases along any renormalization group flow

there is no equivalent statement for c

in this sense, a measures the “degrees of freedom” of the theory

Bounds on a/c

[Hofman, Maldacena]

Unitarity fixes the ratio:

$$\frac{1}{3} \leq \frac{a}{c} \leq \frac{31}{18}$$

free scalar field

free vector field

Bounds on a/c

[Hofman, Maldacena]

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+ supersymmetry $\longrightarrow$ bounds become stronger

Bounds on a/c

[Hofman, Maldacena]

Unitarity fixes the ratio:

+ supersymmetry $\longrightarrow$ bounds become stronger

free scalar field

free vector field

$$\mathcal{N}=0: \quad \frac{1}{3} \le \frac{a}{c} \le \frac{31}{18}$$

free chiral multiplet

free $\mathcal{N}=1$ vector

$$\mathcal{N}=1: \quad \frac{1}{2} \le \frac{a}{c} \le \frac{3}{2}$$

free hypermultiplet

free $\mathcal{N}=2$ vector

$$\mathcal{N}=2: \quad \frac{1}{2} \le \frac{a}{c} \le \frac{5}{4}$$

$$\mathcal{N}=3,4: \quad a=c$$

Class $\mathcal{S}$

[Gaiotto], [Gaiotto, Moore, Nietzke]

$$\mathcal{S}_{\mathfrak{g}}\langle C_{g,n}\rangle\{\cdots\}$$

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$$\mathcal{S}_{\mathfrak{g}}\langle C_{g,n}\rangle\{\cdots\}$$

the 6d $(2,0)$ SCFT
of type $\mathfrak{g}$

String theory: Type IIB on an orbifold $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$

[Witten]

$\mathfrak{g}$ of type ADE:
simple, simply-laced Lie algebra

Class $\mathcal{S}$

[Gaiotto], [Gaiotto, Moore, Nietzke]

$$\mathcal{S}_{\mathfrak{g}}\langle C_{g,n}\rangle\{\cdots\}$$

the 6d $(2,0)$ SCFT
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twisted compactification on
an n-punctured genus g Riemann surface

Class $\mathcal{S}$

[Gaiotto], [Gaiotto, Moore, Nietzke]

data describing punctures =
codimension two defects in the 6d SCFT

typically associated
to nilpotent orbits of $\mathfrak{g}$

$$\mathcal{S}_{\mathfrak{g}}\langle C_{g,n}\rangle\{\cdots\}$$

the 6d $(2,0)$ SCFT
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the 6d $(2,0)$ SCFT
of type $\mathfrak{g}$

= 4d $\mathcal{N}=2$ SCFT

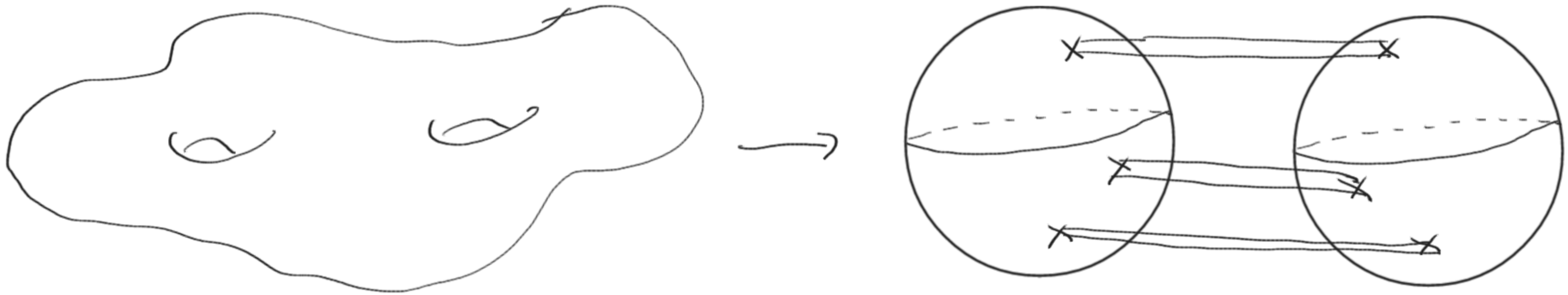
twisted compactification on
an n-punctured genus g Riemann surface

Complicated physical features
(e.g. S-dualities)
captured by this construction

Tinkertoys and Class $\mathcal{S}$

[Gaiotto], [Gaiotto, Moore, Nietzke]

Each punctured Riemann surface has a “pair-of-pants” decomposition

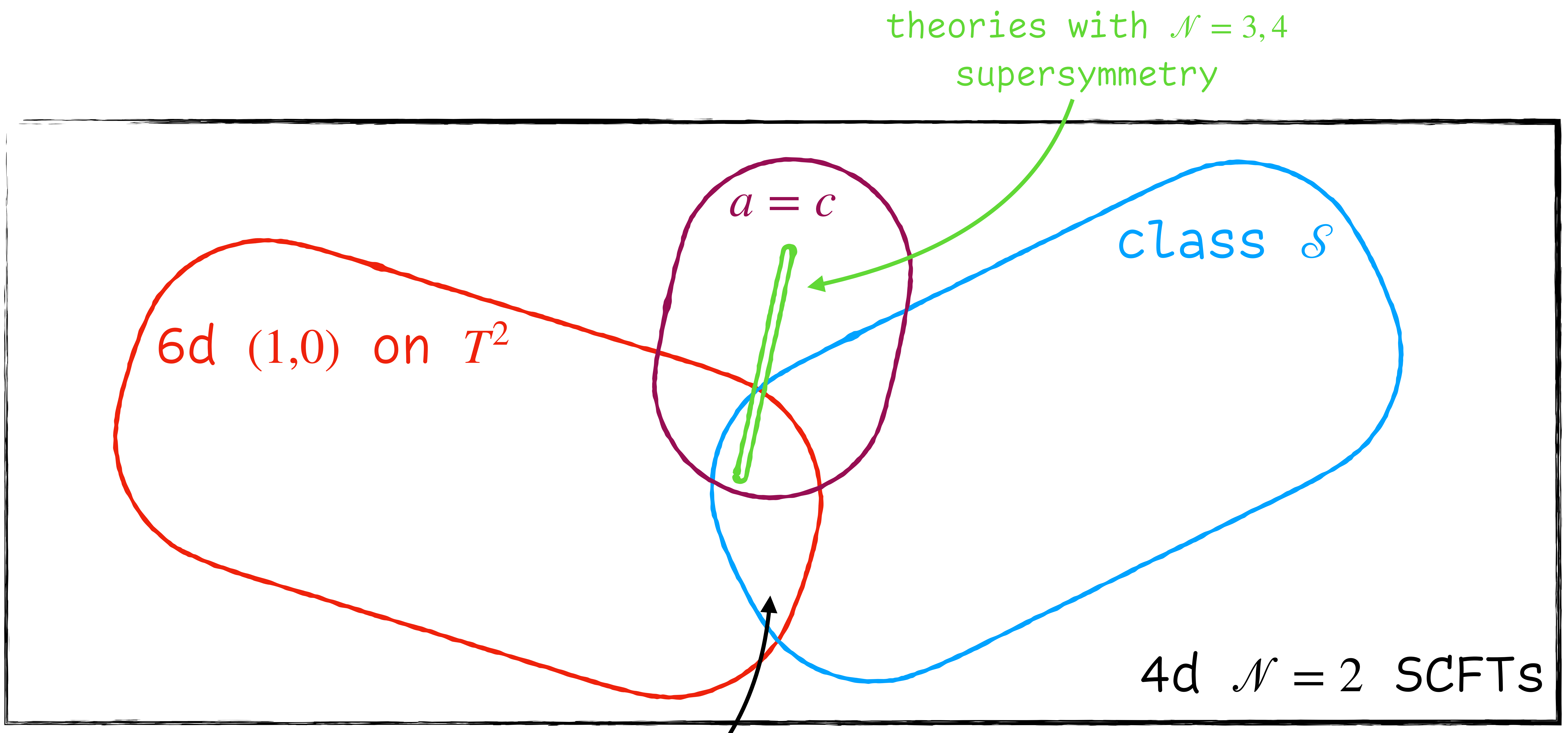


→ gluing three-punctured spheres along punctures

“tinkertoy” theories [Chacaltana, Distler, Trimm, Zhu]

Class $\mathcal{S}$ theories can be constructed from simple building blocks

The 4d $\mathcal{N} = 2$ Landscape



this overlap region is studied in [Baume, Kang, CL]

Today: focus on the $a = c$ region

Why $a = c$?

Holography: if 4d SCFT has $AdS_5 \times X_5$ dual then $a = c \sim O(N^2)$ to leading order in a large N limit

The subleading terms are

$$c - a = \rho N + \sigma$$

if $a = c$ at finite N
then ρ and σ
must conspire to cancel!

open string contributions
i.e. branes

closed string contributions
$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

Why $a = c$?

$c - a$ controls many interesting quantities in a CFT

- Cardy limit of superconformal index: $I \to \exp\left(\frac{16\pi^2}{3\beta}(c-a)\right)$ [di Pietro, Komargodski]

- Entropy-viscosity ratio bound: $\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{c-a}{c} + \cdots\right)$ [Kovtun, Son, Starinet] [Katz, Petrov] [Buchel, Myres, Sinha]

- Mixed current-gravitational anomaly [Anselmi, Freedman, Grisaru, Johansen]

- Single trace higher spin gap for large N [Edelstein, Maldacena, Zhiboedov]

Why $a = c$?

Notice: nothing I just said was specific to $\mathcal{N} = 2$

$\longrightarrow$ equally interesting to consider the subset of the $\mathcal{N} = 1$ landscape with $a = c$

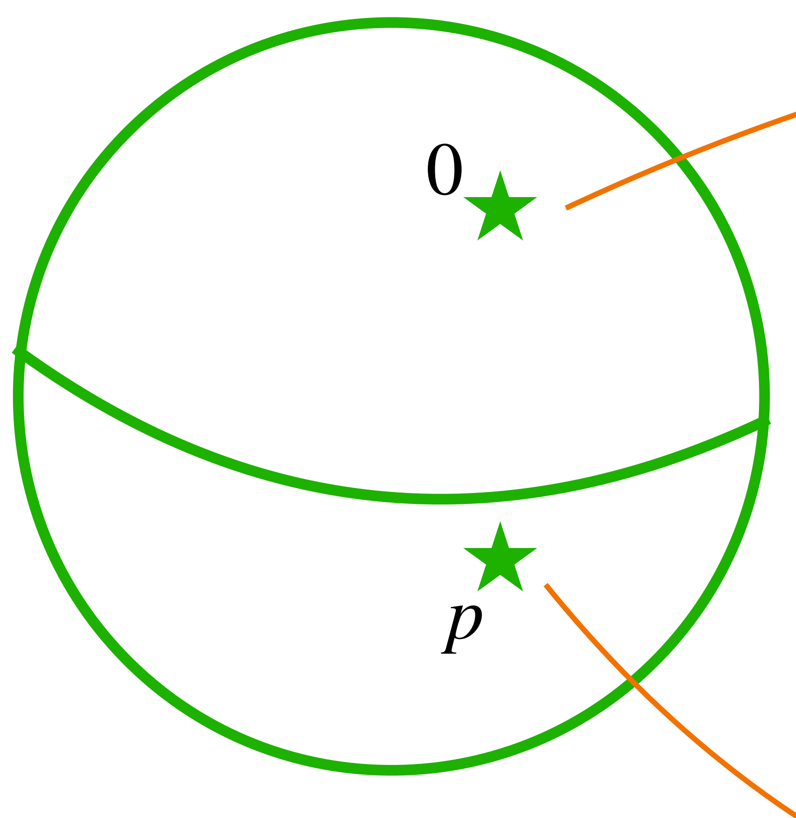
Building Blocks for SCFTs

fractional Coulomb branch scaling dimensions
$\longrightarrow$ "Argyres–Douglas type"

Some known 4d $\mathcal{N}=2$ SCFTs:

1) $\mathscr{D}_p(G)$ – non-Lagrangian [Xie], [del Zotto, Cecotti] [del Zotto, Cecotti, Giaomcelli], [Xie, Wang]

Class $\mathcal{S}$:



regular maximal puncture with flavor symmetry G

irregular puncture

G	$SU(N)$	$SO(2N)$	E_6	E_7	E_8
No additional symmetry	$(p,N)=1$	$p \notin 2\mathbb{Z}_{>0}$	$p \notin 3\mathbb{Z}_{>0}$	$p \notin 2\mathbb{Z}_{>0}$	$p \notin 30\mathbb{Z}_{>0}$

Building Blocks for SCFTs

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1) $\mathcal{D}_p(G)$ – non-Lagrangian [Xie], [del Zotto, Cecotti] [del Zotto, Cecotti, Giaomcelli], [Xie, Wang]

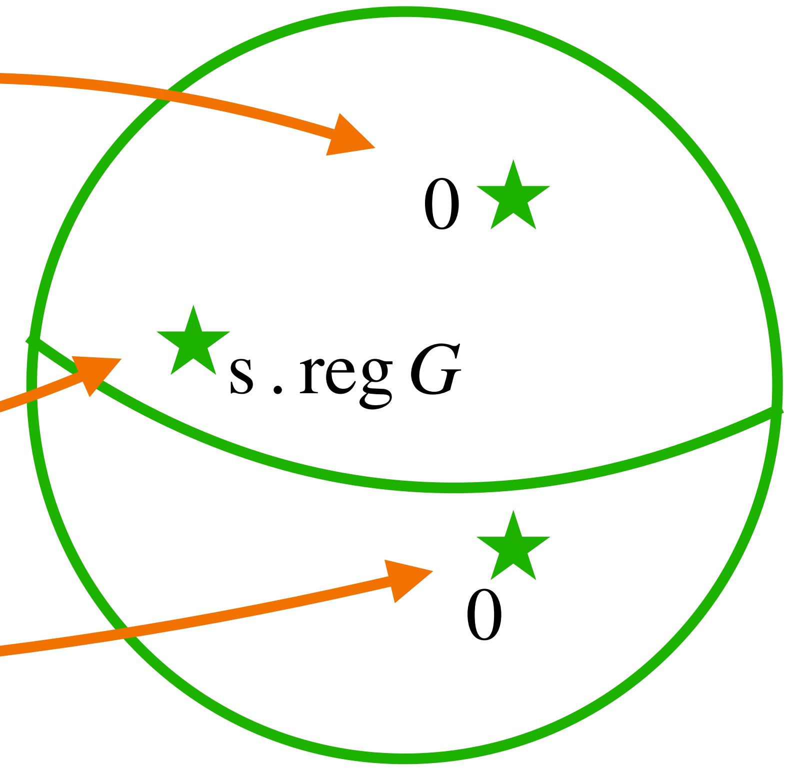
2) (G,G) conformal matter [del Zotto, Heckman, Tomasiello, Vafa]

a strongly-coupled generalization of an $SU(\ell)\times SU(\ell)$ bifundamental hypermultiplet

Class $\mathcal{S}$:

regular maximal punctures each with flavor symmetry G

subregular puncture



[Ohmori, Shimizu, Tachikawa, Yonekura]
[del Zotto, Vafa, Xie]
[Baume, Kang, CL]

Building Blocks for SCFTs

Some known 4d $\mathcal{N}=2$ SCFTs:

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$\longrightarrow$ can we construct new 4d SCFTs using these strongly-coupled theories as building blocks?

$$\boxed{\text{consider } \mathcal{N}=2 \text{ or } \mathcal{N}=1 \text{ gauging of all } G \text{ flavor symmetries}}$$

An $\mathcal{N} = 2$ Classification Problem

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how can we gauge together all G flavor symmetries of a collection of $\mathcal{D}_p(G)$ and (G, G) conformal matter theories such that the result is an $\mathcal{N} = 2$ SCFT?

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it's very restrictive!

The $\mathcal{N} = 2$ Classification Problem

[Kang, CL, Song]

$\mathcal{D}_p(G)$ has flavor G with flavor central charge: $k_G = \frac{2(p-1)}{p} h_G^\vee$

(G, G) conformal matter has flavor $G \times G$ and flavor central charges $k_G = 2h_G^\vee$

Conformal gauging $\longrightarrow$ one-loop β-functions must vanish

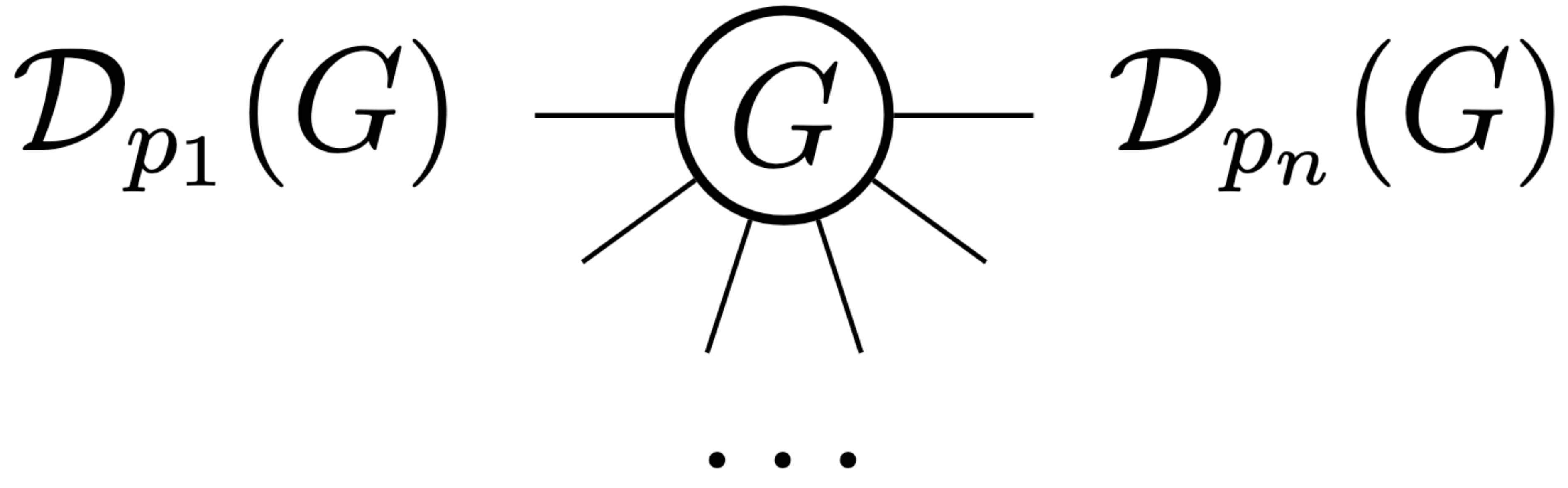
$$\beta_G = 0 \qquad \Longleftrightarrow \qquad \sum k_G = 4h_G^\vee$$

sum over all flavor symmetries
that are gauged together

Without Conformal Matter

[Kang, CL, Song]

First assume there are no copies of conformal matter



$$\sum_{i=1}^{n} \frac{2(p_i - 1)}{p_i} h_G^\vee = 4h_G^\vee$$

equivalent

$$\sum_{i=1}^{n} \frac{1}{p_i} = n - 2$$

There are only four solutions with $p_i \geq 2$: $(2,2,2,2), (3,3,3), (2,4,4), (2,3,6)$

Note: $\mathscr{D}_1(G)$ is the trivial theory

D3-branes on an Orbifold

[Kachru, Silverstein]

before including conformal matter, we take a brief digression

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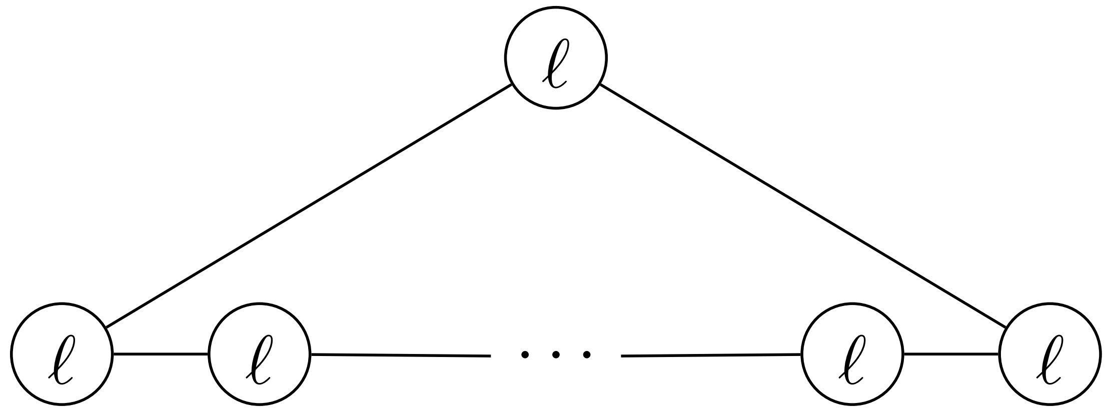
Worldvolume theory on ℓ D3-branes probing a $\mathbb{C}^2/\Gamma$ orbifold

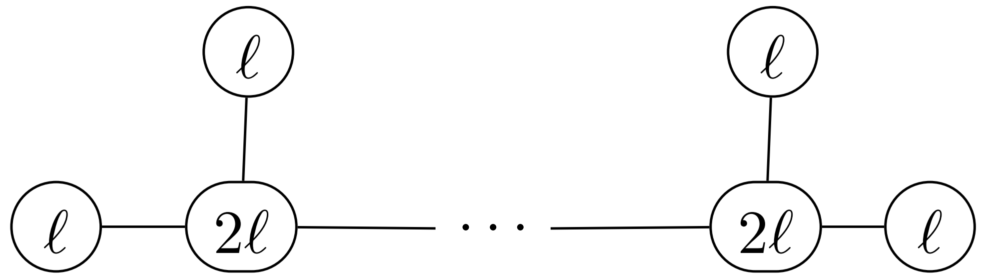
$\longrightarrow$ affine quiver gauge theory

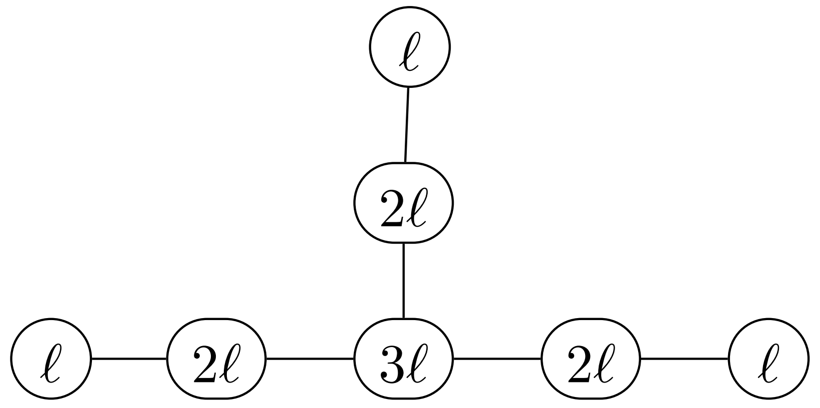
ADE-type of Γ

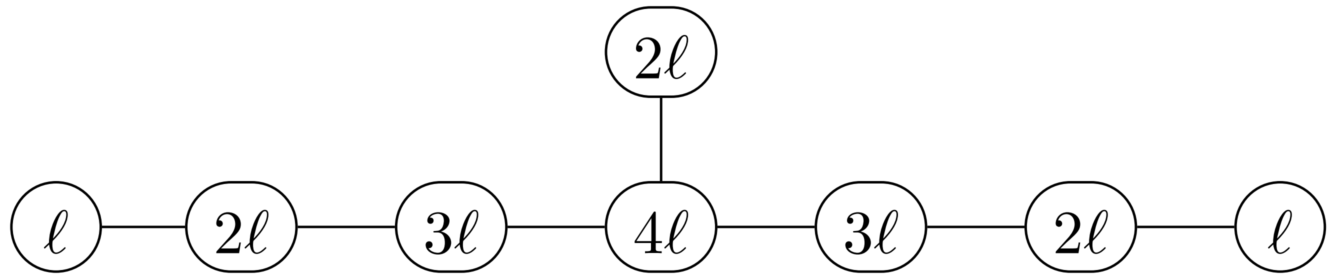
=

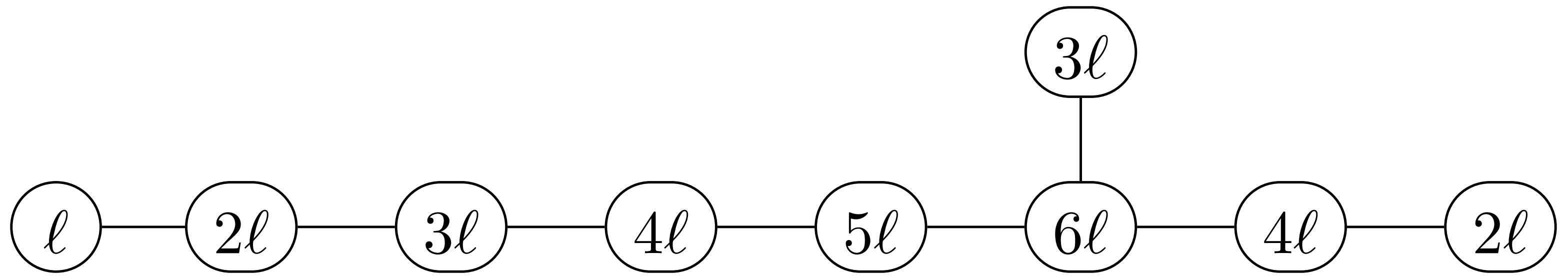
ADE-shape of quiver











What is the generalization beyond SU gauge nodes?

D3-branes on an Orbifold

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What is the generalization beyond SU gauge nodes?

There are two obvious questions:

1) generalization of ℓ — ℓ ?

2) generalization of $m\ell$ — $(m+1)\ell$?

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There are two obvious questions:

1) generalization of ℓ — ℓ ? $\longleftarrow$ (G, G) conformal matter

2) generalization of $m\ell$ — $(m+1)\ell$? $\longleftarrow$ $\mathscr{D}_p(G) - (G)$

[Kang, CL, Song]

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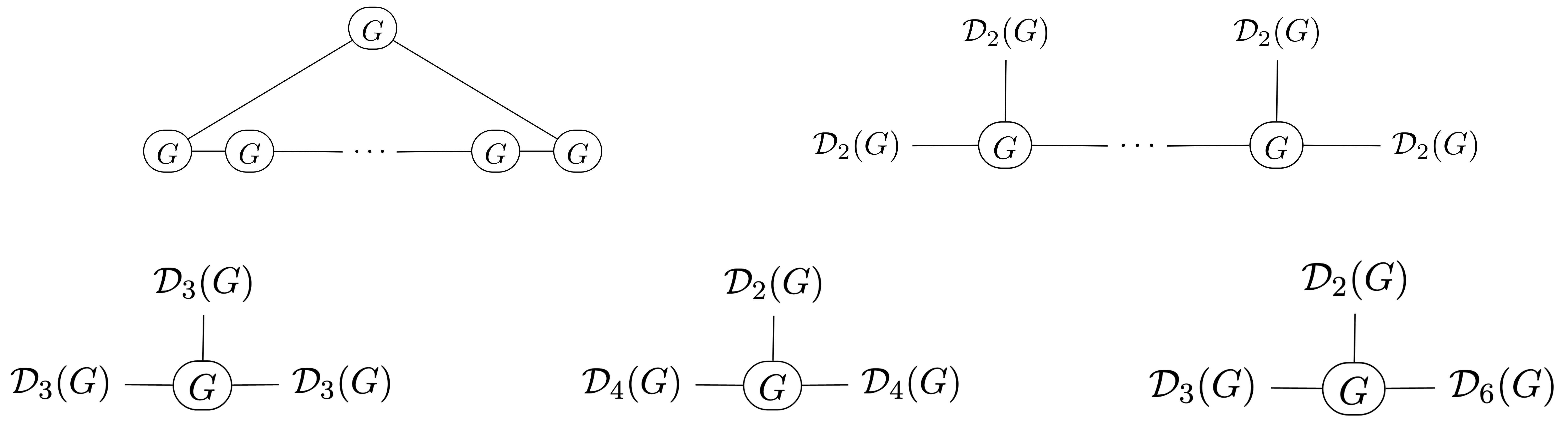
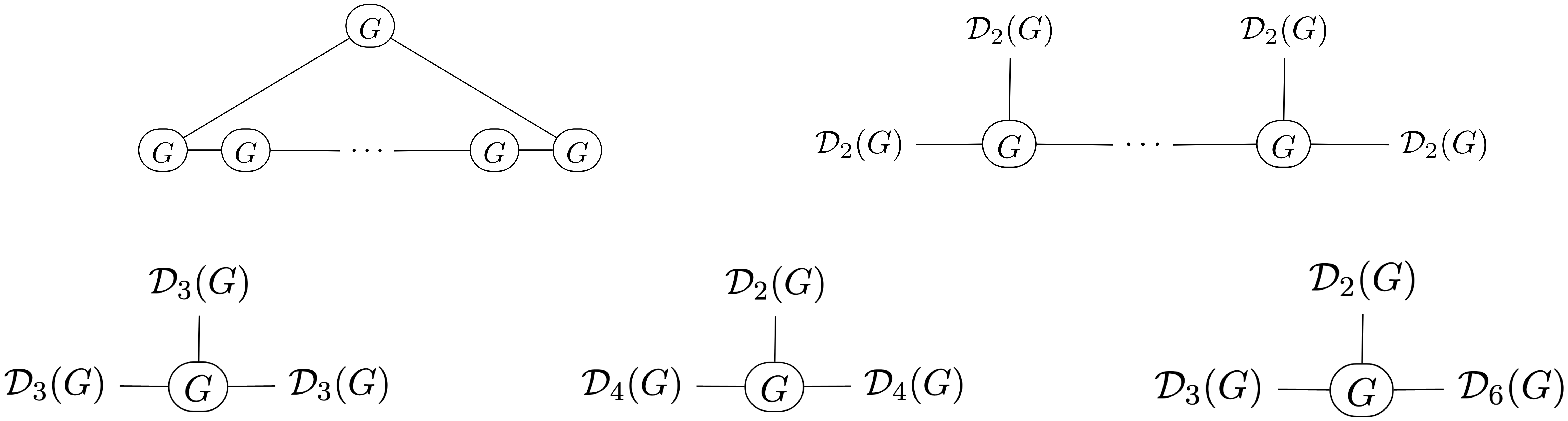


[Kang, CL, Song]

These are exactly the configurations of conformal matter and $\mathscr{D}_p(G)$ solving the classification problem!

The $\mathcal{N} = 2$ Classification Problem

[Kang, CL, Song]



These are exactly the configurations of conformal matter and $\mathscr{D}_p(G)$ solving the classification problem!

ADE Lie group
describing the gauge group
of the quiver

$$\widehat{\Gamma}(G)$$

ADE Lie group
describing the structure
of the quiver

Features of $\widehat{\Gamma}(G)$

Theories with $a = c$

When there is no conformal matter $\longrightarrow$ $\Gamma = D_4, E_6, E_7, E_8$

and if $\gcd(h^\vee_G, \alpha_\Gamma) = 1$

dual Coxeter number of G

largest comark of Γ

i.e., these SCFTs lie in the region of the landscape we want to explore!

then $\widehat{\Gamma}(G)$ has $$a = c = \frac{\alpha_\Gamma - 1}{\alpha_\Gamma}\dim(G)$$

can any of these $a = c$ theories be realized in class $\mathcal{S}$?

Theories with $a = c$ and class $\mathcal{S}$

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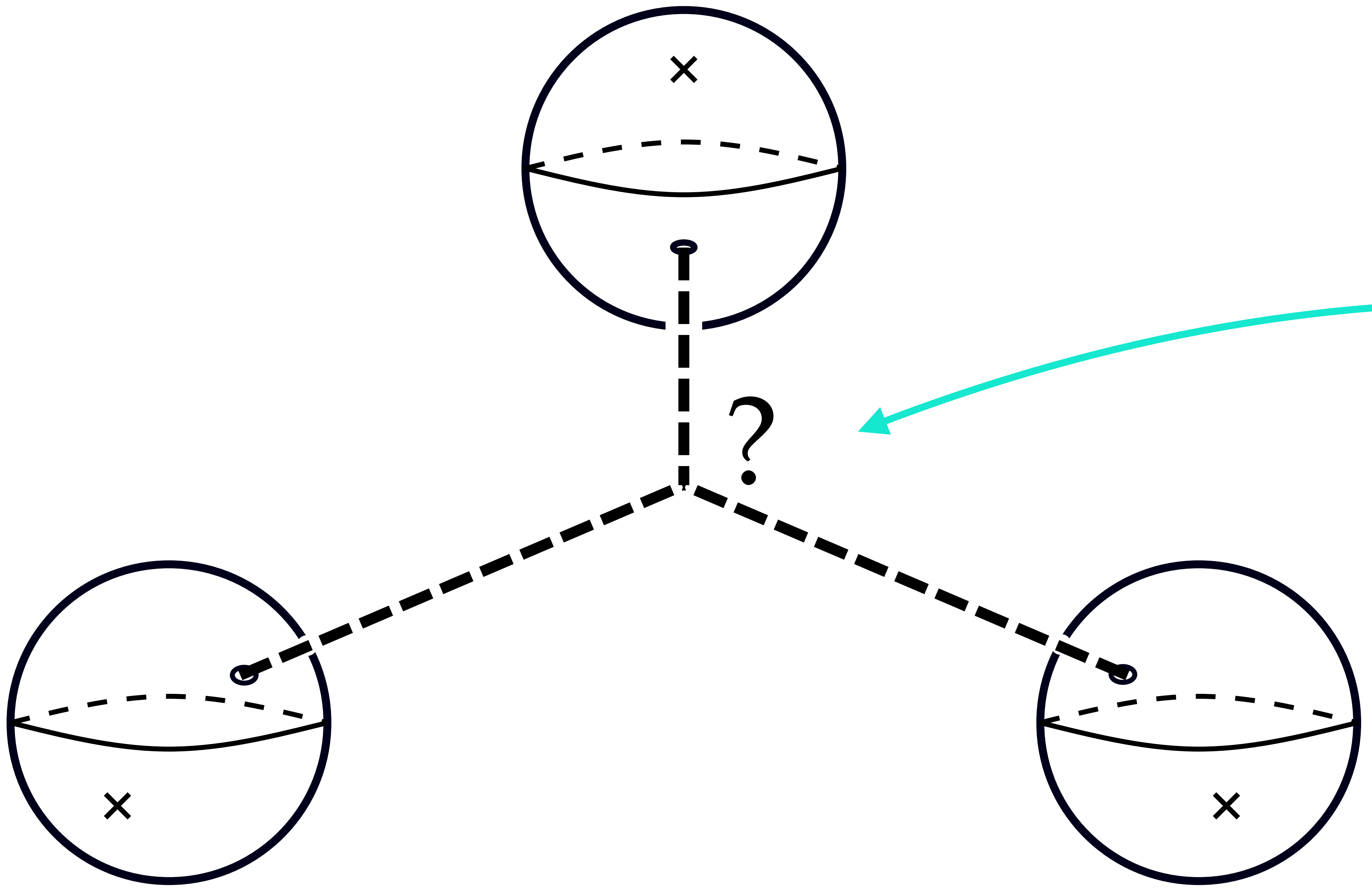


$\longrightarrow$ not obviously!

the multi-valent vertex implies
this is not
a pair-of-pants decomposition
of a punctured Riemann surface

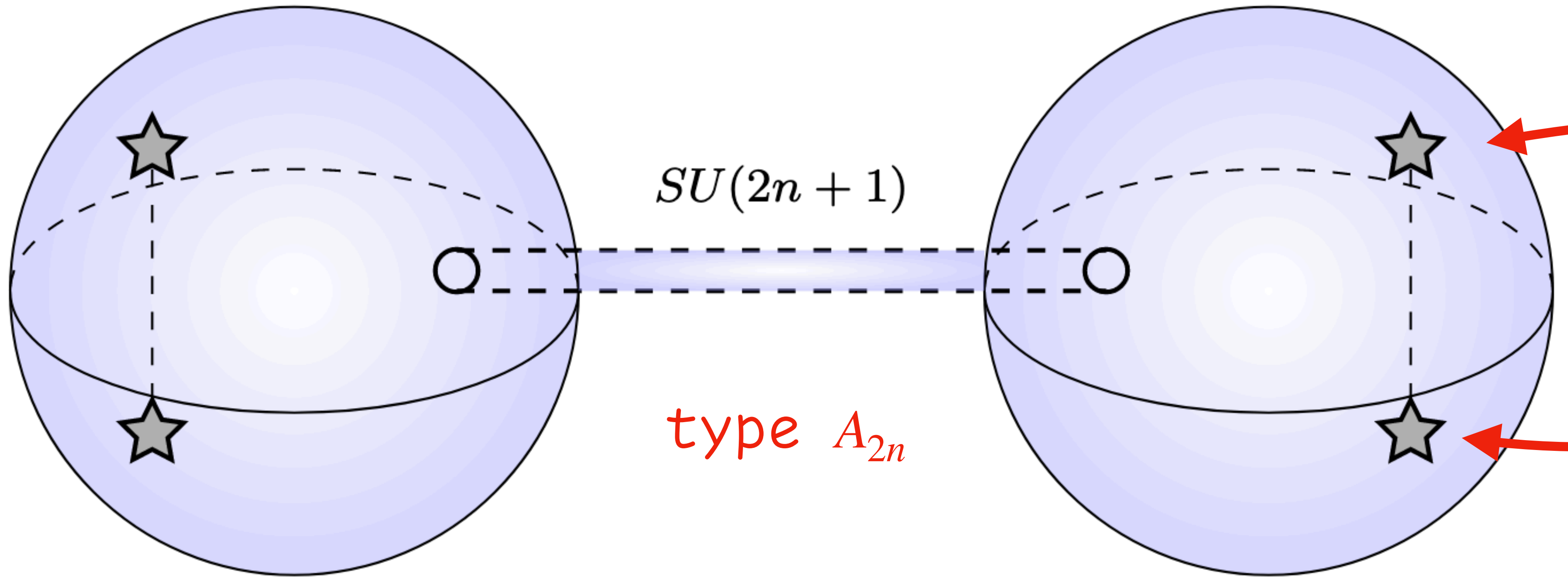
Theories with $a = c$ and class $\mathcal{S}$

can any of these $a = c$ theories be realized in class $\mathcal{S}$?

this three-punctured sphere corresponds to a product SCFT: two copies of $\mathcal{D}_2(SU(2n+1))$

the four-punctured sphere formed by gauging the maximal punctures is exactly $\widehat{D}_4(SU(2n+1))$!

[Beem, Peelaers]
[Berrato, Giacomelli, Mekareeya, Sacchi]



[Kang, CL, Lee, Sacchi, Song *to appear*]

twisted null punctures

how is this class $\mathcal{S}$ description related to the quad-valent gauging of 2-punctured spheres?

The Schur Index of $\widehat{\Gamma}(G)$

The Schur index of a 4d $\mathcal{N}=2$ SCFT is defined as

$$I_S(q) = \mathrm{Tr}(-1)^F q^{\Delta - R}$$

For $\mathscr{D}_p(G)$ without enhanced flavor the index is

$$I_{\mathscr{D}_p(G)}(q, \vec{z}) = \mathrm{PE}\left[\frac{q - q^p}{(1-q)(1-q^p)} \chi^G_{\text{adj}}(\vec{z})\right]$$

[Song, Xie, Yan]
[Kac, Wakimoto]

E.g., for $\widehat{D}_4(G)$ the Schur index is

$$I_{\widehat{D}_4(G)}(q) = \int [d\vec{z}]\ I^G_{\text{vec}}(q, \vec{z}) I_{\mathscr{D}_2(G)}(q, \vec{z})^4 = \int [d\vec{z}]\ \mathrm{PE}\left[\frac{2q - 2q^2}{1 - q^2} \chi^G_{\text{adj}}(\vec{z})\right]$$

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does this
look familiar?

A Connection to the Schur Index of $\mathcal{N} = 4$ SYM

The Schur index for $\widehat{\Gamma}(G)$ with $a = c$ can be written as

$$I_{\widehat{\Gamma}(G)}(q) = \int [d\vec{z}\,]\,\mathrm{PE}\left[\left(\frac{q + q^{\alpha_\Gamma - 1} - 2q^{\alpha_\Gamma}}{1 - q^{\alpha_\Gamma}}\right)\chi^G_{\mathrm{adj}}(\vec{z})\right] = I_G^{\mathcal{N}=4}(q^{\alpha_\Gamma}, q^{\alpha_\Gamma/2-1})$$

this is the Schur index of $\mathcal{N} = 4$ super-Yang–Mills?!

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[Kang, CL, Song]

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fugacity of the $SU(2)$ flavor

is this a special structure implied by $a = c$?

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fugacity of the $SU(2)$ flavor

is this a special structure implied by $a = c$?

For $\widehat{E}_6(SU(2))$ this relation follows from a graded vector space isomorphism between the associated vertex operator algebras

[Buican, Nishinaka]

A Quasi-modular Form

[Kang, CL, Song]

For small G we can evaluate the Schur index explicitly e.g.,

$$I_{\widehat{D}_4(SU(7))} = 1 + 3q^2 + 9q^4 + 22q^6 + 42q^8 + 81q^{10} + 140q^{12} + 231q^{14} + O(q^{16})$$

do these coefficients look familiar?

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do these coefficients look familiar?

In fact, we find

$$I_{\widehat{D}_4(SU(2k+1))}(q) = q^{-k(k+1)} A_k(q^2)$$

where

$$A_k(q) = \sum_{0<m_1<m_2\cdots<m_k} \frac{q^{m_1+\cdots m_k}}{(1-q^{m_1})^2\cdots(1-q^{m_k})^2}$$

MacMahon's generalized sum-of-divisor function
[MacMahon 1921]

this function is quasi-modular [Andrews, Rose] as expected for a character of a VOA

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how can we gauge together all G flavor symmetries of a collection of $\mathcal{D}_p(G)$ and (G, G) conformal matter theories such that the result flows in the infrared to an $\mathcal{N} = 1$ SCFT?

An $\mathcal{N} = 1$ Classification Problem

how can we gauge together all G flavor symmetries of a collection of $\mathcal{D}_p(G)$ and (G, G) conformal matter theories such that the result flows in the infrared to an $\mathcal{N} = 1$ SCFT?

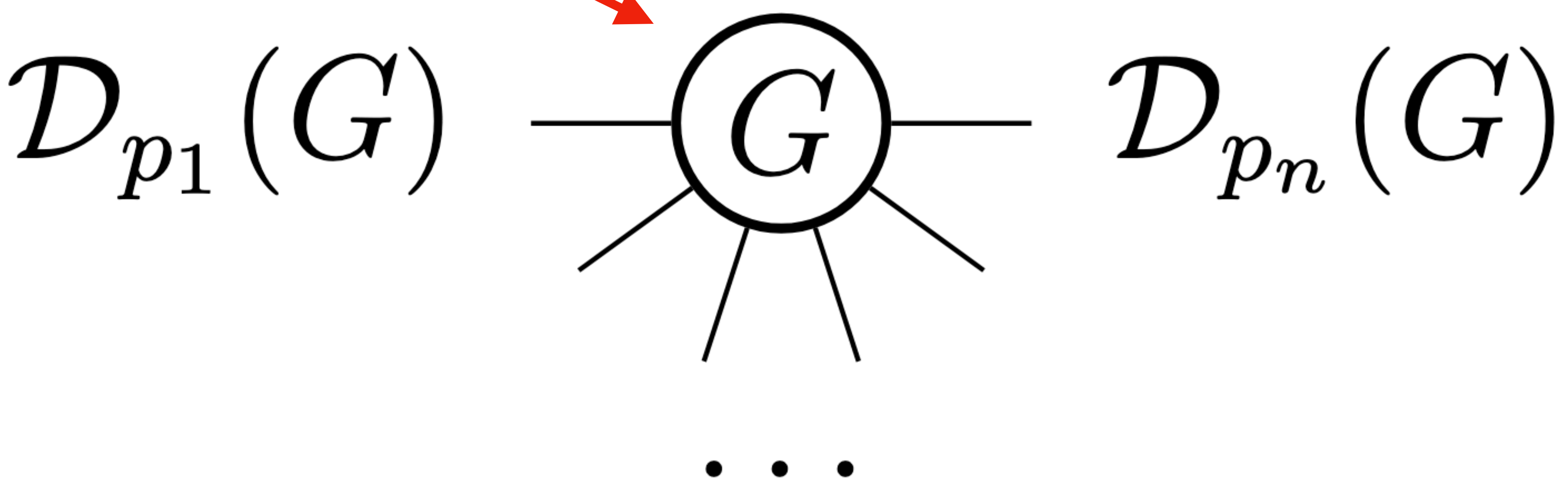
it's much less restrictive than $\mathcal{N} = 2$!

An $\mathcal{N} = 1$ Classification Problem

[Kang, CL, Lee, Song]

First focus on cases likely to have $a = c$ $\longrightarrow$ no conformal matter

this is now an $\mathcal{N} = 1$ gauge node



For an asymptotically-free gauge coupling

$$\sum_{i=1}^{n} \frac{1}{p_i} > n - 3$$

all solutions

equality implies conformal gauging

p_1	p_2	p_3	p_4	p_5
1	1	1	1	p_5
1	1	1	p_4	p_5
1	1	p_3	p_4	p_5
1	2	2	p_4	p_5
1	2	3	≤ 6	p_5
1	2	3	7	≤ 41
1	2	3	8	≤ 23
1	2	3	9	≤ 17

p_1	p_2	p_3	p_4	p_5
1	2	3	10	≤ 14
1	2	3	11	≤ 13
1	2	4	4	p_5
1	2	4	5	≤ 19
1	2	4	6	≤ 11
1	2	4	7	≤ 9
1	2	5	5	≤ 9
1	2	5	6	≤ 7

p_1	p_2	p_3	p_4	p_5
1	3	3	3	p_4
1	3	3	4	≤ 11
1	3	3	5	≤ 7
1	3	4	4	≤ 5
2	2	2	2	p_5
2	2	2	3	3
2	2	2	3	4
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An $\mathcal{N} = 1$ Classification Problem

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Not all such gaugings flow to interacting infrared SCFTs with $a = c$

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1) Use a-maximization to determine the superconformal R-symmetry

[Intriligator, Wecht]

2) Check no operator crosses unitarity bound along the flow

An $\mathcal{N} = 1$ Classification Problem

[Kang, CL, Lee, Song]

Not all such gaugings flow to interacting infrared SCFTs with $a = c$

p_1	p_2	p_3	p_4	p_5
~~1~~	~~1~~	~~1~~	~~1~~	~~p_5~~
1	1	1	p_4	$p_5 > 2$
1	1	p_3	p_4	p_5
1	2	2	p_4	p_5
1	2	3	≤ 6	p_5
1	2	3	7	≤ 41
1	2	3	8	≤ 23
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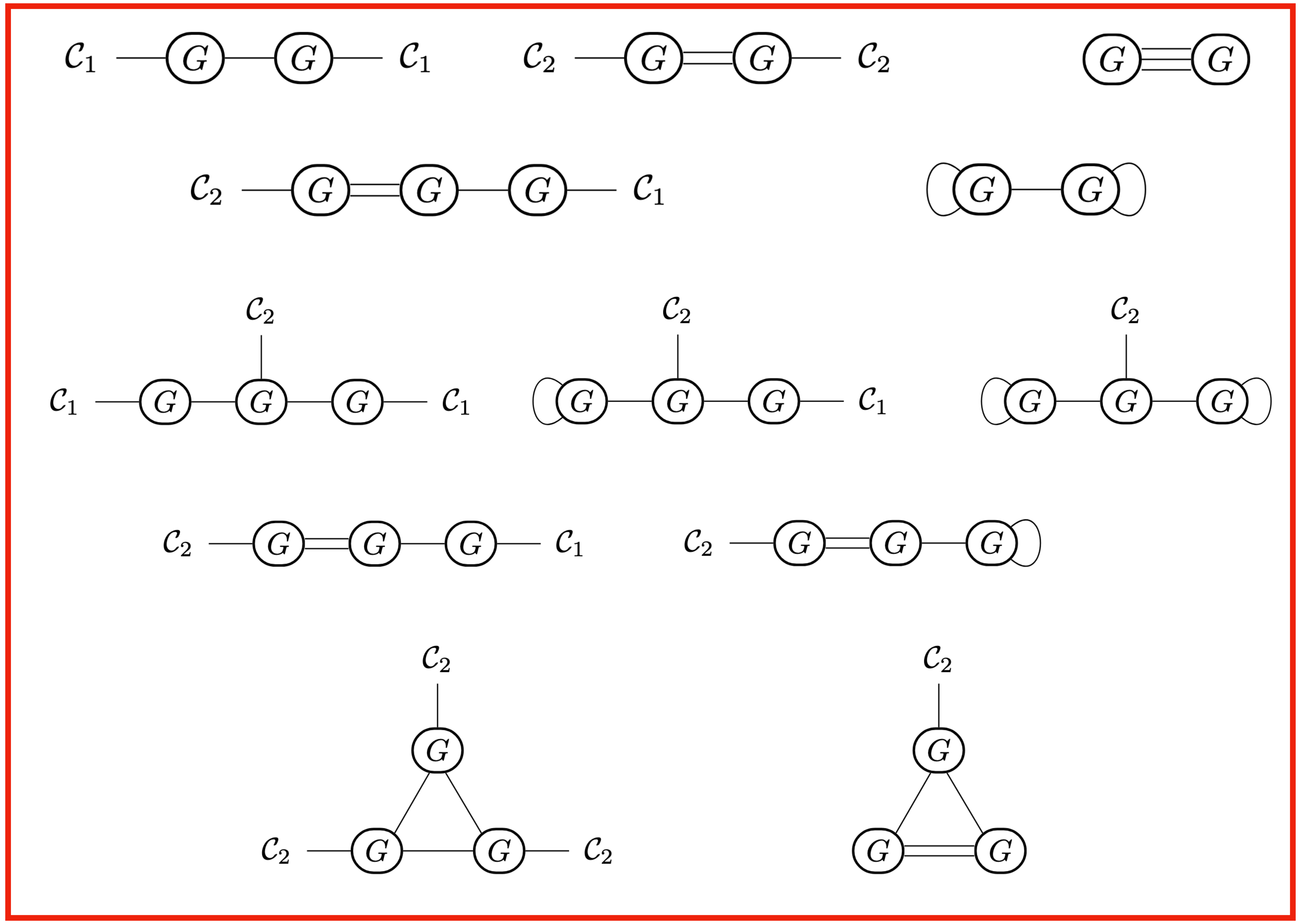
An $\mathscr{N} = 1$ Classification Problem

[Kang, CL, Lee, Song]

To see the ADE classification for $\mathscr{N} = 2$ we needed conformal matter



with 2 or 3 gauge nodes

Does there exist a known classification problem for which this is the answer?

Name	$\{p_i\}$ Asymptotically-free gauging	$\{p_i\}$ Conformal gauging
$\mathcal{C}_1$	$\{p \geq 2\}$ $\{p_1 \geq 2, p_2 \geq 2\}$ $\{2, 2, p \geq 2\}$ $\{2, 3, 4\}$ $\{2, 3, 5\}$	$\{2, 2, 2, 2\}$ $\{3, 3, 3\}$ $\{2, 4, 4\}$ $\{2, 3, 6\}$
$\mathcal{C}_2$	$\{p \geq 2\}$	$\{2, 2\}$

$\mathcal{N} = 1$: Adding Matter

[Kang, CL, Lee, Song]

With $\mathcal{N} = 1$ gauging we can also add one or two adjoint chiral multiplets while preserving $a = c$

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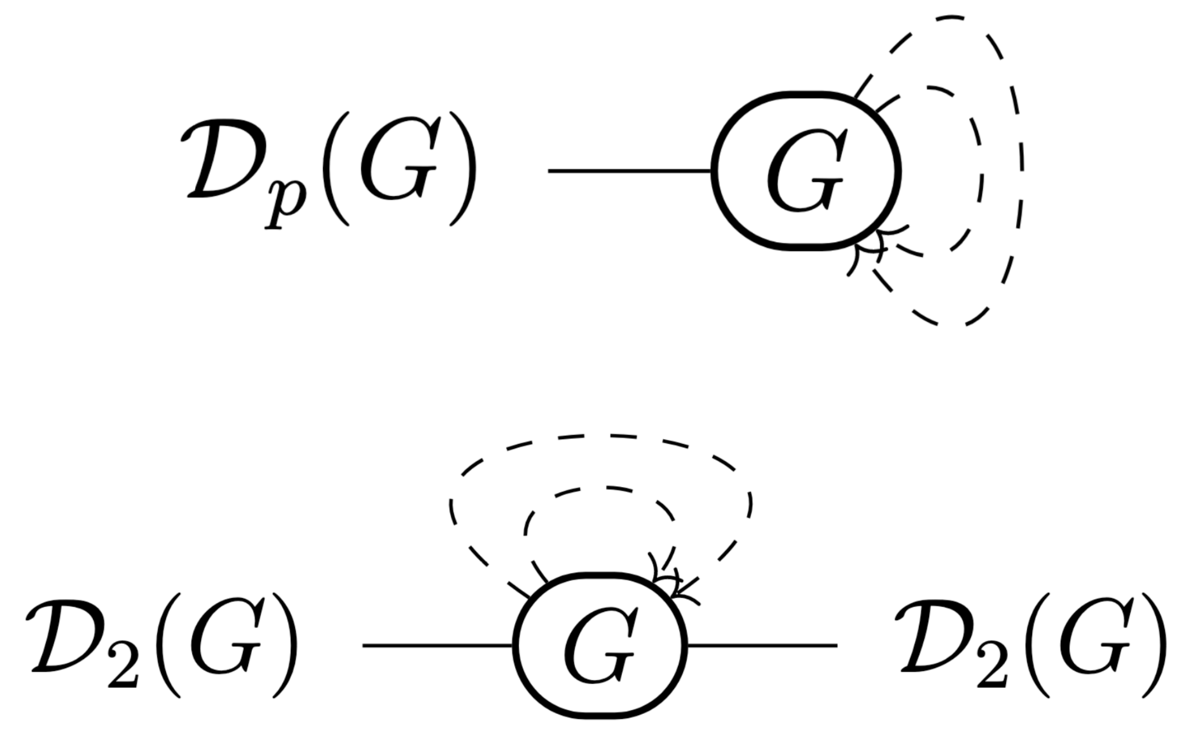
only two options

SCFTs living on the conformal manifold of $\widehat{\Gamma}(G)$ theories



Higgs, Coulomb, and Hall—Littlewood

[Kang, CL, Lee, Sacchi, Song *to appear*]

The Hall—Littlewood Index

[Kang, CL, Lee, Sacchi, Song *to appear*]

use the TQFT approach
[Gadde, Rastelli, Razamat, Yan]
[Lemos, Peelaers, Rastelli]
[Mekareeya, Song, Tachikawa]

We can compute the Hall–Littlewood index for $\widehat{\Gamma}(G)$

	Higgs Branch Sector	Hall–Littlewood Sector
Condition	$\Delta = 2R,\ j_1 = j_2 = r = 0$	$\Delta = 2R + j_2,\ j_1 = 0,\ j_2 = r$
Multiplet contents	$\widehat{\mathcal{B}}_R$	$\widehat{\mathcal{B}}_R,\ \mathcal{D}_{R(0,j_2)}$

Consider $\widehat{D}_4(SU(3))$:

$$\mathrm{HL}(\tau) = \frac{1+\tau^4-\tau^6}{1-\tau^4} = 1 + 2\tau^4 - \tau^6 + 2\tau^8 - \tau^{10} + \mathcal{O}(\tau^{12})$$

there are negative coefficients!

recall $\widehat{D}_4(SU(3))$ can be realized as a genus-zero class $\mathcal{S}$ theory

A Conjecture

Any class $\mathcal{S}$ theory of genus-zero has
Hall–Littlewood index = Higgs branch Hilbert series

[Gadde, Rastelli, Razamat, Yan]

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[Gadde, Rastelli, Razamat, Yan]

however, a Hilbert series cannot have negative coefficients

	Higgs Branch Sector	Hall–Littlewood Sector
Condition	$\Delta = 2R,\ j_1 = j_2 = r = 0$	$\Delta = 2R + j_2,\ j_1 = 0,\ j_2 = r$
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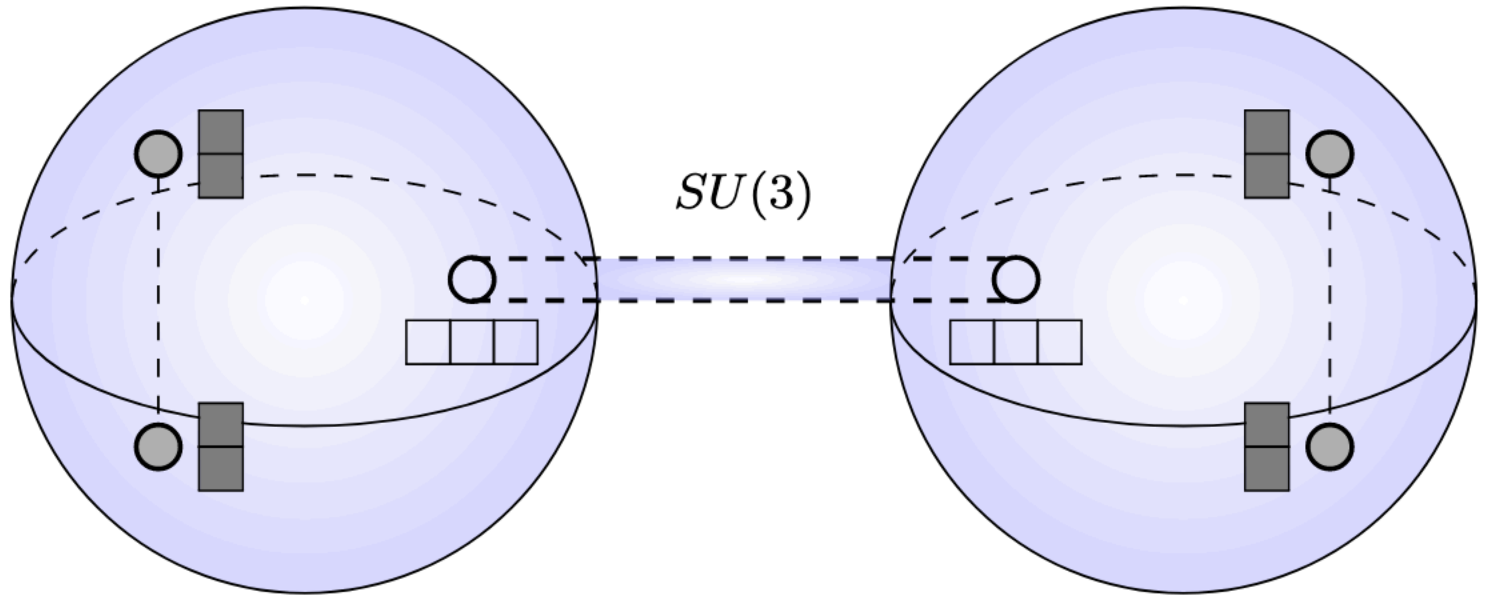
The Higgs Branch Hilbert Series

[Kang, CL, Lee, Sacchi, Song *to appear*]

In fact, we can compute the Higgs branch Hilbert series for $\widehat{D}_4(SU(3))$

$$\mathrm{HS}(\tau) = \frac{1-\tau^2+\tau^4}{(1-\tau^2)(1-\tau^4)} = 1 + 2\tau^4 + \tau^6 + 3\tau^8 + 2\tau^{10} + \mathcal{O}(\tau^{12})$$

no negative coefficients!



class $\mathcal{S}$ of type A_2 with 4 twisted null punctures
is a counterexample to the conjecture

this seems generic when you have more than four twisted punctures

A Conjecture

Conjecture 1.1. *Any class $\mathcal{S}$ theory associated to a genus-zero Riemann surface with at least four $\mathbb{Z}_2$-twisted punctures has a Hall–Littlewood index which is different from the Hilbert series of the Higgs branch.*

[Kang, CL, Lee, Sacchi, Song *to appear*]

Conclusions

We have constructed a broad collection of truly $\mathcal{N}=1$ and $\mathcal{N}=2$ SCFTs with exactly $a=c$

[Kang, CL, Song]
[Kang, CL, Lee, Song]

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We have constructed a broad collection of truly $\mathcal{N}=1$ and $\mathcal{N}=2$ SCFTs with exactly $a=c$

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They form a generalization of affine quivers and have intriguing connections to $\mathcal{N}=4$ super-Yang–Mills

Future Directions

What are the supergravity duals?

Why do higher derivative corrections vanish?

Fractional D3-branes?

Future Directions

$a = c$ theories have many relevant operators

do they trigger flows to new interacting SCFTs?

do they preserve $a = c$?

[Kang, CL, Lee, Song]

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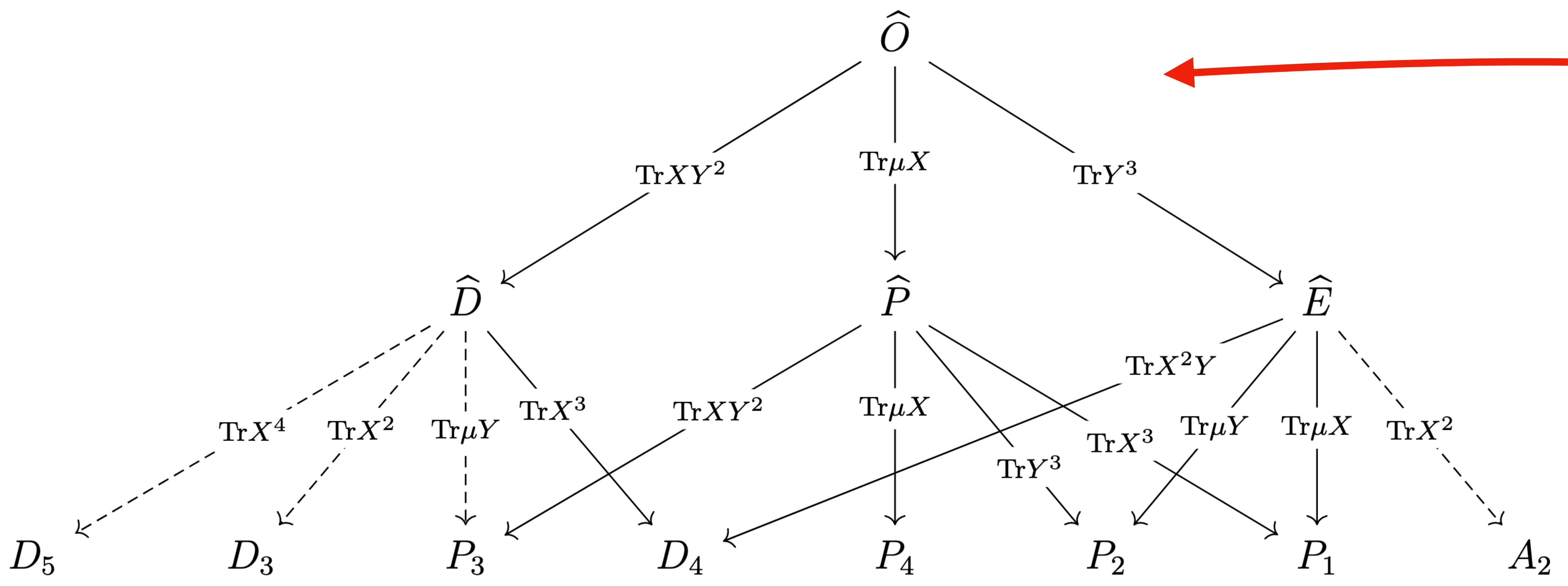
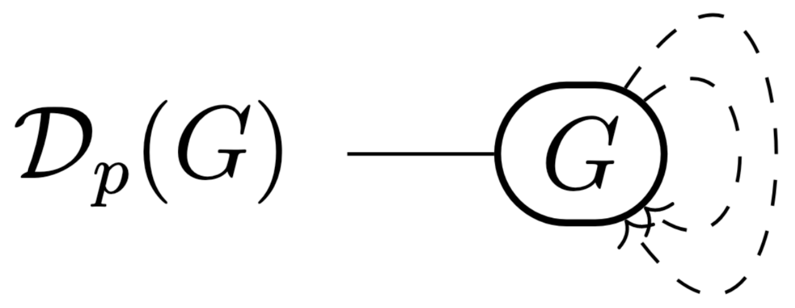


landscape of superpotential deformations of



cf. adjoint SQCD [Intriligator, Wecht]

Future Directions

6d SCFTs provide a powerful approach to “high dimension” SCFTs

are there direct 6d origins of these $a = c$ theories?

Thank you!