A perturbative CFT dual for pure NS-NS AdS_3 strings

Lorenz Eberhardt

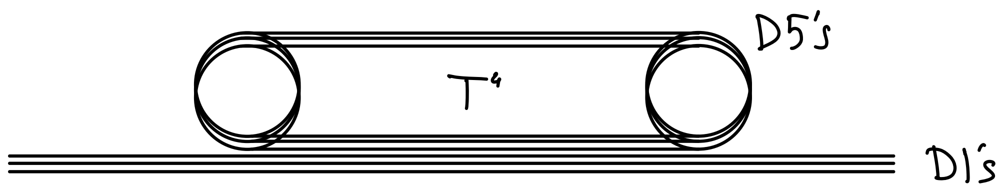
IAS Princeton

Based on: 2110.07535

& related work with Andrea Dei & Matthias Gaberdiel

AdS_3/CFT_2 holography

- AdS_3 backgrounds can be engineered in string theory in the D1 – D5 system with D5 wrapping T^4



$\Rightarrow$ Near horizon limit : $AdS_3 \times S^3 \times T^4$

- From a study of the IR behaviour of the gauge theory on the D1–D5 branes [Maldacena '97] conjectured that the dual CFT is on the same moduli space as the symmetric orbifold $\mathrm{Sym}^N(T^4) = (T^4)^N / S_N$

Moduli spaces

- Both sides have a large number of moduli (20)

$$\frac{O(4,5)}{O(4)\times O(5)} \Big/ O(4,5;\mathbb{Z})$$

(compared to e.g. $\tau \in \mathbb{H}/SL(2,\mathbb{Z})$ for $AdS_5 \times S^5$)

- The map between the bulk & boundary moduli is still somewhat unclear

- Special points in moduli space:
  - * $\mathrm{Sym}^N(T^4)$ on CFT side
  - * pure R-R flux (D1 – D5 system) on string side
  - * pure NS-NS flux (F1 – NS5 system) on string side

Previous checks

– Since the dual theory of say supergravity on $AdS_3 \times S^3 \times T^4$ is not directly known, the matching of bulk/boundary quantities is restricted to protected sectors:

* worldsheet $\frac{1}{2}$-BPS spectrum
* worldsheet elliptic genus
* extremal correlators
* Fuzzball program

many references

Pure NS-NS flux

- Pure NS-NS flux backgrounds are much simpler on the string side, because the B-field couples directly to the string
- Worldsheet σ-model

$$S = \frac{1}{4\pi\alpha'} \int d^2x \sqrt{\gamma} \left(G_{\mu\nu} \gamma^{ab} + i B_{\mu\nu} \varepsilon^{ab} \right) \partial_a X^\mu \partial_b X^\nu$$

$$= S_{WZW} \quad \text{for} \quad \widetilde{SL(2,\mathbb{R})}$$

$k = \dfrac{\ell_{AdS}^2}{4\pi\alpha'}$: AdS size in units of string length

AdS_3 strings

- AdS_3 string theory with pure NS-NS flux is computationally among the most accessible backgrounds
- It is described by the $SL(2,\mathbb{R})_k$ WZW model on the worldsheet.

  [Giveon, Kutasov, Seiberg '98, Maldacena, Ooguri '00, ...]
- But among the least understood in terms of holography!
- Unusual property: Continuous spectrum

  $\Rightarrow$ Non compact spacetime CFT

  "Feature, not a bug"

Tensionless limit

- Parts of the dictionary were recently understood
  [Dei, LE, Gaberdiel, Gopakumar, Knighton, ... '18 – '21]

  $AdS_3 \times S^3 \times T^4$ with one unit of NS-NS flux $(k=1)$ $\Longleftrightarrow$ $\mathrm{Sym}^N(T^4)$

- Also recent progress for some very stringy backgrounds with $k<1$ (different from $AdS_3 \times S^3 \times T^4$).
  [Balthazar, Giveon, Kutasov, Martinec '21]

This talk

- In this talk we answer

$$\text{pure NS-NS } AdS_3 \text{ background} \longleftrightarrow \ ?$$

at least in string perturbation theory directly without just saying that it lies on the same moduli space as $Sym^N(T^4)$.

- ? will be a symmetric orbifold with a precise deformation turned on.

- This is much more realistic than the small k backgrounds understood earlier.

The string side

- Let's start on the known string side
- We consider bosonic strings on $AdS_3 \times X$ (there is a SUSY analog of this story).
- The worldsheet theory is controlled by an $sl(2,\mathbb{R})_k$ algebra

$$[J^3_m, J^\pm_n] = \pm J^\pm_{m+n}$$

$$[J^3_m, J^3_n] = -\frac{k}{2}\delta_{m+n,0}$$

$$[J^+_m, J^-_n] = k\delta_{m+n,0} - 2J^3_{m+n}$$

$k \sim \ell^2_{AdS}/\ell^2_s$ "size of AdS"

Spectrum

- Worldsheet states fall in $sl(2,\mathbb{R})_k$ representations
- These are labelled by a spin j.
- Two sectors:

a) Discrete representations, $j \in \mathbb{R}$

Normalizability: $\frac{1}{2} < j < \frac{k-1}{2}$

$\Big\}$ short strings, lead to a discrete string spectrum

b) Continuous representations, $j \in \frac{1}{2} + i\mathbb{R}$

Reflection symmetry: $j \sim 1-j$.

$\Big\}$ long strings, lead to a continuous string spectrum

Spectral flow

- In AdS_3, we can also have winding strings
- These are described by spectrally flowed representations, which is described by a 'winding number' w.
- The highest weight states are labelled by $|j,h,w\rangle$.

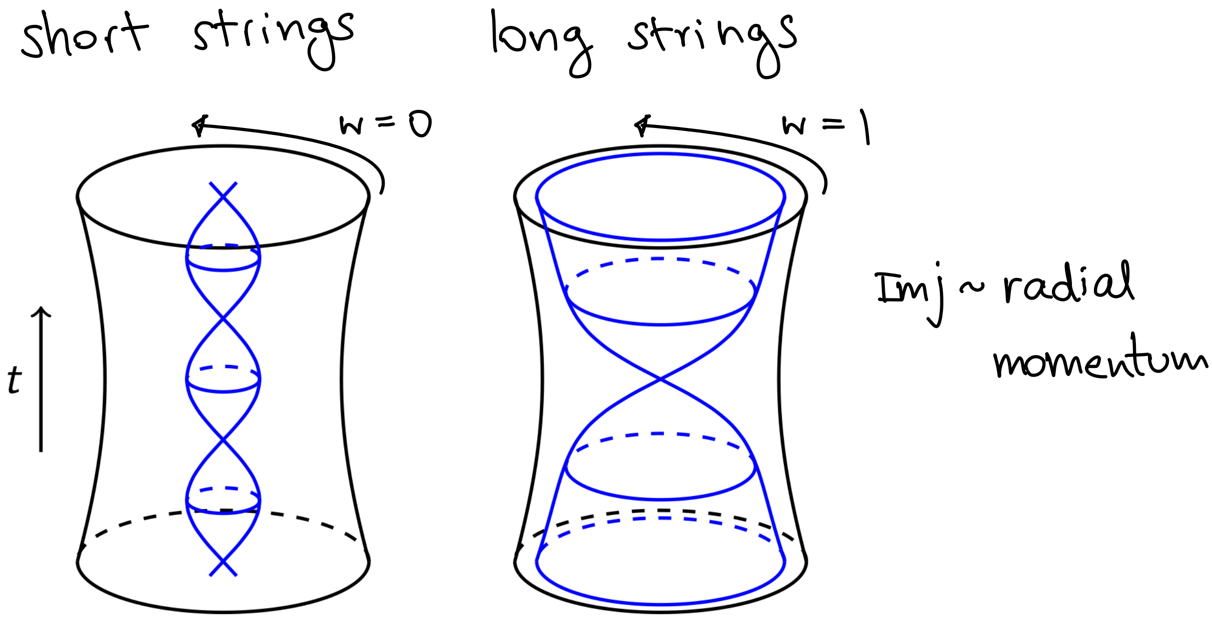
this becomes the spacetime conformal weight

$$J_0^3 |j,h,w\rangle = h |j,h,w\rangle$$

$$J_m^{\pm} |j,h,w\rangle = 0 \qquad m > \pm w.$$

$$J^{\pm}_{\pm w} |j,h,w\rangle = \left(h - \frac{kw}{2} \pm j\right) |j,h\pm 1,w\rangle$$

Short strings are bound states



⟹ Long strings will have a simpler holographic interpretation in terms of scattering states and short strings will emerge as bound states

Vertex operators

- There are corresponding vertex operators
- They are most naturally described in Euclidean AdS using the so-called x-basis

winding / spectral flow

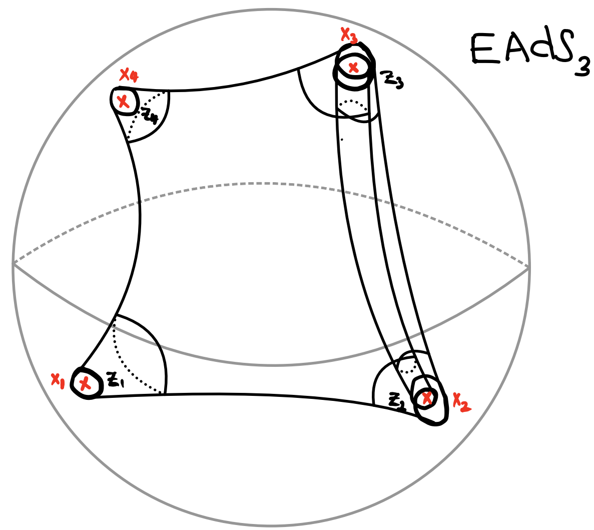
spacetime position

$$V^{w}_{j,h}(x,z)$$

worldsheet position

spacetime conformal weight

$sl(2,\mathbb{R})$ spin



- worldsheet conformal weight : $\Delta = -\dfrac{j(j-1)}{k-2} - wh + \dfrac{kw^2}{4}$.

Two-point functions

- Two point functions are known: [Maldacena & Ooguri '00]

$$\langle V^{w_1}_{j_1,h_1}(0,0)\, V^{w_2}_{j_2,h_2}(\infty,\infty)\rangle = \delta^{(2)}(h_1-h_2)\,\delta_{w_1,w_2}$$

$$\times\left(\delta(j_1+j_2-1) + \underset{\substack{\uparrow \\ \text{reflection coefficient}}}{R_{w_1}(j_1,h_1,\bar{h}_1)}\,\delta(j_1-j_2)\right)$$

$\Rightarrow$ Long strings are like waves scattering at a potential wall, similar to Liouville theory

Reflection coefficient

- The reflection coefficient takes the form

$$R_w(j,h,\bar{h}) = \frac{(k-2)\,\nu^{1-2j}\;\gamma\left(h-\frac{kw}{2}+j\right)}{\gamma\left(\frac{2j-1}{k-2}\right)\,\gamma\left(h-\frac{kw}{2}+1-j\right)\gamma(2j)}\,, \qquad \gamma(x) = \frac{\Gamma(x)}{\Gamma(1-x)}$$

ν: "worldsheet cosmological constant", has no physical significance

- This has poles for

$h - \frac{kw}{2} - j \in \mathbb{Z}_{\geq 0}$, $h - \frac{kw}{2} + j \in \mathbb{Z}_{\leq 0}$ } 'LSZ poles' due to short strings $\Rightarrow$ short strings are bound states

$j \in \frac{1}{2} + \frac{k-2}{2}(n+1)$, $n \in \mathbb{Z}_{\geq 0}$ } 'bulk poles' due to the emergence of a zero mode in the path integral

[Aharony, Giveon, Kutasov '04]

Three point functions

- Three point functions are also known: [Dei, LE '21]

$$\left\langle V^{w_1}_{j_1,h_1}(0,0)\, V^{w_2}_{j_2,h_2}(1,1)\, V^{w_3}_{j_3,h_3}(\infty,\infty)\right\rangle =$$

(holds for $\sum_i w_i \in 2\mathbb{Z}$, there is a similar formula for $\sum_i w_i \in 2\mathbb{Z}+1$)

$$D(j_1,j_2,j_3)\int \prod_{i=1}^{3} \frac{d^2 y_i}{\pi} \prod_{i=1}^{3} \Big| y_i^{\frac{k w_i}{2}+j_i-h_i-1}\, P^{\sum j_i - k}_{(w_1,w_2,w_3)}$$

$$\times \left(P_{(w_1+1,w_2+1,w_3)} + y_1 P_{(w_1-1,w_2+1,w_3)} + y_2 P_{(w_1+1,w_2-1,w_3)} + y_1 y_2 P_{(w_1-1,w_2-1,w_3)}\right)^{-j_1-j_2+j_3} \times \text{cycl.}\Big|^2$$

$D(j_1,j_2,j_3)$: Liouville-like 3pt function [Teschner '99]

$P_{(w_1,w_2,w_3)}$ = explicit numbers

- This function again has bulk poles and LSZ poles corresponding to short strings

The dual CFT - Try 1

- Let's consider a background of bosonic string theory of the form $AdS_3 \times X$, where

  X: compact internal CFT with

  $$c(X) = 26 - \frac{3k}{k-2}$$

- It was observed that the spectrum of long strings on this background matches the CFT [LE, Gaberdiel '19]

$$\mathrm{Sym}^N(\mathbb{R}_Q \times X) \qquad Q = b^{-1} - b = \frac{k-3}{\sqrt{k-2}}, \quad b = \frac{1}{\sqrt{k-2}}$$

in the large N limit.

$$\alpha = \frac{j + \frac{k}{2} - 2}{\sqrt{k-2}} \quad \text{momentum}$$

$\mathbb{R}_Q \sim$ radial direction of AdS_3

This cannot be quite right...

- This cannot be the full dual CFT:
  * This CFT would have an exact $U(1)$ symmetry which is not there on the string side
  * There is no reflection symmetry in this model
  * There are no short string states

$\Rightarrow$ We are missing the wall that makes long strings return to the boundary

- This is similar to the relation of linear dilaton and Liouville theory

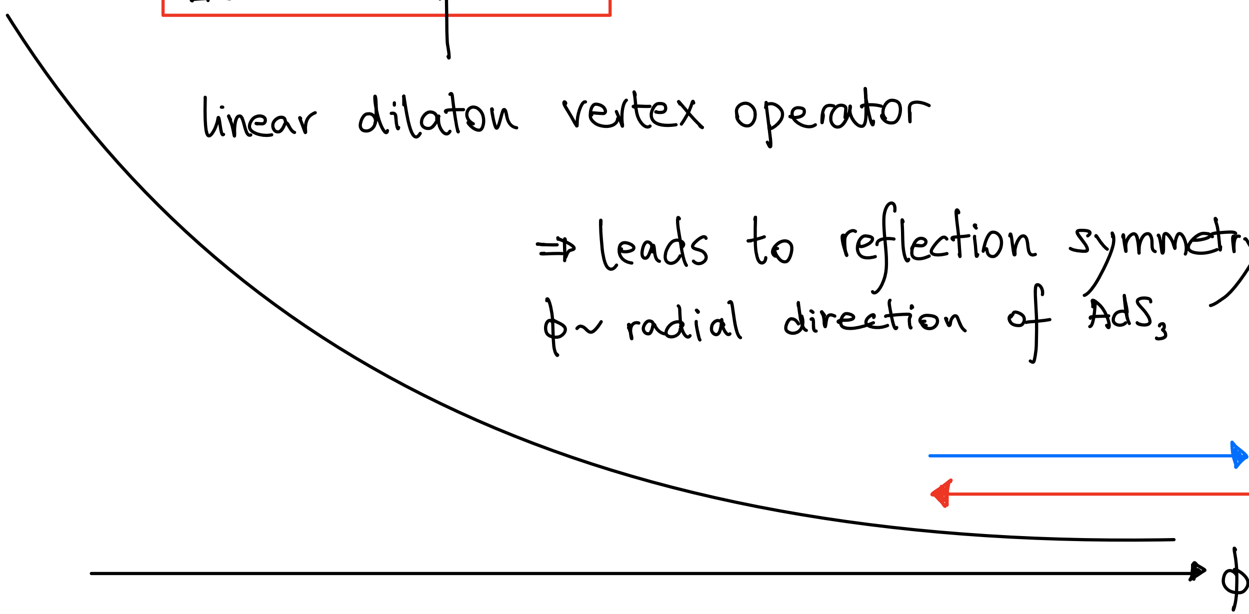
A marginal operator

– The theory possesses a non-normalizable marginal operator

$$\boxed{\Phi = e^{-\frac{1}{2b}\phi}\, O_2}$$

twist-2 operator

linear dilaton vertex operator



⇒ leads to reflection symmetry

$\phi \sim$ radial direction of AdS_3

The dual CFT – Try 2

- We define a dual CFT by perturbing in Φ:

$$\left\langle \prod_{i=1}^{n} \mathcal{O}_{\alpha_i}(x_i) \right\rangle_{\mu} \text{"}=\text{"} \sum_{m=0}^{\infty} \frac{(-\mu)^m}{m!} \int d^2 z_1 \cdots d^2 z_m \left\langle \prod_{i=1}^{n} \mathcal{O}_{\alpha_i}(x_i) \prod_{j=1}^{m} \Phi(z_j) \right\rangle_0$$

sum over momentum-conserving delta-functions

- Like in Liouville theory one has to 'analytically continue' the perturbation series
- The actual correlators $\langle \cdots \rangle_{\mu}$ are meromorphic functions in α_i whose residue is computed by the perturbation series

The dual CFT – Try 2 (contd)

- We assume that there is a CFT with correlators $\langle \dots \rangle_{\nu}$.
- We can only access residues of correlators
- It is not clear to us how to construct this CFT from first principles and whether it exists non-perturbatively
- We conjecture that this is at least perturbatively (in the string coupling) the correct dual CFT! [LE '21]

Genus expansion

- This CFT has a genus expansion [Lunin, Mathur '00]
- The genus g contribution to a correlator goes like

$$\underbrace{\mu^{2b(\sum_i \alpha_i - Q(1-g))}}_{\text{KPZ - scaling}} \underbrace{N^{1-g-\frac{n}{2}}}_{\text{combinatorics}}$$

[KPZ '88]

$$\Rightarrow g_s \sim \mu^{\frac{k-3}{k-2}} N^{-1/2}.$$

- In the large N limit, our CFT only depends non-trivially on this combination, so both sides of the correspondence depend only on (k, g_s, X).

Matching of correlation functions

- We want to check that

normalization string path integral

$$\left\langle \mathcal{O}^{w_1}_{j_1}(0)\, \mathcal{O}^{w_2}_{j_2}(1)\, \mathcal{O}^{w_3}_{j_3}(\infty) \right\rangle_{\text{string}} = C_{S^2} \left\langle V^{w_1}_{j_1}(0,0)\, V^{w_2}_{j_2}(1,1)\, V^{w_3}_{j_3}(\infty,\infty) \right\rangle$$

$$\overset{!}{=} \prod_{i=1}^{3} N(w_i, j_i) \left\langle \mathcal{O}^{w_1}_{\alpha_1}(0)\, \mathcal{O}^{w_2}_{\alpha_2}(1)\, \mathcal{O}^{w_3}_{\alpha_3}(\infty) \right\rangle_{\text{CFT}}$$

relative normalization of vertex operators

- The poles in the CFT correlators translate to bulk poles in the string correlator and one can show that the set of poles agrees.
- We confirmed that the residues agree up to 4^{th} order in conformal perturbation theory. [LE'21]

Matching of correlation functions

- We also have the string 4pt functions under good control and recently showed that this matching extends to 4pt functions [Dei, LE '21, '22].

A test beyond conformal perturbation theory

- The CFT makes an exact prediction for string correlators. Since

$$\partial_\mu \langle \mathcal{O}_{\alpha_1}(0)\,\mathcal{O}_{\alpha_2}(\infty)\rangle_{CFT} = \int d^2z\, \langle \Phi(z)\, \mathcal{O}_{\alpha_1}(0)\,\mathcal{O}_{\alpha_2}(\infty)\rangle_{CFT}$$

and $\Phi \sim V^{w=2}_{j=3-k,h=1}$ we get the prediction that

$$\langle V^{w_1}_{j_1,h_1}(0)\, V^{w_2}_{j_2,h_2}(\infty)\, V^{w_3=2}_{j_3=3-k,h_3=1}(1)\rangle \sim \partial_\nu \langle V^{w_1}_{j_1,h_1}(0)\, V^{w_2}_{j_2}(\infty)\rangle$$

normalization of Φ & relation between μ & ν

- This identity works out miraculously!

Short strings in the CFT

- We can find bound states by looking at LSZ-type poles in correlators $h - \frac{k w}{2} - j \in \mathbb{Z}_{\geq 0}$
- Since the correlators of the string & the proposed CFT are identical, we can deduce also the short string spectrum from the CFT side.

$\Rightarrow$ Full match of the spectrum, both discrete & continuous

Conclusion

- We proposed

$$\mathrm{Sym}^N(\mathbb{R}_Q \times X) \qquad Q = \frac{k-3}{\sqrt{k-2}},$$

  deformed by $\Phi = e^{-\frac{1}{2b}\phi}\sigma_2$ as a dual CFT to perturbative NS-NS strings on $AdS_3 \times X$.

- Checked matching of two- and three-point functions in conformal perturbation theory

- Full spectrum gets reproduced correctly.