On the Stability of String Theory Vacua

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based on [2112.10795] with Suvendu Giri & Luca Martucci

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Introduction

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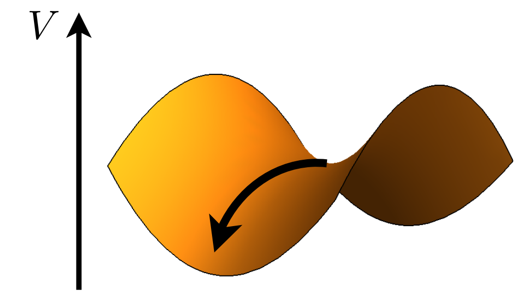
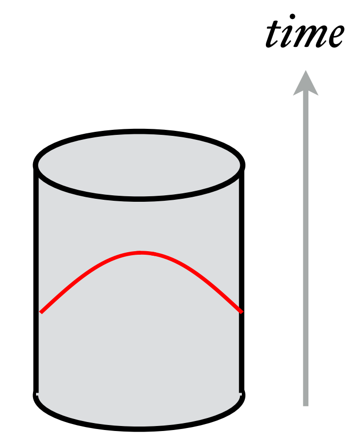
- Most immediate challenge: 'tachyons'



  - instability: once excited, they grow exponentially.

  - but in AdS$_d$, fields with $m^2 > -\frac{(d-1)^2}{4L_{\text{AdS}}}$ [Breitenlohner, Freedman '82]

    form a standing wave by reflecting off the boundary
    and thus are stable



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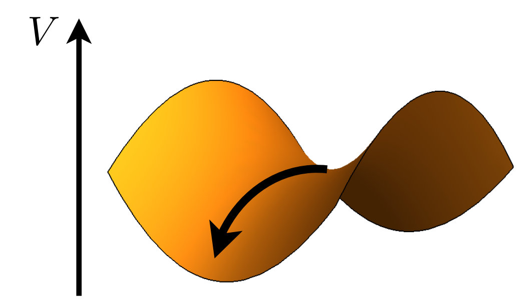

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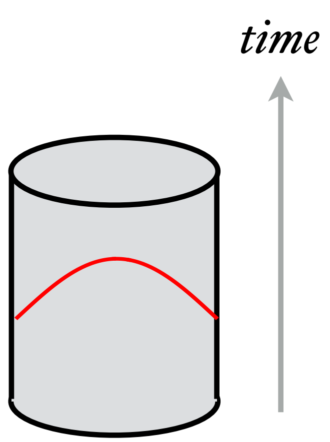
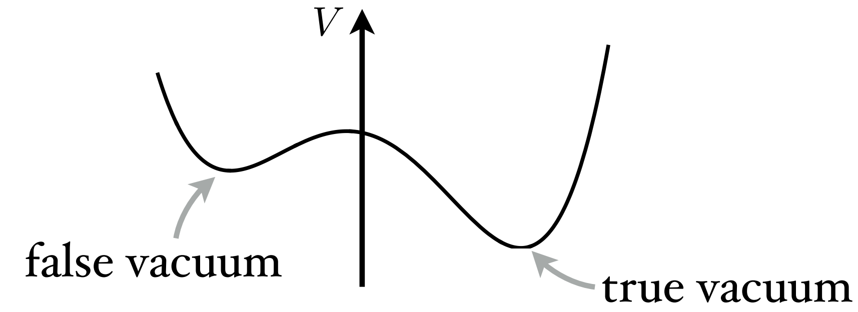


    [dual to relevant operators in the boundary CFT]

  - in vacua with extra dimensions, instabilities detected via KK analysis

    very complicated, but doable in principle. [Kim, Romans, Van Nieuwenhuizen '85; Fabbri, Fré, Gualtieri, Termonia '99; ... Malek, Samtleben, '19; Malek, Nicolai, Samtleben, '20]

- More subtle issue: tunnel effects



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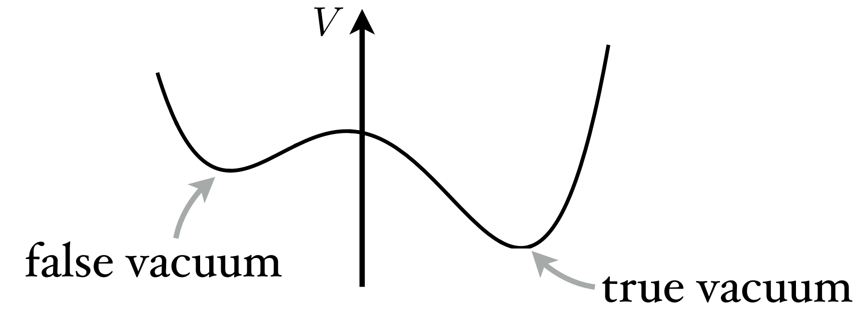

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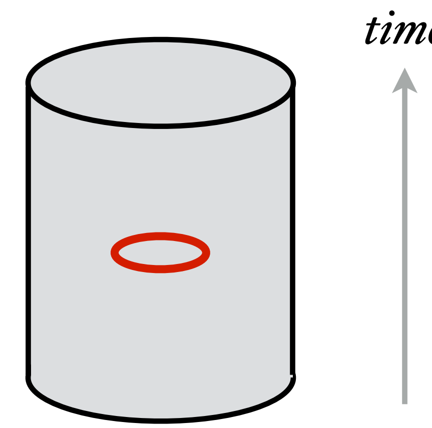

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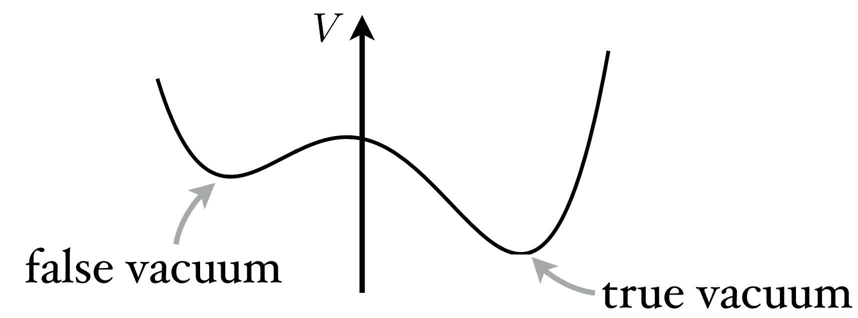

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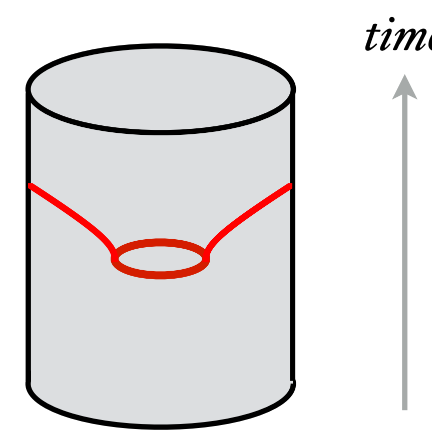

examples abound [Maldacena, Michelson, Strominger '98; Gaiotto, AT '09; Apruzzi, De Luca, Gnecchi, Lo Monaco, AT '19; Bena, Pilch, Warner '20...]

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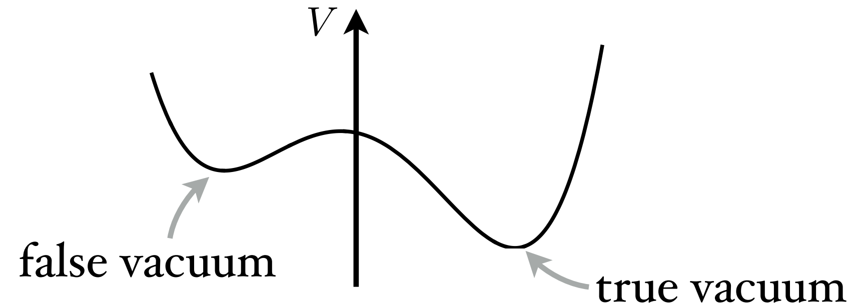

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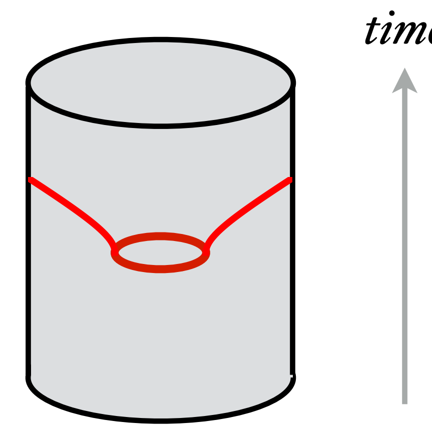

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- This doesn't always happen. Suppose for example bubble is a D-brane. Does it expand?

$$S = \int \underbrace{-T\sqrt{-g}}_{\text{makes it shrink}} + \underbrace{qC_{d-1}}_{\text{makes it expand}}$$

for supersymmetric vacua, they compensate and bubble doesn't expand.

[In fact it doesn't even nucleate]

- Weak Gravity conjecture: there is always a particle for which gravity is weakest force

analogue for branes: there is always a brane for which gravity is weakest force ⇨ expansion wins ⇨ instability

so maybe all non-susy AdS vacua are unstable? [Ooguri, Vafa '16; Freivogel, Kleban '16]

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so maybe all non-susy AdS vacua are unstable? [Ooguri, Vafa '16; Freivogel, Kleban '16]

- difficult to check in theories with many vacua, such as string theory

alternative protection against bubbles?

- AdS8 with O8-planes [Cordova, De Luca, AT '18]
- S-fold AdS [Giambrone, Guarino, Malek, Samtleben, Sterckx, Trigiante '21]

Inspiration: earlier questions about stability of Minkowski in GR

[Schoen, Yau '79; Witten '81]

- Rough idea:
  - energy $\sim \langle \{Q, Q\} \rangle \geq 0$
  - then show energy $= 0 \;\Rightarrow\;$ Minkowski

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⇨ stability for susy vacua in pure gauged $\mathcal{N} = 8, \mathcal{N} = 4$ sugra

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- in 4d models, proof can be extended to some non-susy vacua!

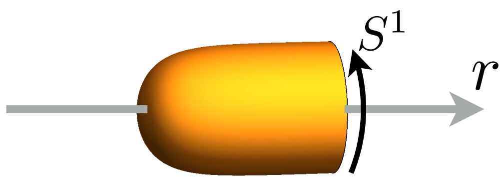
  idea: 'fake' supersymmetry that still implies EoM

  [Boucher '84; Townsend '84;...
  Amsel, Hertog, Hollands, Marolf '07]

This talk:

- Show how stability argument works for susy vacua directly in $d = 10, 11$

extra dimensions expected to introduce subtleties:
recall infamous 'bubble of nothing' for $\mathrm{Mink}_4 \times S^1$



- Try to adapt argument to find some stable non-susy AdS vacua

Plan

- Review of stability in 4d theories
- Stability of supersymmetric compactifications
- Supersymmetry breaking

Stability in 4d

- Toy model: minimal sugra

$$\mathcal{L} = R + \mathrm{i}\bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho \text{ [+matter]}$$

$$\delta_\epsilon \psi_\mu = D_\mu \epsilon$$

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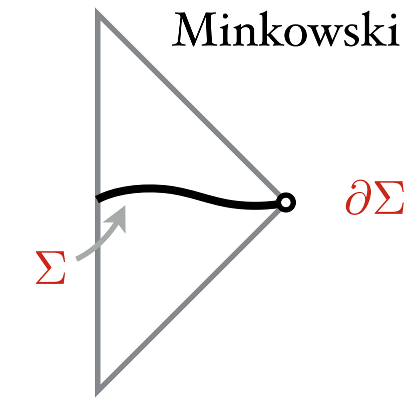

Noether $\Rightarrow$ supercharge $Q = \int_{\partial\Sigma} \bar{\epsilon}\gamma_{\mu\nu\rho}\psi^\rho \, * \, \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$

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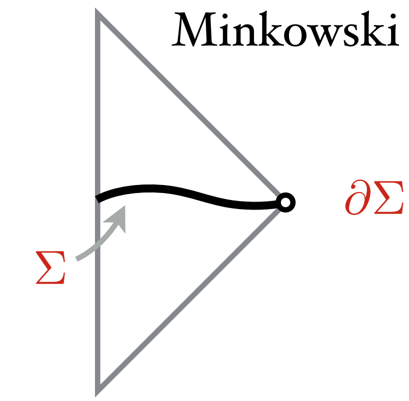
energy $E \sim \{Q, Q\} = \int_{\partial\Sigma} \bar{\epsilon}\gamma_{\mu\nu\rho} D^\rho \epsilon \, * \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$

same as ADM mass
if $\epsilon \to \epsilon_0 + O(1/r)$



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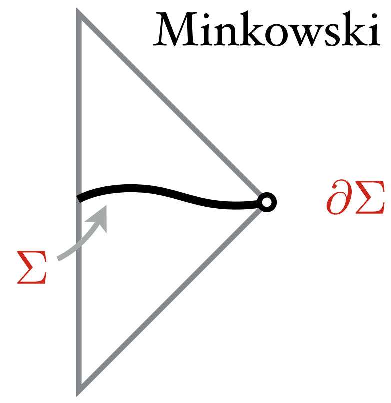

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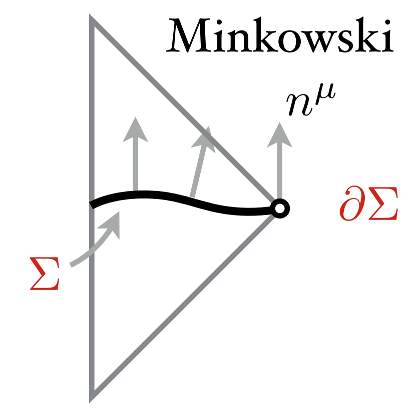

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$$\int_\Sigma (D^\nu \bar{\epsilon}\gamma_{\mu\nu\rho} D^\rho \epsilon + T_{\mu\nu}\bar{\epsilon}\gamma^\nu \epsilon) n^\mu \mathrm{vol}_\Sigma$$

- a fun identity:

$$\gamma^{\mu\nu\rho}[D_\nu, D_\rho] = \tfrac{1}{4} R^{\alpha\beta}{}_{\nu\rho}\gamma^{\mu\nu\rho}\gamma_{\alpha\beta}$$

$$= \left(R^{\mu\nu} - \tfrac{1}{2} R g^{\mu\nu}\right)\gamma_\nu$$

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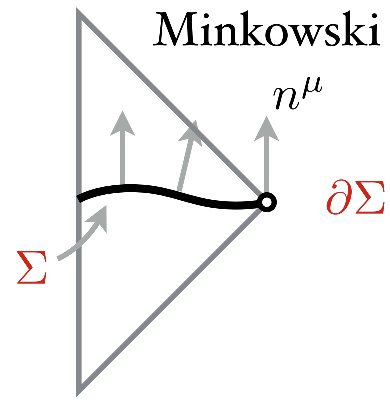

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$$\int_\Sigma (D^\nu \bar{\epsilon}\gamma_{\mu\nu\rho} D^\rho \epsilon + T_{\mu\nu} \underbrace{\bar{\epsilon}\gamma^\nu \epsilon}_{\text{null}}) n^\mu \mathrm{vol}_\Sigma$$

$$\geqslant 0$$

if Dominant Energy Condition holds

- a fun identity:

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- pick frame such that $n = e_{\underline{0}}$
flat index

$$D^a \bar{\epsilon} \gamma_{a\underline{0}b} D^b \epsilon = (D^a \epsilon)^\dagger D_a \epsilon - |\gamma^a D_a \epsilon|^2$$

flat along Σ

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- as. constant ϵ can always be chosen to satisfy $\gamma^a D_a \epsilon = 0$

  'Witten condition'

  [$\sim$ existence of Green's function for $\gamma^a D_a$]

  [Witten '82; more formal proof in Parker, Taubes '82]

- all in all $E = \int_\Sigma (D^a\epsilon)^\dagger D_a \epsilon + T_{\mu\underline{0}} \bar{\epsilon} \gamma^\mu \epsilon \geqslant 0$

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- all in all $E = \int_\Sigma (D^a \epsilon)^\dagger D_a \epsilon + T_{\mu\underline{0}} \bar{\epsilon} \gamma^\mu \epsilon \geqslant 0$

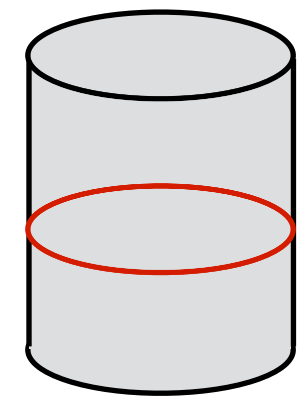
- moreover $E = 0 \;\Rightarrow\; D_a \epsilon = 0 \;\forall \Sigma$ $\Rightarrow$ Minkowski is the only spacetime with zero energy; it can't decay to anything

$\Downarrow$

Minkowski vacuum is stable

- Similar arguments for AdS

[Gibbons, Hull, Warner '83]



  - minimal case: $D_\mu \to D_\mu + W_0 \gamma_\mu$
  - as. AdS boundary conditions $\Rightarrow$ our energy coincides with covariant phase space formalism

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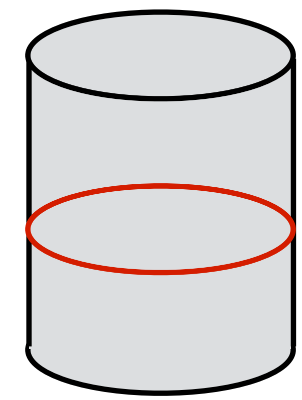

- More realistic theories: same strategy
  - minimal gauged $\mathcal{N} = 4$ sugra:

dilatino transf.

$$\nabla^\mu(\bar{\epsilon}_i\gamma_{\mu\nu\rho}D^\rho\epsilon^i) = D^\mu\bar{\epsilon}_i\gamma_{\mu\nu\rho}D^\rho\epsilon^i + \overline{\delta\lambda_i}\gamma_\nu\delta\lambda^i + T^{\rm mat}_{\mu\nu}\bar{\epsilon}_i\gamma^\mu\epsilon^i$$

matter outside the sugra multiplet

  - other models: slightly different details

- **Susy-breaking:**

[Boucher '84; Townsend '84]

for ex. $\mathcal{L} = R - \partial_\mu \phi \partial^\mu \phi - 2V(\phi)$ **no susy**

- introduce auxiliary spinors ϵ such that $\mathcal{D}_\mu \epsilon = 0$ $\quad \mathcal{D}_\mu \equiv D_\mu + W(\phi)\gamma_\mu$

  earlier argument works if we assume usual $V = 2(\partial_\phi W)^2 - 3W^2$

- however, subtleties in boundary conditions for ϕ

$\Rightarrow$ 'energy' doesn't always coincide with other definitions, not guaranteed to be conserved

[Amsel, Hertog, Hollands, Marolf '07]

M-theory & Type II

Four-dimensional approach shows the way. But:

- some model dependence; should be worked out
- higher KK modes are not considered; could be important, recall BoN!

So we will work directly in $d = 10, 11$

Warm-up: a simple argument in type II in the probe approximation

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$$S = \int -T\sqrt{-g} + qC_{d-1}$$

makes it shrink — makes it expand

evaluate on an S^2 in global coordinates:

$$\Rightarrow \quad V_{\text{eff}} = \left(T\,\text{ch}\rho - \tfrac{1}{3}q\,\text{sh}\rho\right)\text{sh}^2\rho$$

bubble expands iff $q > 3T$ [actually it never even nucleates!]

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$$F = f + \underbrace{\text{vol}_4 \wedge \text{e}^{4A} * \overbrace{\lambda}^{\text{a sign}} f}_{\text{external RR flux}}$$

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one of the 'pure spinor equations' for bulk susy

[Graña, Minasian, Petrini, AT '06]

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$$/\!\!\wedge \quad \text{calibration}$$

[Martucci, Smyth '05; Koerber, Martucci '06]

$$3\int_{B_{p-2}} \text{e}^{3A-\phi}\text{vol} = T$$

no bubble ✓

Beyond the probe approximation: back to positive energy

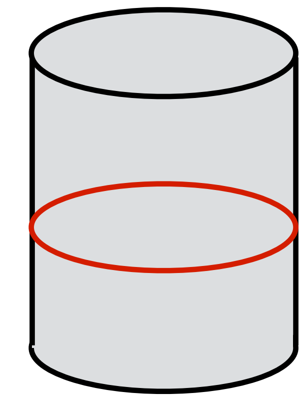
M-theory:

- again energy $= \int_{S^2 \times M_7} *E_2$

$$\delta\psi_M = \mathcal{D}_M \epsilon$$

$$\|$$

$$E_{MN} = \bar{\epsilon}\gamma_{MN}{}^P \mathcal{D}_P \epsilon \qquad\qquad D_P + \frac{1}{24}(-\Gamma_P G + G\Gamma_P)$$



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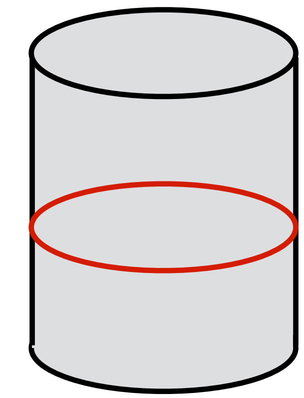

$\partial\Sigma \cong S^2 \times M_7$

$\times M_7$

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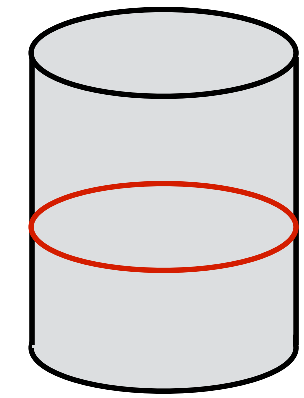

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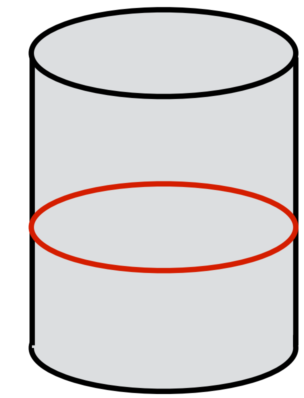

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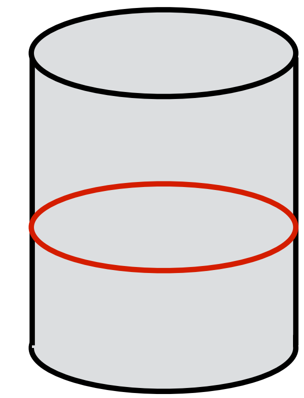

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can be handled as in 4d

Einstein equation

M5 sources

M2 sources

Using calibrations we can prove energy positivity even in presence of sources.

[Martucci '11]

rest of the stability argument similar to 4d. Need:

- math lemma about Witten condition
- in AdS case, comparison with conserved energy from covariant phase space formalism

type II:

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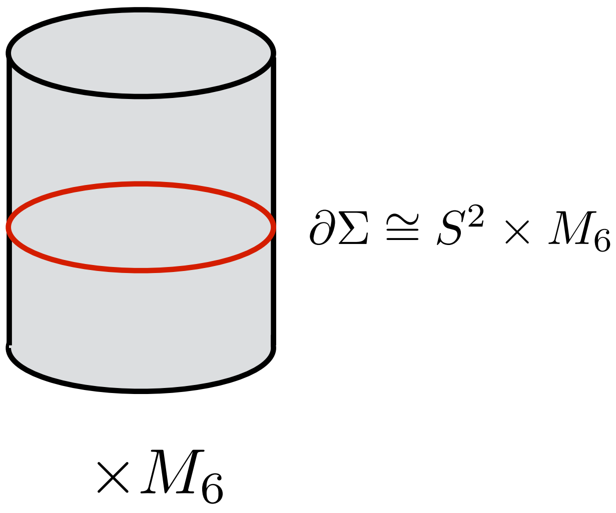

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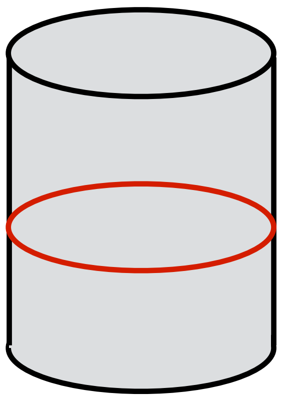

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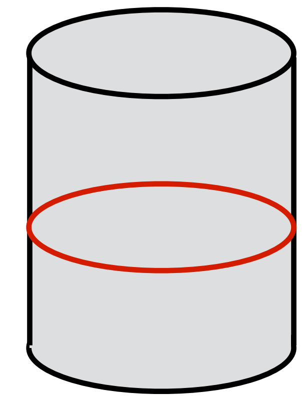

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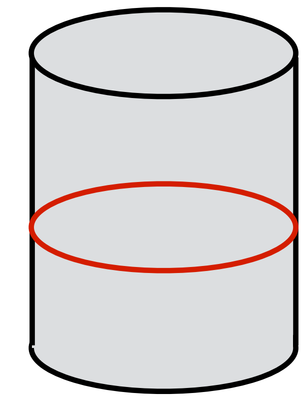

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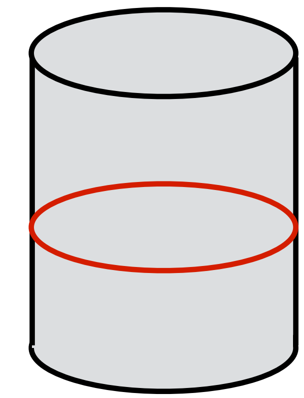

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calibrations

[Martucci '11]

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This is roughly the same 'fake susy' idea that works in 4d.

[Boucher '84; Townsend '84;...
Amsel, Hertog, Hollands, Marolf '07]

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- Englert vacua: $AdS_4 \times$ (weak-G_2) [Englert '82]

  $\exists$ single Killing spinor, $\nabla_m \eta = \frac{1}{2} \gamma_m \eta$

  or: G_2-structure $\mid d\phi = -4 * \phi$

  with also internal flux: $G = g_0 \mathrm{vol}_{AdS_4} + g_1 * \phi$

  $* \phi \cdot \eta = 7\eta$ $\Rightarrow$ purely algebraic equations $\Rightarrow$ $\exists$ solution with $a_2 = \frac{3 \pm \sqrt{114}}{10}$, $a_1 = 3 - a_2$, $a_3 = \pm \frac{21 - 2a_2}{3}$

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This already excludes Englert!

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sign? $\qquad$ sign?

- So we don't find any positivity

We could restart the same strategy in type II.

first-order in derivatives,
at most linear in flux $\Rightarrow$ lots of possibilities!

systematic analysis initiated in
[Lüst, Marchesano, Martucci, Tsimpis '08]

$$\delta_\epsilon \psi_M = \mathcal{D}_M \epsilon \qquad \mathcal{D}_M \equiv D_M \otimes \mathbf{1}_2 - \frac{1}{4} H_M \otimes \sigma_3 + \mathcal{F}\,\Gamma_M$$

$$\delta_\epsilon \lambda = \mathcal{O}\epsilon \qquad \mathcal{O} \equiv \mathrm{d}\phi \otimes \mathbf{1}_2 - \frac{1}{2} H \otimes \sigma_3 + \Gamma^M \mathcal{F}\,\Gamma_M$$

$$\mathcal{F} \equiv \frac{\mathrm{e}^{\phi}}{16} \begin{pmatrix} 0 & F \\ \pm\lambda(F) & 0 \end{pmatrix}$$

- change coefficients
- add new terms $\sim H\Gamma_M$, $\Gamma_M \mathcal{F}$, $\partial_M \phi$, ... $\otimes\ 2 \times 2$ matrix

Conclusions

- As we all expected, susy compactifications are stable

- Modification of stability argument in M-theory doesn't succeed

  but there could of course be another stability argument

- Type II wide open; computation doable in principle