

Dark matter in the exponential growth scenarios

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Null results of dark matter

Rethink:

Production mechanisms

Detection prospects

Various production mechanisms of dark matter

Thermal Freeze-out

Freeze-out to SM states

$$\chi\chi \rightarrow \text{SM SM}$$

via SM particles: Higgs, Z

$$(m_\chi > O(104\text{GeV}))$$

(Lee-Weinberg)

via BSM particles

DM can be lighter

Freeze-out to new states

$$\chi\chi \rightarrow \phi\phi$$

$$\chi\chi \rightarrow \chi\phi$$

Cannibalism

$$n\chi \rightarrow 2\chi$$

$$(n > 2)$$

Various production mechanisms of dark matter

Non-Thermal Freeze-out

Scatterings with the bath particles

Freeze
in
(Decay /
Scattering)

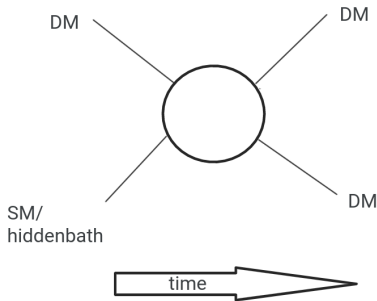
Exp.
growth

Phenomena
unrelated with
bath particles

→ inflation
→ PBHs
→ gravitational

Exponential growth mechanism

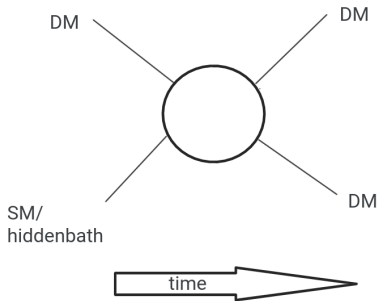
T. Bringmann, J.T. Rudermann et al, PRL 21, A. Hryczuk, M. Laletin, JHEP, 21



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- Initial dark matter density must be non-zero.

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Boltzmann equation for dark matter abundance

- For scattering: $\chi(p) + \phi(p') \rightarrow \chi(k) + \chi(k')$
- Abundance through Boltzmann equation i.e. $\mathcal{L}[f_\chi] = \mathcal{C}[f_\chi]$

$$\mathcal{L}[f_\chi(p, t)] = \left(\frac{\partial}{\partial t} - Hp \frac{\partial}{\partial p} \right) f_\chi(p, t),$$

$$\begin{aligned} \mathcal{C}[f_\chi(p, t)] = & \frac{1}{2E_p S} \int \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 k}{(2\pi)^3 2E_k} \frac{d^3 k'}{(2\pi)^3 2E_{k'}} \times (2\pi)^4 \delta^4(\vec{P}_i - \vec{P}_f) \\ & \times |M|^2 \left[f_\chi(p, t) f_\phi^{\text{eq}}(p', t) (1 \pm f_\chi(k, t)) (1 \pm f_\chi(k', t)) \right. \\ & \left. - f_\chi(k, t) f_\chi(k', t) (1 \pm f_\chi(p, t)) (1 \pm f_\phi^{\text{eq}}(p', t)) \right]. \end{aligned}$$

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Dark matter abundance continued...

- For thermal scenarios,

$$f_{\chi}(p, T) = A(T) f_{\chi}^{\text{eq}}(p, T), \quad \text{where} \quad A(T) = n_{\chi}(T) / n_{\chi}^{\text{eq}}(T).$$

i.e. dark matter is in (*kinetic*) equilibrium

(Gondolo and Gelmini, 91)

- With this, the dark matter density can be estimated by considering the zeroth moment of the Boltzmann equation:

$$\frac{1}{a^3} \frac{d}{dt} (n_{\chi} a^3) = \langle \sigma v \rangle n_{\phi}^{\text{eq}} n_{\chi} \left(1 - \frac{n_{\chi}}{n_{\chi}^{\text{eq}}} \right)$$

Standard case for semi-annihilations

- Or coupled Boltzmann equation in n_{χ} and T'

(Binder, Bringmann, PRD 2017)

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Exponential production

- Solved using f_χ tracing equilibrium pattern
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Since the production is non-thermal, does the equilibrium assumption hold true?

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Boltzmann equation again

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Which can be solved in the comoving frame

$$(p, t) \rightarrow (q, t)$$

$$\text{where } \frac{dq}{dt} = 0$$

$$\left[\begin{aligned} q &= p a \\ &= \frac{p}{a^{1/3}} \end{aligned} \right]$$

$$\text{hence } \frac{d}{dt} f_x(q, t) = \frac{\partial}{\partial t} f_x(q, t)$$

Boltzmann equation reduces to infinitely many ordinary differential equations

Generalised solutions to exponential growth scenarios

$$f_{\chi}(q, x) = f_{\chi}(q, x_{\text{init}}) \exp \left(\int_{x_{\text{init}}}^x dx P'(q, x) \right) \quad \text{where ,}$$
$$P'(q, x) = \frac{h_{\text{eff}}(x) g_{\chi} g_{\phi}}{S H(x) x} \int \frac{d^3 p'}{(2\pi)^3} (\sigma v_{\text{mol}}(q, p')) f_{\phi}^{\text{eq}}(p', x) .$$

The above equation is similar to any growth/decay equation
 $\Rightarrow P'(q, x)$ is a growth function.

$$P'(a, x) > 0$$

$$\rightarrow 0$$

For DM production

To avoid DM
overproduction



$m_\phi > m_\chi \rightarrow f_\phi^{eq} \rightarrow \text{Boltzmann}$
supp.

$m_\chi > m_\phi \rightarrow (6v) \text{ is}$
Boltzmann
suppressed.

Growth function

- Since the growth function depends on q , not all momentum modes grow similarly with the expansion of the universe.
- DM growth in the universe is purely exponential if $P'(q, x)$ is constant in x .
- In general the growth function can be a complicated function in x and it is hard to parameterise the growth as simple exponential.
- The growth of the distribution in general can scale as a factor of exponential $\exp[A(x)]$.
- We still call growth as exponential to avoid new names.

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Comparison between the generalised approach and the equilibrium assumption:

Generalised approach for distribution function:

$$Y_{\chi}(x) = g_{\chi} \int \frac{d^3 q}{(2\pi)^3} \left[f_{\chi}(q, x_{\text{init}}) \exp \left(\int_{x_{\text{init}}}^x dx P'(q, x) \right) \right] \quad \text{where ,}$$

$$P'(q, x) = \frac{h_{\text{eff}}(x) g_{\chi} g_{\phi}}{S H(x) x} \int \frac{d^3 p'}{(2\pi)^3} (\sigma v_{\text{mol}}(q, p')) f_{\phi}^{\text{eq}}(p', x) .$$

Simplified approach for distribution function:

$$Y_{\chi}(x) = Y_{\chi}(x = x_{\text{init}}) \exp \left(\int_{x_{\text{init}}}^x dx P(x) \right) , \quad \text{where}$$

$$P(x) = \frac{h_{\text{eff}}(x) n_{\phi}^{\text{eq}} \langle \sigma v \rangle}{S x H(x)}$$

Comparison between the generalized approach and the equilibrium assumption:

- Simplified approach:

- The dependence on initial conditions is through appearance of $Y_\chi(x = x_{\text{init}})$.
- Do not need to specify the process of generation of the initial density.

- Generalised approach:

- The dependence on initial conditions is through appearance of $f_\chi(q, x = x_{\text{init}})$
- The final relic intrinsically depends on the initial process populating dark matter and the momentum modes.

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What kind of models?

- Basically all models explaining semi-annihilations in the low coupling regions.
- A Z_3 symmetric dark matter interaction with a singlet scalar $\Rightarrow \chi^3 \phi + \text{h.c.}$
- A mass-mixed fermion having self interactions via scalar/gauge boson $\Rightarrow \Delta m \bar{\chi}_1 \chi_2$ with $\bar{\chi}_1 \chi_1 \phi$ or $\bar{\chi}_1 \gamma^\mu \chi_1 Z'_\mu$
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First: Toy model for exponential growth mechanism

- The models with fermionic mass mixings/ or Z_3 symmetric dark matter can give rise to scatterings of the form: $\chi + \phi \rightarrow \chi + \chi$.
- We consider a scalar DM coupled with a bath particle ϕ .
- For simplicity, we assume ϕ to be coupled with the SM bath.
- We consider the case where $m_\phi > m_\chi$ but $m_\phi < 3m_\chi$ to avoid $\phi \rightarrow \chi\chi\chi$.
- The growth function in this case:

$$P'(q, x) = \frac{|\lambda_{\text{tr}}|^2 h_{\text{eff}}(x)}{32\pi E_q x H(x)} \int \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{\sqrt{s - 4m_\chi^2}}{\sqrt{s}} f_\phi^{\text{eq}}(p', x)$$

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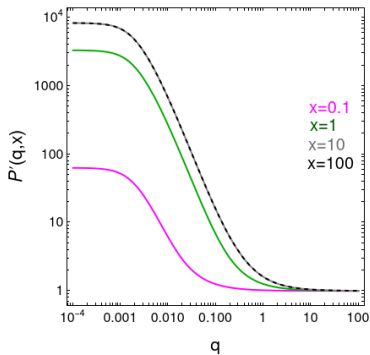
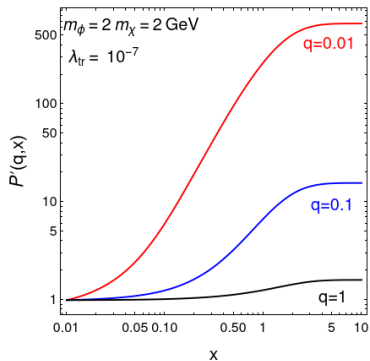
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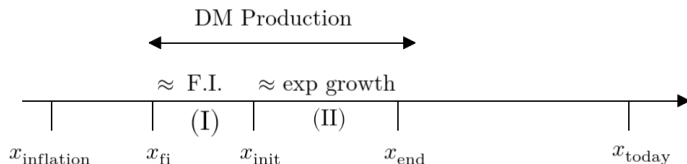
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$P'(q, x) \propto 1/E_q \Rightarrow$ low momentum modes populate more than high momentum modes.

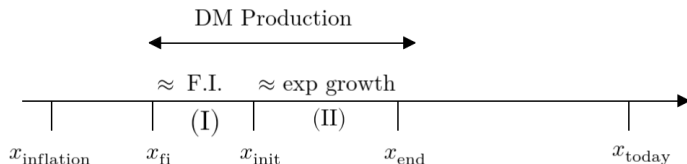
Initial conditions



- The collision operator in region (I) can be approximated as:

$$\mathcal{C}^{(I)}[f_\chi(q, x)] \approx \frac{1}{2E_q S} \int \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 k}{(2\pi)^3 2E_k} \frac{d^3 k'}{(2\pi)^3 2E_{k'}} (2\pi)^4 \delta^4(P_i - P_f) \\ \times f_\chi^{\text{eq}}(q, x) f_\chi^{\text{eq}}(p', x) |M|_{\phi\phi \rightarrow \chi\chi}^2$$

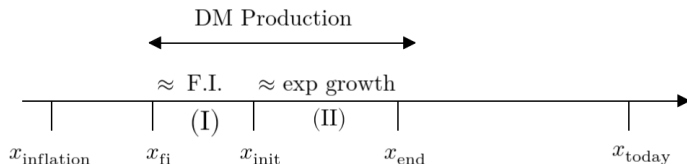
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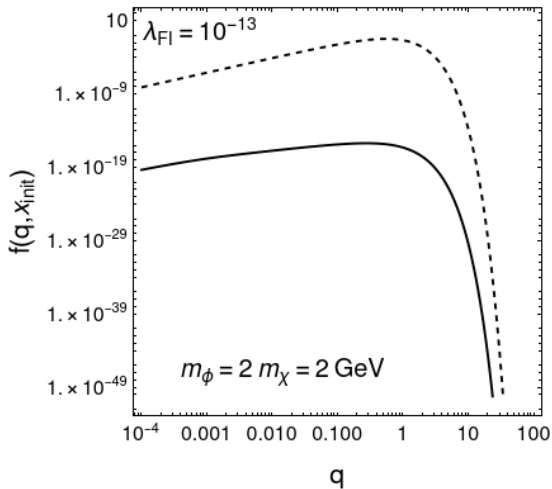
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Out-of-equilibrium decay of a heavy non-relativistic particle

- The initial distribution function can be solved by considering: $\Phi \rightarrow \chi\chi$
- Ideally it depends on the f_Φ .
- Simplification when $m_\Phi \gg m_\chi \Rightarrow p_\chi \approx m_\Phi/2$
- The initial distribution function in this case (heuristically):

$$f_\chi(q, x) = A \delta(|q| - q_{\text{init}}) \Theta(x - x_{\text{init}}) .$$

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Out-of-equilibrium decay of a heavy non-relativistic particle

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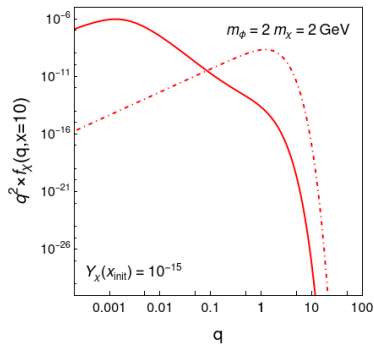
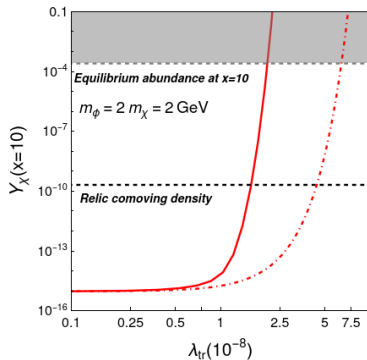
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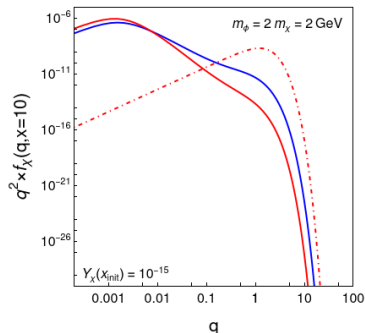
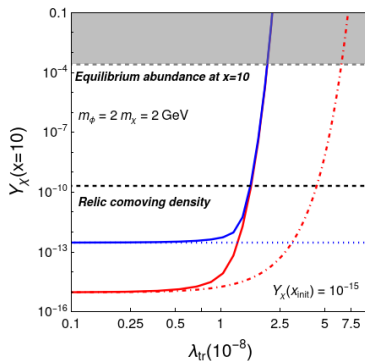
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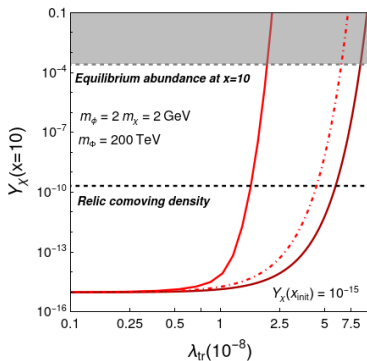
Freeze-in results:



Freeze-in results:



Out-of-equilibrium decay of a heavy non-relativistic particle continued...



Summary and concluding remarks

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- Large number of possibilities to explore.
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