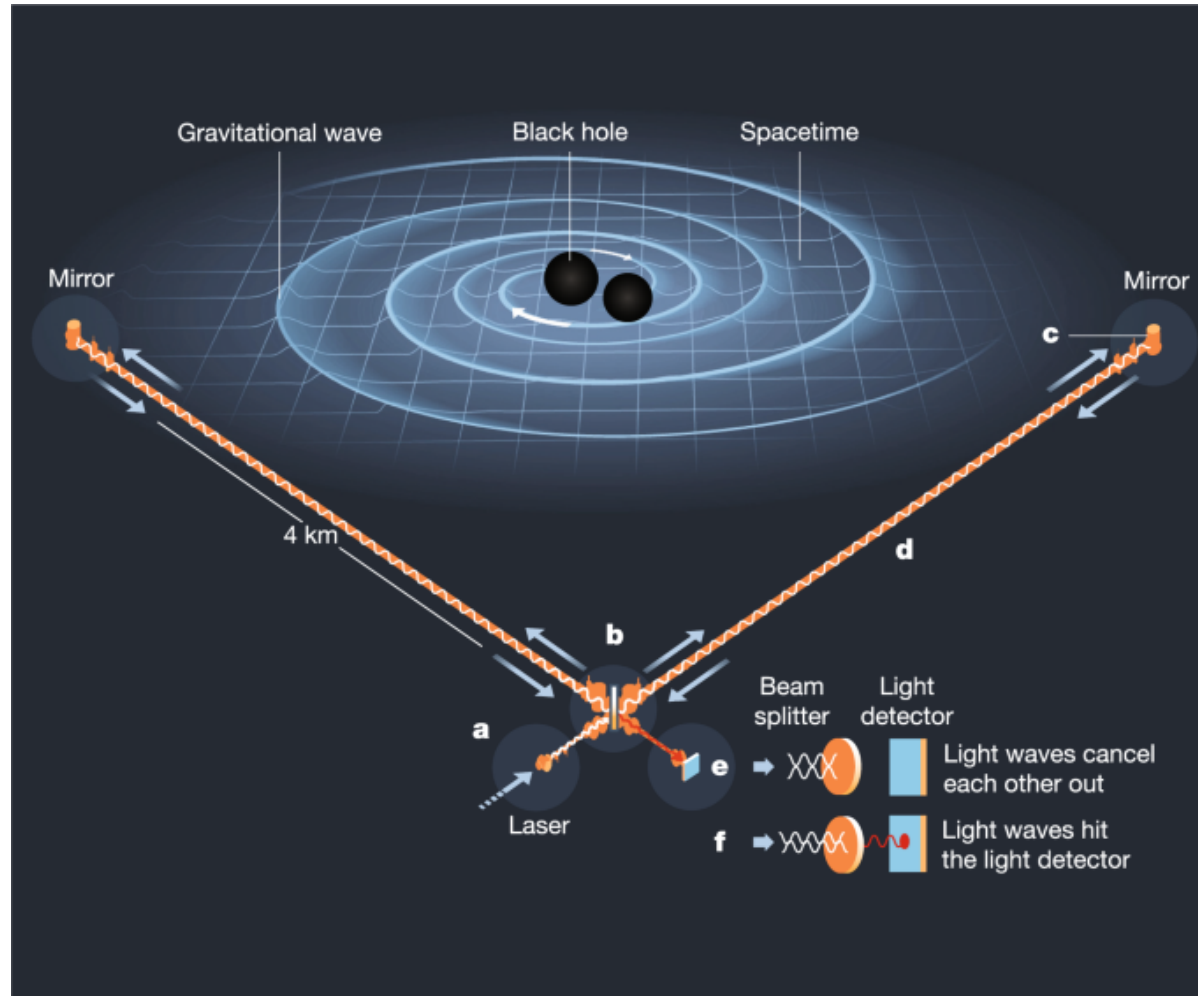
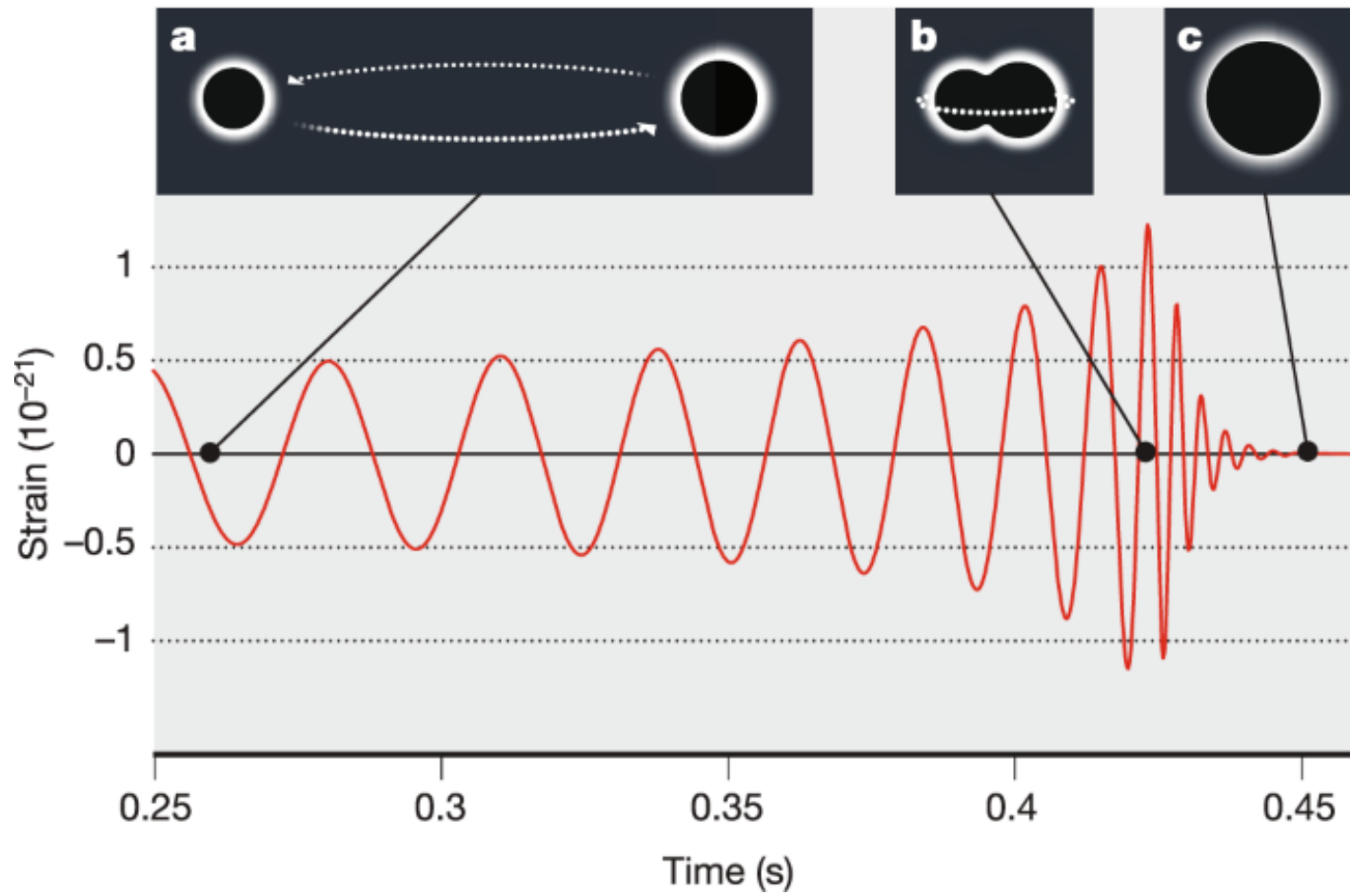


# Primordial Black Holes in the era of Gravitational Wave Astronomy

Antonio Riotto  
University of Geneva

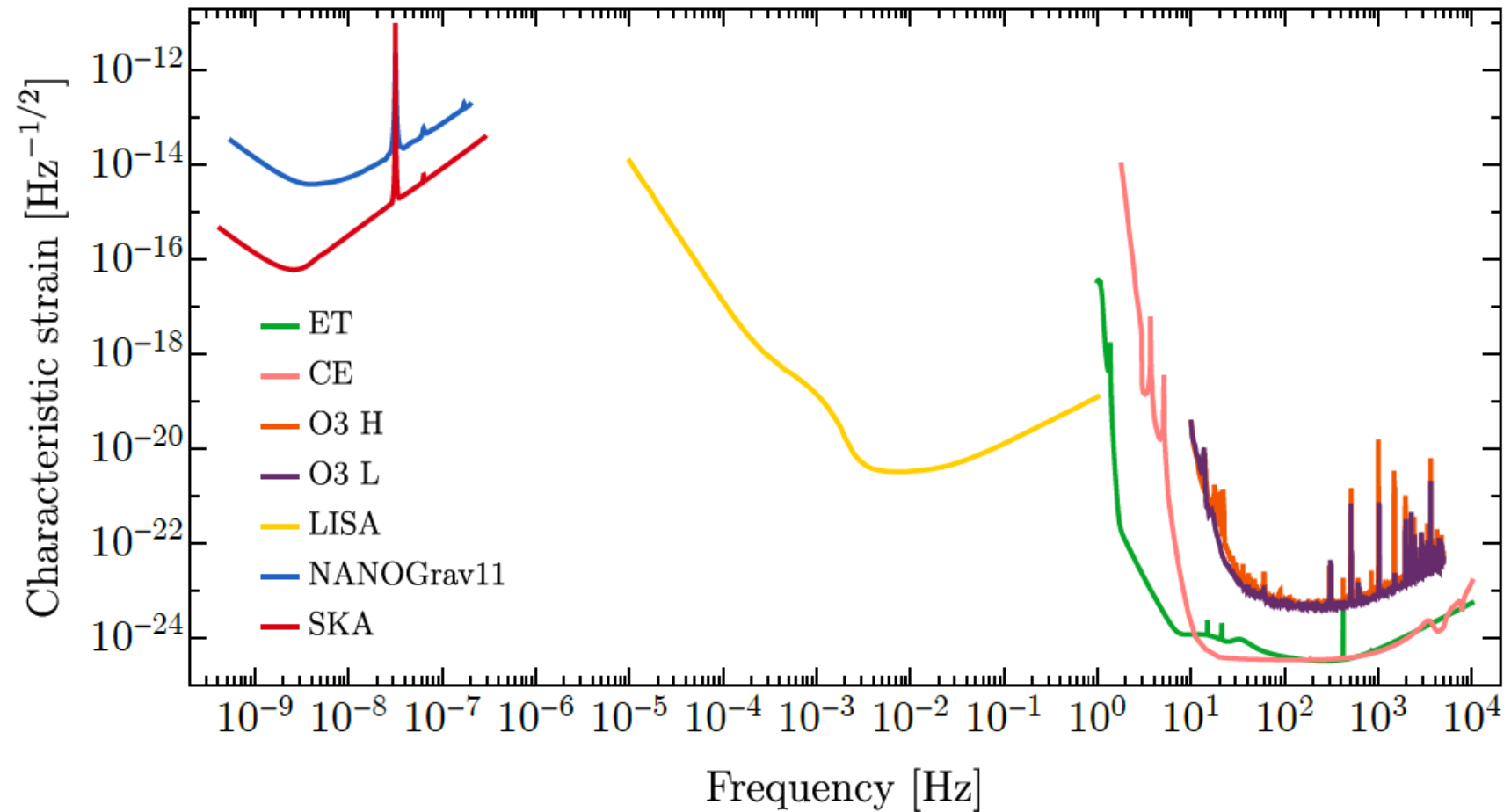
# Gravitational waves and Black Holes are key predictions of General Relativity





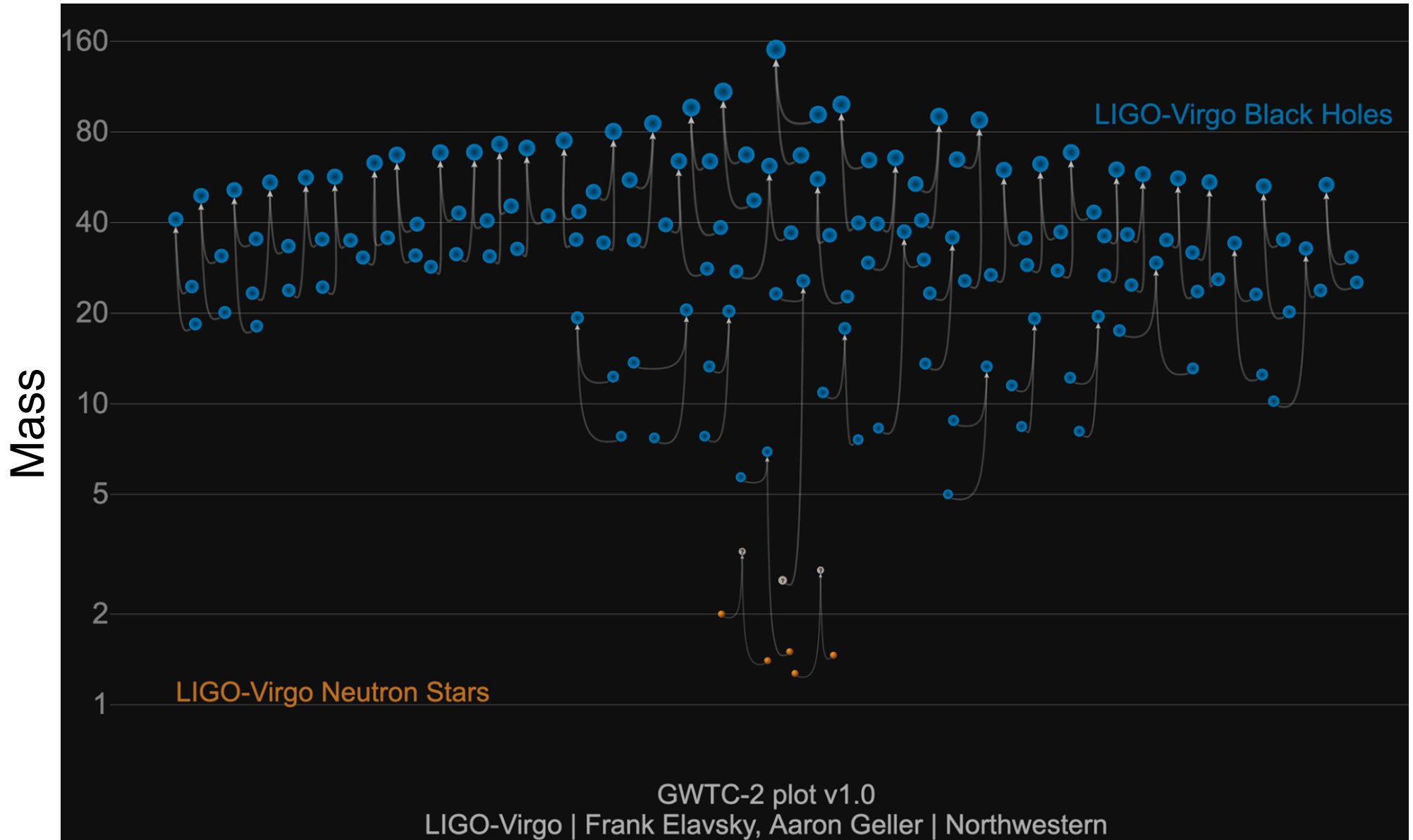
$$\text{Strain} \sim \frac{\delta L}{L} \sim h$$

# Current and future sensitivities





# Era of Gravitational Wave Astronomy



# Era of Gravitational Wave Astronomy

- **Astrophysics:** black holes, neutron stars, multi-messenger astrophysics, ...
- **Fundamental physics and cosmology:** tests of GR, *primordial black holes*, the nature of dark matter and dark energy, towards the Big Bang, ...

# Black Holes

- Astrophysical BHs forms from the gravitational collapse of a star. We know they exist. Their mass must be above the Chandrasekhar limit,

$$M > \mathcal{O}(1) M_{\odot}$$

- PBHs are formed in the early universe. Their mass can be small and they can still be around as long as they do not evaporate within the age of the universe

$$M > 10^{-18} M_{\odot}$$

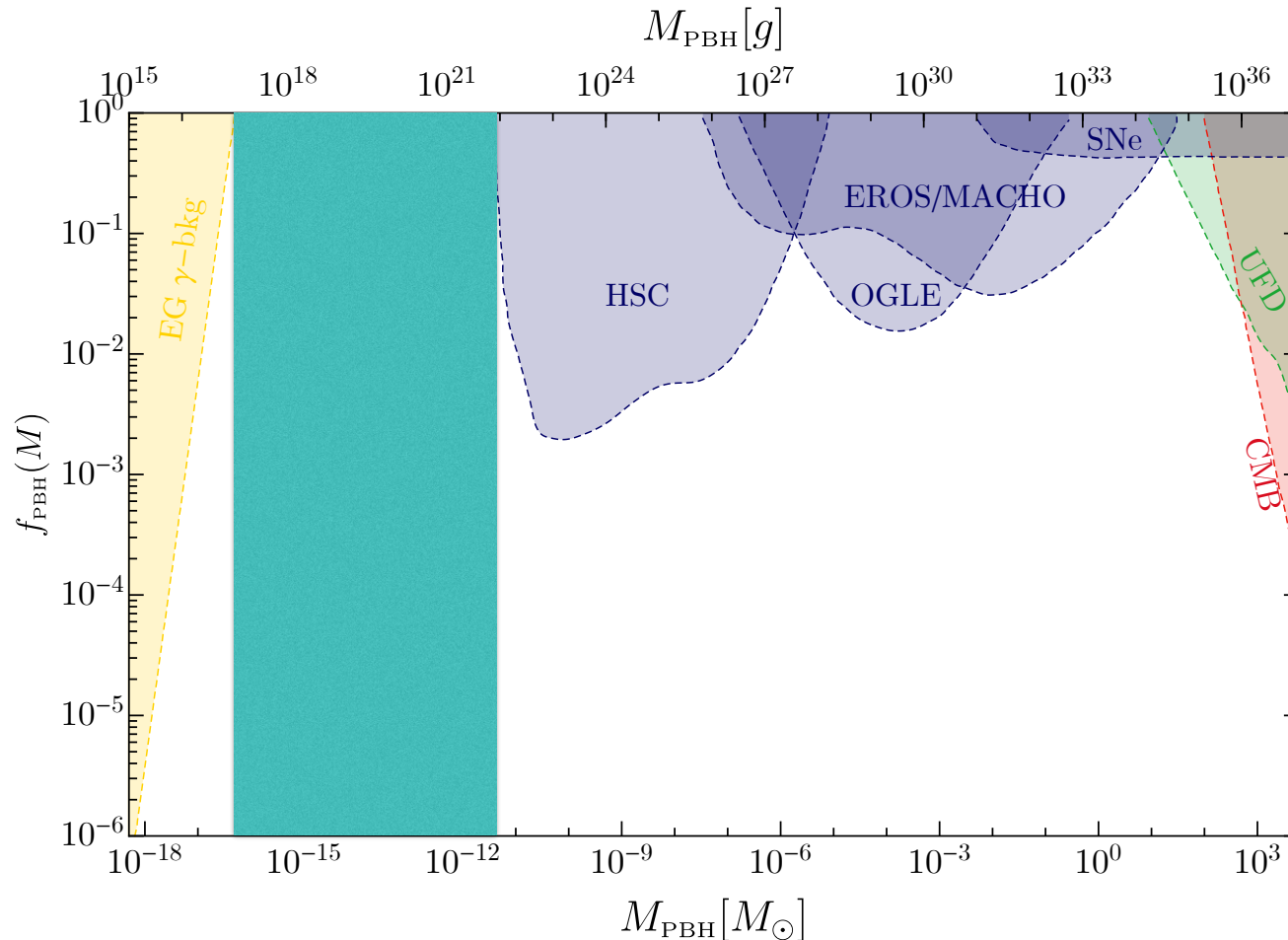
# Key Questions on PBHs in the GW era

- Do (will) PBHs contribute to current (future) GW signals?
- What are the smoking-gun signals of PBHs and how to distinguish them from astrophysical sources?
- Can PBHs account for all the dark matter in the universe?

# PBHs

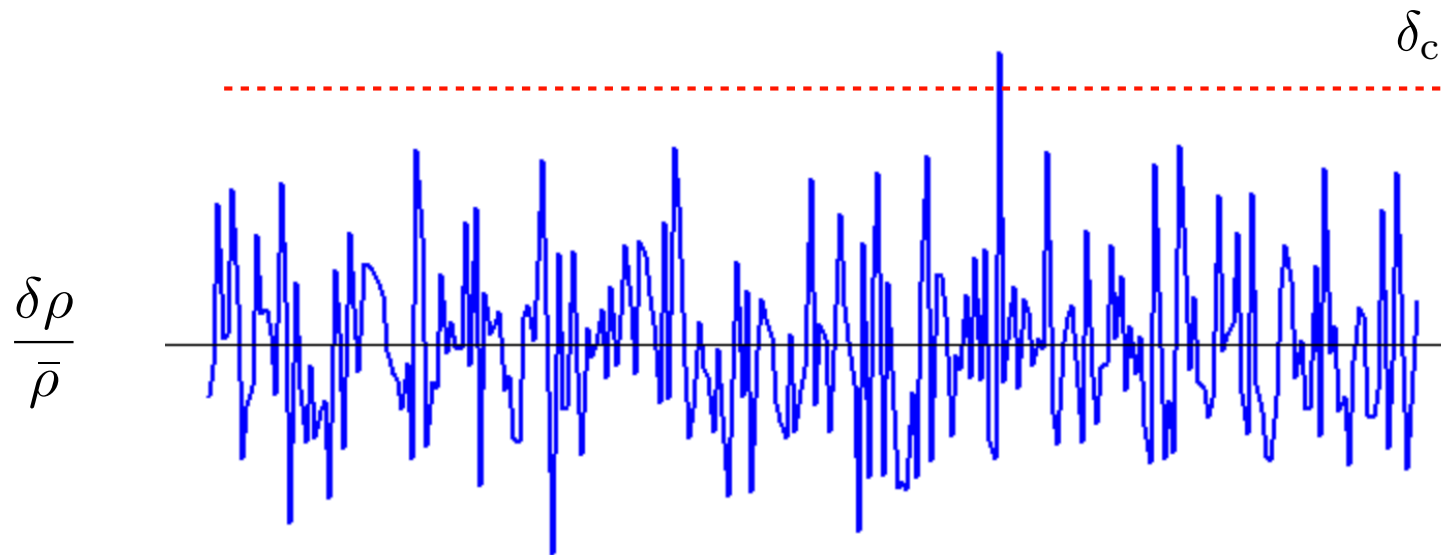
Primordial black holes can compose all the dark matter (or a fraction of it)

$$f_{\text{PBH}} = \Omega_{\text{PBH}} / \Omega_{\text{DM}}$$



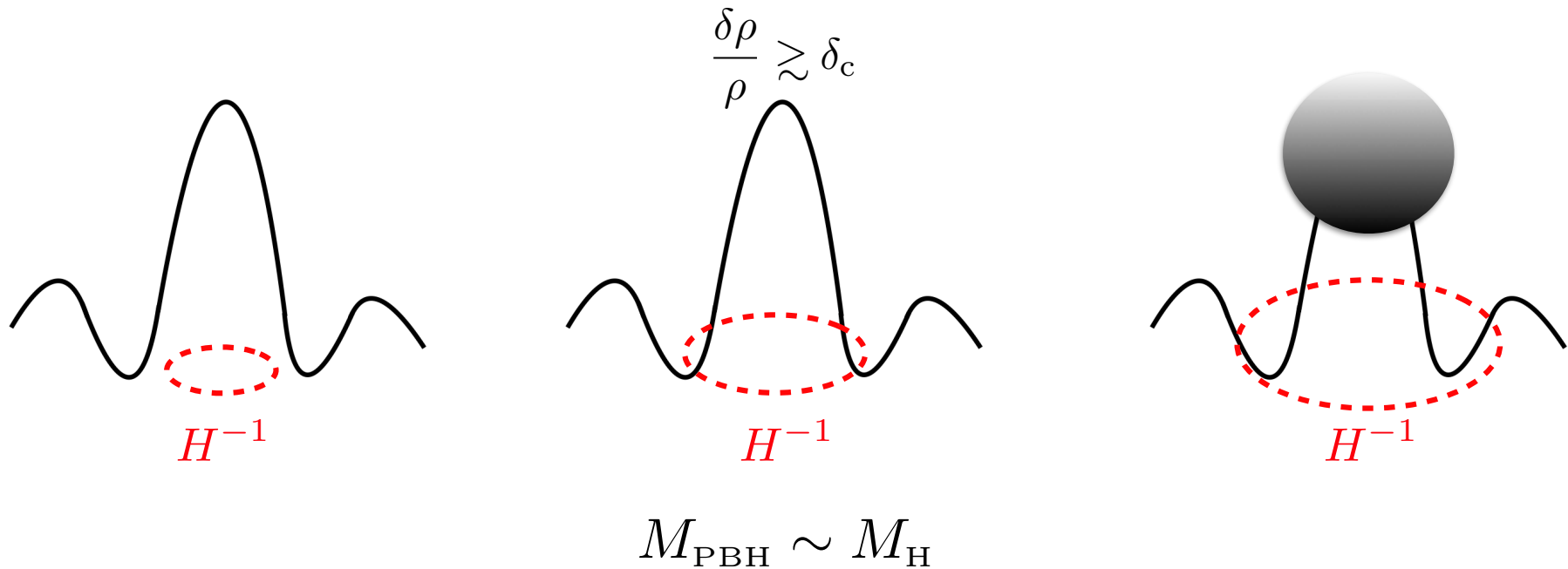
# Where the PBHs may come from?

PBHs may be originated from peaks of the density perturbations generated in the early universe



# Where the PBHs may come from?

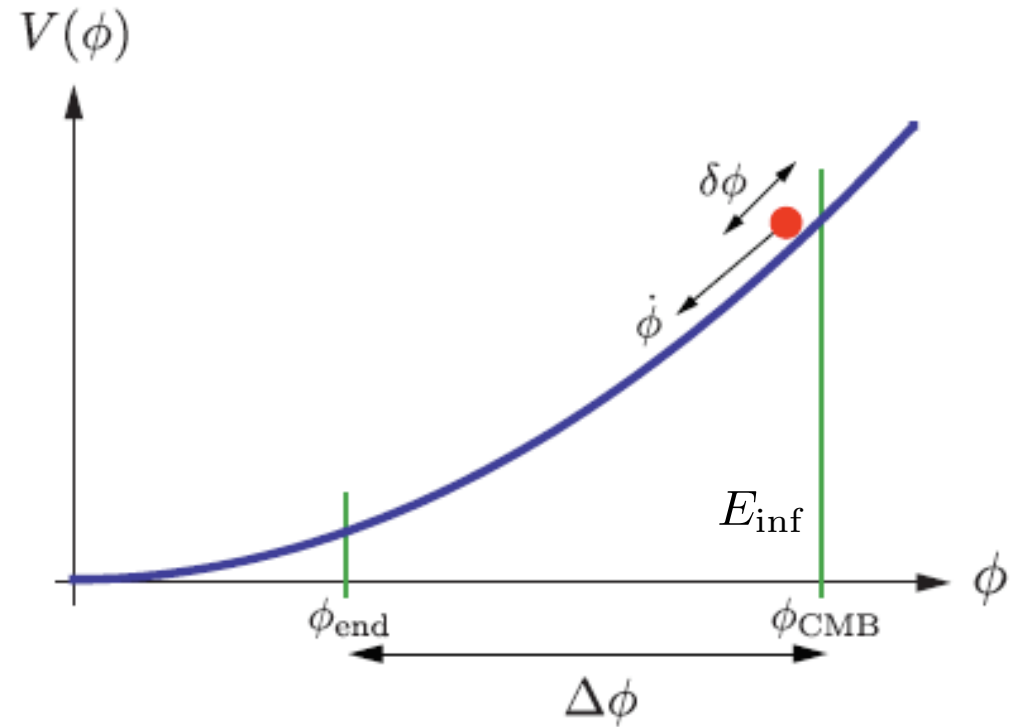
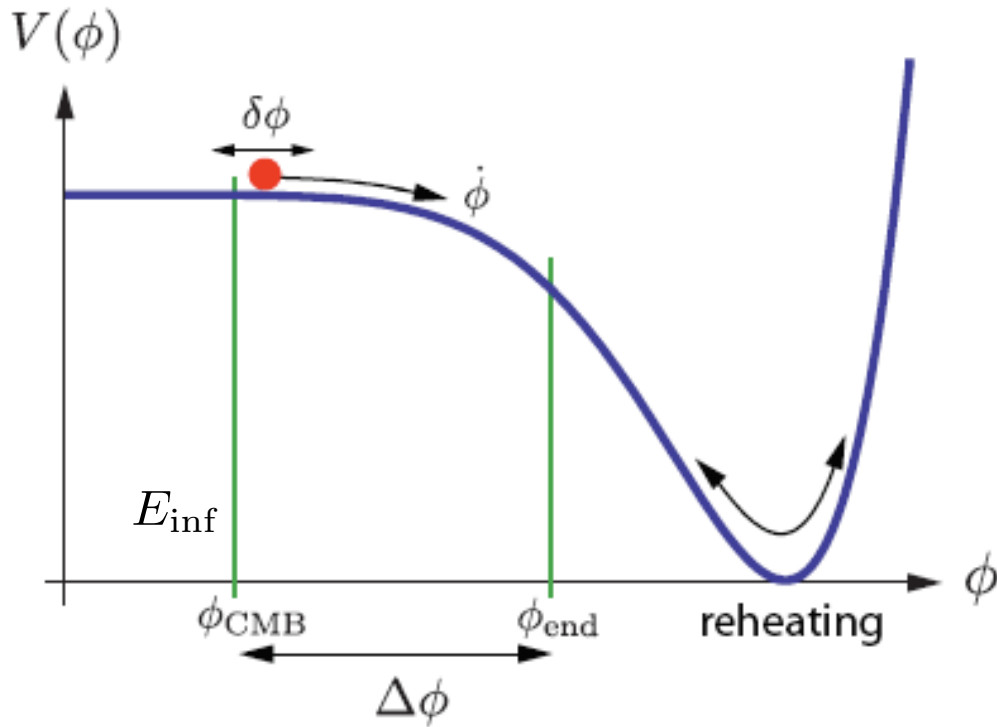
PBHs may be originated from peaks of the density perturbations generated in the early universe



PBHs are rare events, tail of the distribution

One possible mechanism: large fluctuations from inflation

# Inflation

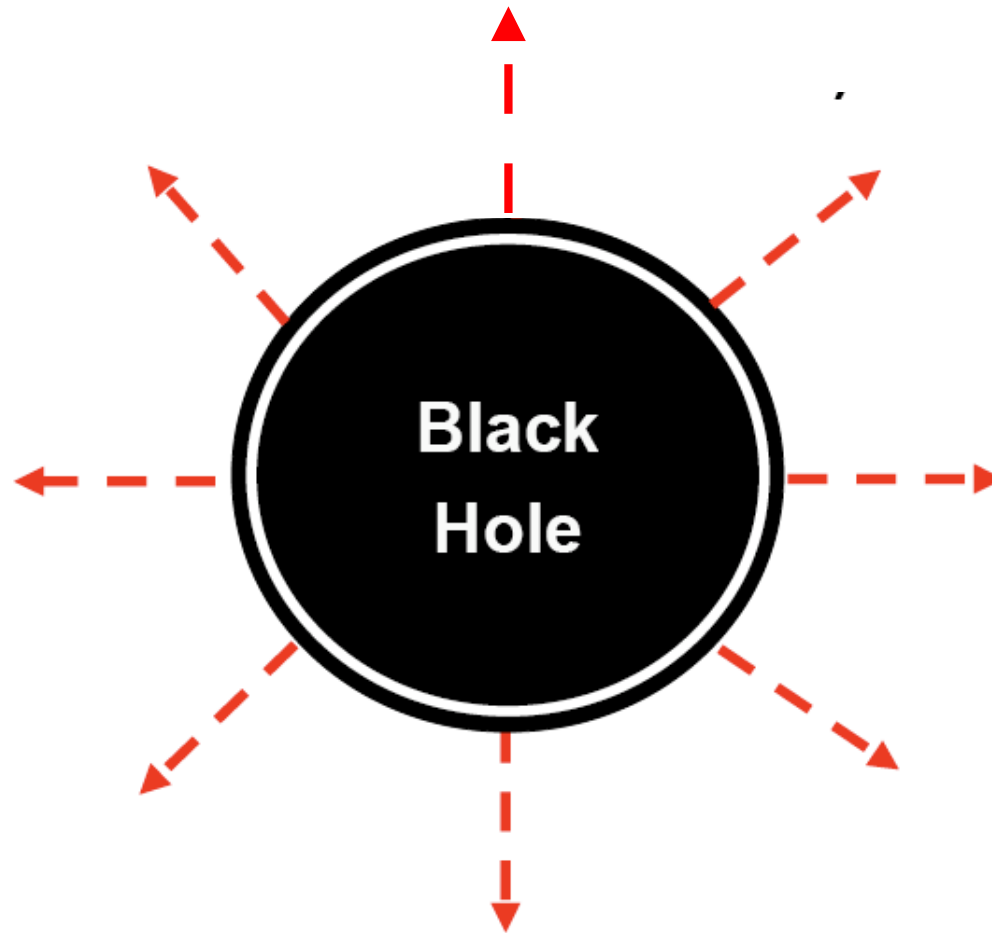


$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{\rho}{3m_{\text{P}}^2} = \frac{E_{\text{inf}}^4}{3m_{\text{P}}^2} \Rightarrow a(t) \sim e^{Ht}$$

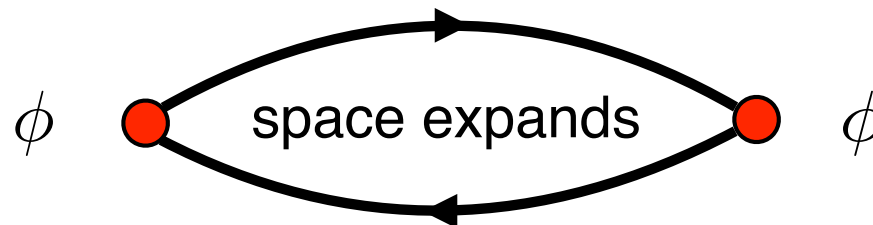
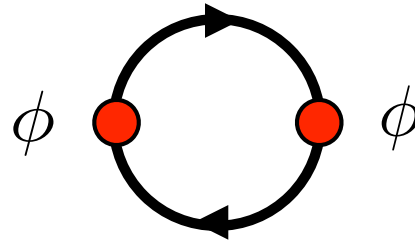


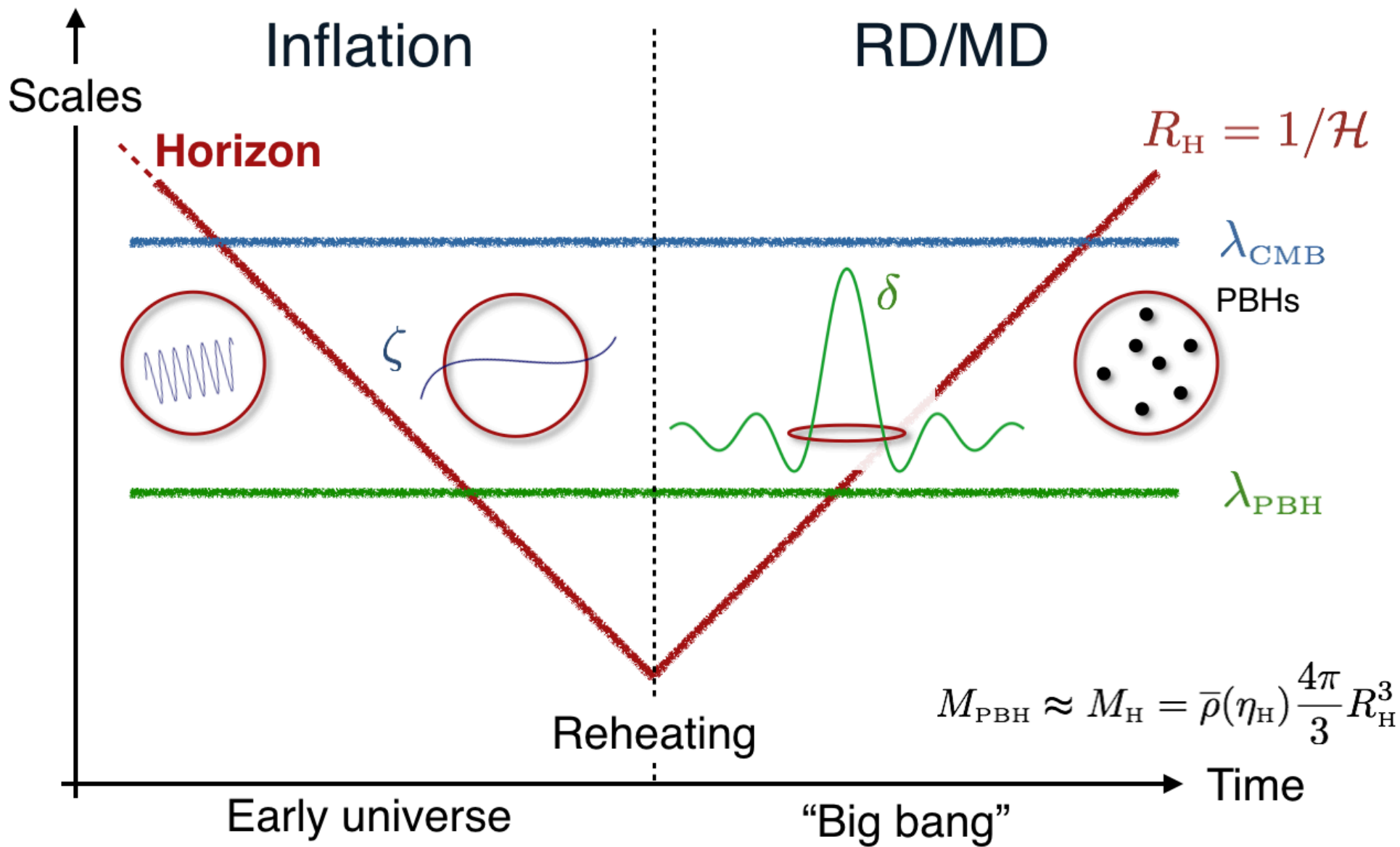
# Where is the structure coming from?

Hawking radiation

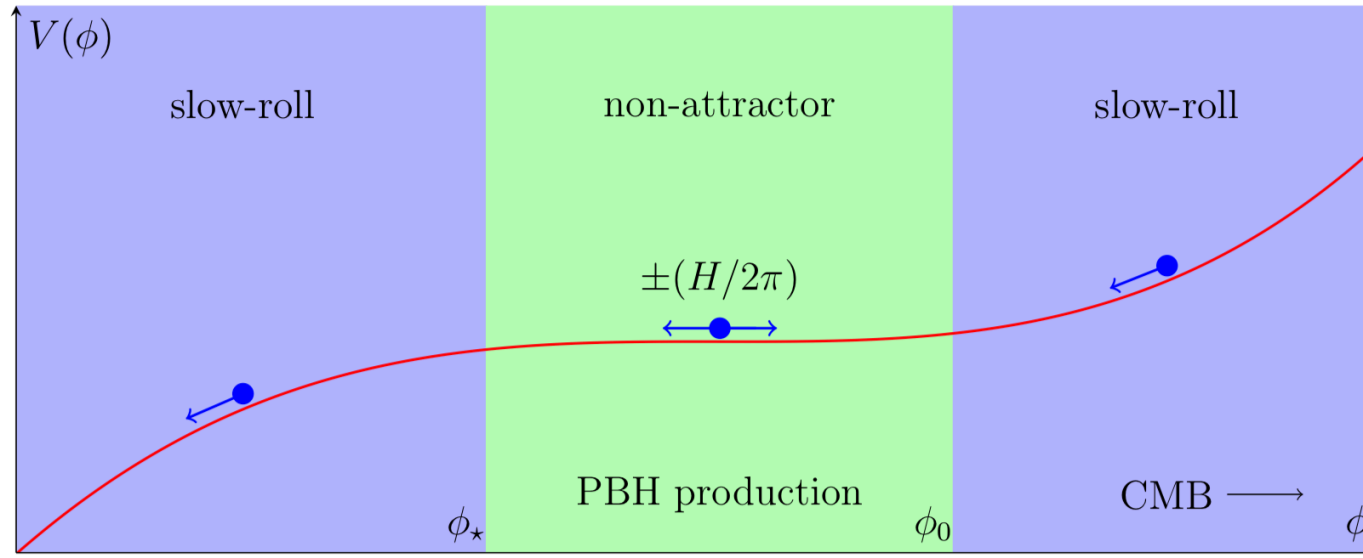


# Heuristic argument for the inflationary perturbations



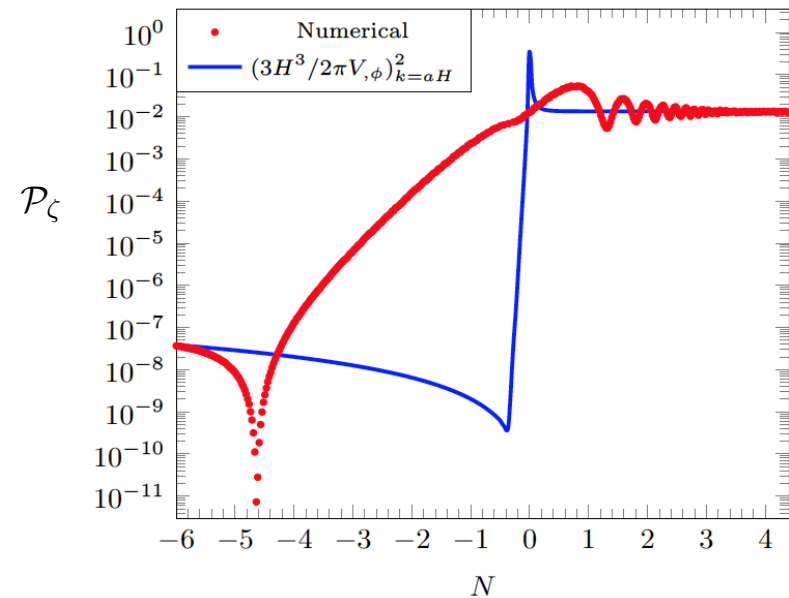


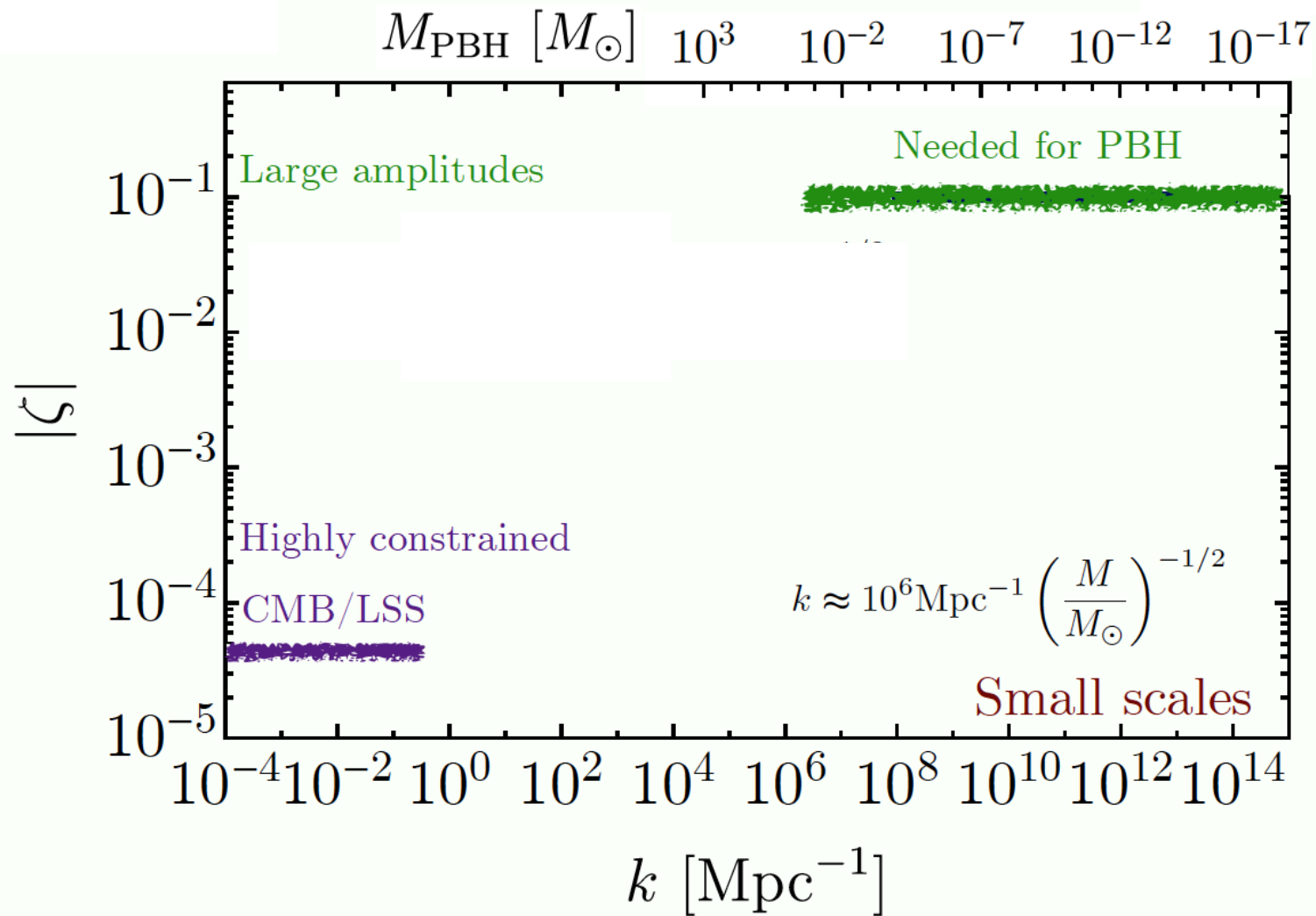
# Ultra-slow-roll during inflation



$$\mathcal{P}_\zeta^{1/2} = \frac{H^2}{2\pi|\dot{\phi}|}$$

$$\frac{d\phi}{dN} \sim e^{-3N} \Rightarrow \mathcal{P}_\zeta^{1/2} \sim e^{3N}$$





# Properties of PBHs at formation

# The PBH mass function at formation

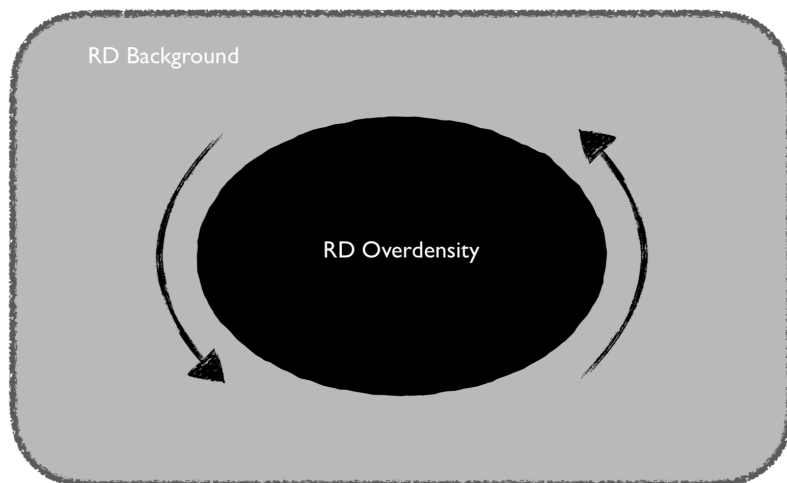
Mass distribution dependent on the overdensity perturbation spectrum and statistical properties

Standard parametrisation

$$\psi(M_{\text{PBH}}) = \frac{1}{\sqrt{2\pi}M_{\text{PBH}}} \exp\left(-\frac{\ln^2(M_{\text{PBH}}/M_c)}{2\sigma^2}\right)$$

# The spin of PBHs at formation is small

- PBHs originate from peaks, that is from *maxima* of the local density contrast.
- The spin results from the action of the torques generated by the gravitational tidal forces upon horizon crossing



$$\vec{\chi} = \vec{S} / G_N M_{\text{PBH}}^2$$

$$\chi_i \sim 10^{-2} \sqrt{1 - \gamma^2}$$

Shape of the density power spectrum

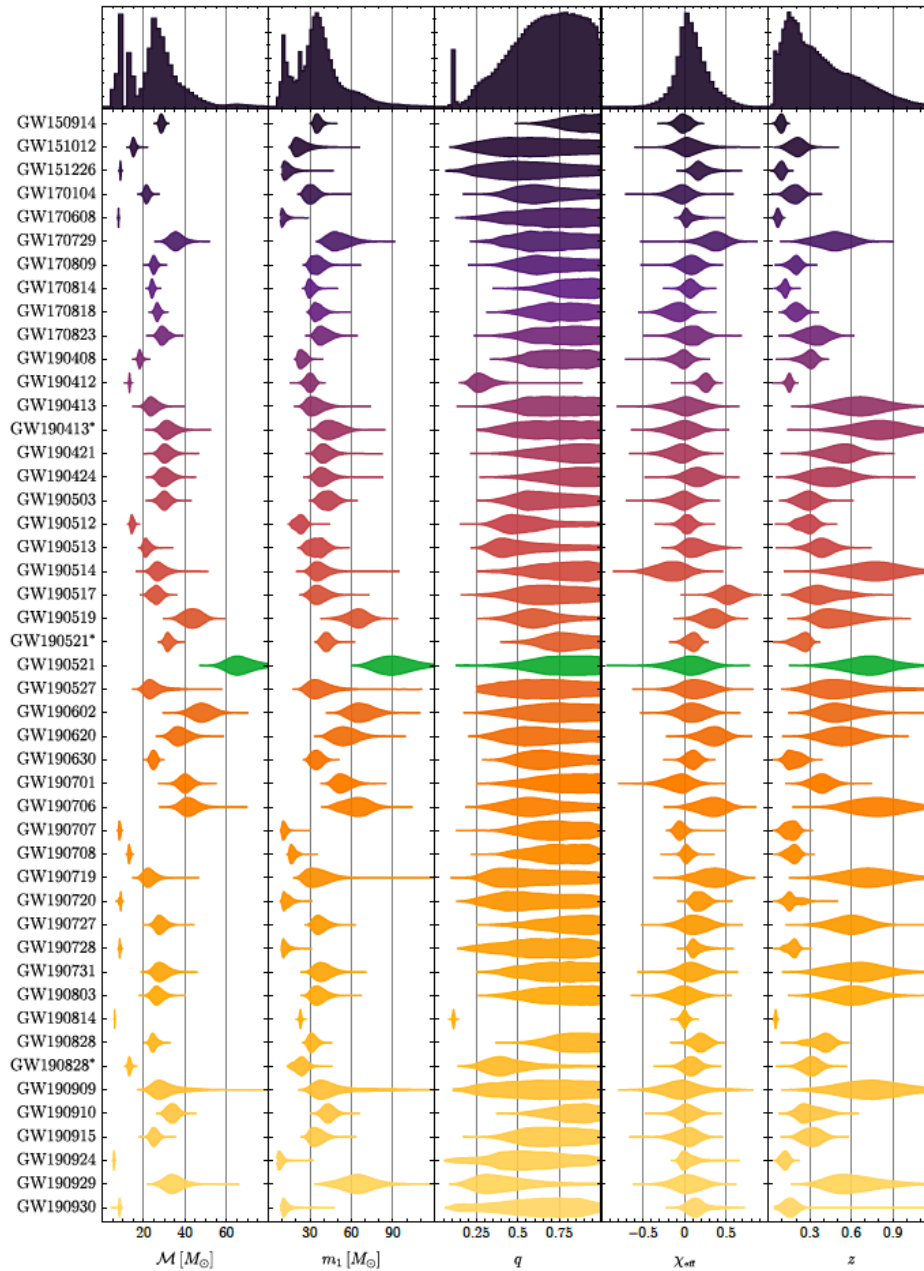
De Luca et al. (2018)



# Key Questions on PBHs in the GW era

- Do (will) PBHs contribute to current (future) GW signals?
- What are the smoking-gun evidences for PBHs and how to distinguish them from astrophysical sources?
- Can PBHs account for all the dark matter in the universe?

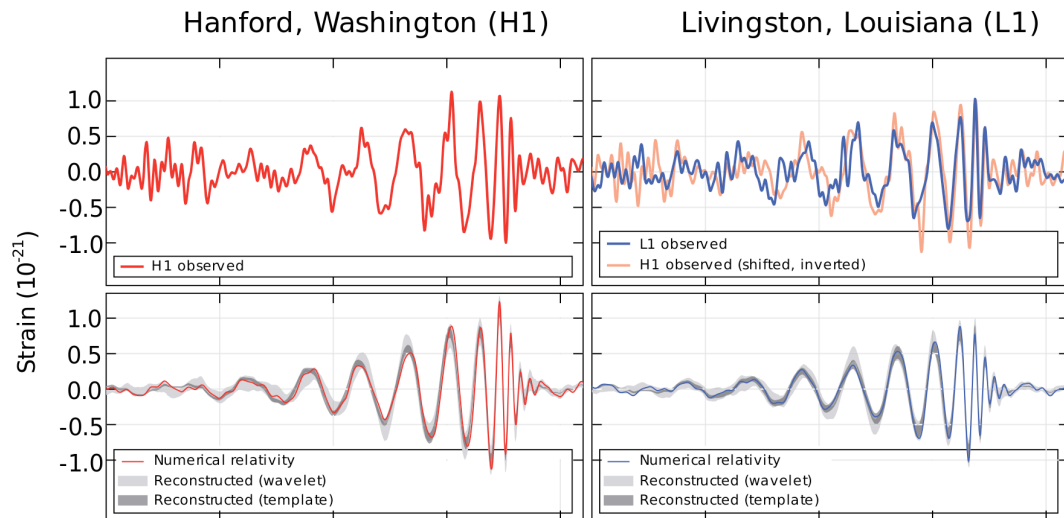
# GWTC-2 catalogue



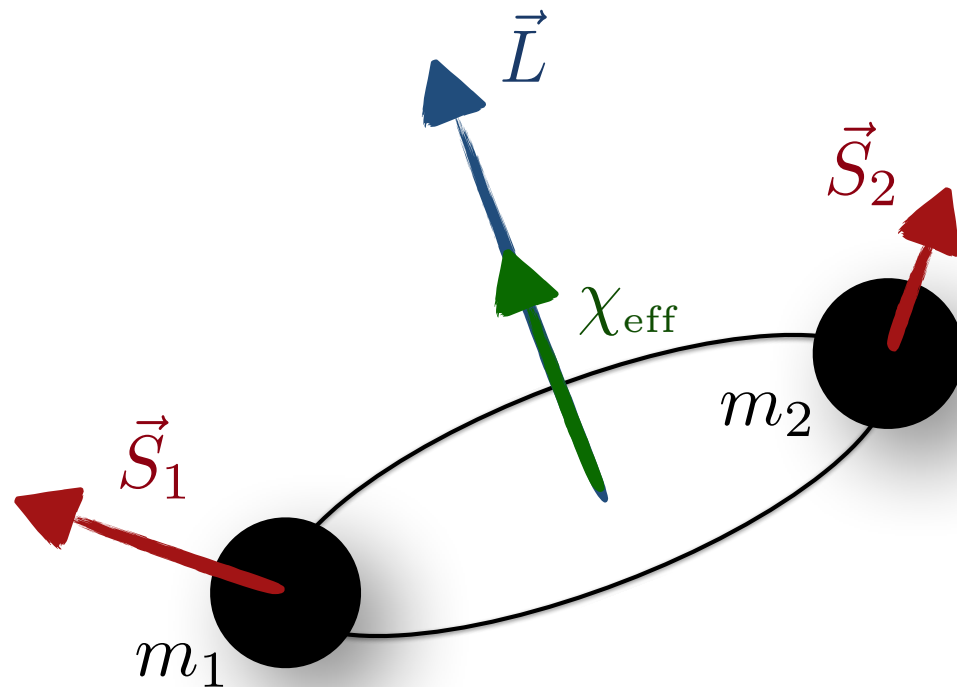
Most events consistent with equal masses

About 10 events with large spins

# BH binary



GW150914, LIGO (2016)



Waveforms dependent on the binary event parameters

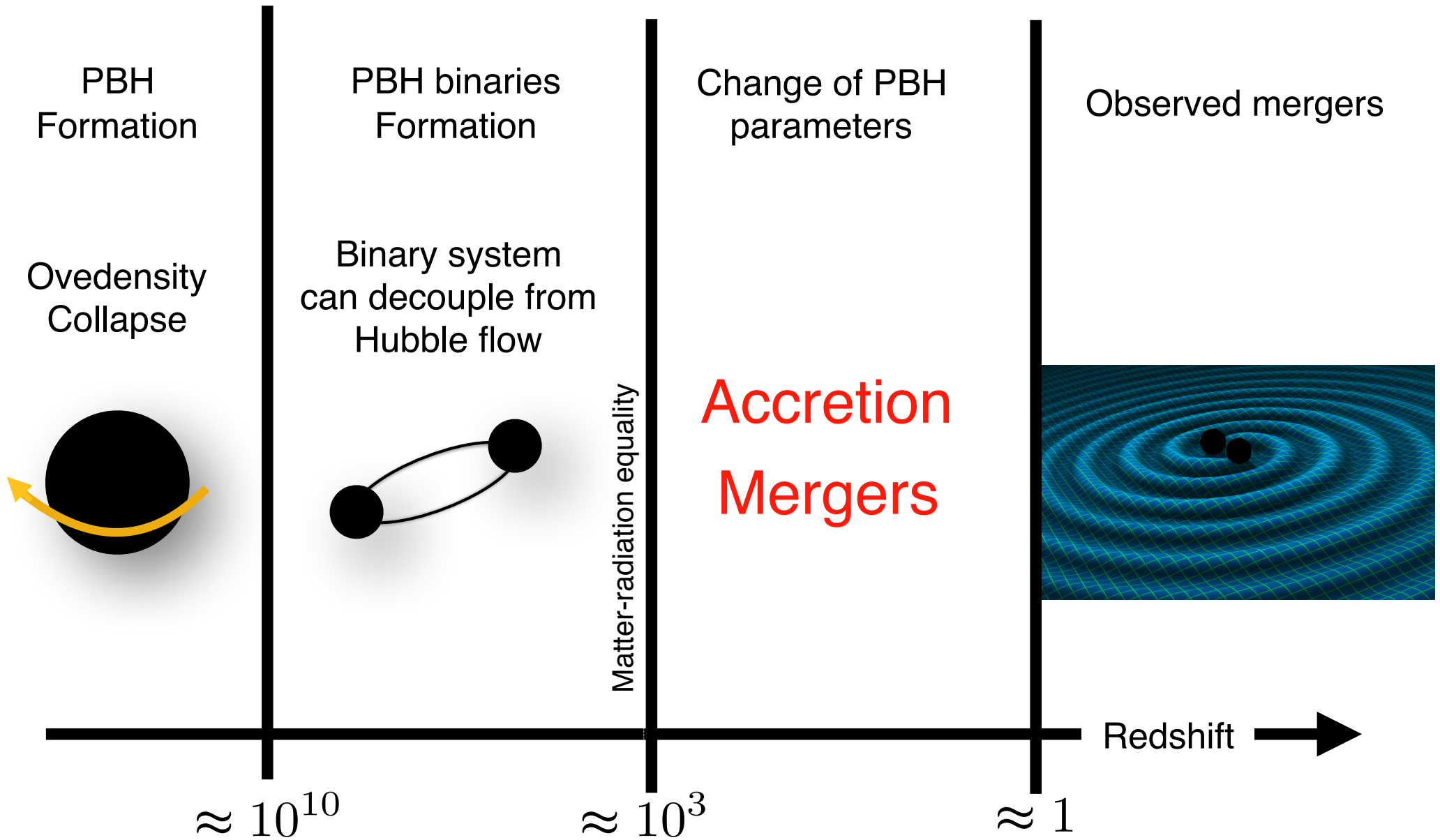
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$q = m_2 / m_1$$

$$\chi_{\text{eff}} = \frac{\vec{S}_1 / m_1 + \vec{S}_2 / m_2}{m_1 + m_2} \cdot \hat{L}$$

...

# PBH evolution



# Accretion onto isolated PBHs

For  $f_{\text{PBH}} < 1$  PBHs coexist with another DM component in the universe

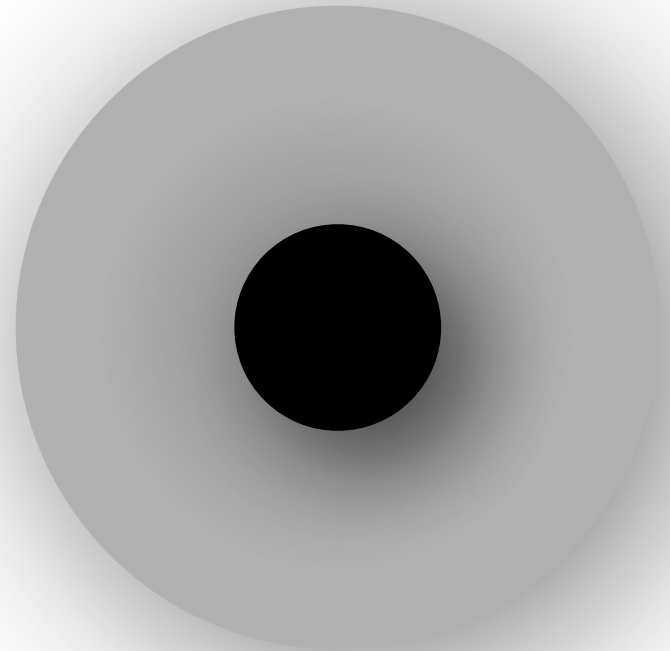
A DM halo builds up around the PBHs  
enhancing accretion

(larger gravitational potential well)

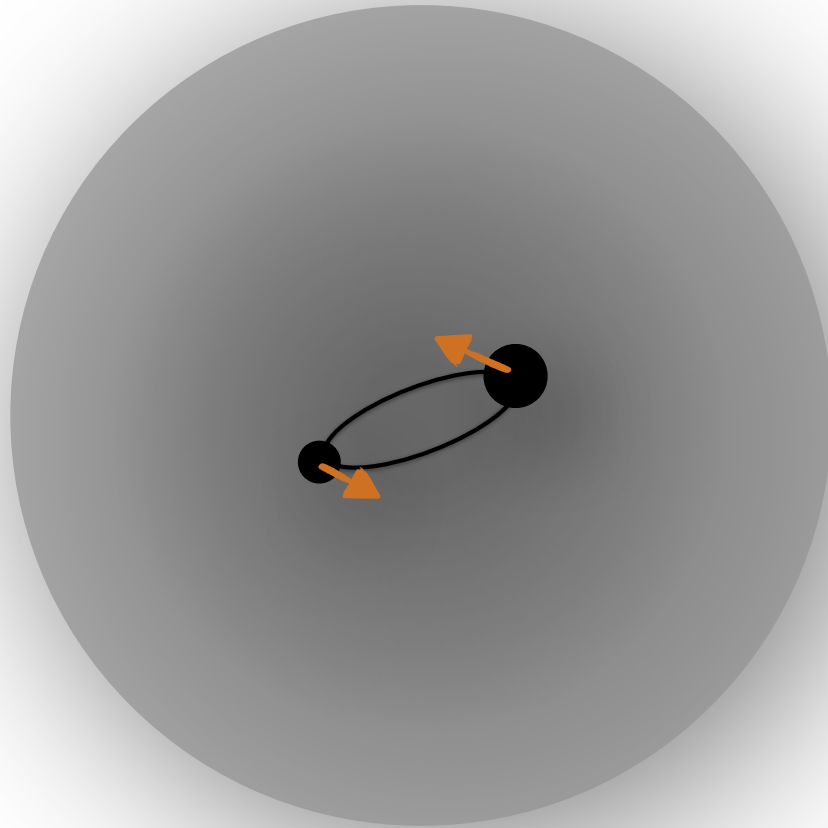
$$M_h(z) \approx 3M_{\text{PBH}} \left( \frac{1000}{1+z} \right)$$

Bondi-Hoyle accretion from the  
surrounding baryonic fluid

$$\dot{M} = 4\pi\lambda m_H n_{\text{gas}} v_{\text{eff}}^{-3} M^2$$



# Accretion onto PBH binaries



Accretion on the system enhances the gas density around the PBH binary

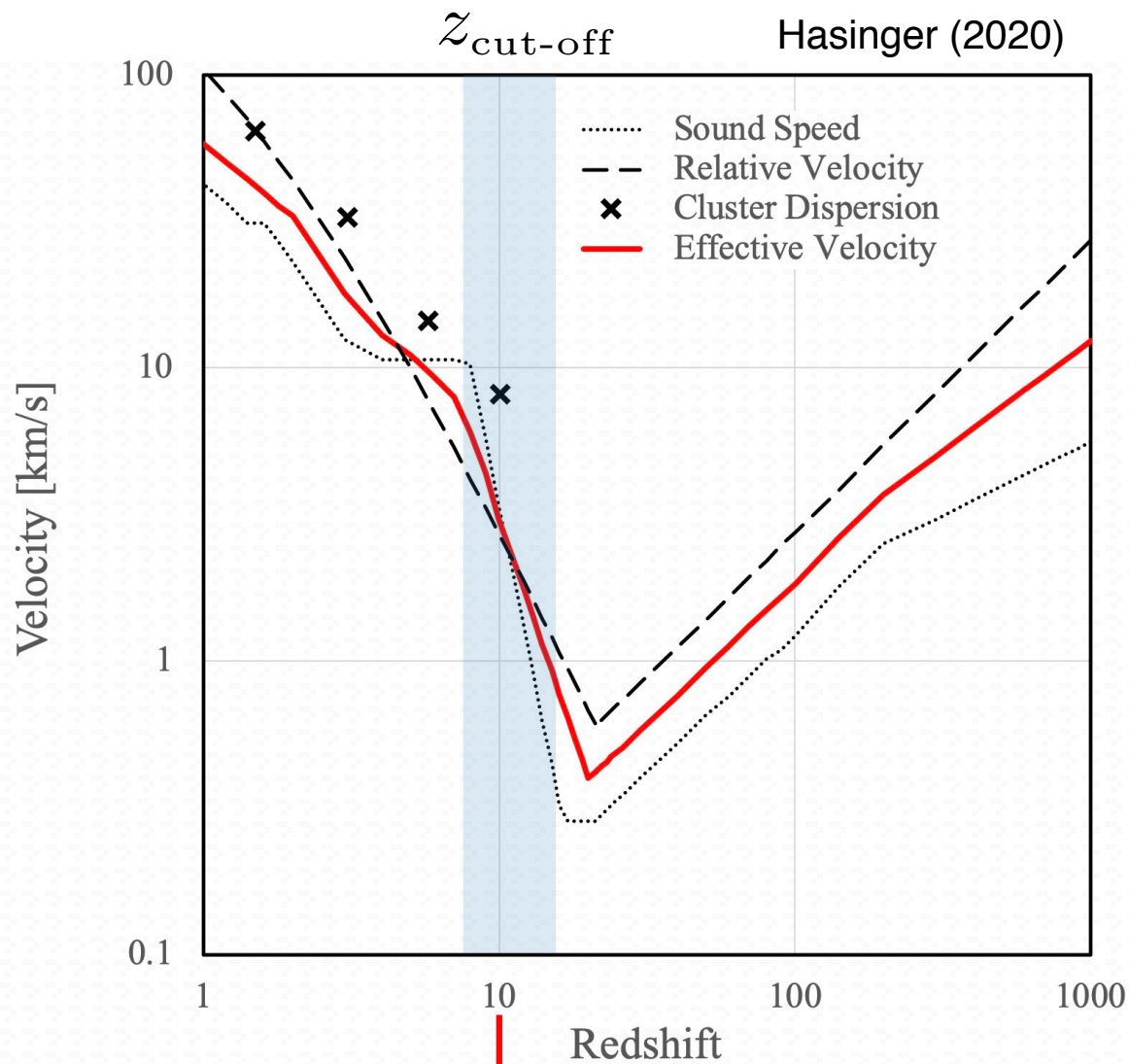
Accretion on the single PBH modulated by masses and orbital velocities

$$\dot{M}_1 = \dot{M} \frac{1}{\sqrt{2(1+q)}}$$

$$\dot{M}_2 = \dot{M} \sqrt{\frac{q}{2(1+q)}}$$

$$q = 1 \quad \text{fixed point}$$

- The smaller PBH always experiences a larger relative accretion
- PBH can experience accretion for  $M \gtrsim \mathcal{O}(10)M_\odot$



Structure formation  
reionization epoch

- Virialised velocities
- Higher temperatures

$$\dot{M} \approx (v_{\text{rel}}^2 + c_s^2)^{-3/2}$$

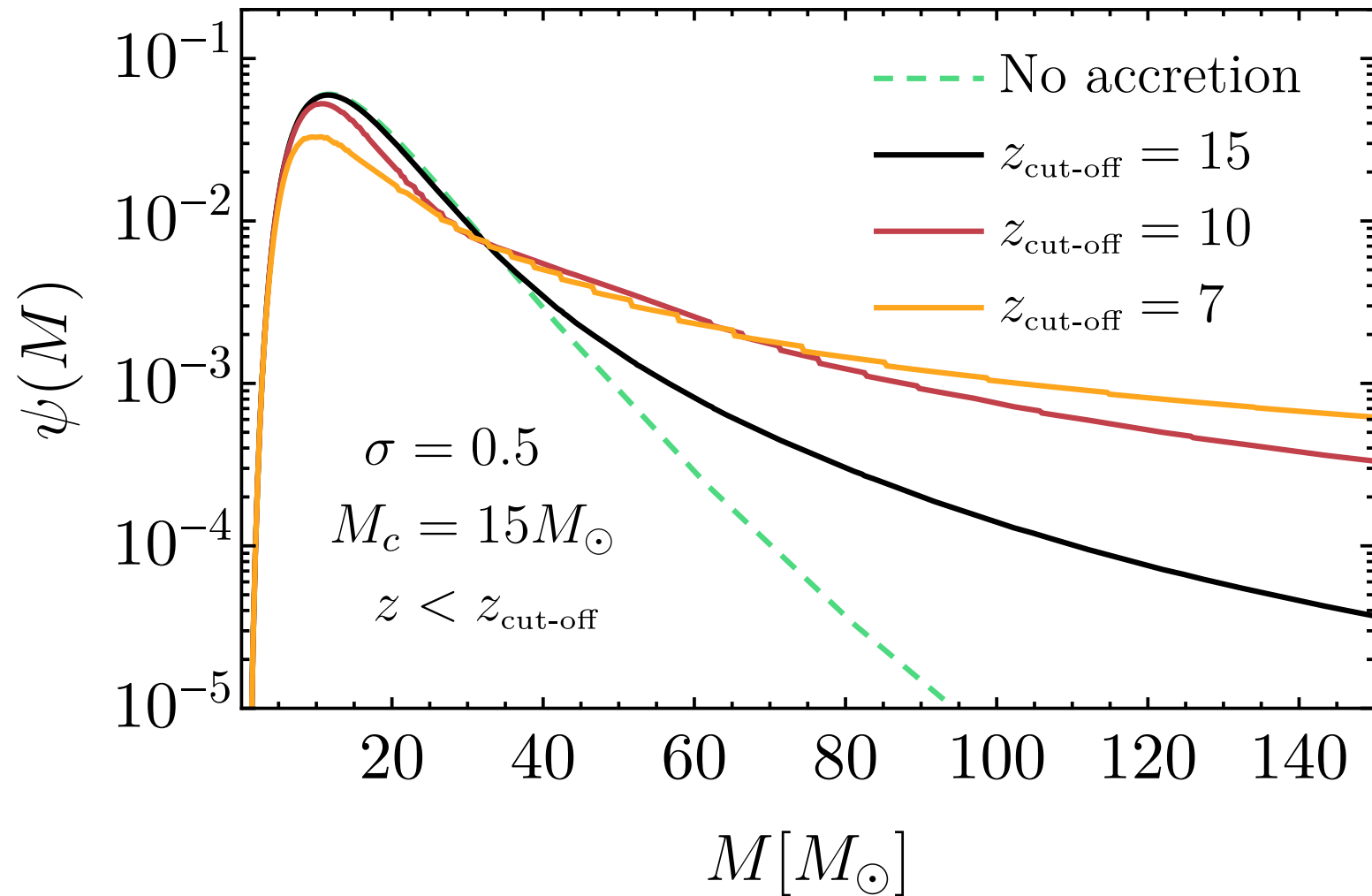


Strong suppression around

$z_{\text{cut-off}}$

Uncertainties  
in the accretion model  
accounted for by  
varying the cut-off

# PBH mass function evolution

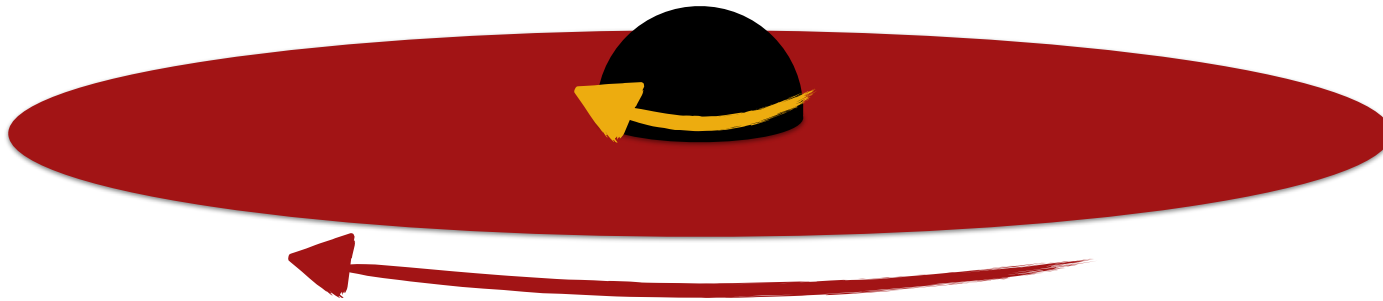


Non-linear mass evolution enhances large-mass tails



# PBH spin evolution

If matter angular momentum is large enough, an accreting disk forms, leading to a spin growth

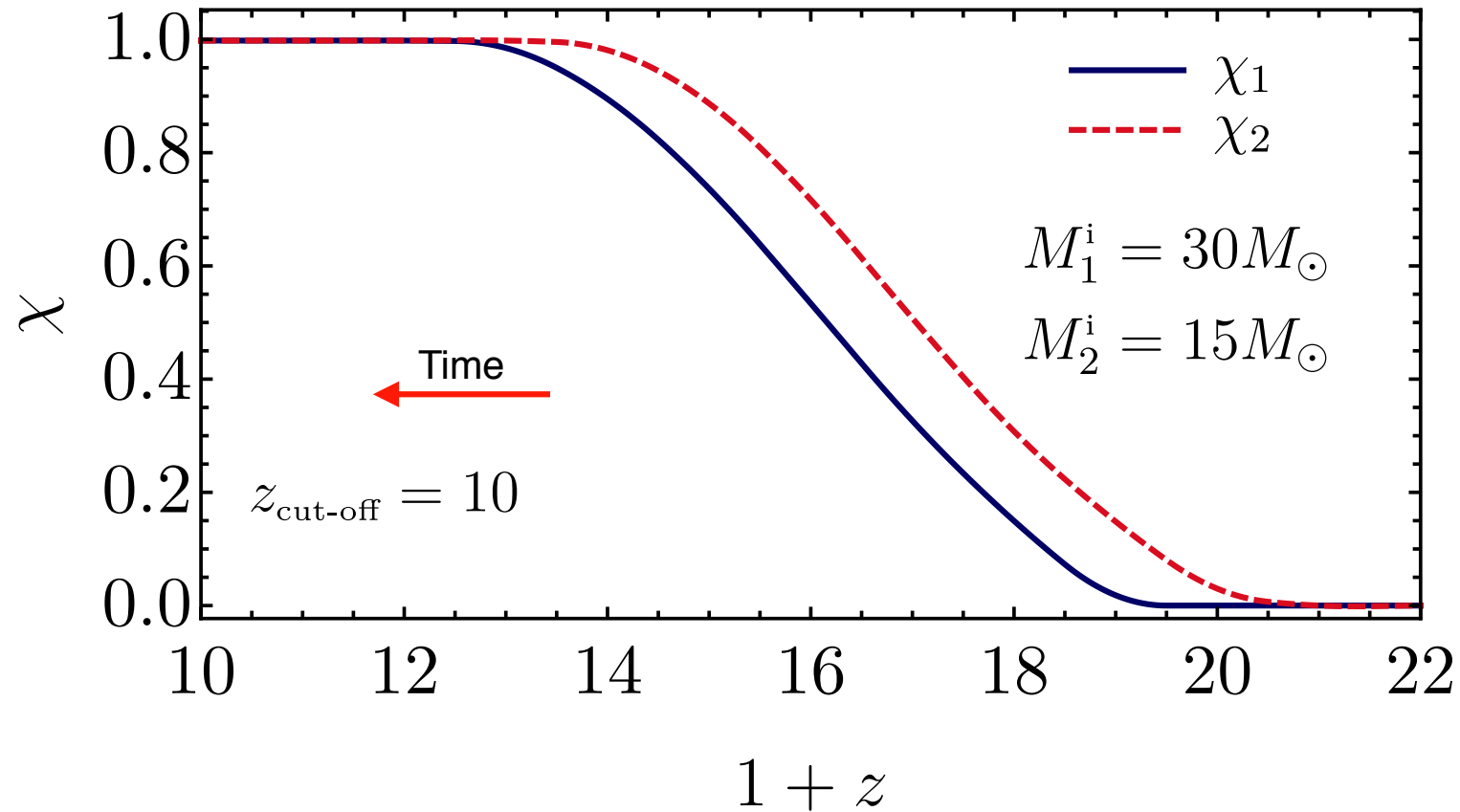


Angular momentum transfer between gas and PBH

$$\dot{\chi} = g(\chi) \frac{\dot{M}}{M}$$

by solving the geodesic model of disk accretion

# Spins pushed towards extremality



- Uncorrelated spin orientation
- Effective spin spreads around zero
- Accretion: low/large mass - low/large spin correlation

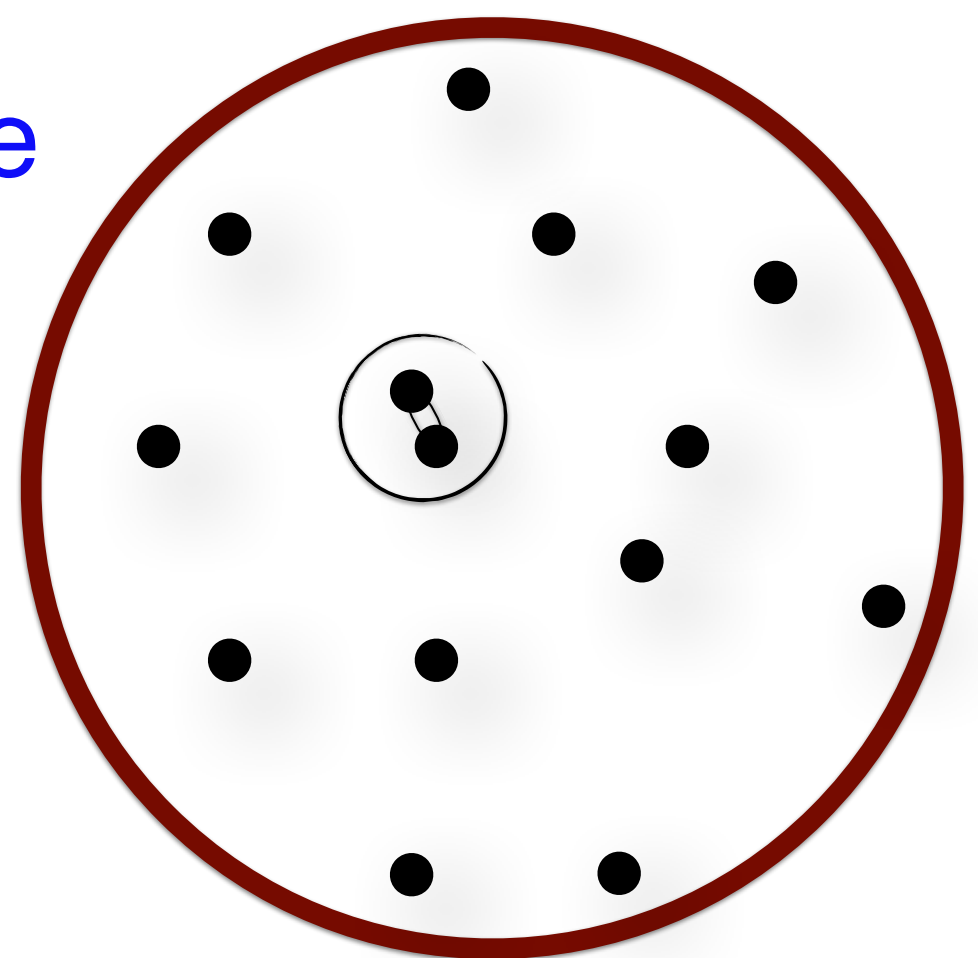
# Merger rate

- Initial spatial Poisson distribution
- Random decoupling of binary systems



Compute probability of decoupling and the binary initial geometry

- Semi-major axis
- Eccentricity



Raidal et al (2018)

$$\frac{dR}{dm_1 dm_2} = \frac{1.6 \times 10^6}{\text{Gpc}^3 \text{ yr}} f_{\text{PBH}}^{\frac{53}{37}} \eta^{-\frac{34}{37}} \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} \left(\frac{M_{\text{tot}}}{M_{\odot}}\right)^{-\frac{32}{37}} S(M_{\text{tot}}, f_{\text{PBH}}) \mathcal{A}_{\text{acc}}(m_j) \psi(m_1) \psi(m_2)$$

- Accretion hardens the binaries
- Larger masses leads to shorter mergers



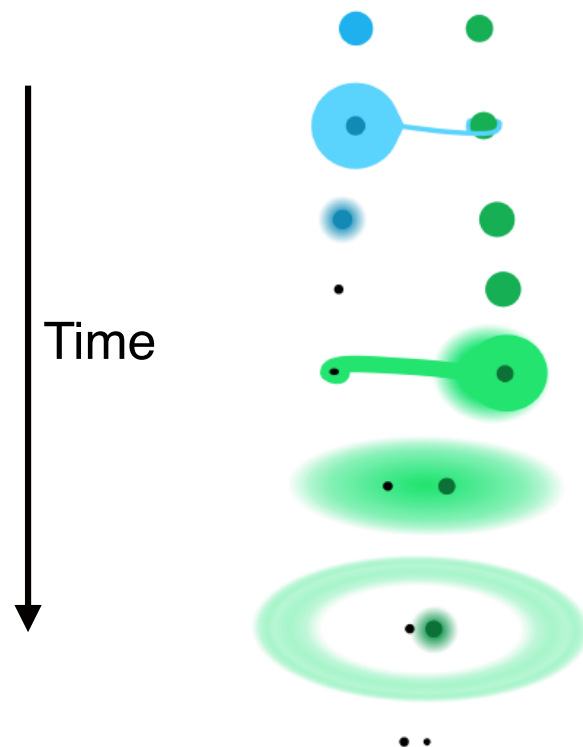
De Luca et al. (2020)

# Astrophysical populations

Zevin et al. (2021)

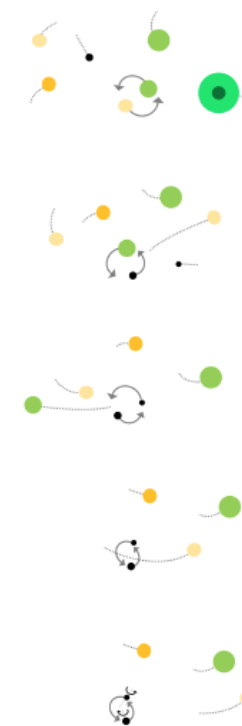
## Isolated formation

Binary formation in galactic fields through a Stable Mass Transfer (SMT) or Common-Envelope (CE) phase



## Dynamical formation

Binary formation in Globular Cluster (GC) or Nuclear Star Clusters (NCS) through encounters and GW captures



For a review, Mandel and Farmer (2018)

# Bayesian evidence in GWTC-2

Event parameters  $\vec{\theta}$

$m_1$        $m_2$        $\chi_{\text{eff}}$        $z$

Population Hyperparameters  $\vec{\lambda}$

$M_c$   $\sigma$   $f_{\text{PBH}}$   $z_{\text{cut-off}}$        $\alpha_{\text{CE}}$   $\chi_b$   $N_{\text{CE}}$   $N_{\text{GC}}$   $N_{\text{NSC}}$

$$p(\vec{\lambda}|\vec{d}) \propto p(\vec{\lambda}) \int d\vec{\theta} p(\vec{d}|\vec{\theta}) p_{\text{pop}}(\vec{\theta}|\vec{\lambda})$$

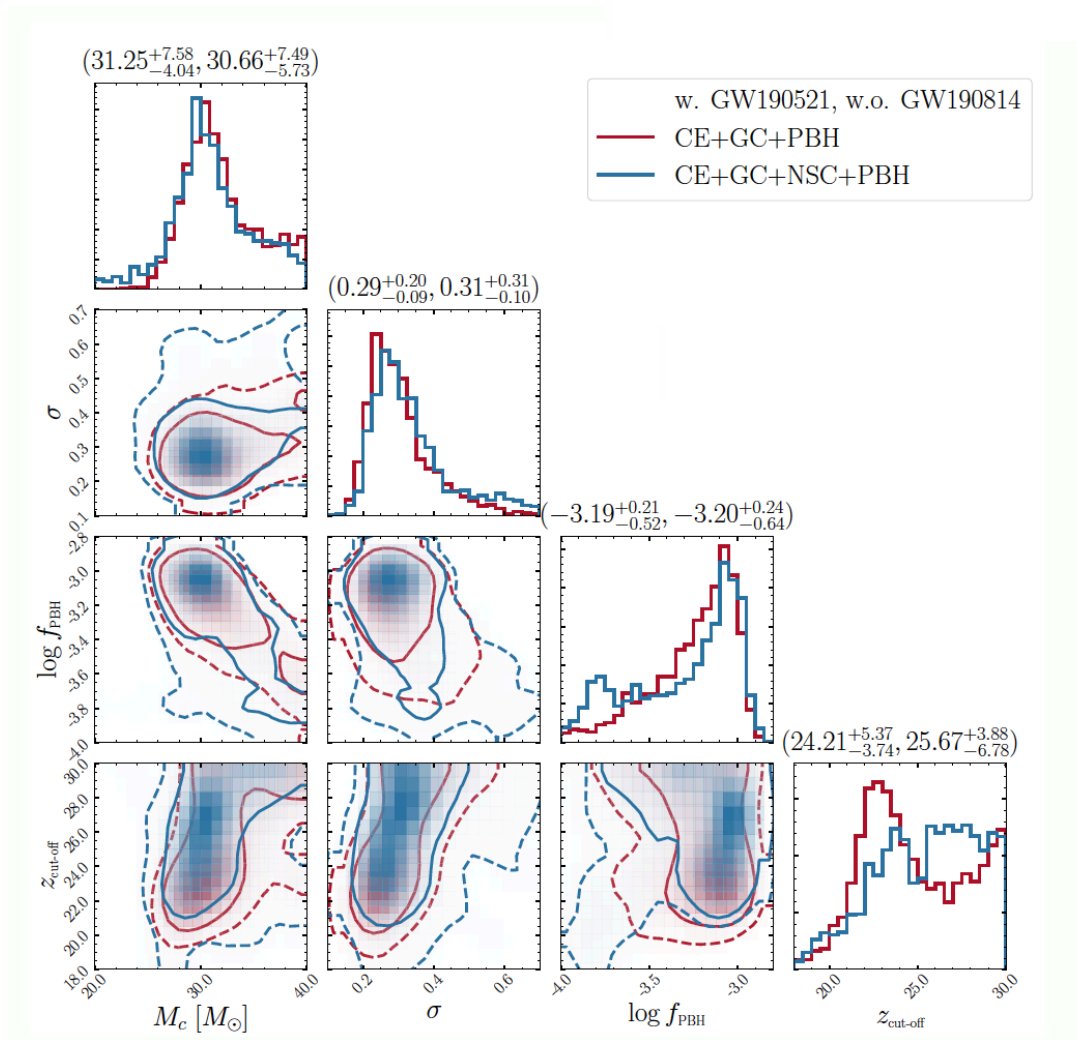
Posterior  
distribution

Hyperparameter  
prior

Single event  
likelihood

Population  
likelihood (ML)

# Population posterior distributions



$$M_c \simeq 30M_\odot$$

$$\sigma \simeq 0.3$$

$$f_{\text{PBH}} \simeq 6 \cdot 10^{-4}$$

$$z_{\text{cut-off}} \simeq 25$$

PBH not the dark matter

Moderate accretion

# Key Questions on PBHs in the GW era

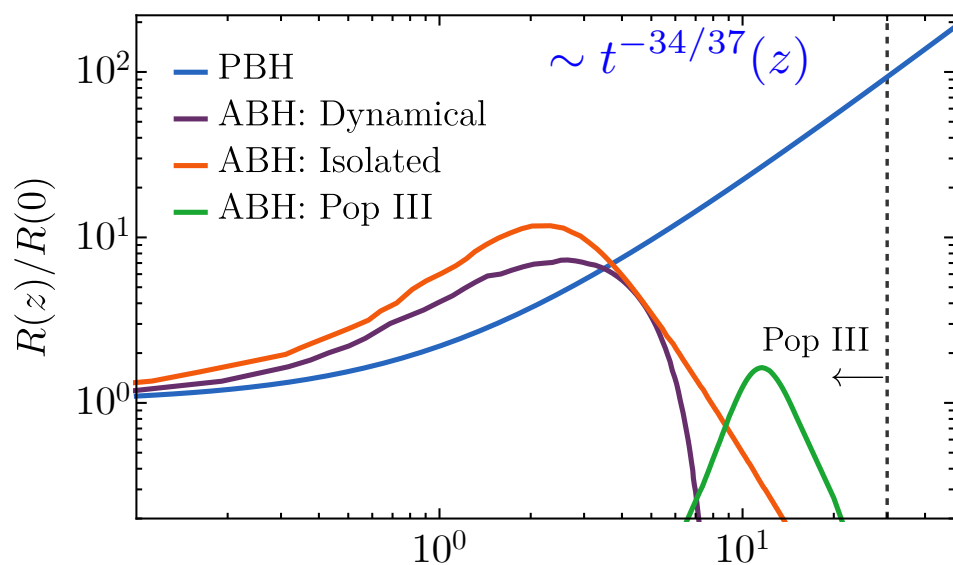
- Do (will) PBHs contribute to current (future) GW signals?
- What are the smoking-gun signals of PBHs and how to distinguish them from astrophysical sources?
- Can PBHs account for all the dark matter in the universe?

# Smoking-gun signals of PBHs

- Merger rate time evolution at high redshifts
- Spin of PBHs (large spins for large masses)
- Stochastic GW background from PBHs at high redshifts



K. Ng et al. (2020)

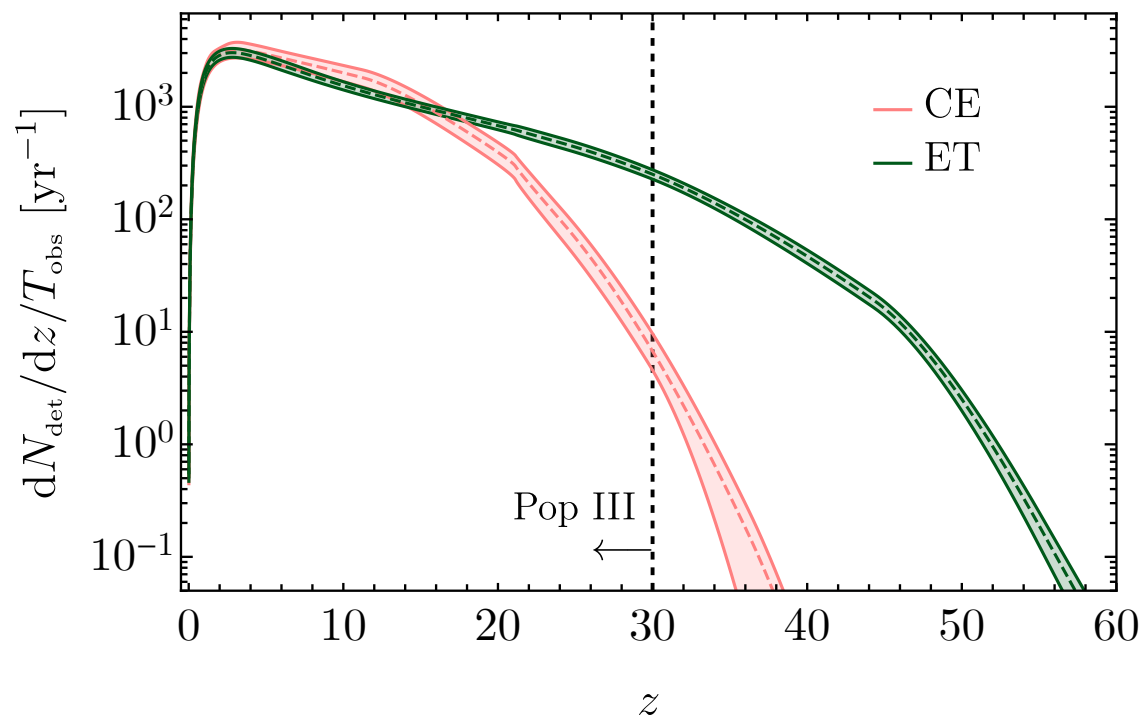
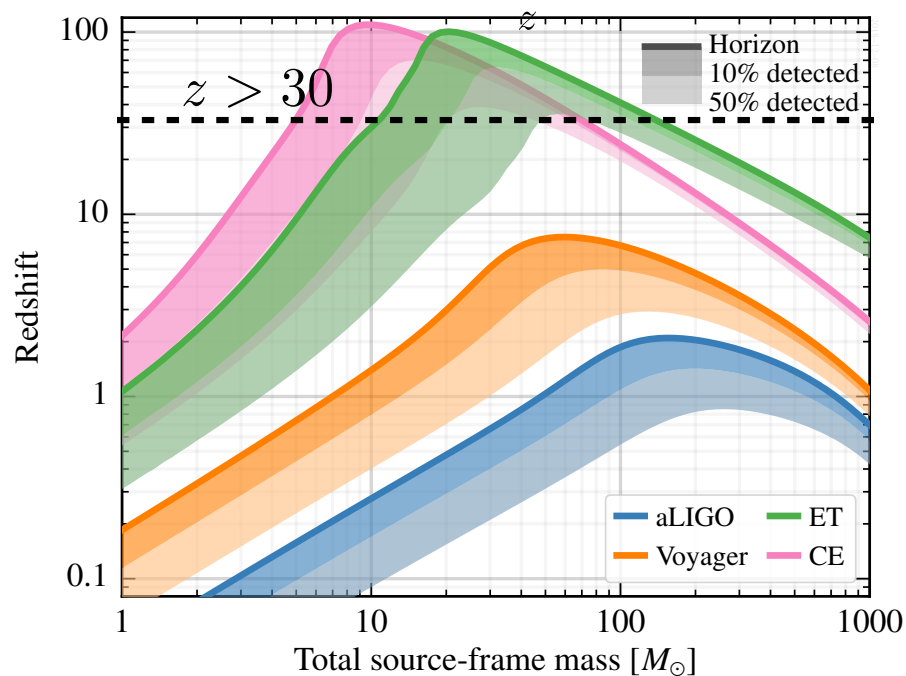


- The PBH population would imply high-redshift observations:

$$N_{\text{det}}^{\text{ET}}(z > 30) = 1315_{-168}^{+305} / \text{yr}$$

No astrophysical contamination

V. De Luca et al. (2021)



# Key Questions on PBHs in the GW era

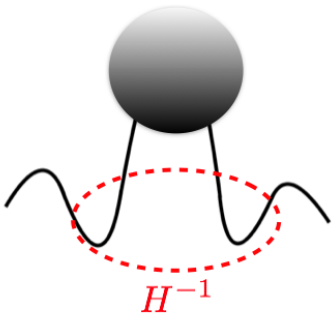
- Do (will) PBHs contribute to current (future) GW signals?
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# PBHs and the stochastic background of GWs

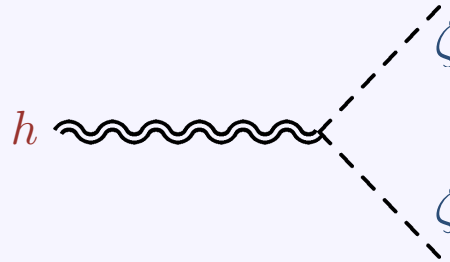
# GWs from PBHs

The same curvature perturbations giving rise to PBHs are unavoidably a source for GWs at *second-order* in perturbation theory

$$\frac{\delta\rho}{\bar{\rho}} \sim \frac{\nabla^2\zeta}{a^2H^2}$$



$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = \mathcal{O}(\partial_i\zeta\partial_j\zeta)$$



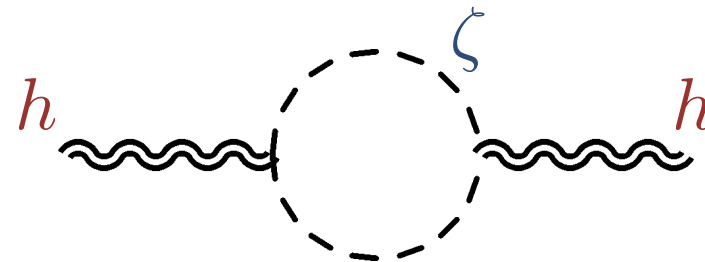
Potentially observable at current and future GW observatories



# GW Power Spectrum

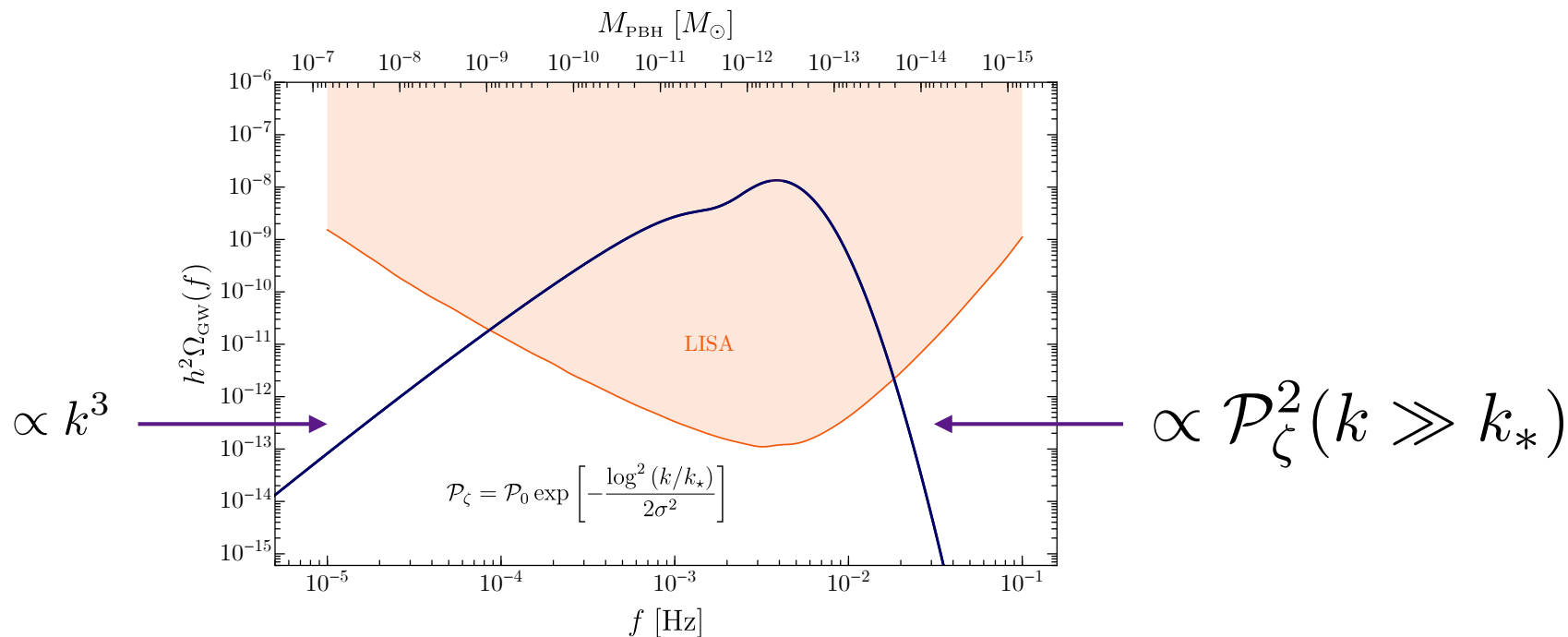
Power spectrum of GWs:

$$\left\langle h^{\lambda_1}(\eta, \vec{k}_1) h^{\lambda_2}(\eta, \vec{k}_2) \right\rangle' \approx \mathcal{P}_\zeta \mathcal{P}_\zeta$$

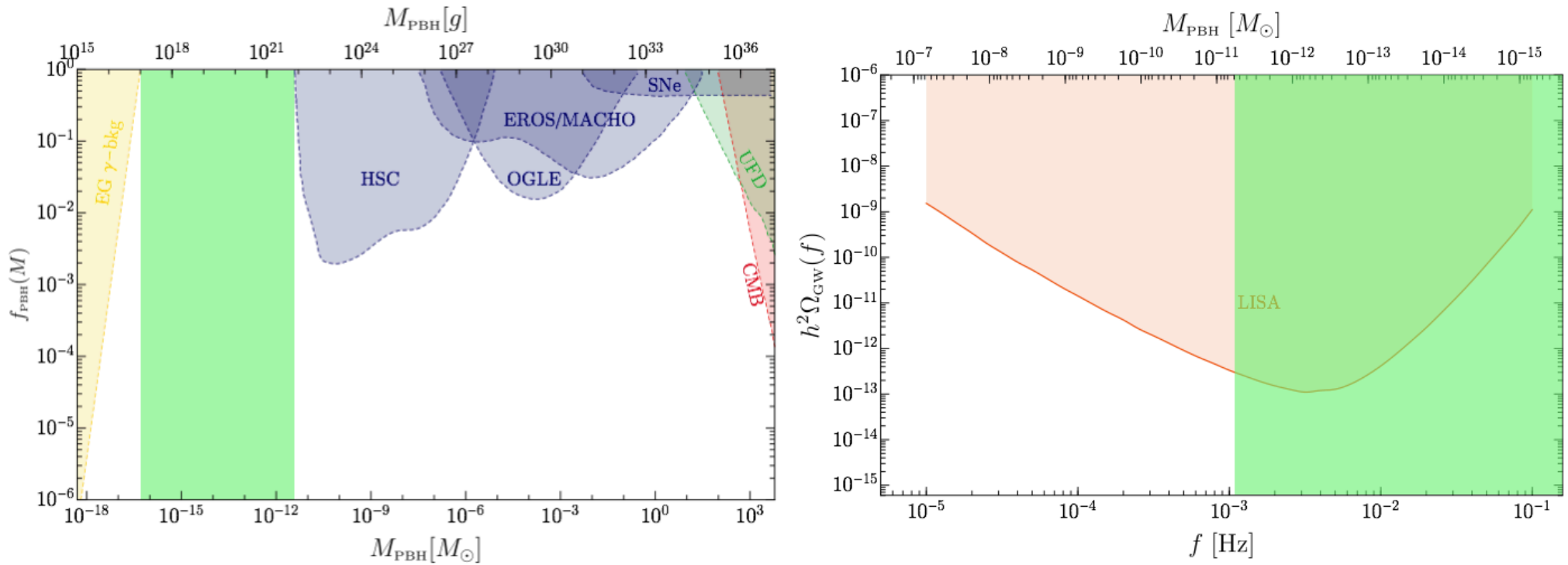


At second order in comoving curvature perturbation, after averaging over the fast oscillating pieces

$$\Omega_{\text{GW}}(\eta, k) = \frac{\pi^2}{243\mathcal{H}^2\eta^2} \int \frac{d^3p}{(2\pi)^3} \frac{p^4 [1 - \mu^2]^2}{p^3 |\vec{k} - \vec{p}|^3} \mathcal{P}_\zeta(p) \mathcal{P}_\zeta(|\vec{k} - \vec{p}|) \mathcal{I}^2(\vec{k}, \vec{p})$$



# The PBH dark matter-LISA serendipity



$$M \simeq 10^{-12} M_{\odot} \left( \frac{f_{\text{LISA}}}{f} \right)^2$$

$$f_{\text{LISA}} = 3.4 \text{ mHz}$$

$$M \approx 10^{-12} M_{\odot}$$

Bartolo et al. PRL (2019)

# Nano-Grav 12.5 year



Millisecond pulsars whose signal sensitive to the stochastic GW background

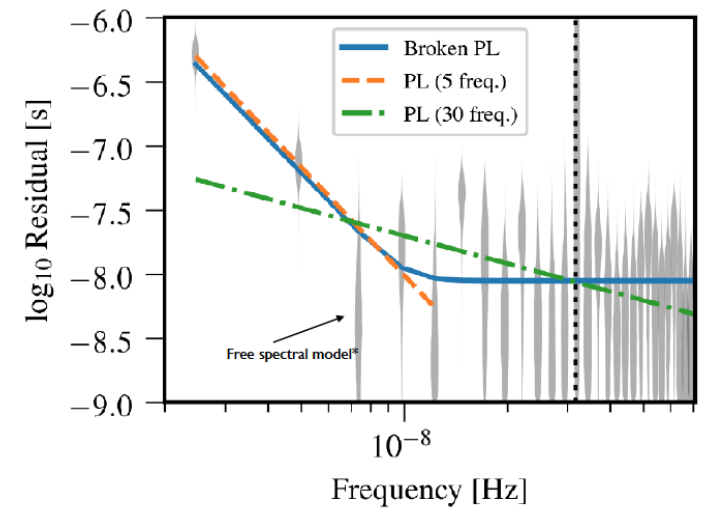
Cross-correlation of  
timing residuals

$$S_{ab} = \Gamma_{ab} \frac{h_c^2}{12\pi^2 f^3}$$

# Nano-Grav 12.5 year

Strong evidence for a stochastic common process across 45 pulsars

$$\Omega(f) = \frac{2\pi^2}{3H_0^2} A^2 f_{\text{yr}}^2 \left( \frac{f}{f_{\text{yr}}} \right)^{5-\gamma}$$



Possible flat spectrum with amplitude

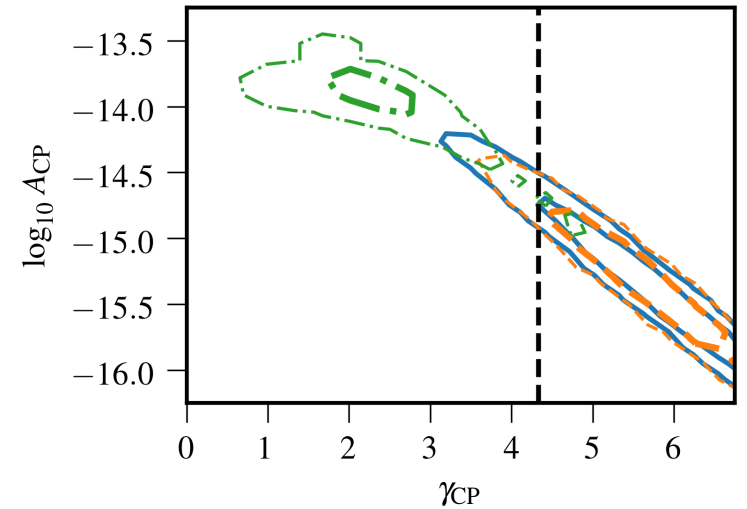
$$\Omega(f) \sim 5 \cdot 10^{-10}$$



# Nano-Grav 12.5 year

Strong evidence for a stochastic common process across 45 pulsars

$$\Omega(f) = \frac{2\pi^2}{3H_0^2} A^2 f_{\text{yr}}^2 \left( \frac{f}{f_{\text{yr}}} \right)^{5-\gamma}$$

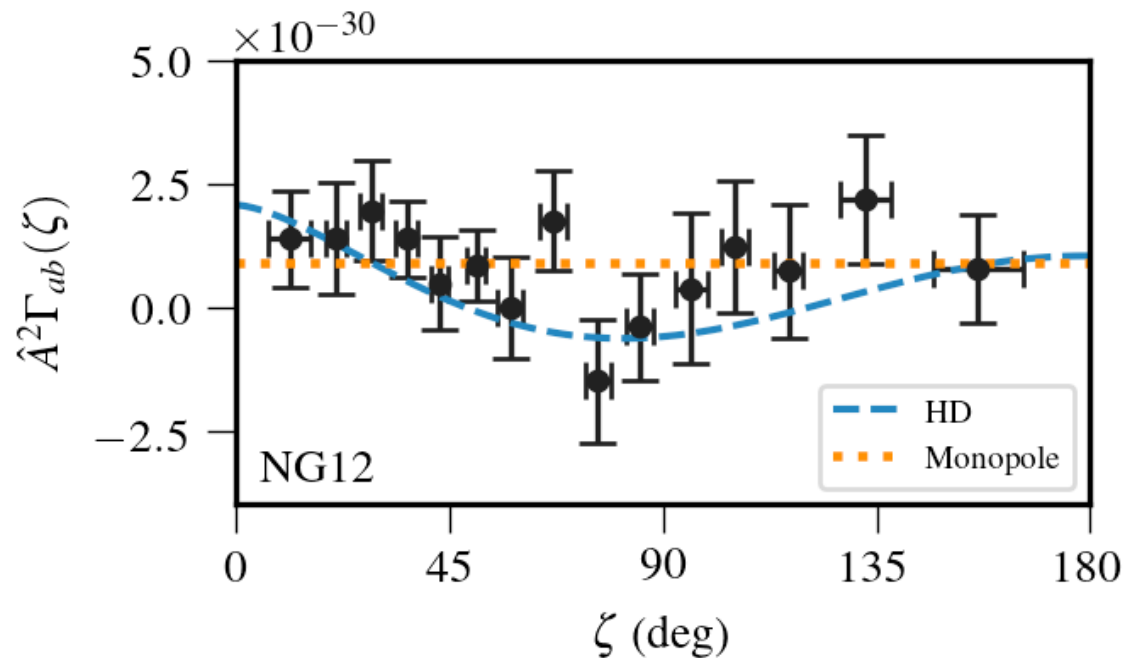


Possible flat spectrum with amplitude

$$\Omega(f) \sim 5 \cdot 10^{-10}$$

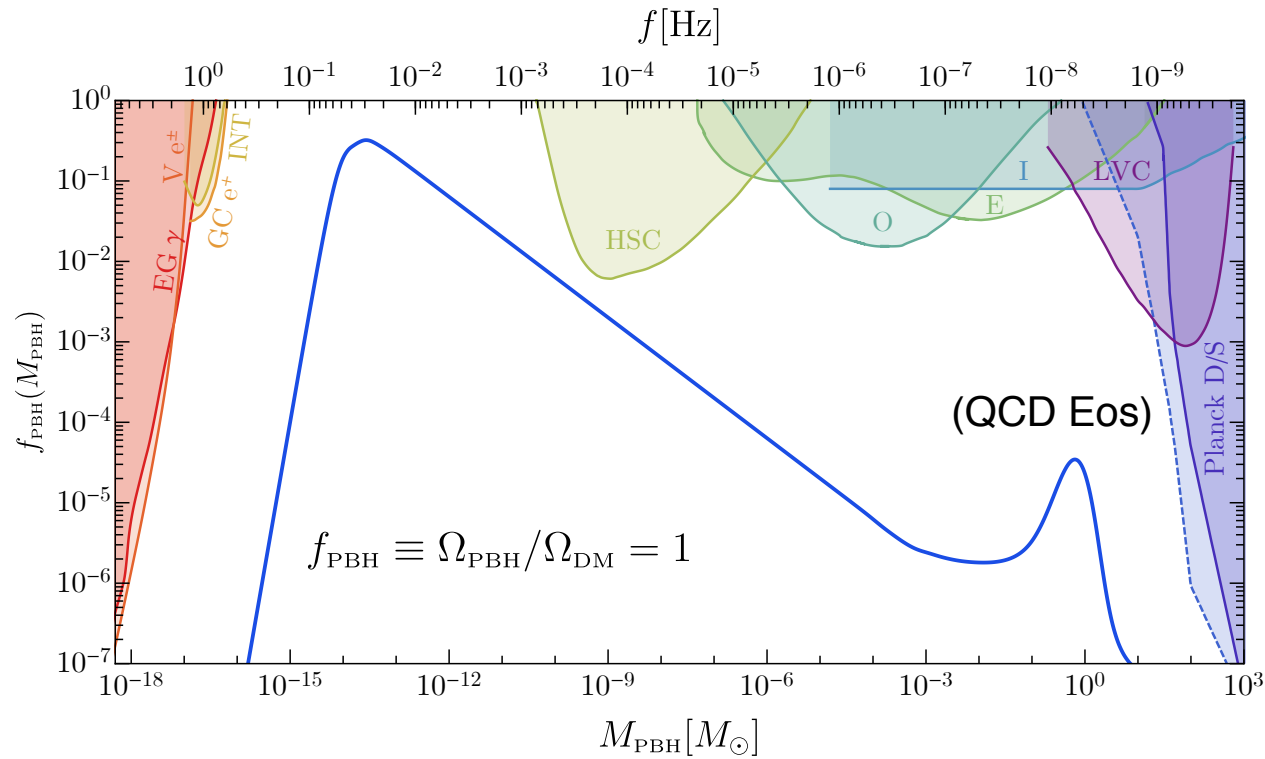
# Nano-Grav 12.5 year

Non-conclusive evidence for quadrupolar Hellings-Downs (HD) correlation pattern (GW footprint)



Need to wait for more data (two years on)

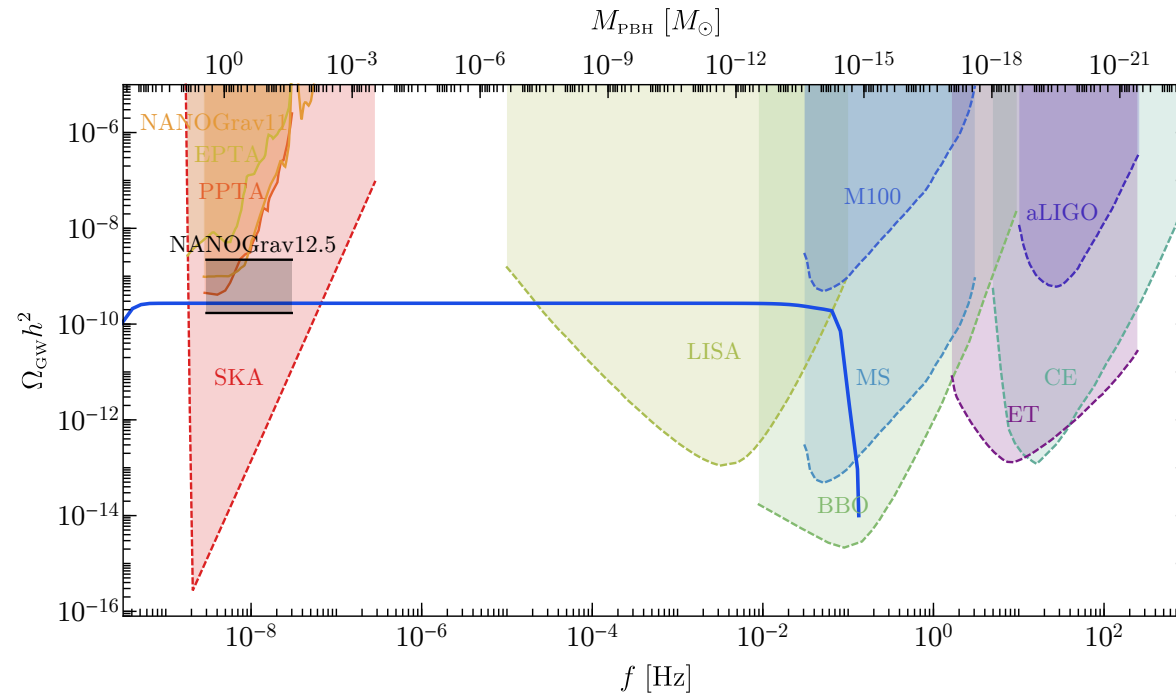
# Can be consistent with a PBH = DM scenario



From flat curvature power spectrum

$$\mathcal{P}_{\zeta}(k) = A_{\zeta} \Theta(k_s - k) \Theta(k - k_l) \quad k_s \gg k_l$$

# Can be consistent with a PBH = DM scenario



May be confirmed by LISA

# Conclusions

- The era of gravitational wave astronomy has begun opening a new window into fundamental physics and cosmology
- PBHs may exist and comprise the totality of the dark matter, future data will tell us