# Phases of $\mathcal{N} = 2$ SYK Models

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Paris
June 2022

# Motivations

- Complex SYK is an interesting quantum mechanical model displaying features similar to black holes

Sachdev Ye, Kitaev

Gu Kitaev Sachdev Tarnopolsky 19

$$H = \sum_{ijkl} J_{ijkl}\, \bar{\psi}^i \bar{\psi}^j \psi^k \psi^l,$$

$$\langle J_{ijkl}^2 \rangle \sim J^2/N^3$$

$$i = 1, \ldots, N$$

- It develops an emergent conformal symmetry at low temperatures. Can be studied at non-zero background charge (electric field in bulk)

- Displays maximal chaos, its compressible, and has a large zero temperature entropy

- Phase transitions to low-entropy gapped phases were found at non-zero charge, even though the conformal solution has no pathology.

Azeyanagi, Ferrari, Schaposnik-Massolo 18

# Motivations

- A deceptively simple modification of complex SYK is to take the random couplings to be
  Fu Gaiotto Maldacena Sachdev 16

$$J_{ijkl} \sim \sum_a \bar{C}_{aij} C_{kla} \qquad \langle C_{ijk}^2 \rangle \sim J/N^2$$

- This has drastic consequences in the behavior of the model, developing $\mathcal{N} = 2$ supersymmetry with $\mathcal{Q} \sim C_{ijk} \psi^i \psi^j \psi^j$

- This model develops a Superconformal symmetry in the IR $SU(1,1\,|\,1) \supset SL(2,\mathbb{R}) \times U(1)$

- Study high- to low-entropy (or conformal to gapped) transition in this model

# Motivations

- As a separate motivation, the Fu et al model has fractional R-charge fermions. All these models have vanishing index, but large ground state degeneracy

- We will construct models with multiple fermions allowing us to realize theories integer R-charge

- The new models are described in the IR by the same Super-Schwarzian modes appearing in the description of near-BPS black holes in $AdS_5$

See M. Heydeman talk

# $\mathcal{N} = 2$ SYK Model

- System of $N$ complex fermions $\psi^i$, with $i = 1,\ldots,N$, satisfying $\{\psi^i, \bar{\psi}^j\} = \delta^{ij}$ while other combinations vanish $\{\psi^i, \psi^j\} = 0$

- When $\mathcal{N} = 2$ supersymmetry is preserved, the Hamiltonian can be written as $H = \{\mathcal{Q}, \bar{\mathcal{Q}}\}$. We choose the supercharge to be, for $q = 3,5,\ldots$

$$\mathcal{Q} = i^{\frac{q-1}{2}} \sum_{i_1\ldots i_q} C_{i_1\ldots i_q}\, \psi^{i_1}\ldots\ \psi^{i_q}$$

Random coupling:

$$\langle C_{i_1 i_2 \ldots i_q} \bar{C}^{i_1 i_2 \ldots i_q} \rangle \sim J/N^{q-1}$$

- The theory has a $U(1)_R$ symmetry, generated by

$$Q = \sum_j \bar{\psi}^j \psi^j - \frac{N}{2}, \quad Q_R = \frac{1}{q} Q$$

For N odd charge is half-integer

# The Index of the model

- The fermion number of the theory is $(-1)^F = e^{i\pi Q_R}$

- The Index of $\mathcal{N} = 2$ SYK vanishes $\mathrm{Tr}\ (-1)^F e^{-\beta H} = 0$. This can be easily computed in the free fermion limit. Is supersymmetry broken?

- No, there is a large number of BPS ground states. To see this, compute the refined index $\mathcal{I}(r) = \mathrm{Tr}\left[(-1)^F\ e^{2\pi i r Q_R}\ e^{-\beta H}\right]$ for $r \in \mathbb{Z}_q$. This is protected since $[e^{2\pi i r Q_R}, \mathcal{Q}] = 0$

$$\mathcal{I}(r) = \left(2 \sin \frac{\pi r}{q}\right)^N$$

Total number of ground states

Maximized for $r = (q \pm 1)/2$ and gives: $\max\ \mathcal{I} = e^{N \log\left(2 \cos \frac{\pi}{2q}\right)}$

# $\mathcal{N} = 2$ SYK at Strong Coupling

- Focus on $q = 3$. To derive the mean field theory to solve at strong coupling, it is necessary to introduce an auxiliary boson $b_i \sim \{\bar{\mathcal{Q}}, \psi_i\} \sim \bar{\psi}^2$

- Introduce superspace coordinates $Z = (\tau, \theta, \bar{\theta})$ and covariant derivative $D = \partial_\theta + \bar{\theta}\partial_\tau$. Package the fundamental fields in a chiral super field:

$$\overline{D}\Psi^i = 0, \quad \Rightarrow \quad \Psi^i = \underbrace{\psi^i(\tau + \theta\bar{\theta})}_{\text{Fundamental Fermion}} + \underbrace{\theta b^i(\tau)}_{\text{Auxiliary Boson}}$$

- The action is (implicit summation)

$$\mathscr{L} = \int d^2\theta\, \overline{\Psi}^i\Psi^i + i\int d\theta\, C_{ijk}\Psi^i\Psi^j\Psi^k + \mathrm{h.c.}$$

$$= \bar{\psi}^i\partial_\tau\psi^i - \bar{b}^i b^i + C_{ijk}b^i\psi^j\psi^k + \mathrm{h.c.}$$

# $\mathcal{N} = 2$ SYK at Strong Coupling

- Introduce bi-local 2pt function $\mathcal{G}(Z_1, Z_2) = \dfrac{1}{N}\langle \bar{\Psi}^i(Z_1)\Psi^i(Z_2)\rangle$. And similarly self-energy $\Sigma(Z_1, Z_2)$. In components:

$$\mathcal{G}(Z_1, Z_2) = G_{\psi\psi}(\tau_1 - \theta_1\bar{\theta}_1, \tau_2 + \theta_2\bar{\theta}_2) + \bar{\theta}_1\theta_2 G_{bb}(\tau_1, \tau_2)$$

$$+\bar{\theta}_1 G_{b\psi}(\tau_1, \tau_2 + \theta_2\bar{\theta}_2) - \theta_2 G_{\psi b}(\tau_1 - \theta_1\bar{\theta}_1, \tau_2)$$

- Procedure: Introduce $\mathcal{G}(Z_1, Z_2)$ using $\Sigma(Z_1, Z_2)$ as a Lagrange multiplier. Integrate out disorder, integrate out $\Psi(Z)$. Ends up with mean field action

Schwinger-Dyson Equations

$$\boxed{\begin{gathered}\Sigma(Z_2, Z_3) = J\mathcal{G}(Z_2, Z_3)^{q-1} \\ D_3\mathcal{G}(Z_1, Z_3) + \int dZ_2\mathcal{G}(Z_1, Z_2)\Sigma(Z_3, Z_2) = \delta(Z_1 - Z_3)\end{gathered}}$$

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Schwinger-Dyson Equations

IR Approx

$$\Sigma(Z_2, Z_3) = J\mathcal{G}(Z_2, Z_3)^{q-1}$$

$$\cancel{D_3\mathcal{G}}(Z_1, Z_3) + \int dZ_2 \mathcal{G}(Z_1, Z_2)\Sigma(Z_3, Z_2) = \delta(Z_1 - Z_3)$$

# Schwinger-Dyson Equations

- Consider a solution with $G_{\psi b} = G_{b\psi} = 0$, and with time translation. We need to solve

$$\Sigma_{\psi\psi}(\tau) = J(q-1)G_{\psi\psi}(\tau)^{q-2}G_{bb}(\tau), \qquad \Sigma_{bb}(\tau) = JG_{\psi\psi}(\tau)^{q-1}$$

$$G_{\psi\psi}(\omega) = \frac{1}{-i\omega + \mu + \Sigma_{\psi\psi}(-\omega)}, \qquad G_{bb}(\omega) = \frac{1}{-1 - \Sigma_{bb}(-\omega)}$$

- We will solve these equations in the IR $|J\tau| \gg 1$ with a conformal ansatz

$$G_{\psi\psi}(\tau) = g_{\psi\psi}\frac{-e^{\pi\mathscr{E}}\Theta_{-\tau} + e^{-\pi\mathscr{E}}\Theta_{\tau}}{|\tau|^{2\Delta}}, \qquad G_{bb}(\tau) = g_{bb}\frac{e^{\pi\mathscr{E}_b}\Theta_{-\tau} + e^{-\pi\mathscr{E}_b}\Theta_{\tau}}{|\tau|^{2\Delta_b}}$$

Turning on $\mathscr{E}$ corresponds to turning on background $U(1)$ charge

# Conformal Solution

- We begin with $G\Sigma \sim 1$ which gives

$$\left[\begin{aligned} \Sigma_{\psi\psi} &= \frac{1}{g_{\psi\psi}} \frac{(1-2\Delta)\sin 2\pi\Delta}{\cosh 2\pi\mathcal{E} + \cos 2\pi\Delta} \frac{-e^{\pi\mathcal{E}}\Theta_{-\tau} + e^{-\pi\mathcal{E}}\Theta_{\tau}}{|\tau|^{2(1-\Delta)}} \\ \Sigma_{bb} &= \frac{1}{g_{bb}} \frac{(1-2\Delta_b)\sin 2\pi\Delta_b}{\cosh 2\pi\mathcal{E}_b - \cos 2\pi\Delta_b} \frac{-e^{\pi\mathcal{E}_b}\Theta_{-\tau} - e^{-\pi\mathcal{E}_b}\Theta_{\tau}}{|\tau|^{2(1-\Delta_b)}} \end{aligned}\right.$$

- Inserting this in the other equations gives:

$$\left[\begin{aligned} &\mathcal{E}_b = -(q-1)\mathcal{E} \\ &(q-1)\Delta + \Delta_b = 1 \end{aligned}\right.$$

- The scaling dimension is NOT determined by dimensional analysis (usual case with Yukawa interactions). In $\mathcal{N}=2$ SYK this is determined by looking at prefactors

# Scaling Dimensions

- Matching the prefactors in the Schwinger-Dyson equations gives the redundant equations

$$\frac{(1-2\Delta)\sin 2\pi\Delta}{\cosh 2\pi\mathscr{E} + \cos 2\pi\Delta} = (q-1)Jg_{\psi\psi}^{q-1}g_{bb} \qquad \frac{(1-2\Delta_b)\sin 2\pi\Delta_b}{\cosh 2\pi\mathscr{E}_b - \cos 2\pi\Delta_b} = Jg_{\psi\psi}^{q-1}g_{bb}$$

- This gives a self-consistency equation for the scaling dimension as a function of spectral asymmetry

$$\boxed{\frac{(1-2\Delta)\sin 2\pi\Delta}{\cosh 2\pi\mathscr{E} + \cos 2\pi\Delta} = (q-1)\frac{(1-2(q-1)\Delta)\sin 2\pi(q-1)\Delta}{\cosh 2\pi(q-1)\mathscr{E} - \cos 2\pi(q-1)\Delta} \quad \Rightarrow \quad \Delta(\mathscr{E})}$$

- The conformal solution is NOT fully determined, only $g_{\psi\psi}^{q-1}g_{bb}$ , but not e.g. $g_{\psi\psi}/g_{bb}$

# Superconformal Solution

- Supersymmetric solutions are found when:

$$\mathcal{E}_{\text{susy}} = \frac{ir}{q}, \quad r \in \mathbb{Z}_q, \quad \Rightarrow \quad \Delta = \frac{1}{2q}, \;\; \Delta_b = \frac{1}{2q} + \frac{1}{2}$$

- Using that $G_{bb} = -\partial_\tau G_{\psi\psi}$ we can find the full solution using $g_{bb} = 2\Delta g_{\psi\psi}$. We focus on $r = 0$

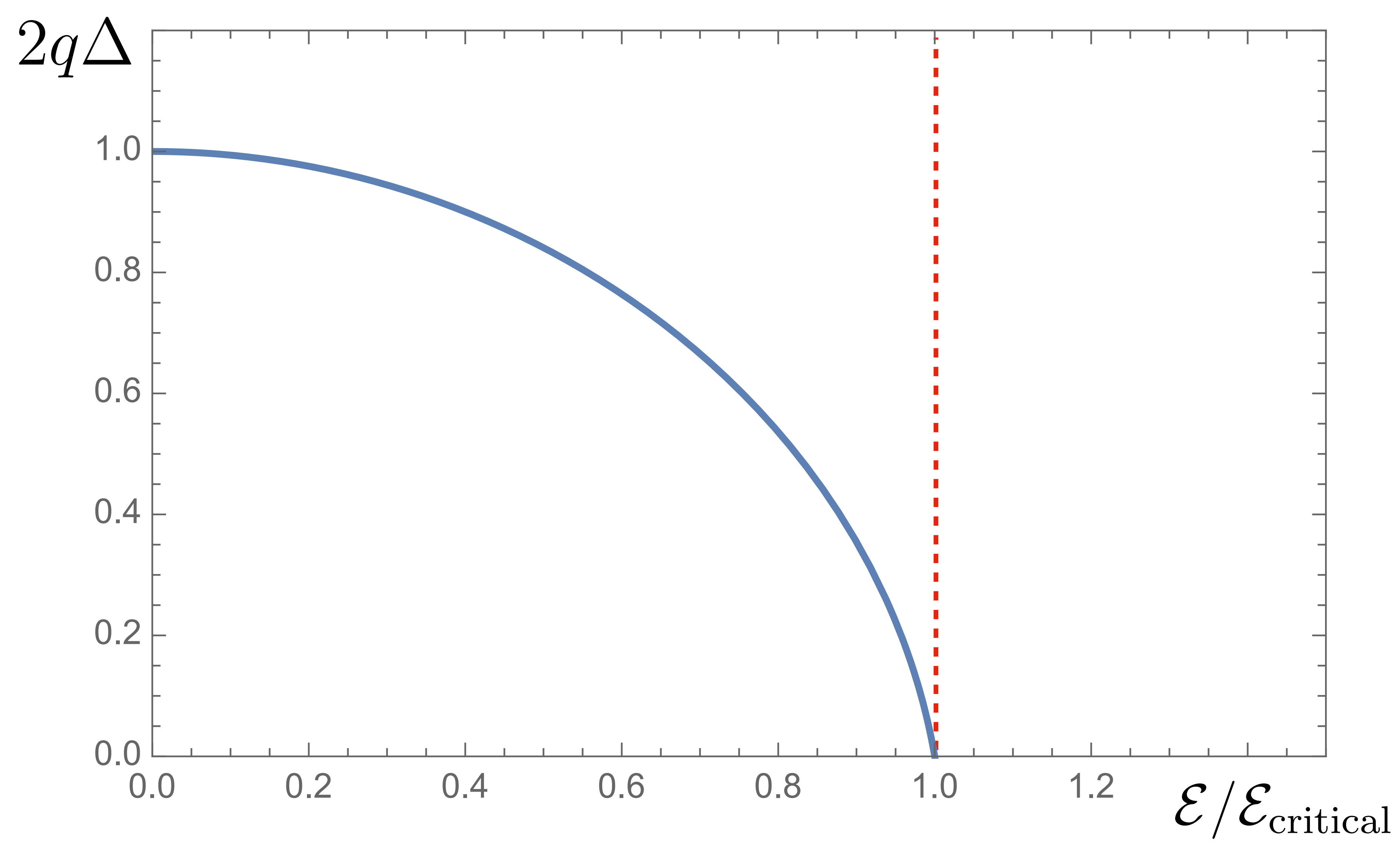
- This solution has an emergent $SU(1,1\,|\,1) \supset SL(2,\mathbb{R}) \times U(1)_R$ symmetry

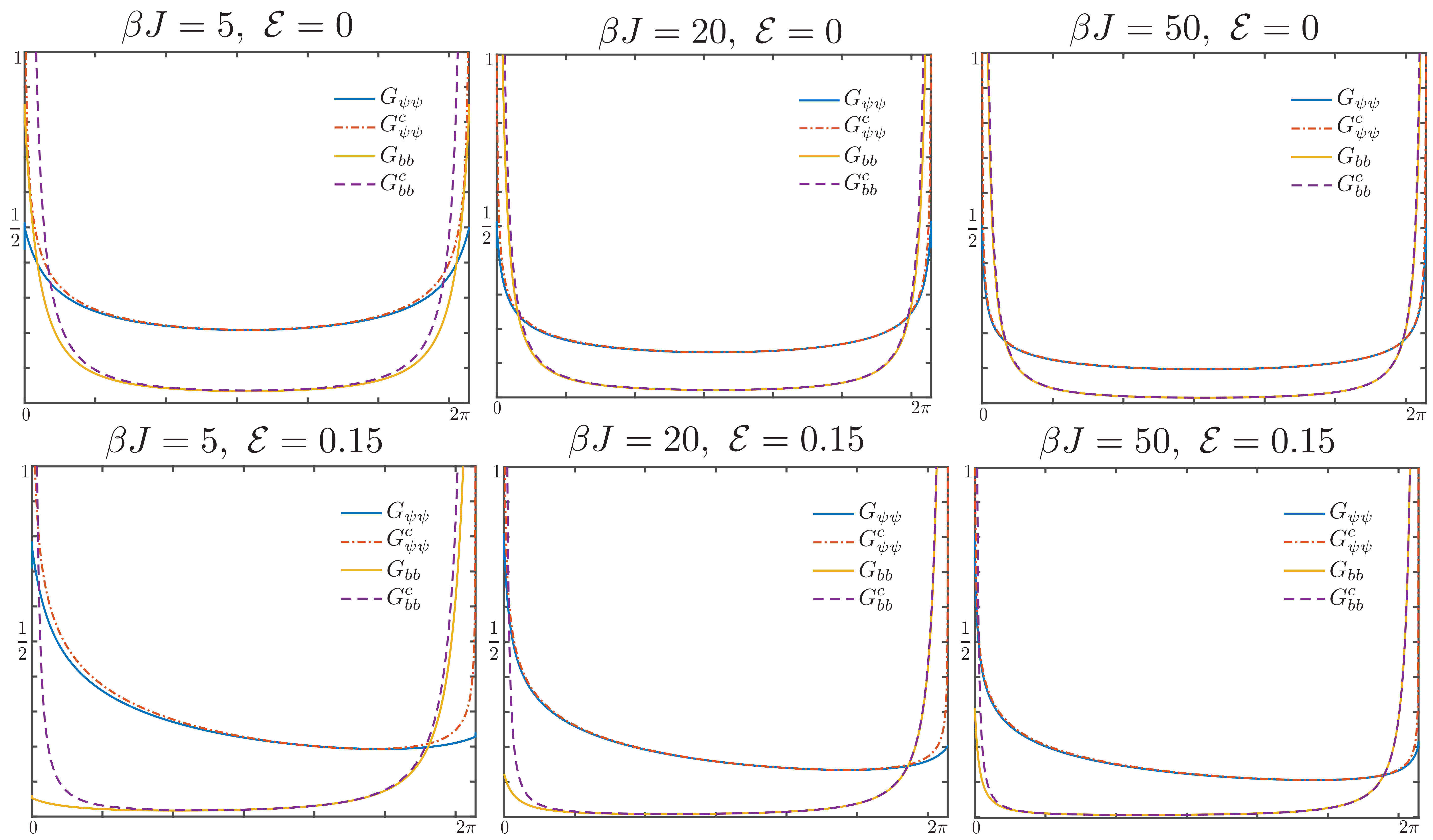
$$\left.\begin{aligned} \tau &\to \tau' = f(\tau) + \dots \\ \theta &\to \theta' = e^{ia(\tau)}\sqrt{f'(\tau)}\,\theta + \eta(\tau) + \dots \end{aligned}\right] \longrightarrow \quad \boxed{\mathcal{G}(Z_1, Z_2) \to (D_{\theta_1}\theta_1')^{1/q}\,(D_{\bar{\theta}_2}\bar{\theta}_2')^{1/q}\,\mathcal{G}(Z_1', Z_2')}$$

# Conformal Solution at non-Zero Charge

- $q = 3$ behavior of scaling dimension:

$$\mathscr{E}_{\text{critical}} = 0.28055$$



# Conformal Solution at non-Zero Charge

- We checked the UV boundary conditions fix the undetermined coefficients:

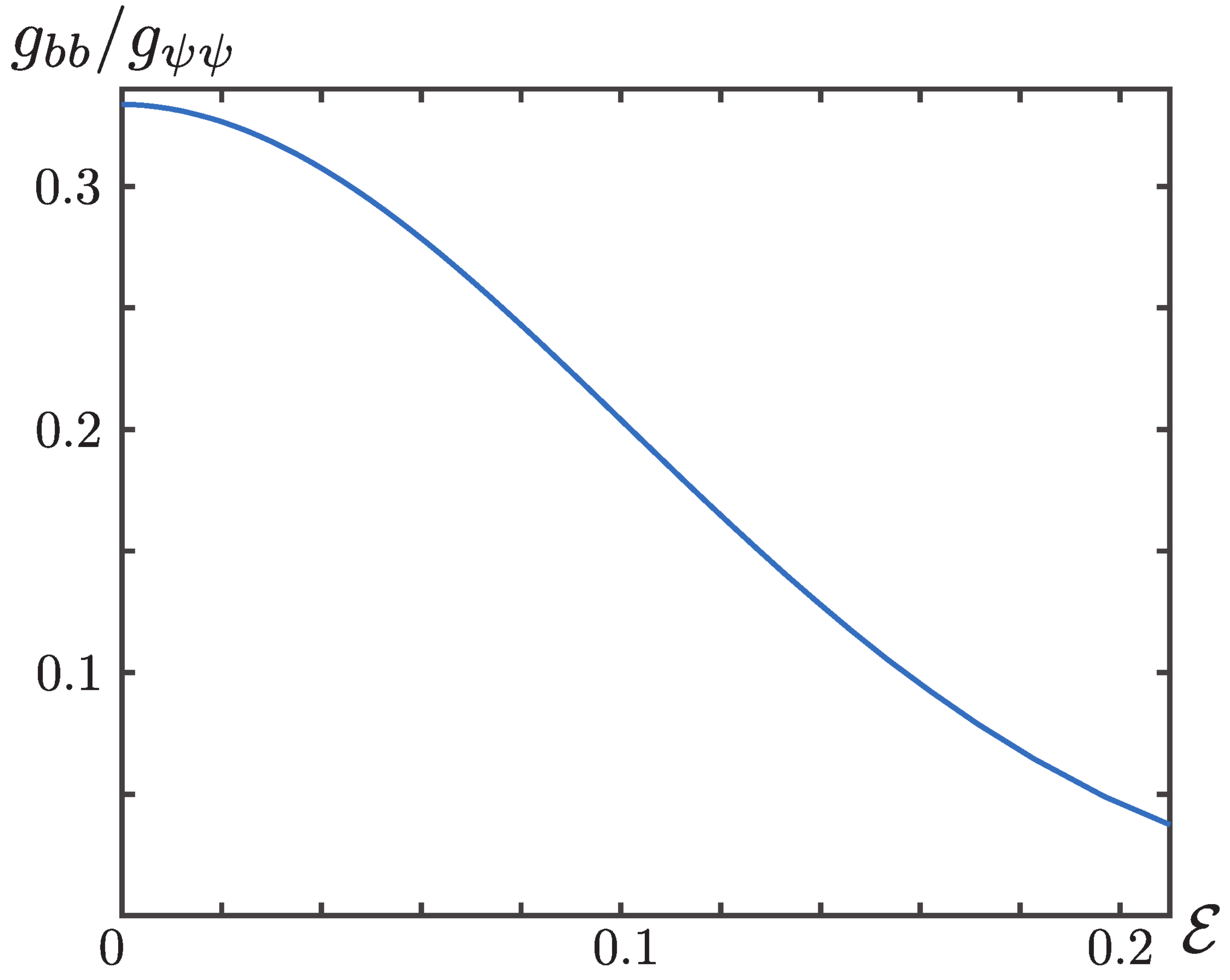


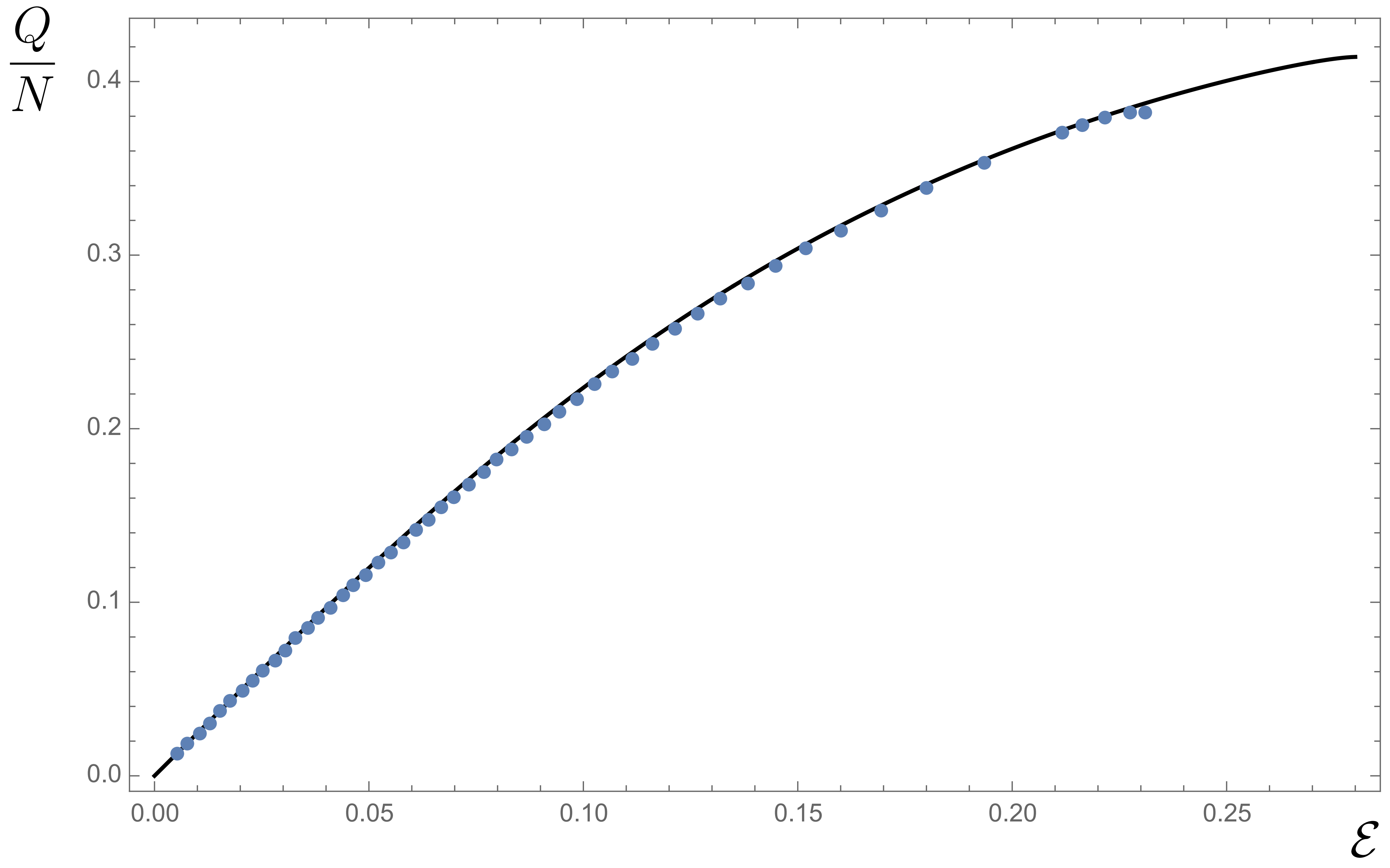

**Figure 3:** Plot of $g_{bb}/g_{\psi\psi}$ computed from the numerical solution for $q = 3$ at different values of $\mathcal{E}$. Note the infrared Schwinger-Dyson equations can only determine the combination $g_{\psi\psi}^{q-1} g_{bb}$, but the individual values are determined by the full solution. At $\mathcal{E} = 0$, it agrees with the supersymmetric answer $g_{bb} = 2\Delta g_{\psi\psi} = \frac{1}{3} g_{\psi\psi}$.

- This scaling symmetry does not generate a genuine IR mode

# Luttinger-Ward Relation

- The Luttinger-Ward relation gives the charge expectation value (a UV quantity) in terms of the conformal spectral asymmetry (an IR quantity). Georges Parcollet Sachdev 00

- We derived and verified numerically:




$$Q/N = \overbrace{\frac{\left(\frac{1}{2}-\Delta\right)\sinh 2\pi\mathcal{E}}{\cosh 2\pi\mathcal{E} + \cos 2\pi\Delta} + \frac{i \log \frac{\cos \pi(\Delta + i\mathcal{E})}{\cos \pi\Delta - i\mathcal{E}}}{2\pi}}^{\text{Fermion Contribution (same as cSYK)}}$$

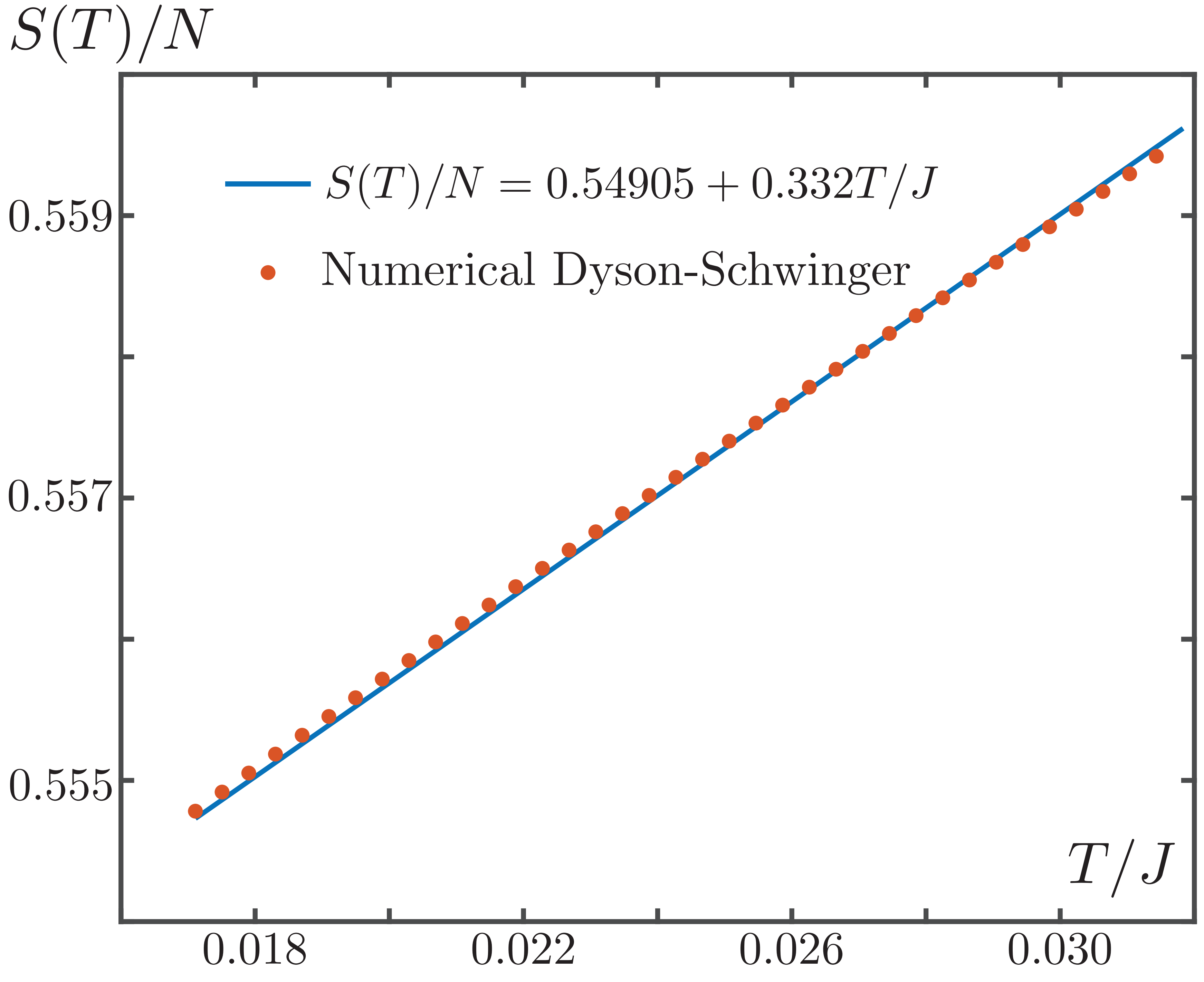
$$+(q-1)\underbrace{\frac{\left(\frac{1}{2}-\Delta_b\right)\sinh 2\pi\mathcal{E}_b}{\cosh 2\pi\mathcal{E}_b - \cos 2\pi\Delta_b} + \frac{i \log \frac{\sin \pi(\Delta_b + i\mathcal{E}_b)}{\sin \pi\Delta_b - i\mathcal{E}_b}}{2\pi}}_{\text{Aux Boson Contribution}}$$

Important: $\mathcal{E}_{\text{critical}}$ corresponds to less than maximal charge $Q_{\text{critical}} = 0.414N \Rightarrow$ Phase Transition

# $\mathcal{N} = 2$ Schwarzian Theory

- At low temperatures the Schwarzian dominates. The Bosonic sector has reparametrization and U(1) modes with action

$$I_{\mathcal{N}=2\ \mathrm{Sch}} = \frac{\alpha_S N}{J}\int d\tau \left(\{f,\tau\} + 2q^2(\partial_\tau a)^2\right) + \text{fermions}$$




- The free energy is given by

$$-\beta F = S_0 + \frac{c}{2\beta} + \ldots, \qquad c = \frac{4\pi^2 \alpha_S N}{J} \approx 0.332\frac{N}{J}$$

- SUSY ties the Schwarzian coupling to compressibility

$$\left(\frac{\partial S}{\partial T}\right)_{T=0} = \frac{\pi^2}{q^2}\left(\frac{\partial Q}{\partial \mu}\right)_{T=0} \qquad \Rightarrow \qquad K = \left(\frac{\partial Q}{\partial \mu}\right)_{T=0} \approx 0.303 N/J$$

# $\mathcal{N} = 2$ Schwarzian Theory

- The quantum BPS spectrum of the Schwarzian is

$$Z(\beta,\mu) = \sum_{Q\in\mathbb{Z},|Q|<q/2} e^{\beta\mu Q}\frac{2e^{Ns_0}\cos(\frac{\pi Q}{q})}{q} + Z_{\text{non-BPS}}$$

- For $q = 3$ we have $s_0 = \log\sqrt{3}$ . Then the Schwarzian exactly matches the SYK answer computed in [Fu et al '16]

$$D(N,0) = 2\,3^{N/2-1}\,, \qquad D(N,\pm\frac{1}{3}) = 3^{N/2-1}\,, \qquad \text{for} \quad N \quad \text{even}$$
$$D(N,\pm\frac{1}{6}) = 3^{(N-1)/2}\,, \qquad \text{for} \quad N = 3 \bmod 4$$
$$D(N,\pm\frac{1}{6}) = 3^{(N-1)/2}\,, \qquad D(N,\pm\frac{3}{6}) = 1 \text{ or } \ 3\,, \qquad \text{for} \quad N = 1 \bmod 4$$

- Can be a good setup to understand better microstates from gravity [Lin Maldacena Rosenberg Shan, WIP]

# Solution for $Q > Q_{\text{critical}}$

- We checked numerically the fate of the instability we found. The answer is

## Solution develops a Gap

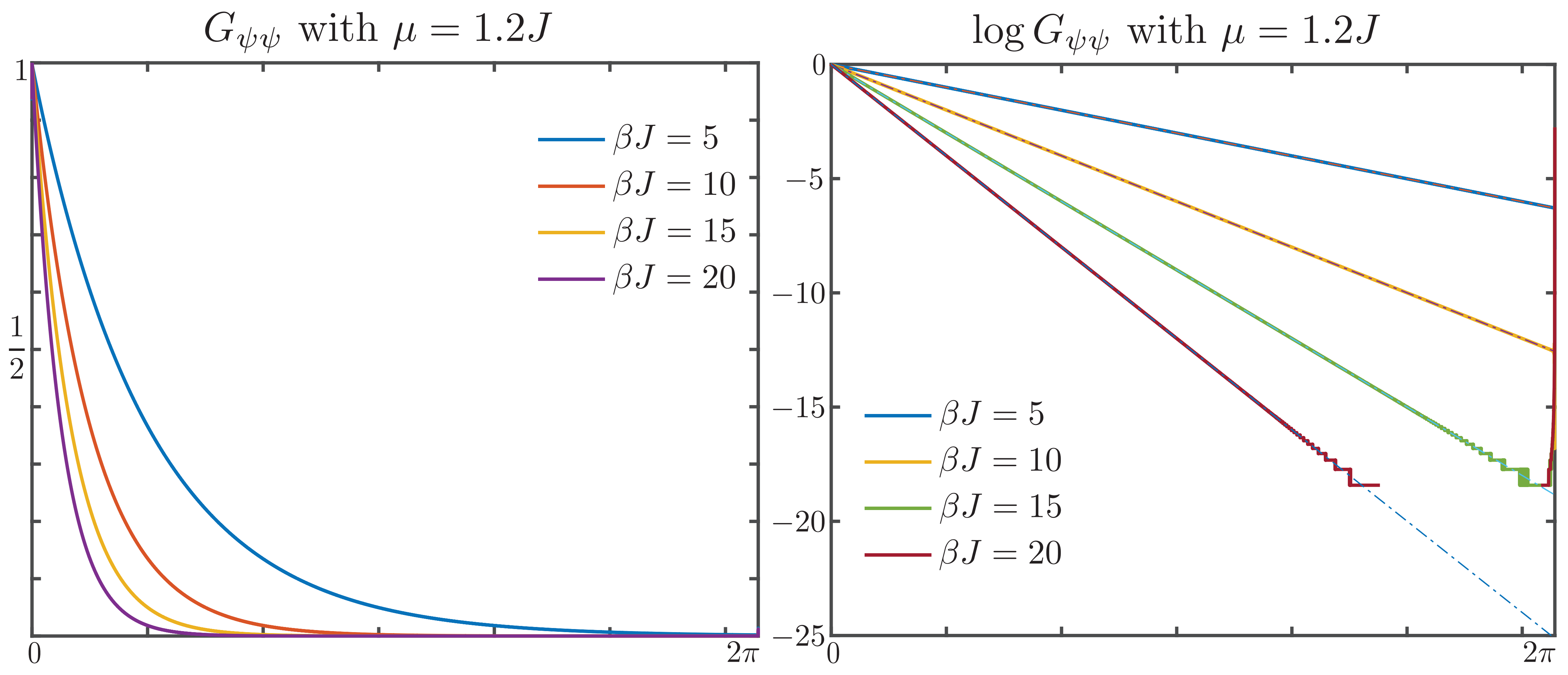



**Figure 6:** Left: Numerical solutions of $G_{\psi\psi}$ in the region where $\mathcal{E} > \mathcal{E}_{\text{critical}}$. We observe exponential decay solutions. Since the solution ceases to be conformal, the infrared parameter $\mathcal{E}$ is no longer meaningful. The solutions depend on both $\beta J$ and $\mu/J$. Right: log plot of $G_{\psi\psi}$ at various values of $\beta J$, where dashed lines are linear fits. We observe that the exponent is linear in $\mu$.

## High- to Low-Entropy Transition

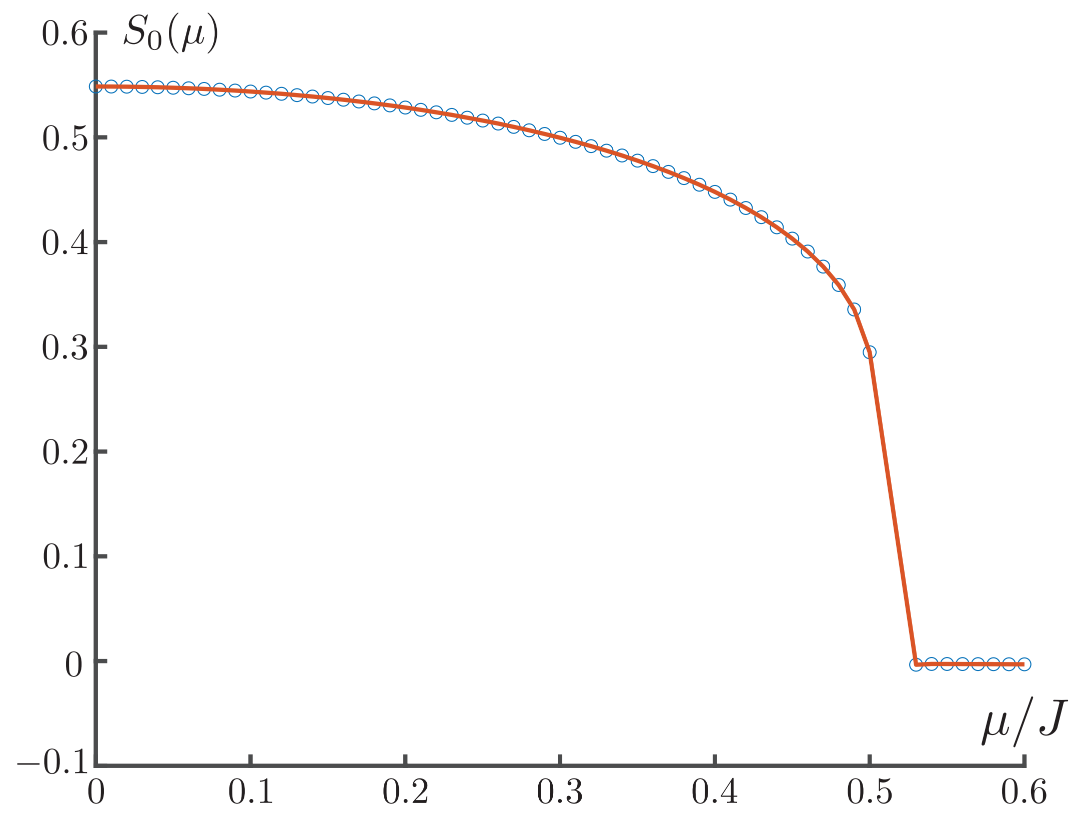
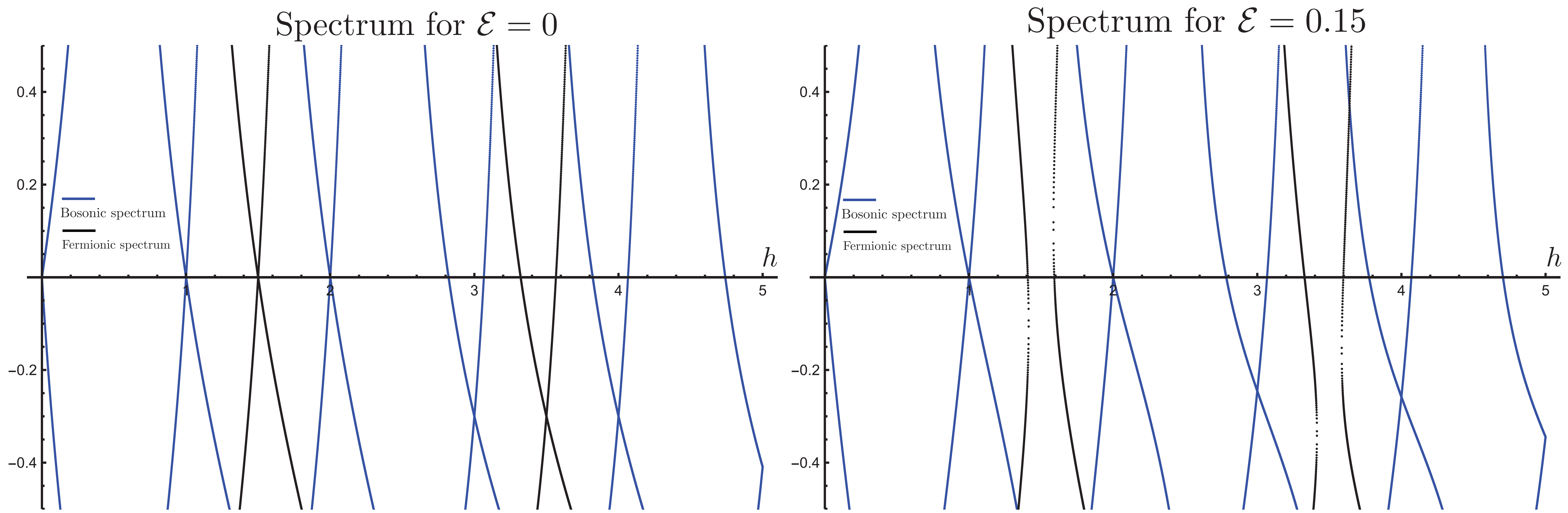



**Figure 7:** The entropy computed numerically as a function of the chemical potential $\frac{\mu}{J}$. For each value of $\mu$, we compute the free energy and entropy on a numerical grid of size $2^{25}$ at small temperatures and extrapolate the zero temperature entropy. Note at $\mu = 0$, we obtain $S_0(0) \approx 0.5484$ which is close to the value predicted by the index $\log(2\cos\frac{\pi}{6}) \approx 0.5493$. The transition happens at around $\mu = \mu_c = 0.5J$.

# Some features of bilinear spectrum




- $\mathscr{E} = 0$ : We find two sets of $h = (1) + 2 \times (3/2) + (2)$. One is the $\mathscr{N} = 2$ Super-Schwarzian, the other is spurious

- $\mathscr{E} \neq 0$ : Fermionic modes develop a mass, suggests one ends up with Schwarizan + U(1)

- $\mathscr{E} > \mathscr{E}_{\text{critical}}$ : Complex solutions appear and a continuum of real ones. We already saw it is unstable even for fundamental fermion

# Holographic Interpretation

- Picture 1: $\mathcal{N} = 2$ Jackiw-Teitelboim gravity coupled to fermion multiplets. At large enough electric field the fermion becomes unstable.

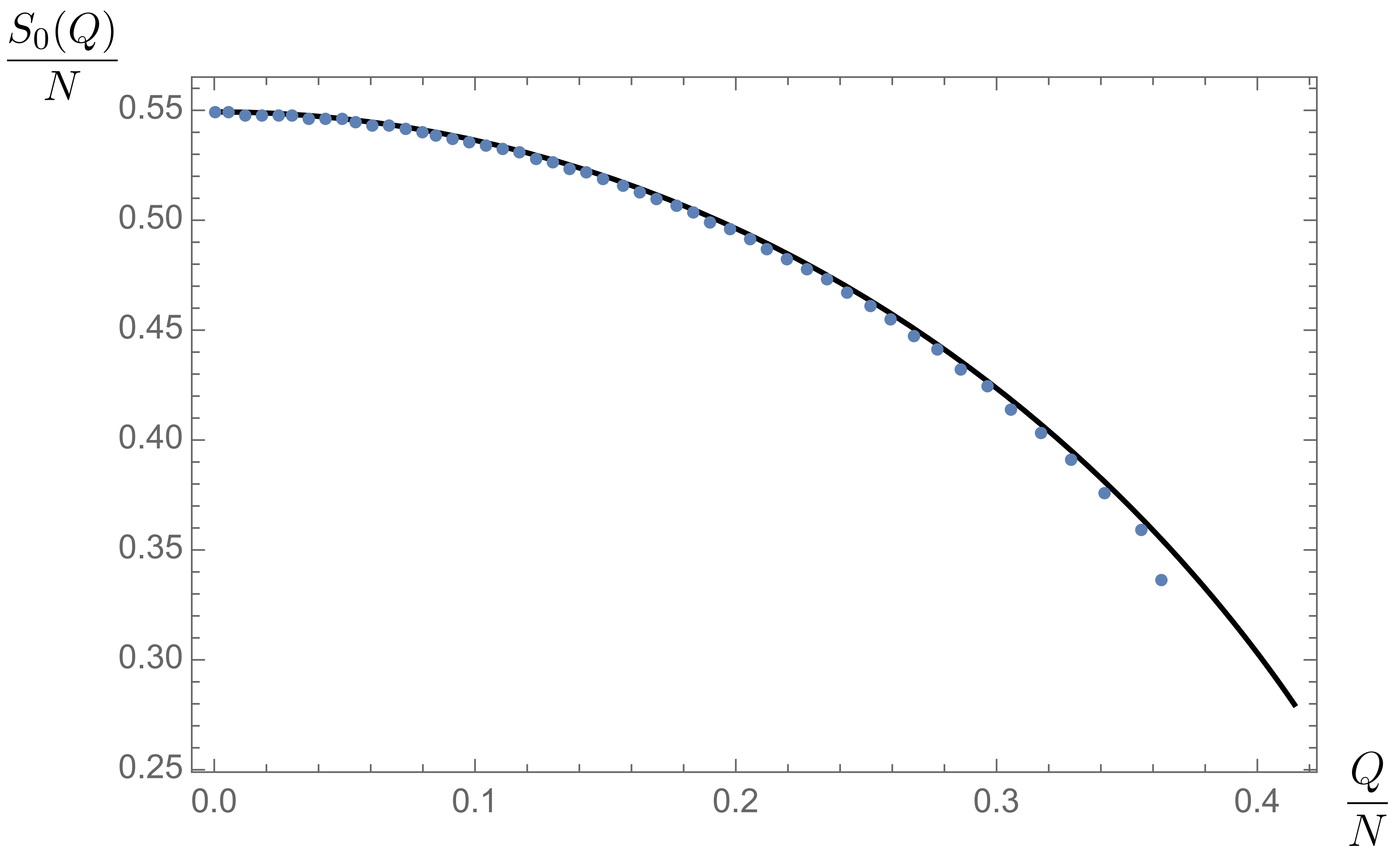
$$\Delta = \frac{1}{2} - \sqrt{M^2 - \mathcal{E}^2}$$

- Picture 2: Gravity is emergent from $N = 2$ “spooky” fermions + “spooky” bosons. Gives concrete formula for grand potential

cSYK: Gu Kitaev Sachdev Tarnopolsky 19

$$G(\mathcal{E}) = \int_{\Delta}^{1/2} \frac{dx\pi(1-2x)\sin 2\pi x}{\cosh 2\pi\mathcal{E} + \cos 2\pi x} + \int_{1/2}^{1-(q-1)\Delta} \frac{dx\pi(2x-1)\sin 2\pi x}{\cosh 2\pi(q-1)\mathcal{E} - \cos 2\pi x}$$

$\Delta(\mathcal{E})$ extremizes $G(\Delta, \mathcal{E})$



# New class of $\mathcal{N} = 2$ SYK Models

- System of $2N$ complex fermions $\psi^i$ and $\chi^i$. The Hamiltonian is given in terms of the supercharge by

$$\mathcal{Q} = i \sum_{ijk} C_{ijk} \, \psi^i \psi^j \chi^k$$

- Two $U(1)$ symmetries

$$Q_\psi = \sum_j \bar{\psi}^j \psi^j - \frac{N}{2}, \quad Q_\chi = \sum_j \bar{\chi}^j \chi^j - \frac{N}{2}$$

- A special role is played by a flavor symmetry that commutes with the supercharge

$$Q_F = Q_\psi - 2Q_\chi, \qquad [\mathcal{Q}, Q_F] = 0$$

- Emergent $SU(1,1\,|\,1)$ in the IR with Superconformal R-symmetry $\quad Q_R = Q_\chi + \alpha \, Q_F$

# The Index of the model

- The ‘fermion’ number of the theory is $(-1)^F = e^{i\pi Q_\chi}$. The index is:

$$\mathcal{I}(y) = \mathrm{Tr}\left[(-1)^F e^{iyQ_F} e^{-\beta H}\right]$$
$$= \left(2\cos\left(\frac{y}{2}\right) 2\sin\left(\frac{q-1}{2}y\right)\right)^N$$

- In a fixed $Q_F$ sector the index is non-vanishing

$$\mathrm{Tr}_{Q_F}\left[(-1)^F\right] = \int_0^{2\pi} \frac{dy}{2\pi} e^{-iyQ_F} \mathcal{I}(y)$$
$$= e^{N s_0(Q_F)}$$

- Take for simplicity $Q_F = 0$, then

$$\tan\left(\frac{y_c}{2}\right) = (q-1)\cot\left(\frac{(q-1)y_c}{2}\right), \qquad \Rightarrow \qquad \frac{d}{dq} s_0 = \frac{y_c}{2}\cot\left(\frac{(q-1)y_c}{2}\right)$$

# $\mathcal{N} = 2$ SYK at Strong Coupling

- Introduce two chiral super fields made of fermions and auxiliary bosons

$$\Psi^i = \psi^i(\tau + \theta\bar{\theta}) + \theta b^i_\psi(\tau), \quad X^i = \chi^i(\tau + \theta\bar{\theta}) + \theta b^i_\chi(\tau)$$

- The action includes the interaction term now

$$\mathscr{L} \supset i \int d\theta \; C_{ijk} \Psi^i \Psi^j X^k + \mathrm{h.c.}$$

- Derive Schwinger-Dyson equations in terms of two superfields $\mathscr{G}_\psi(Z_1, Z_2)$ and $\mathscr{G}_\chi(Z_1, Z_2)$

# Schwinger-Dyson Equations

- Consider a solution with $G_{\psi b} = G_{b\psi} = 0$, and with time translation. We need to solve

$$\Sigma_{\psi\psi} = J(q-1)(G_{\psi\psi}^{q-2} G_{b_\chi b_\chi} + (q-2) G_{b_\psi b_\psi} G_{\chi\chi} G_{\psi\psi}^{q-3}), \qquad \Sigma_{b_\psi b_\psi} = J(q-1) G_{\psi\psi}^{q-2} G_{\chi\chi}$$

$$\Sigma_{\chi\chi} = J(q-1) G_{b_\psi b_\psi} G_{\psi\psi}^{q-2}, \qquad \Sigma_{b_\chi b_\chi} = J G_{\psi\psi}^{q-1}$$

- We will solve these equations in the IR $|J\tau| \gg 1$ with a conformal ansatz ($A = \psi, \chi$)

$$G_{AA}(\tau) = g_{AA} \frac{-e^{\pi \mathscr{E}_A} \Theta_{-\tau} + e^{-\pi \mathscr{E}_A} \Theta_\tau}{|\tau|^{2\Delta_A}}, \qquad G_{b_A b_A}(\tau) = g_{b_A b_A} \frac{e^{\pi \mathscr{E}_{b_A}} \Theta_{-\tau} + e^{-\pi \mathscr{E}_{b_A}} \Theta_\tau}{|\tau|^{2\Delta_{b_A}}}$$

# Conformal Solution

- Boson spectral asymmetries:

$$\begin{cases} \mathscr{E}_{b_\psi} = -(q-2)\mathscr{E}_\psi - \mathscr{E}_\chi \\ \mathscr{E}_{b_\chi} = -(q-1)\mathscr{E}_\psi \end{cases}$$

$$b_\psi \sim \bar{\psi}^{q-2}\bar{\chi}$$

$$b_\chi \sim \bar{\psi}^{q-1}$$

- Scaling Dimensions constrain:

$$\begin{cases} \Delta_{b_\chi} + (q-1)\Delta_\psi = 1 \\ \Delta_{b_\psi} + (q-2)\Delta_\psi + \Delta_\chi = 1 \end{cases}$$

- The four equations determine only two prefactors $g_{b_\chi b_\chi} g_{\psi\psi}^{q-1}$ and $g_{\psi\psi}^{q-2} g_{\chi\chi} g_{b_\psi b_\psi}$ and consistency determines

$$\Delta_\psi(\mathscr{E}_\psi, \mathscr{E}_\chi), \qquad \Delta_\chi(\mathscr{E}_\psi, \mathscr{E}_\chi)$$

# Superconformal Solutions

- Supersymmetry imposes $G_{b_A b_A} = -\partial_\tau G_{AA}$, for $A = \psi, \chi$. This gives two further constrains:

$$\Delta_\chi + (q-1)\Delta_\psi = \frac{1}{2}$$

$$\mathscr{E}_\chi + (q-1)\mathscr{E}_\psi = 0$$

- For simplicity take $\mathscr{E}_\psi = 0$, then

$$\left[\begin{array}{l} \tan(\pi\Delta_\psi) = (q-1)\cot(\pi(q-1)\Delta_\psi) \\ \dfrac{d}{dq}s_0 = \pi\Delta_\psi \cot(\pi(q-1)\Delta_\psi) \end{array}\right.$$

The equation for $\Delta_\psi$ is the same as for $y_c \to 2\pi\Delta_\psi$ in the extremization of the index!

# Superconformal Solutions

- Under Superconformal transformations the fermions transform as chiral primaries

$$\mathcal{G}_\psi(Z_1, Z_2) \to (D_{\theta_1}\theta_1')^{\Delta_\psi}(D_{\bar{\theta}_2}\bar{\theta}_2')^{\Delta_\psi}\ \mathcal{G}_\psi(Z_1', Z_2')$$

$$\mathcal{G}_\chi(Z_1, Z_2) \to (D_{\theta_1}\theta_1')^{\Delta_\chi}(D_{\bar{\theta}_2}\bar{\theta}_2')^{\Delta_\chi}\ \mathcal{G}_\chi(Z_1', Z_2')$$

- This gives an assignment $Q_R[\psi] = 2\Delta_\psi$, $Q_R[\chi] = 2\Delta_\chi$, and completely determines the R-charge in the IR

$$Q_R = Q_\chi + 2\Delta_\psi Q_F$$

- Since the R-charge of the supercharge is one, we get the simplest Schwarzian with only $Q_R = 0$ ground states

# Index Maximization

- Why does the fermion dimension match with the saddle point for $y$? $y_c = \pi\alpha$

- Index-maximization in 1D nSCFT: Define $R = R_0 + \alpha F$, then the coefficient is picked by maximizing the absolute value of

[Bah, Heydeman, GJT, Zhao, WIP]

Stated in [Benini Hristov Zaffaroni 15]

$$\mathrm{Tr}\ e^{i\pi R}$$

- Moreover, in the large N limit the maximal value of the index matches the ground state entropy

# $\mathcal{N} = 2$ Schwarzian Theory

- At low temperatures the Schwarzian dominates. The Bosonic sector has a reparametrization and U(1) modes with action

$$I_{\mathcal{N}=2\ \mathrm{Sch}} = \frac{\alpha_S N}{J} \int d\tau \left( \{F, \tau\} + 2(\partial_\tau a)^2 \right) + \text{fermions}$$

- The quantum Schwarzian spectrum of this model is

$$Z(\beta, \mu) = e^{N s_0} + Z_{\mathrm{non-BPS}}$$

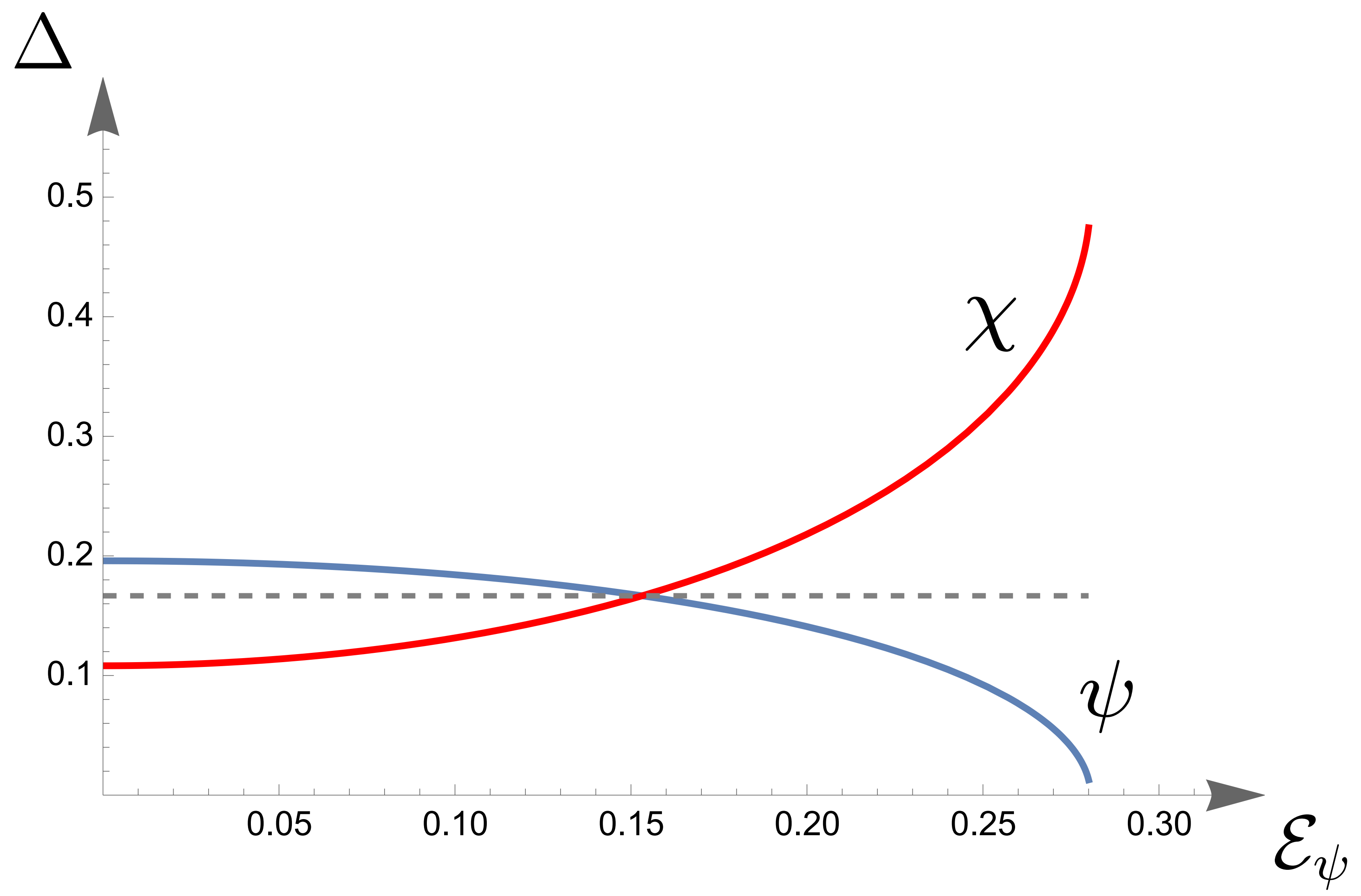
- This is the Schwarizan theory appearing in the near BPS limit of black holes in $AdS_5$

[Boruch Heydeman Iliesiu GJT 22]

# Stability of Superconformal solution

- At arbitrary charges there are instabilities of the conformal solution, just like before

- It is surprising that potential instabilities also appear for Superconformal solution




The Luttinger-Ward relation resolves this issue since at the potential transition point the charge is maximal

$$Q_\chi(\mathscr{E}_{\text{critical}})/N = 1/2$$

# Summary

- We studied the phase structure of $\mathcal{N}=2$ supersymmetric SYK at non zero charge

- There are phase transitions to gapped low-entropy phases. Similar to complex SYK but simpler to interpret

- We constructed models with several charges, realizing new types of Schwarizan theories at strong coupling

- Role of index maximization in nearly Superconformal quantum mechanics

# Thank you for your attention!