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Null Polygons

from N=4 SYM theory to Fishnets and back

Rencontres Theoriciennes 28 March 2024

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Plan of the Talk

- *Tessellation*: Integrability beyond the spectrum

- *Null Polygon* limit of 1/2-BPS correlators in *N=4 SYM*

- *Fishnet toy model*: multi-point Feynman Diagrams inside a Null Polygon

- Worked out *examples:* 6-pt split Ladders and 6-pt correlators

Integrability in 4d CFTs

N=4 super Yang-Mills theory:

- Maximally supersymmetric CFT in 4d spacetime, PSU(2,2|4)
- Mixing of scaling dimensions at weak-coupling: integrable (next-to) nearest-neighbour Hamiltonian

...

- Scaling dimensions at strong coupling: energy of string excitations on $AdS_5 \times S^5$ (free, classical)

1) Integrability equations at finite coupling: *Quantum Spectral Curve* $\Rightarrow$ scaling dimensions at finite g^2

[Gromov, Kazakov, Vieira]

$g^2 = g^2_{YM}$

2) AdS/CFT correspondence: *"Holographic dual"* integrable non-linear sigma model on $AdS_5 \times S^5$

[Maldacena '97]

Integrable deformations: non-gauge non-unitary CFTs

- Marginal deformations to N=4 SYM: twisting the SU(4) symmetry

$q_A = q(\varphi_A) = (q_{A,1}, q_{A,2}, q_{A,3})$ Cartan charges

$\varphi_A = \{ \Phi_I , \Psi_a , A_\mu \}$;

$q_A \wedge q_B = q_A^t \cdot \begin{bmatrix} 0 & \gamma_1 & -\gamma_2 \\ -\gamma_1 & 0 & \gamma_3 \\ \gamma_2 & -\gamma_3 & 0 \end{bmatrix} \cdot q_B$; $\boxed{\varphi_A \cdot \varphi_B \xmapsto[\gamma\text{-def}]{} e^{i\, q_A \wedge q_B}\, \varphi_A \cdot \varphi_B}$ [Leigh, Strassler]

- Integrability is preserved (*Quantum Spectral Curve*, *Baxter equations*, *spin-chain* at weak coupling)

[Beisert, Roiban] [Kazakov] [Levkovich-M., Preti] [Gromov, Kazakov, Negro, Sizov]

- Double-Scaling: weak-coupling and strong twists (*non-unitary*, *non-gauge* theories)

$\gamma_k \to -i\infty$; $g^2 e^{i\gamma_k} = \xi_k^2 < \infty$; $\mathrm{Tr}\,[X,Y][\bar{X},\bar{Y}] \longmapsto X Y \bar{X} \bar{Y}\, e^{2i\gamma_3}$ + subleading

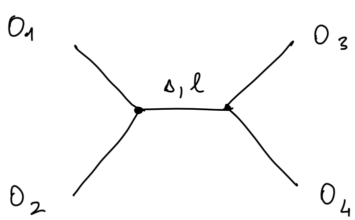
Bi-scalar Fishnet: $\mathcal{L} = N\,\mathrm{Tr}\left(-\bar{X}\,\partial^2 X - \bar{Y}\,\partial^2 Y + \xi^2\, X Y \bar{X} \bar{Y}\right)$ [Kazakov, Gurdogan]

What about 3-point functions?

What about 3-point functions?

Operator Product Expansion

$$O_1(x)\,O_2(0) \sim \sum_{\Delta,\ell} \Big[C_{12}(\Delta,\ell)\,(x^2)^{\frac{\Delta-\Delta_1-\Delta_2-\ell}{2}}\; \underbrace{O_{\Delta,\ell}^{\mu_1\ldots\mu_\ell}(y)\;\hat{x}_{\mu_1}\cdots\hat{x}_{\mu_\ell}}_{} \;+\; \text{descendants} \Big]$$



$$\sim \sum_{\Delta,\ell} \Big[C_{12}(\Delta,\ell)\,C_{34}(\Delta,\ell)\,|z|^{2(\Delta-\ell)} \times g_{\Delta,\ell}(z,\bar{z}) \Big];$$

$$z\bar{z} = x_{12}^2\, x_{34}^2 / x_{13}^2\, x_{24}^2$$

$$(1-z)(1-\bar{z}) = x_{14}^2\, x_{23}^2 / x_{13}^2\, x_{24}^2$$

What about *n*-point correlators?

(VERY HARD PROBLEM!)

Multiple Operator Product Expansion: more data!

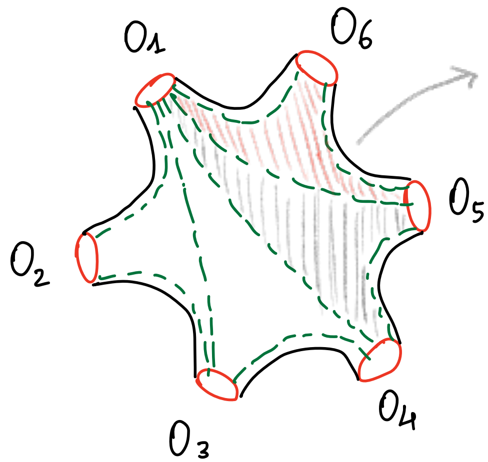
- Chiral primaries $\quad O_K(x,y) = \mathrm{Tr}\,(y\cdot\Phi)^{L_K}\ ; \quad y_{I=1,\dots,6}\ \Big|\ \sum_{I=1}^{6} y_I\, y_I = 0\ ; \quad \frac{1}{2}$-BPS
- protected 2-pt and 3-pt functions
- planar limit : $g^2 \to 0$, $N \to \infty$ (t'Hooft)

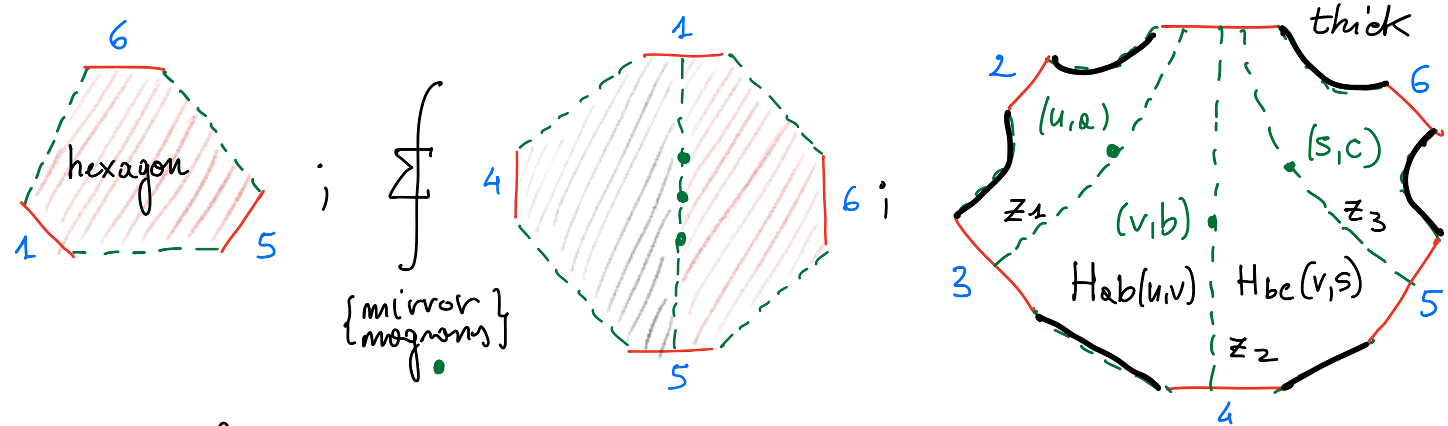
$$\boxed{\langle O_1(x_1,y_1)\, O_2(x_2,y_2) \cdots O_n(x_n,y_n) \rangle}$$

Tessellation of the punctured Sphere

[Basso, Komatsu, Vieira]

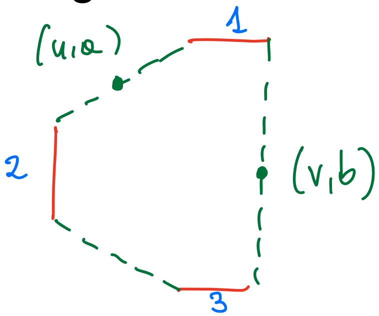
Factorization along cuts = completeness sum over *mirror excitations*





$\{u, a, I\}$ rapidity, bound-state index $\to$ a-antisymm irrep $su(2|2)$

Integrable *mirror-mirror* **scattering**



$$\sim {}_{I}\langle u,a| \otimes {}_{J}\langle v,b|\ \hat{S}\ |v,b\rangle_{I'} \otimes |u,a\rangle_{J'} = S_{ab}(u,v)_{IJ}^{I'J'}$$

$\hat{S}$ mirror-mirror scattering : [• unitarity • Yang-Baxter • Crossing-symmetry]

Tessellation of the punctured Sphere

Mellin-like **integrals at weak-coupling** 6 pts (planar kinematics); $\{(u,a),(v,b),(s,c)\}$ magnons

$$\sum_{a,b,c=1}^{\infty}\int_{-\infty}^{+\infty}du\int dv\int_{-\infty}^{+\infty}ds\left[\mu_a(u)\mu_b(v)\mu_c(s)\,|z_1|^{2ip_a}|z_2|^{2ip_b}|z_3|^{2ip_c}\underbrace{F_{abc}\left(z_k/\bar{z}_k\right)}_{\text{Matrix part}}H_{ab}(u,v)H_{b,c}(v,s)\;E_a(u)^{\ell_1}E_b(v)^{\ell_2}E_c(s)^{\ell_3}\right];$$

$$\mu_a(u)=\frac{a g^2}{(a^2/4+u^2)^2}+O(g^4)\qquad p_a(u)=u+O(g)$$

$$E_a(u)=\pi^2g^2\Big/\left(u^2+\frac{a^2}{4}\right)+O(g^4)\qquad H_{ab}(u,v)=\frac{1}{g^2}\frac{\Gamma\left(1-\frac{a}{2}+iu\right)\Gamma\left(1+\frac{b}{2}-iv\right)\Gamma\left(\frac{a-b}{2}+iv-iu\right)}{\Gamma\left(-\frac{a}{2}-iu\right)\Gamma\left(\frac{b}{2}+iv\right)\Gamma\left(1+\frac{a+b}{2}+iu-iv\right)}\left(\frac{a}{2}-iu\right)\left(\frac{b}{2}+iv\right)+\cdots$$

All-loop quantities via ***Zhukovsky variables***

$$x(u)+x(u)^{-1}=u/g\qquad\mu_a(u)=\frac{a\left(x^{[+a]}\,x^{[-a]}\right)^2}{g^2\left(x^{[+a]}x^{[-a]}-1\right)^2\left(\left(x^{[+a]}\right)^2-1\right)\left(\left(x^{[-a]}\right)^2-1\right)};$$

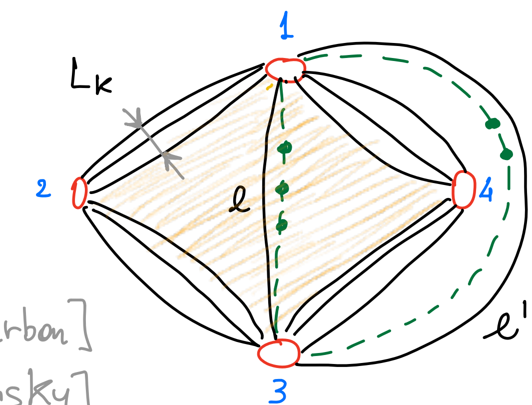
$$x^{\pm}(u)=x\left(u\pm\frac{i}{2}\right);\qquad p_a(u)=-i\frac{a}{2}-g\left(\frac{1}{x^{[-a]}}-x^{[+a]}\right);$$

Lorentzian Regime: Null Polygons

``Null Square" (aka *null octagon*)

$$\langle O_1(x_1,y_1)\, O_2(x_2,y_2)\, O_3(x_3,y_3)\, O_4(x_4,y_4)\rangle \sim \mathbb{O}_\ell(z,\bar z) \times \mathbb{O}_{\ell'}(z,\bar z)$$

$O_k = \mathrm{tr}(\Phi\cdot y_k)^{L_k}$; $L_k \gg 1$; appropriate choice of $\{y_k\}$ forms a thick frame @ tree-level.



[Coronado]
[Kostov, Petkova, Serban]
[Belitsky, Korchemsky]

Leading UV divergence: 1D radial Toda Equation

$$z\to 0,\ \bar z \to \infty \ \Big|\ s^2 = -g^2 \log z \log \bar z$$

$$\mathbb{O}_\ell(z,\bar z) \sim e^{-s^2}\,\tau_\ell(s) \ \Big|\ \boxed{(s\partial_s)^2 \log \tau_\ell = \frac{\tau_{\ell+1}\,\tau_{\ell-1}}{\tau_\ell^2}} \Rightarrow \tau_\ell = \det_{i,j}\big(I_{|i-j|}(s)\big)$$

Leading twist, subleading logs: 2D Toda Field Theory [E.O., Vieira '21]

$s_1 = -g^2 \log z$: leading logs OPE

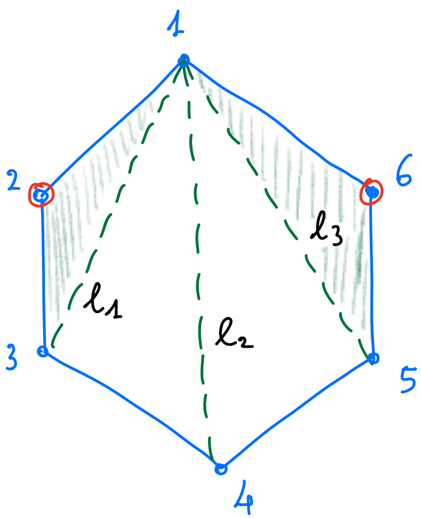
$s_2 = \log \bar z$: any logs ; $\boxed{s_1 s_2\, \partial_{s_1}\partial_{s_2} \log \tau_\ell(s_1,s_2) = \frac{\tau_{\ell+1}\,\tau_{\ell-1}}{\tau_\ell^2}}$

Lorentzian Regime: Null Polygons

n-points leading UV behaviour**: Toda FT equations + *Null Square* boundary conditions**

[E.O., Vieira '22]

$t_j^2 = \log(x^2_{j-1,j})\,\log(x^2_{j,j+1})$ double-scaling $x^2_{j,j+1} \to 0$



ℓ_j = "thickness" of the j-th cut = # of propagators @ tree level.

$$\sim e^{-(t_1^2+t_2^2+\dots+t_6^2)}\, X_{\ell_1\ell_2\ell_3}(t_1^2,\dots,t_6^2)\,;$$

- boundary condition: $t_2^2 = t_6^2 = 0$, $X_{\ell_1\ell_2\ell_3} = \tau_{\ell_1+\ell_2+\ell_3}(t_1)\,\tau_{\ell_1}(t_3)\,\tau_{\ell_3}(t_5)$
- 2 coupled Toda FT equations:

$$\boxed{(t_1\partial_{t_1}+t_2\partial_{t_2})(t_2\partial_{t_2}+t_3\partial_{t_3})\log X_{\ell_1\ell_2\ell_3} = \frac{X_{\ell_1+1,\ell_2,\ell_3}\,X_{\ell_1-1,\ell_2,\ell_3}}{X^2_{\ell_1,\ell_2,\ell_3}}\,t_2^2}$$

What about subLeading behaviour?

*(that is: subleading Logs, constants; dropping **only** powers-suppressed terms)*

Null Polygons from Tessellation

4-points: *analytic continuation* **and** *power-law truncation*

$$\sum_a \int du\ \mu_a(u)\ |z|^{2ip_a(u)} E_a(u)^{\ell_1} F_a(z/\bar{z}); \qquad F_a = \sum_{j=0}^{a-1} \left(\frac{z}{\bar{z}}\right)^{\frac{a-1}{2}-j} \sim \left(\frac{\bar{z}}{z}\right)^{\frac{a-1}{2}} + \cancel{\text{subleading powers}}$$

Sommerfeld-Watson trick:

$$\sum_{a=1}^{\infty} f(a) = \int_{\epsilon-i\infty}^{\epsilon+i\infty} da\ \frac{\pi(-1)^a}{\sin(\pi a)} \hat{f}(a)\ ; \qquad \hat{f}(a) = f(a) \quad \forall\ a \in \mathbb{N}^*$$

Asymptotics from a $\boxed{\text{single residue:}}$

e.g. 1 magnon @ weak coupling

$$\int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{da\ \pi(-1)^a}{\sin(\pi a)} \int_{-\infty}^{+\infty} du\ \frac{z^{iu-\frac{a}{2}}\ \bar{z}^{iu+\frac{a}{2}}}{\left(\frac{a^2}{4}+u^2\right)^{\ell+2}}$$

$$\begin{cases} z \to 0 \Rightarrow |z|<1 \Rightarrow iu = \frac{a}{2} \\ \bar{z} \to \infty \Rightarrow \operatorname{Re} a \leq 0 \end{cases}$$

$$\boxed{iu = a/2\ ;\ a = 0}$$

Null Polygons from Tessellation

n>4 **points:** ***analytic continuation? power-law truncation?***

For higher polygons the integrand is more complicated due to non trivial hexagons :

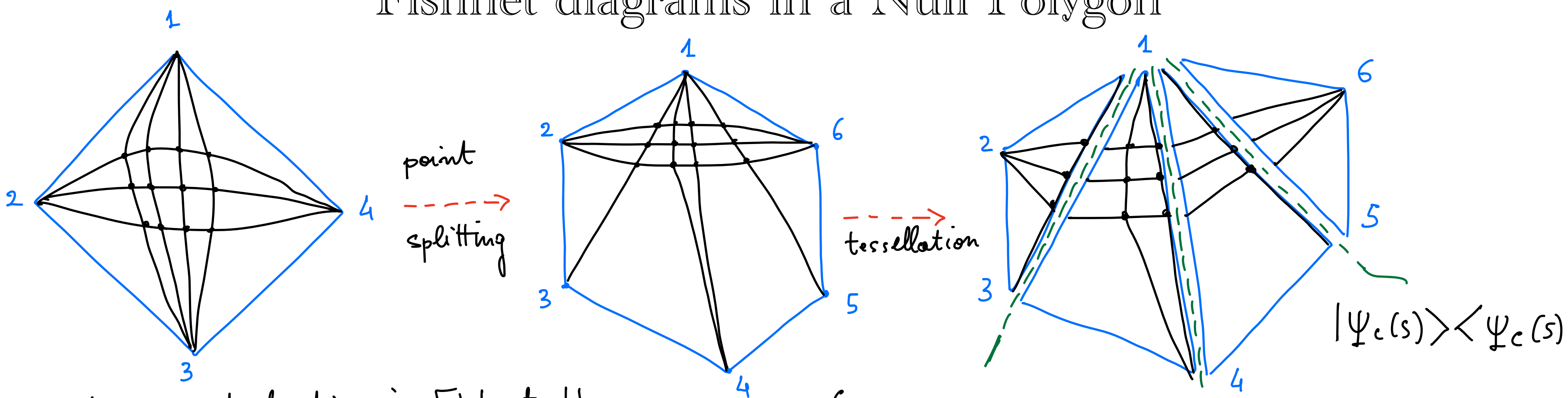
$$g^2 H_{ab}(u,v) = \frac{\Gamma(1-\frac{a}{2}+iu)\Gamma(1+\frac{b}{2}-iv)\Gamma(\frac{a-b}{2}+iv-iu)}{\Gamma(-\frac{a}{2}-iu)\Gamma(\frac{b}{2}+iv)\Gamma(1+\frac{a+b}{2}+iu-iv)} \overset{?}{=} \frac{\Gamma(1+\frac{a}{2}+iu)\Gamma(1-\frac{b}{2}-iv)\Gamma(\frac{a-b}{2}+iv-iu)}{\Gamma(\frac{a}{2}-iu)\Gamma(\frac{b}{2}+iv)\Gamma(1+\frac{a+b}{2}+iu-iv)}(-1)^a$$

$$\pi/\sin(\pi x) = \Gamma(x)\Gamma(1-x)\ ; \quad (-1)^x \overset{?}{=} \sin(\pi x)$$

Different Integrands lead to different results even at leading order

$\Rightarrow$ We need a criteria for analytic continuation made right!

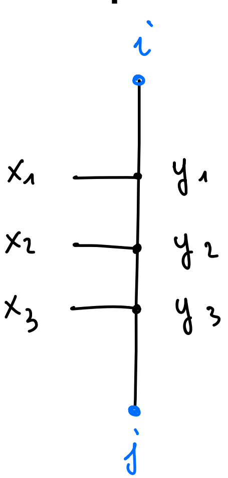
Fishnet diagrams in a Null Polygon



single-trace 4-pt function in Fishnet theory

$$\sum\!\!\!\!\int d\mu_a(u)\, \underbrace{|\psi_a(u)\rangle\langle\psi_a(u)|}_{\text{basis of eigenfunctions of the Fishnet lattice}} \qquad |\Psi_b(v)\rangle\langle\Psi_b(v)|$$

Separation of Variables (SoV) integrand:



Graph building Kernel $B(\underline{x}\,|\,\underline{y})$

$\equiv$

Conserved charge of $SO(1,5)$ spin chain;

irrep $(\Delta, s, \dot{s}) = (1,0,0)$

$$|\Psi_{\underline{a}}(\underline{u})\rangle = |\Psi_{a_1 a_2 a_3}(u_1,u_2,u_3)\rangle =$$

$$= \Lambda^{(3)}_{a_1}(u_1)\cdot\Lambda^{(2)}_{a_2}(u_2)\cdot\Lambda^{(1)}_{a_3}(u_3)$$

SoV:

$$\boxed{B^{(m)}\circ\Lambda^{(m)}_a(u) = \frac{\pi^2}{\left(u^2+\frac{a^2}{4}\right)}\,\Lambda^{(m)}_a(u)\cdot B^{(m-1)}}$$

Separation of Variables SO(1,5) spin chain

Excitations of the Fishnet: 1-excitation: $\left[\Psi_a(u)\right]_{\alpha}^{\dot\alpha}$; $\alpha = \alpha_1,\dots,\alpha_{a-1}$; $\dot\alpha = \dot\alpha_1,\dots,\dot\alpha_{a-1}$; $\alpha_j,\dot\alpha_j \in \{1,2\}$

symmetric product of $(a-1)$ spin vectors

Exchange of 2 excitations inside an eigenfunction.

$$\Psi_{a_2a_1}(u_2,u_1) = R_{a_1a_2}(iu_1-iu_2)\,\Psi_{a_1a_2}(u_1,u_2)\,R_{a_1a_2}(iu_2-iu_1)\,;$$

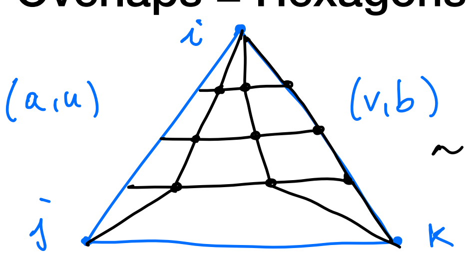
$R_{ab}(u)$ fused $SU(2)$ [Kulish, Reshetikin]

Orthogonality/Completeness:

$$\langle\Psi_{\underline{a}}(\underline{u})|\Psi_{\underline{b}}(\underline{v})\rangle \sim \prod_{k=1}^{m}\Big(\delta_{a_k,b_k}\,\delta(u_k-v_k)\,\Delta_+(a,u)\Delta_-(a,u)\Big) + \text{permutations}$$

$$\Delta_\pm(a,u) = \prod_{k\neq h}\left(\frac{(a_k+a_h)^2}{4}+(u_h-u_k)^2\right)\left(\frac{(a_k-a_h)^2}{4}+(u_k-u_h)^2\right);$$

Overlaps = Hexagons:



$$\sim \langle\Psi_a(u)|\Psi_b(v)\rangle_{ij,\,ik} = H_{a_1b_1}(u_1,v_1)\,H_{a_2b_2}(u_2,v_2)\dots H_{a_3,b_3}(u_3,v_3)\,\times$$

$$\times\left[R_{a_1b_1}(u_1-v_1)\,R_{a_1b_2}(u_1-v_2)\cdots R_{a_3b_3}(u_3-v_3)\right]$$

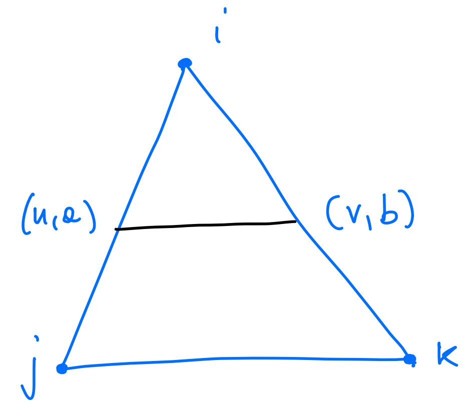
matrix part.

Separation of Variables SO(1,5) spin chain

***Fishnet Hexagons*:**

$$\boxed{p^{2\alpha}\, x^{2(\alpha+\beta)}\, p^{2\beta} = x^{2\beta}\, p^{2(\alpha+\beta)}\, x^{2\alpha}}$$ braid relation "star-triangle"

$$H_{a,b}(u,v) = \frac{(-1)^{b-1}\,\Gamma\left(1+\frac{b}{2}-iv\right)\Gamma\left(1+\frac{a}{2}+iu\right)\Gamma\left(\frac{a-b}{2}+iv-iu\right)}{\Gamma\left(\frac{b}{2}+iv\right)\Gamma\left(\frac{a}{2}-iu\right)\Gamma\left(1+\frac{a-b}{2}+iu-iv\right)\left(\frac{a+b}{2}+iu-iv\right)}\;;$$



Matrix Part:

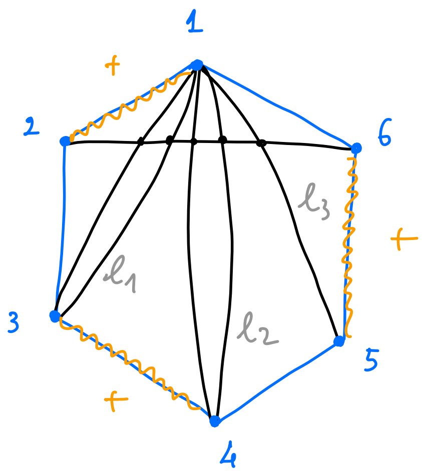
$$R_{ab}(iu-iv)^{\alpha\beta}_{\dot\alpha\dot\beta}\;\underbrace{\left(\sigma_\mu\bar\sigma_\nu\,\hat x^\mu_{jk}\,\hat x^\nu_{ki}\right)^{\dot\alpha}_{\alpha}}_{\text{rank }(a-1)}\;\overbrace{\left(\sigma_\rho\bar\sigma_\gamma\,\hat x^\rho_{ij}\,\hat x^\gamma_{jk}\right)^{\dot\beta}_{\beta}}^{\text{rank }(b-1)}$$

symmetric, traceless tensor (Fierz identities)

$$F_{a,b} = \mathrm{Tr}_a\,\mathrm{Tr}_b\left[R_{ab}(iu-iv)\left(\frac{z_1}{\bar z_1}\right)^{\hat J_{3,a}}\otimes\left(\frac{z_2}{\bar z_2}\right)^{\hat J_{3,b}}\right]$$

Eigenfunctions and Matrix part are known (and simple). Great toy model for N=4 SYM!

Example: 6-point split-Ladders



$$\sum_{a,b,c} a\,b\,c \int du\,dv\,ds \Big[|z_1|^{2iu}\, |z_2|^{2iv}\, |z_3|^{2is}\, H_{ab}(u,v)\, H_{b,c}(v,s)\; E_a(u)^{1+\ell_1}\, E_b(v)^{1+\ell_2}\, E_c(s)^{1+\ell_3} \times$$

$$\times\, \mathrm{Tr}\left(R_{ab}(iu-iv)\, R_{bc}(iv-is) \left(\frac{z_1}{\bar z_1}\right)^{\hat J_{3a}} \otimes \left(\frac{z_2}{\bar z_2}\right)^{\hat J_{3b}} \otimes \left(\frac{z_3}{\bar z_3}\right)^{\hat J_{3c}} \right) \Big]$$

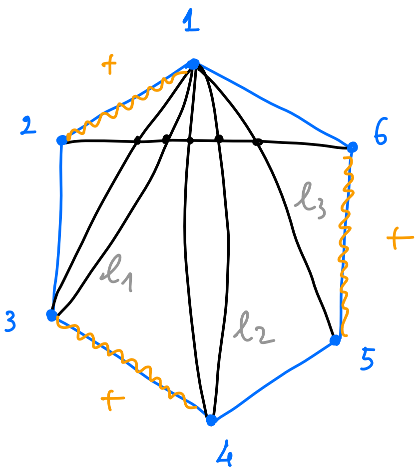
OPE channels: bound-states vs interaction poles

$|z_j| > 1$ or $|z_j| < 1$ $\Rightarrow$ different contour closure (Cauchy's thm.)

$|z_1| < 1,\ |z_2|, |z_3| > 1$ $\Rightarrow$ signature $(+,-,-)$

$$\begin{cases} iu = \dfrac{k}{2} \quad \text{"bound state"} \;;\quad k \in \mathbb{N} \;\rightarrow\; |z_2|^{iv} \\[2ex] iu = iv + \dfrac{|a-b|}{2} + k \quad \text{"interaction"} \;\rightarrow\; |z_1 z_2|^{iv} \;; \qquad |z_1 z_2| < 1 \Rightarrow \mathrm{Re}(iv) > 0 \end{cases}$$

Example: 6-point split-Ladders



$$\sum_{a,b,c} a\, b\, c \int du\, dv\, ds \Big[|z_1|^{2iu} |z_2|^{2iv} |z_3|^{2is} H_{ab}(u,v) H_{b,c}(v,s)\; E_a(u)^{1+\ell_1} E_b(v)^{1+\ell_2} E_c(s)^{1+\ell_3}$$

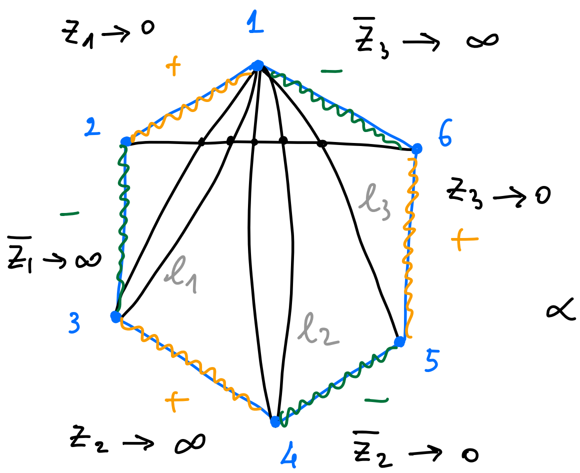
$$\times \mathrm{Tr}\left(R_{ab}(iu-iv)\, R_{bc}(iv-is) \left(\frac{z_1}{\bar{z}_1}\right)^{\hat{J}_{3a}} \otimes \left(\frac{z_2}{\bar{z}_2}\right)^{\hat{J}_{3b}} \otimes \left(\frac{z_3}{\bar{z}_3}\right)^{\hat{J}_{3c}} \right) \Big]$$

OPE channels: bound-states vs interaction poles

$|z_1| < 1,\ |z_2|, |z_3| > 1 \Rightarrow$ signature $(+,-,-)$; $|z_1 z_2| < 1$, $|z_1 z_2 z_3| < 1$

$iu = \frac{k}{2}$	$iu = iv + \frac{\lvert a-b\rvert}{2} + k$		3 sets of poles $\forall\, a,b,c$
$iv = -\frac{h}{2}$	$iv = \frac{h}{2}$	$iv = is + \frac{\lvert b-c\rvert}{2} + h$	
$is = -\frac{\ell}{2}$	$is = -\frac{\ell}{2}$	$is = \frac{\ell}{2}$	

Power truncation & Analytic continuation



$$\mathrm{Tr}\left(R_{ab}(iu-iv)\, R_{bc}(iv-is) \left(\frac{z_1}{\bar z_1}\right)^{\hat J_{3a}} \otimes \left(\frac{z_2}{\bar z_2}\right)^{\hat J_{3b}} \otimes \left(\frac{z_3}{\bar z_3}\right)^{\hat J_{3c}} \right) \propto$$

$$\propto \sum_{m=1}^{a}\sum_{m=1}^{b}\sum_{\ell=1}^{c}\left[z_1^{\,a+1-m}\,\bar z_1^{\,m}\, z_2^{\,b+1-m}\,\bar z_2^{\,m}\, z_3^{\,c+1-\ell}\,\bar z_3^{\,\ell}\, \big(r_{ab}(u,v)\big)_{mm}\big(r_{bc}(v,s)\big)_{m\ell}\right]$$

$\sim \bar z_1^{\,a-1}\, z_2^{\,b-1}\, \bar z_3^{\,c-1}\, (r_{ab})_{a1}(r_{bc})_{b1}$ + subleading powers WRONG!

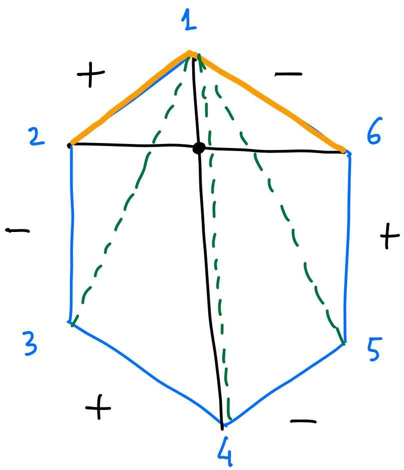
At interaction poles: $iu = iv + \frac{|a-b|}{2}$; $|z_1|^{iu}\,|z_2|^{iv} \longrightarrow |z_1 z_2|^{iv}$

competition of limits: $\left(\underbrace{\frac{z_1}{\bar z_1}}_{\downarrow\, 0} \times \underbrace{\frac{z_2}{\bar z_2}}_{\downarrow\, \infty}\right) \longrightarrow 0 \quad \Rightarrow \quad \bar z_1^{\,a-1}\,\bar z_2^{\,b-1}\,\bar z_3^{\,c-1}\,(r_{ab})_{a,b}(r_{bc})_{b,c}$ RIGHT

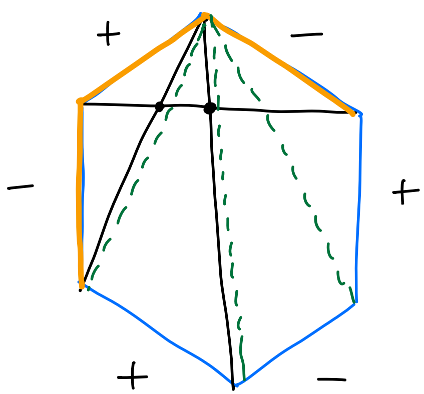
Tests: point-split Ladders

$|z_1| < 1$; $|z_2| \sim 1$; $|z_3| > 1$; $|z_1 z_2| < 1$; $|z_2 z_3| > 1$

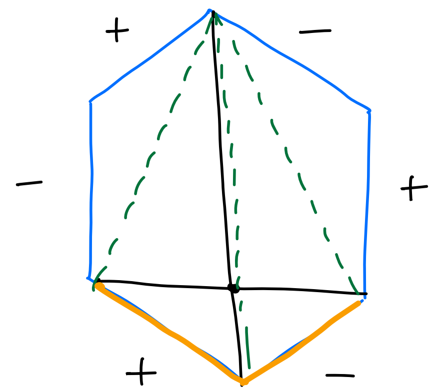
$L_K = g \log\left(x^2_{K\,K+1}\right)$



$\sim L_1 \bar{L}_6$



$\sim L_1^2 \left(\bar{L}_6^2 + 2 \bar{L}_6 L_2\right)$



$\sim L_3 \bar{L}_4$

poles :

$$iu = iv \pm \frac{a-b}{2}$$

$$iv = is + \frac{b-c}{2}$$

$$is = -\frac{c}{2}$$

order of integration

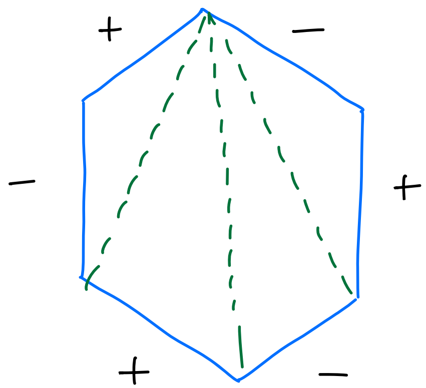
$a = 0$

$b = c$

$c = 0$

"Fishnet prescription"

Back to N=4 SYM: 6-pt Integrand at weak coupling



$$\sim e^{-(L_1 L_2 + L_2 L_3 + \cdots + L_6 L_1)} + (\text{subleading logs, consts, } \cancel{\text{powers}})$$

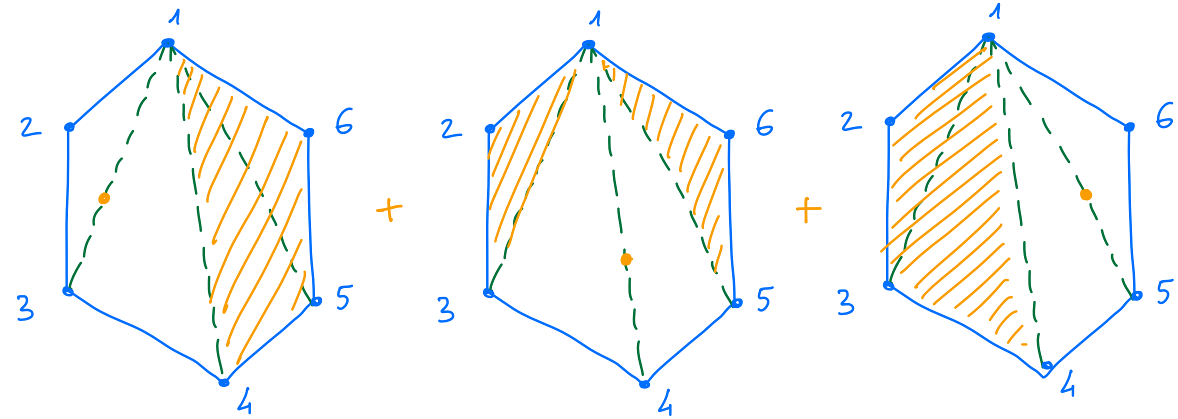
Fishnet prescription: **poles, truncations and analytic continuation**

$$\boxed{\frac{\Gamma\left(1-\frac{a}{2}+iu\right)}{\Gamma\left(-\frac{a}{2}-iu\right)}(-1)^a} = \frac{\Gamma\left(1+\frac{a}{2}+iu\right)}{\Gamma\left(\frac{a}{2}-iu\right)}\;\frac{\sin\left(\pi\left(1+iu+\frac{a}{2}\right)\right)}{\sin\left(\pi\left(1+iu-\frac{a}{2}\right)\right)} = \boxed{\frac{\Gamma\left(1+\frac{a}{2}+iu\right)}{\Gamma\left(\frac{a}{2}-iu\right)}}$$

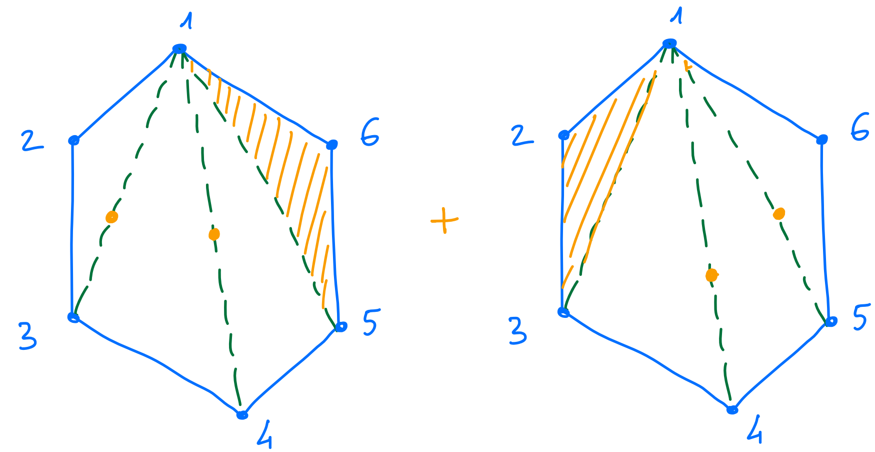
1-excitation

$$(L_1+L_3)\bar{L}_2 + L_3\bar{L}_4 + (\bar{L}_6+\bar{L}_4)L_5 +$$

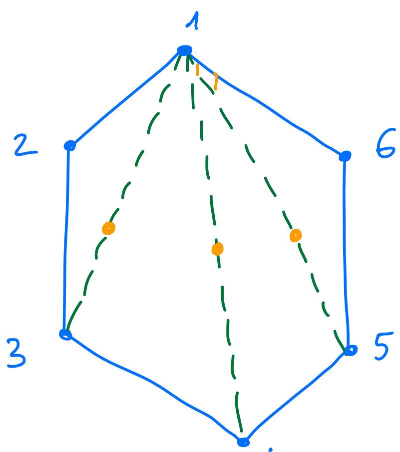
$$+\frac{1}{2}\left[\left(\bar{L}_2(L_1+L_3)\right)^2 + \left(L_3\bar{L}_4\right)^2 + \left(L_5(\bar{L}_6+\bar{L}_4)\right)^2\right] =$$



$$L_1\bar{L}_2 L_3\bar{L}_4 + L_1\bar{L}_2^2 L_3 + \bar{L}_6 L_5\bar{L}_4 L_3 + \bar{L}_6\bar{L}_4 L_5^2 =$$



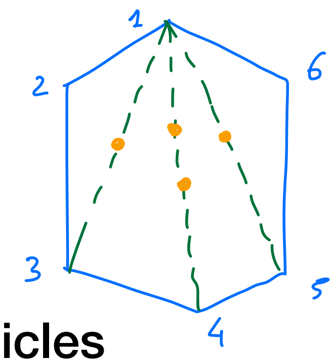
3-excitations



$$= L_1\bar{L}_6 + L_1 L_2 L_5 L_6;$$

$$e^{-(L_1\bar{L}_2+\dots+\bar{L}_6 L_1)} = 1 + (1-\log p) + \frac{(1-\log p)^2}{2} + \dots$$

Open Problems



Missing term @ 2 loops

$\sim L_1 \bar{L}_6 L_3 \bar{L}_4$

How to treat 2-excitations?

[Subtleties for analytic continuation and truncation]

All-loop formulae for a given # of particles

How to fix analyt. continuation prescription @ all loops from Zhukovsky variables?

New Integrable Hierarchies? (Generalization of Toda Equations?)

$\exists$ differential / finite-difference equation beyond LC leading-logs for null polygons?

Symbol Bootstrap of multiloop, multipoint Fishnet Feynman Integrals

We have a fast algorithm to evaluate LC OPE of multipt, multiloop Feynman diagrams

Can we fix an ansatz in terms of symbols (MPLs, etc.)


Merci