New probes of the string spectrum

Chrysoula Markou

LPTHE, Sorbonne Université
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Université de Mons

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Generalities of the spectrum

- two universal string parameters: α' and g_S

- infinitely many *physical* states

$$M^2 = \text{integer} \times \frac{1}{\alpha'}$$

that are thought of as:

1. mass eigenstates $\Rightarrow$ on–shell mass

2. irreps of $SO(D-1)$ or $SO(D-2)$ $\Rightarrow$ TT

   1–particle states à la Bargmann and Wigner ?

- What does the spectrum look like? Is there a bigger *symmetry*?

The physicality condition

- Polyakov action: the string field can be expanded in *modes* α_n^μ
- energy–momentum tensor on the worldsheet $\Rightarrow$ modes:

$$L_n = \frac{1}{2} \sum_{m=-\infty}^{+\infty} :\alpha_{n-m} \cdot \alpha_m: \quad , \quad \alpha_0^\mu \sim p^\mu$$

- algebra: Virasoro

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12} m(m^2-1)\delta_{m+n}$$

- physicality condition: Virasoro constraints

$$L_{n>0}|\text{phys}\rangle = 0 \quad , \quad (L_0 - 1)|\text{phys}\rangle = 0$$

on any state need only three constraints:

see e.g. Sasaki, Yamanaka 1985

$$L_0|\text{phys}\rangle = 0 \quad , \quad L_1|\text{phys}\rangle = 0 \quad , \quad L_2|\text{phys}\rangle = 0$$

Example: the leading Regge trajectory

- oscillator algebra: $[\alpha_n^\mu, \alpha_m^\nu] = \delta_{m+n}\eta^{\mu\nu}$
- Fock space: $\alpha_{-k} \sim (a^\dagger)_k^\mu$ and $\alpha_{+k} \sim a_k^\mu$, $k > 0$

$$\alpha_{+k}^\mu |p\rangle = 0$$

- leading Regge trajectory:

$$\epsilon_{\mu_1 \dots \mu_s}(p)\, \alpha_{-1}^{\mu_1} \dots \alpha_{-1}^{\mu_s} |p\rangle$$

- Virasoro constraints: L_0 fixes p^2 and

$$L_1 \sim p \cdot \alpha_1 + \dots \qquad\qquad L_2 \sim \alpha_1 \cdot \alpha_1 + \dots$$

$\Rightarrow$ L_1 checks transversality, $p^\nu \epsilon_{\nu\mu_2 \dots \mu_s} = 0$
and L_2 checks tracelessness, $\eta^{\nu\sigma}\epsilon_{\nu\sigma\mu_3 \dots \mu_s} = 0$

Building the spectrum 1

- Method 1: light–cone gauge
  - e.g. bosonic strings: use transverse oscillators α^i_{-n}
  - *non*–covariant, leads to *superposition* or *fraction* of states

Goddard, Thorn 1972
Goddard, Goldstone, Rebbi, Thorn 1973

N	$gl(24)$ tensors	$so(24)$ irreps	little group irreps
0	$\vert k\rangle$ $\bullet$	$\bullet$	$\bullet$
1	$\alpha^i_{-1}\vert k\rangle$ $\square$	$\square$	$\square$
2	$\alpha^{i_1}_{-1}\alpha^{i_2}_{-1}\vert k\rangle$ $\alpha^i_{-2}\vert k\rangle$ $\square\square$ $\square$	$\square\square \oplus \square \oplus \bullet$	$\square\square$
3	$\alpha^{i_1}_{-1}\alpha^{i_2}_{-1}\alpha^{i_3}_{-1}\vert k\rangle$ $\alpha^{i_1}_{-2}\alpha^{i_2}_{-1}\vert k\rangle$ $\alpha^i_{-3}\vert k\rangle$ $\square\square\square$ $\square \otimes \square$ $\square$	$\square\square\square \oplus \square\square \oplus \square \oplus \bullet$ $\oplus \begin{smallmatrix}\square\\ \square\end{smallmatrix} \oplus \square$	$\square\square\square$ $\oplus \begin{smallmatrix}\square\\ \square\end{smallmatrix}$

see e.g. Blumenhagen, Lüst, Theisen book

A covariant way: vertex operators

- state–operator correspondence, e.g. for open bosonic strings:

$$\alpha^{\mu_1}_{-n_1} \dots \alpha^{\mu_k}_{-n_k} |0;p\rangle \quad \leftrightarrow \quad \partial^{n_1} X^{\mu_1} \dots \partial^{n_k} X^{\mu_k} \, e^{ip\cdot X}$$

  - operator ingredients: primary ∂X and its descendants

$$V(z) = F\Big(\partial X^\mu, \partial^2 X^\mu, \dots, \partial^k X^\mu\Big)\, e^{ip\cdot X}$$

  - impose the physical state condition:

$$[Q,V] \stackrel{!}{=} \text{tot. deriv.} \quad \Rightarrow \quad h_V = 1 \quad , \quad \alpha' p^2 = N-1 \quad , \quad h_F = N$$

- first few levels:
  1. $N=0$: $V_{\text{tachyon}}(p,z) = e^{ip\cdot X}$
  2. $N=1$: $V_\epsilon(p,z) = \epsilon_\mu \, \partial X^\mu \, e^{ip\cdot X}$
  3. $N=2$: $V_B(p,z) = B_{\mu\nu} \, \partial X^\mu \, \partial X^\nu \, e^{ip\cdot X}$

[g_o, T^a and normalizations in front of v.o. omitted for simplicity]

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Building the spectrum 2

- Method 2: DDF
  - scatter k photons, each of momentum $-n_i q$, off of tachyon

$$\Rightarrow \text{generic state:} \quad \big(\epsilon_k \cdot A_{-n_k}\big) \dots \big(\epsilon_1 \cdot A_{-n_1}\big) e^{i\widetilde{p}\cdot X} \quad , \quad N = \sum_{i=1}^{k} n_i$$

where A^{μ}_{-n}: DDF operators, with $A^{i}_{-n} \leftrightarrow \alpha^{i}_{-n}$

  - condition on the reference momentum: $\widetilde{p} \cdot q \overset{!}{=} 1$

Del Giudice, Di Vecchia, Fubini 1972
Brower 1972
Skliros, Hindmarsh '11

  - disparate decays of states at very high N, *chaos*?

Gross, Rosenhaus '21
Rosenhaus '21
Firrotta, Rosenhaus '22
Bianchi, Firrotta, Sonnenschein, Weissman '22–'23

- Method 3: construct $SO(D-1)$ irreps from partition function

Forcella, Hanany, J. Troost '10

Main challenge: how do excited states look like?

Visualisation: Regge *trajectories*

N	decomposition in physical states
0	$\bullet$
1	$\ydiagram{1}_{so(d-2)}$
2	$\ydiagram{2}$
3	$\ydiagram{3} \oplus \ydiagram{1,1}$
4	$\ydiagram{4} \oplus \ydiagram{2,1} \oplus \ydiagram{2} \oplus \bullet$
5	$\ydiagram{5} \oplus \ydiagram{3,1} \oplus \ydiagram{3} \oplus \ydiagram{2,1} \oplus \ydiagram{1,1} \oplus \ydiagram{1}$
6	$\ydiagram{6} \oplus \ydiagram{4,1} \oplus \ydiagram{4} \oplus \ydiagram{3,1} \oplus \ydiagram{3} \oplus \ydiagram{2,2} \oplus \ldots$

see e.g. Weinberg 1985, Mañes, Vozmediano 1989

the spectrum seems repetitive, but is there a certain *pattern*?

Visualisation: Regge *trajectories*

- *leading* Regge: highest spins $\forall$ level
  1. bosonic strings:

$$V(p,z) = \sum_s \epsilon_{\mu_1\ldots\mu_s}(p)\,\partial X^{\mu_1}\ldots\partial X^{\mu_s}\,e^{ip\cdot X} \quad , \quad p^2 = -\frac{s-1}{\alpha'}$$

$$p^\mu \epsilon_{\mu\mu_2\ldots\mu_s} = 0 \quad , \quad \epsilon^\mu{}_{\mu\mu_3\ldots\mu_s} = 0 \quad \text{or} \quad p\cdot\frac{\delta F_1}{\delta\partial X} = 0 \quad , \quad \frac{\delta^2 F_1}{\delta\partial X\cdot\delta\partial X} = 0$$

  $\Rightarrow \mathcal{A}_3, \mathcal{A}_4$ and s_1–s_2–s_3 couplings

  Sagnotti, Taronna '10

  2. superstrings, heterotic strings

  Schlotteter '10
  Bianchi, Lopez, Richter '10
  Bianchi, Teresi '11
  for string field theory see Benakli, Berkovits, Daniel, Lize '21
  Benakli, Daniel, Ke '22

- What does the spectrum look like beyond the leading Regge? Is it concealing a *bigger organizing symmetry*?

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What does a state look like?

- *any* state has a polarization depicted by a Young diagram

$$\epsilon^{\mu(s_1),\,\lambda(s_2),\ldots,\,\nu(s_K)}(p): \quad \begin{array}{|l|} \hline \quad\quad s_1 \quad\quad \\ \hline \quad\quad s_2 \quad \\ \hline \quad \ldots \quad \\ \hline s_K \\ \hline \end{array} \quad , \quad s_1 \geq s_2 \geq \ldots \geq s_K$$

- for *physical* states: *dress* polarization by suitable polynomial

- notation: $X_\mu^{(k)} \equiv \partial^k X_\mu$, simplest physical example:

$$F = \epsilon^{\mu(s_1),\,\lambda(s_2),\ldots,\,\nu(s_K)}\, X^{(1)}_{\mu_1} \ldots X^{(1)}_{\mu_{s_1}} \ldots X^{(K)}_{\nu_1} \ldots X^{(K)}_{\nu_{s_K}}$$

lowest possible level: $N_{\text{min}} = \sum_{i=1}^{K} s_i\, i$

- leading Regge: 1 row, spin–s at $N = s \Rightarrow$ simplest subexample

CM, Skvortsov '23

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Where does a state appear?

- there are *infinitely* many ways to render a given diagram physical

- *any* state appears at N_{min} and at higher levels $N = N_{\text{min}} + w$
  $\Rightarrow$ let's call w "*depth*"

- let's call trajectory a *fixed* number of rows at *fixed* w
  example: at $w = 0$, each value of K corresponds to a new trajectory ($K = 1$: leading Regge)

- example: *shifted* clone of leading Regge that starts at $N = 4$

trajectory	Young shape	spin	N	w	lowest member
leading Regge	$\boxed{\quad s \quad}$	$s \geq 0$	s	0	•
first "clone"	$\boxed{\quad s \quad}$	$s \geq 2$	$s + 2$	2	$\boxed{}\boxed{}$

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Let's reorganize the spectrum

- instead of N, let's **use w to organize the spectrum**

w	trajectories
0	$\boxed{s} \oplus \left(\begin{array}{l}\boxed{\quad s-2 \quad}\\ \boxed{}\end{array} \oplus \begin{array}{l}\boxed{\quad s-4 \quad}\\ \boxed{}\boxed{}\end{array} \oplus \ldots \right) \oplus \begin{array}{l}\boxed{\quad s-5 \quad}\\ \boxed{}\\ \boxed{}\end{array} \oplus \ldots$
1	$\left(\begin{array}{l}\boxed{\quad s-3 \quad}\\ \boxed{}\end{array} \oplus \begin{array}{l}\boxed{\quad s-5 \quad}\\ \boxed{}\boxed{}\end{array} \oplus \ldots \right) \oplus \ldots$
2	$\boxed{s-2} \oplus 2^* \begin{array}{l}\boxed{\quad s-4 \quad}\\ \boxed{}\end{array} \oplus \ldots$
3	$\boxed{s-3} \oplus \ldots$
4	$3^* \boxed{s-4} \oplus \ldots$

- *complexity*: measured by w (and number of rows)
- remember: finite K for every trajectory!

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All trajectories at once

- let's consider the *most general* vertex operator:

$$\mathbb{V}_F(z,p) = F\big[X^{(1)}, X^{(2)}, ...\big]e^{ip\cdot X^{(0)}}$$

- let's impose $[Q, \mathbb{V}_F(z,p)] =$ tot. deriv. $\Rightarrow$ obtain:
  - 1 on–shell condition

$$(L_0 - 1)F = \left(\sum_{n=0} n\, X^{(n)} \cdot \frac{\delta}{\delta X^{(n)}} + \alpha' p^2 - 1\right) F = 0$$

  - n differential constraints

$$L_{n>0}F \quad = \quad \Bigg[2\alpha'\, n!\, i\, p \cdot \tfrac{\delta}{\delta X^{(n)}} + \alpha' \sum_{m=1}^{m=n-1} m!(n-m)! \frac{\delta^2}{\delta X^{(m)} \cdot \delta X^{(n-m)}} - \sum_{m=0} \frac{(n+m+1)!}{m!} X^{(m+1)} \cdot \frac{\delta}{\delta X^{(n+m+1)}}\Bigg] F = 0$$

- now leading Regge is the special case with no descendants

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Howe duality and a bigger organizing symmetry

- let's define the operators

$$T^k{}_l \equiv X^{(k)} \cdot \frac{\delta}{\delta X^{(l)}} \quad , \quad T_{kl} \equiv \frac{\delta^2}{\delta X^{(k)} \cdot \delta X^{(l)}} \quad , \quad T^{kl} \equiv X^{(k)} \cdot X^{(l)}$$

  which check Young symmetry & tracelessness

$$T^k{}_l \, F \;\; = 0 \quad (k < l) \quad , \quad T_{kl} F = 0$$

  and act on the k indices, whose range is *infinite*

- simplification: transverse subspace is sufficient
  $\Rightarrow p$ appears only within the transverse metric

  see e.g. Mañes, Vozmediano 1989

- $\{T^k{}_l \, , T_{kl}, T^{kl}\}$ form $sp(2\bullet) \Rightarrow$ *Howe dual* to $so(D-1,1)$

  for the duality of commuting algebras see Howe 1989

idea: use this *bigger* symmetry to construct trajectories

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Howe duality and a bigger organizing symmetry

- let's rewrite our constraints as

$$(L_0 - 1)F = \Big(\textstyle\sum_{n=0} n\, T^n{}_n \; + \alpha' p^2 - 1 \Big) F = 0$$

$$L^\perp_{n>0} F = \left[\alpha' \sum_{m=1}^{n-1} m!(n-m)!\, T^\perp_{m,n-m} - \sum_{m=0} \frac{(n+m+1)!}{m!} T^{m+1}{}_{n+m+1} \right] F = 0$$

- on any function F need only

$$L_0 F = 0 \quad , \quad L^\perp_1 F = 0 \quad , \quad L^\perp_2 F = 0$$

- Howe duality now implies that:

  the lowest weight states of $sp(2\bullet)$ *solve* the Virasoro constraints

  - example: $w = 0$

$$F = \epsilon^{\mu(s_1),\, \lambda(s_2), \dots,\, \nu(s_K)}\, X^{(1)}_{\mu_1} \dots X^{(1)}_{\mu_{s_1}} \dots X^{(K)}_{\nu_1} \dots X^{(K)}_{\nu_{s_K}}$$

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Building the spectrum: a new technology

- idea: use $sp(2\bullet)$ creation operators to construct trajectories

$$F^f \equiv f(T_\perp^{mn}, T^k{}_l)\, F \quad , \quad k > l\,,$$

where f: *trajectory–shifting* operator of weight w

- can distinguish 2 kinds of embeddings:
"*principal*" ($w = 0$) and "*non–principal*" ($w > 0$)

- example: take leading Regge ($w = 0$)

$$F \equiv \epsilon^{a(s)} X^{(1)}_{a(s)} \quad , \quad T^\perp_{11} F = 0$$

and dress it to create a subleading trajectory at $w = 2$

$$F^f = \left[\frac{\gamma_1}{\alpha'} T_\perp^{11} + \gamma_2 T^3{}_1 + \gamma_3 (T^2{}_1)^2\right] F$$

$$\boxed{\quad s \quad} = f \times \boxed{\quad s \quad}$$

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Building the spectrum: a new technology

- for the trajectory $\boxed{\ s\ }$ at $w = 2$, solve the Virasoro constraints:

$$\Rightarrow \quad \gamma_2 = \gamma_1 \frac{D+2s-1}{3s} \quad , \quad \gamma_3 = -\gamma_1 \frac{D+2s-1}{2(s-1)}$$

1 free parameter (other than spin)

- singularity at $s \to 1$: determines lightest member–state

- spin is not fixed $\Rightarrow$ full trajectory is known!

- example: the lightest member–state $\boxed{\ }\boxed{\ }$ of the clone

$$\begin{aligned} F_B &= B_{\mu\nu}\, i\partial X^\mu i\partial X^\nu \\ F_{\widetilde{B}} &= \widetilde{B}_{\mu\nu} \Big[-\tfrac{3}{\alpha'}\, i\partial X^\mu i\partial X^\nu i\partial X^\kappa i\partial X_\kappa + 29\, i\partial X^\mu i\partial^3 X^\nu - 87\, i\partial^2 X^\mu i\partial^2 X^\nu \Big] \end{aligned}$$

shifting the massive spin–2 from $N = 2$, $w = 0$ to $N = 4$, $w = 2$

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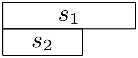
Building the spectrum: a new technology

- 2–row example:

$$F = \epsilon^{\mu(s_1),\,\nu(s_2)} X^{(1)}_{\mu_1} \dots X^{(1)}_{\mu_{s_1}} X^{(2)}_{\nu_1} \dots X^{(2)}_{\nu_{s_2}}$$

so

$$\begin{array}{lcllllll} f_{w=1} & : & T^2{}_1\,, & T^3{}_2 & & & & \\ f_{w=2} & : & T^{11}_{\perp}\,, & (T^2{}_1)^2\,, & T^2{}_1 T^3{}_2\,, & T^3{}_2 T^3{}_2\,, & T^3{}_1\,, & T^4{}_2 \end{array}$$

Young shape	w	N	possible f terms	# of free parameters	lightest member
	0	$s_1 + 2s_2$	trivial	1	
	1	$s_1 + 2s_2 + 1$	2	1	
	2	$s_1 + 2s_2 + 2$	6	3	

- multiplicity = number of free parameters (other than spin)
- other deep trajectories available

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Interactions with the new technology

- factorize polarizations and "exponentiate" polynomials
  leading Regge example: $\epsilon^{\mu(s)} = \epsilon^{\mu_1} \dots \epsilon^{\mu_s} \quad , \quad \epsilon \cdot \epsilon = 0$

$$\mathbb{V}_{F_1}(z,p) = \big(\epsilon \cdot \partial X\big)^s e^{ip\cdot X} = \big(-i\epsilon \cdot \partial_\xi\big)^s \exp\big(ip\cdot X + i\,\xi \cdot \partial X\big)\Big|_{\xi=0}$$

  then the N–point function can be used

$$\mathcal{Z}_N^{\text{leading}} = \mathcal{E}_N \exp\left[\sum_{i\neq j}^{N} \alpha' \Big(2\frac{\xi_i \cdot p_j}{z_{ij}} + \frac{\xi_i \cdot \xi_j}{z_{ij}^2}\Big)\right]$$

see e.g. Kawai, Lewellen, Tye 1986

- for *any* trajectory: generalize to

$$\mathbf{V}(z,p,\vec{\xi}) = \exp\left(ip\cdot X + i\sum_{n=1}^{\infty} \xi^{(n)} \cdot X^{(n)}\right)$$

- amplitudes for entire subleading trajectories accessible!
  example: scattering of tachyon off a spin–s

$$\mathcal{A}^{s00}_{w=2} \sim \mathcal{A}^{s00}_{w=0}(-D+4s+31) \sim (p_2\cdot\epsilon)^s(-D+4s+31)$$

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Is the *complete* spectrum within reach?

What can the complete spectrum teach us about fields and strings and, ultimately, about quantum gravity?

Other examples of subleading states

- let's look at massive spin–2 superstring states in 4D

- *lightest* such brane state: leading

$$V^{(-1)}_{M,\text{open}}(z,p) = g_o\, T^a\, e^{-\phi}\, M_{\mu\nu}\, \partial X^\mu \psi^\nu\, e^{ipX} \quad , \quad p^2 = -\tfrac{1}{\alpha'}$$

- (some of the) *lightest* such bulk states: subleading

$$V^{(-1,-1)}_{M,\text{closed}}(z,\bar{z},k) = g_c\, e^{-\phi-\widetilde{\phi}}\, M_{\mu\nu}\, \mathcal{J}\, \widetilde{\mathcal{J}}\, \psi^\mu\, \widetilde{\psi}^\nu\, e^{ik\cdot X} \quad , \quad k^2 = -\tfrac{4}{\alpha'}$$

  $\Rightarrow$ *mimics* graviton but is *not* highest spin of supermultiplet
  ($\mathcal{J}$: Kac–Moody current due to compactification)

- can compute e.g. 3–point string amplitudes
  bulk massive spin–2: manifest *double copy* structure

Lüst, CM, Mazloumi, Stieberger '21, '23

Other examples of subleading interactions

- $\exists$ consistent *field* theory of massless and massive spin–2 states: bimetric theory around flat backgrounds (kinetic term for massive spin–2: Einstein–Hilbert)

de Rham, Gabadadze, Tolley '10
Hassan, Rosen '11
Hassan, Schmidt–May, von Strauss '12
Babichev, Marzola, Raidal, Schmidt–May, Urban, Veermäe, von Strauss '16

- extract "effective" vertices from our *finite cubic* amplitudes, e.g.:

$$\mathcal{L}^{\text{eff}}_{\text{M}^3} \quad = \quad \frac{g_c}{\alpha'}\left\{y\left[M^3\right]+2\alpha' M^{\mu\nu}\left[\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma}-x\,\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\right]\right\}$$

$$(x,y)=\begin{cases}(2,0) & \text{if} \quad M_{\mu\nu} \quad \text{is a bulk state of} \quad \mathcal{N}=\widetilde{4},\widetilde{8}\\ (3,1) & \text{if} \quad M_{\mu\nu} \quad \text{is a brane state}\end{cases}$$

- *match* for bulk state with 1–parameter subfamily of (on–shell, cubic) bimetric theory

Lüst, CM, Mazloumi, Stieberger '21 and '23

curiously same family as:
Bonifacio, Hinterbichler, Joyce, Rosen '17
Momeni, Rumbutis, Tolley '20 and Johnson, Jones, Paranjape '20
Engelbrecht, Jones, Paranjape '22

Concluding remarks

- massive strings and fields: only cubic level similarities

Sagnotti, Taronna '10
Lüst, CM, Mazloumi, Stieberger '21, '23

- we have a new technology that excavates entire trajectories
  - key observation: Howe duality between $sp(2\bullet)$ and $so(D-1,1)$
  - idea: use this bigger than the Virasoro symmetry
  - gearwheel: $sp(2\bullet)$ creation operators

CM, Skvortsov '23

- field–theory amplitudes for spinning black holes

Guevara, Ochirov, Vines '18
Maybee, O'Connell, Vines '19

"massive" higher–spin symmetry $\Rightarrow$ 3–point Kerr amplitudes!

Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov '22

leading Regge: no BH features

Pichini, Cangemi '22

- Can strings yield Kerr amplitudes? can we treat the *entire* spectrum? chaos? holography?