The NANOGrav Bound on Ultralight Dark Matter

Alessandro Dondarini
(Pisa University & INFN)

Based on: M. Aghaie, G. Armando, AD, P. Panci, arXiv: 2308.04590 (to appear in PRD)

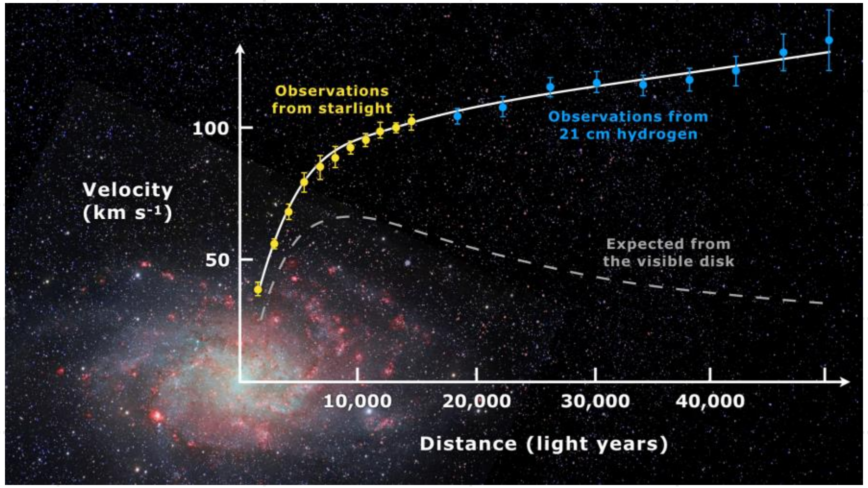
Outline of the Talk

- Introduction: the ΛCDM success and its shortcomings
- Fuzzy Dark Matter: a viable alternative
- Properties of Fuzzy Dark Matter: solitons and all that

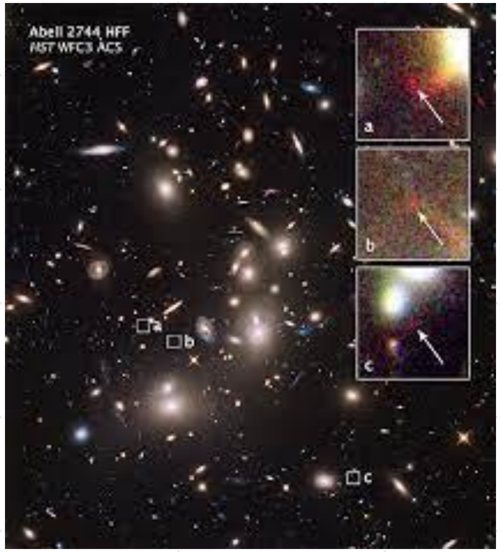
- Recent advancements in GW detection: PTAs and the Stochastic GW Background (SGWB)
- Behind the SGWB: Astrophysical Origin (SMBH binaries) or New Physics?

- NANOGrav limits on Fuzzy Dark Matter: combining new physics with the Astrophysical interpretation

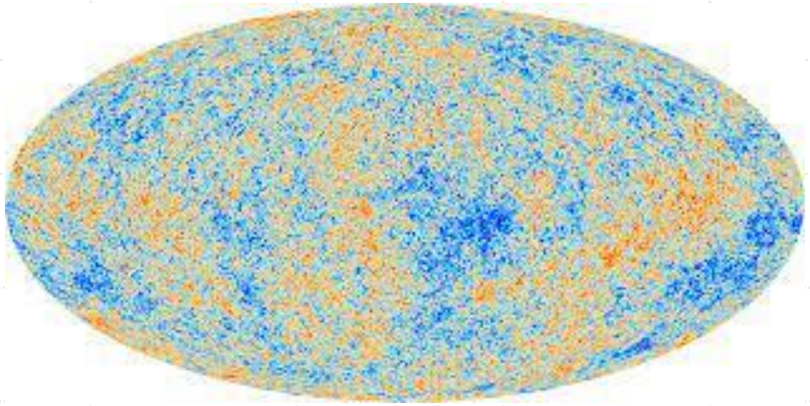
Cold Dark Matter: Motivations at (almost) Every Scale

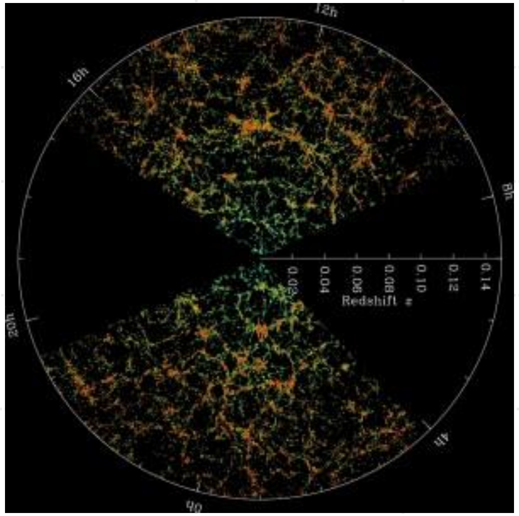


Rotation Curves (~ 10 kpc)



Clusters of Galaxies (~ few Mpc)

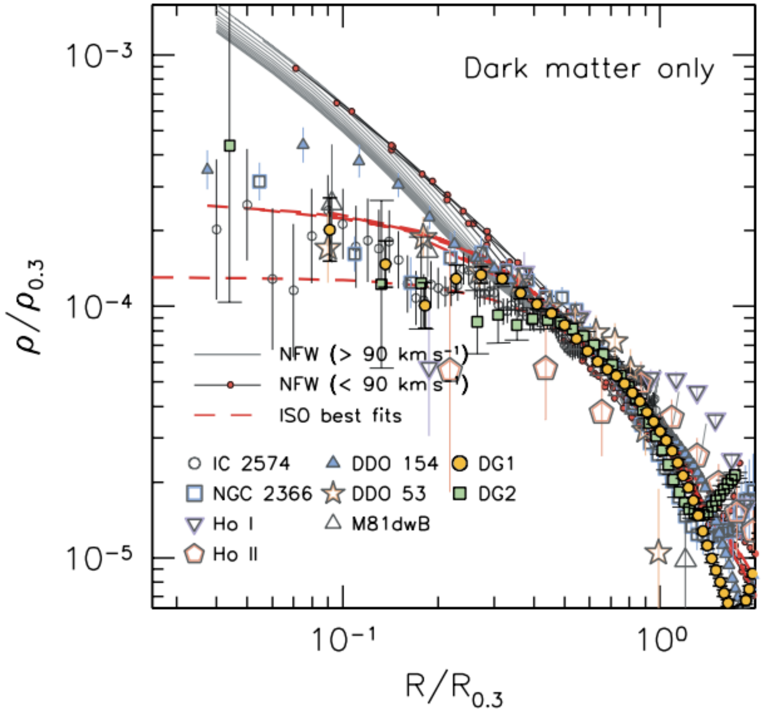




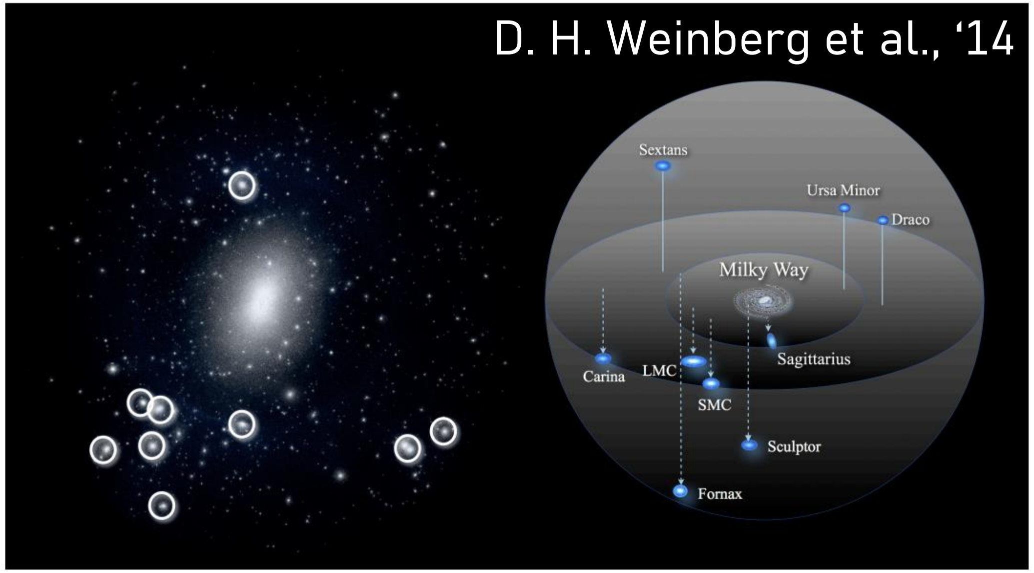
CMB & Large Scale Structures (~ the Universe)

Cold Dark Matter: Shortcomings (?)

Se-Heon Oh et al., '10



«Cusp-Core» problem

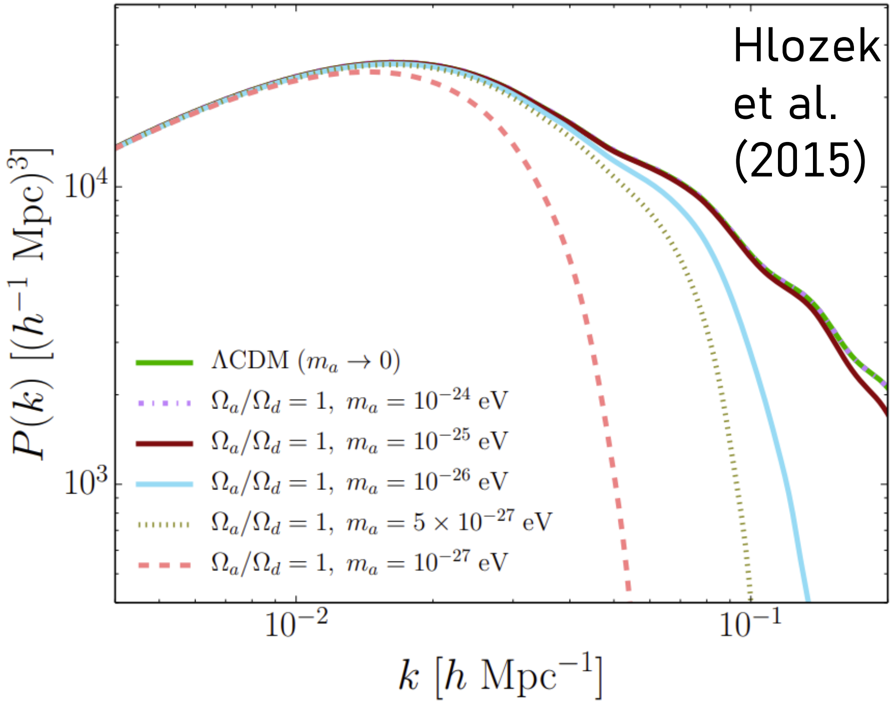


«Missing satellite» and «too big to fail» problems

(Or Baryonic effects??)

DM goes Fuzzy: appealing alternative…

- Cored profile predicted! (We'll see how…)
- … and less (sub)haloes: FDM reduces the matter power spectrum!



$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 P(k)\, \delta^3(\boldsymbol{k} - \boldsymbol{k}')$$

It seems that ultralight masses are crucial for this suppression. Why?

Properties of ULDM – Quantum pressure

A sizable de Broglie wavelength

$$\frac{\lambda}{2\pi} = \frac{\hbar}{mv} = 1.92\,\mathrm{kpc}\left(\frac{10^{-22}\,\mathrm{eV}}{m}\right)\left(\frac{10\,\mathrm{km\ s^{-1}}}{v}\right)$$

Galactic size for velocity dispersion of Dwarf spheroidal galaxies !

$$k_J = \frac{2(\pi G\rho)^{1/4} m^{1/2}}{\hbar^{1/2}}$$

Jeans length ~ de Broglie: fluctuations mitigated by quantum pressure!

Properties of ULDM- Gravitational Relaxation

Bosonic nature of ULDM
+
Quasi-Particle description

$$m_{\text{eff}} \sim \rho(\tfrac{1}{2}\lambda)^3 \quad \sim 3 \times 10^5\, M_\odot (10^{-22}\,\text{eV}/m)^3$$

(MW-like galaxies)

Relaxation time comparable to the age of the Universe

Relaxation into what?

$$t_{\text{relax}}(r) \sim \frac{0.4}{f_{\text{relax}}} \frac{m^3 v^2 r^4}{\pi^3 \hbar^3} \sim \frac{1 \times 10^{10}\,\text{yr}}{f_{\text{relax}}} \left(\frac{v}{100\,\text{km s}^{-1}}\right)^2 \left(\frac{r}{5\,\text{kpc}}\right)^4 \left(\frac{m}{10^{-22}\,\text{eV}}\right)^3$$

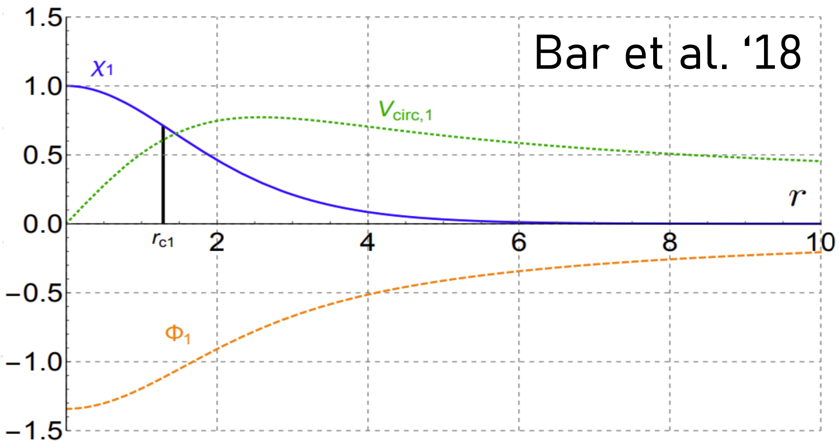
The ULDM Soliton: Field Treatment

Schrödinger-Poisson Equation

$$\begin{cases} i\partial_t \psi = -\frac{1}{2m}\nabla^2\psi + m\Phi\psi, \\ \nabla^2\Phi = 4\pi G|\psi|^2. \end{cases}$$

$$\psi(x,t) = \left(\frac{mM_{pl}}{\sqrt{4\pi}}\right) e^{-i\gamma m t}\chi(x)$$

$$\partial_r^2(r\chi) = 2r(\Phi - \gamma)\chi,$$
$$\partial_r^2(r\Phi) = r\chi^2.$$



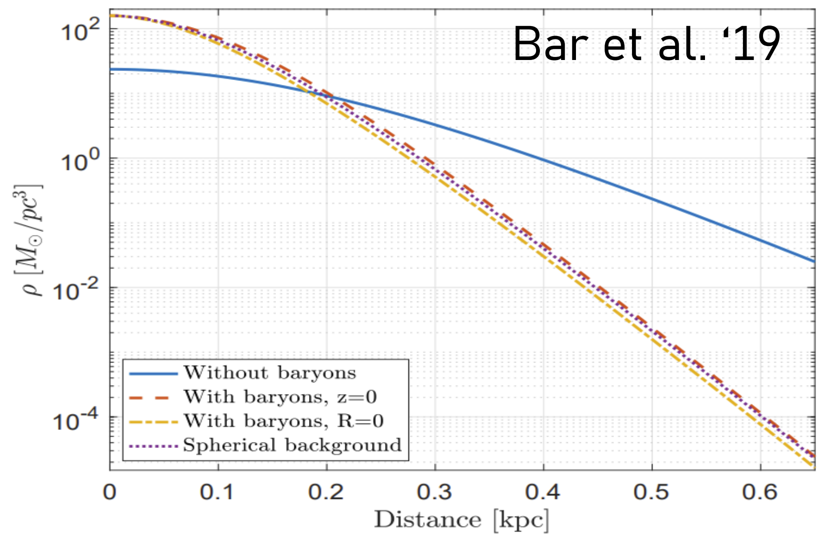
Cored
solutions for χ
(solitons)

Soliton(s): some analytics…

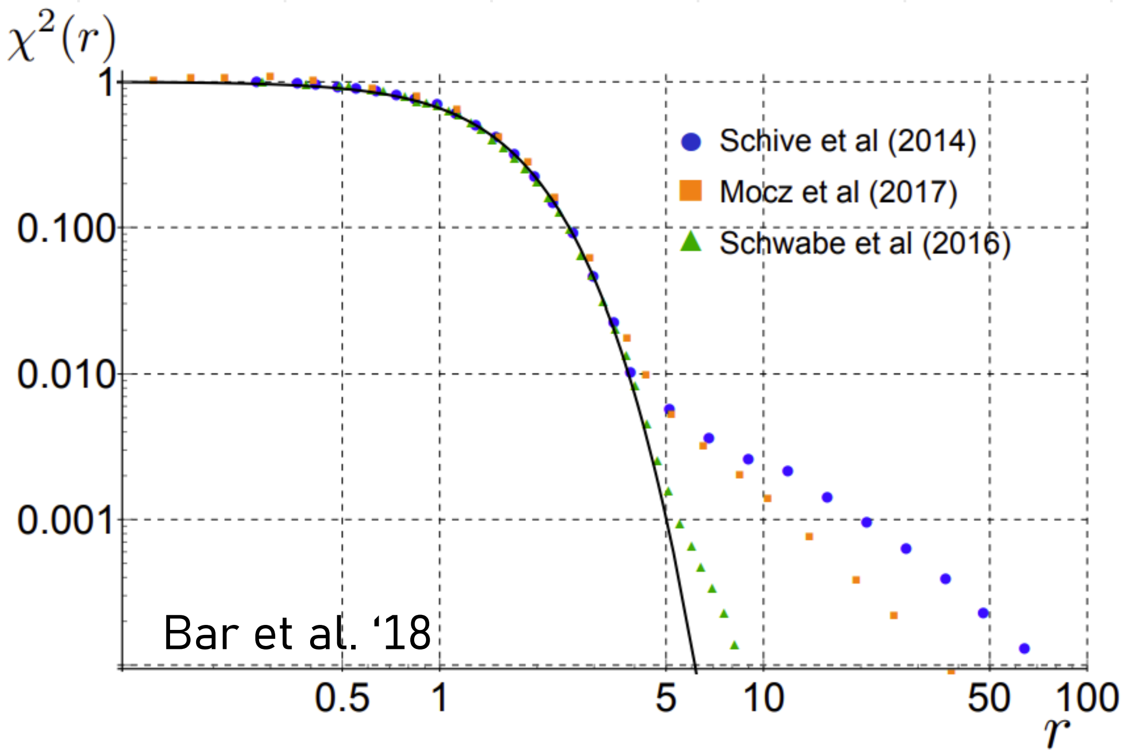
$$\chi_\lambda(r) = \lambda^2 \chi_1(\lambda r),$$
$$\Phi_\lambda(r) = \lambda^2 \Phi_1(\lambda r),$$
$$\gamma_\lambda = \lambda^2 \gamma_1,$$

There are many solitons due to the scaling symmetries of the SP equation

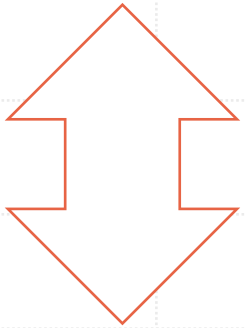
$$x_{c1} \approx 0.082 \left(\frac{m}{10^{-22}\ \mathrm{eV}}\right)^{-1} \mathrm{pc}.$$
$$M_1 = \frac{M_{pl}^2}{m} \int_0^\infty dr r^2 \chi_1^2(r) \approx 2.79 \times 10^{12} \left(\frac{m}{10^{-22}\ \mathrm{eV}}\right)^{-1} \mathrm{M}_\odot.$$



Contact with ULDM Simulations



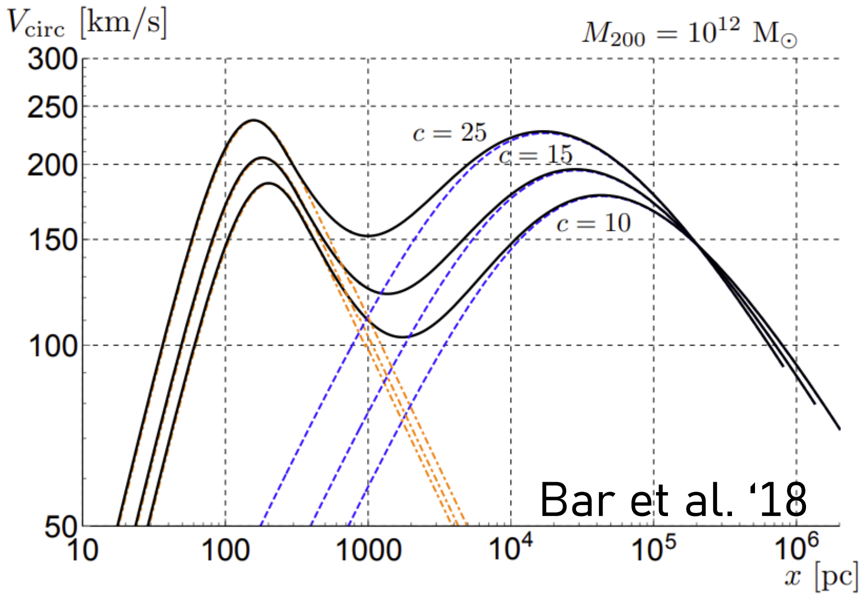
$$M_{\rm sol}^{\rm NS}[M_\odot] \simeq \frac{1.4\times 10^8}{m[10^{-21}\,{\rm eV}]} M_h^{\frac{1}{3}}[10^{12}M_\odot]$$



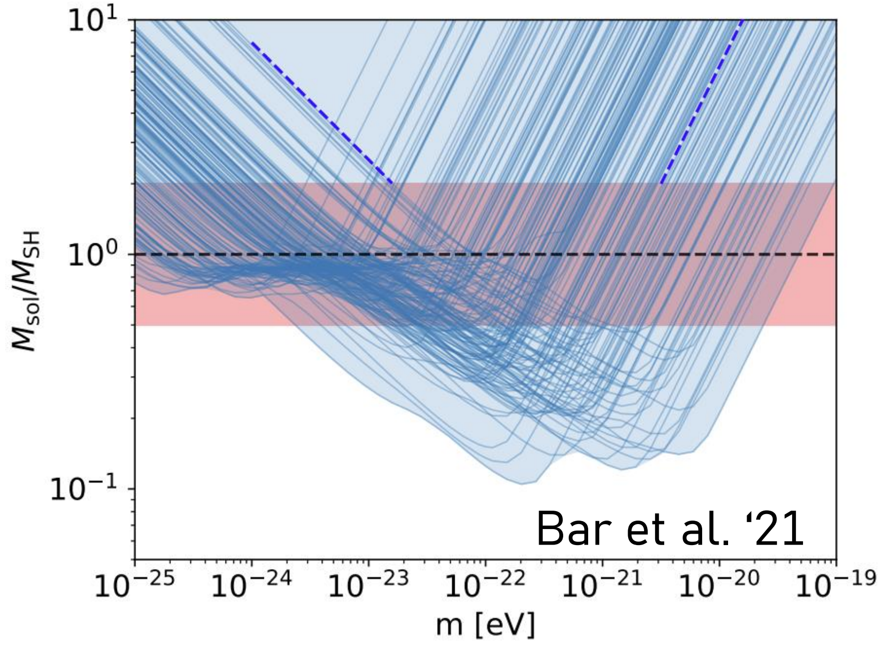
$$\left.\frac{K}{M}\right|_{\rm soliton} \approx \left.\frac{K}{M}\right|_{\rm halo}$$

Kinetic equilibration is a sharp prediction of ULDM!

Constraints on ULDM Solitons



Impact on rotation curves!!

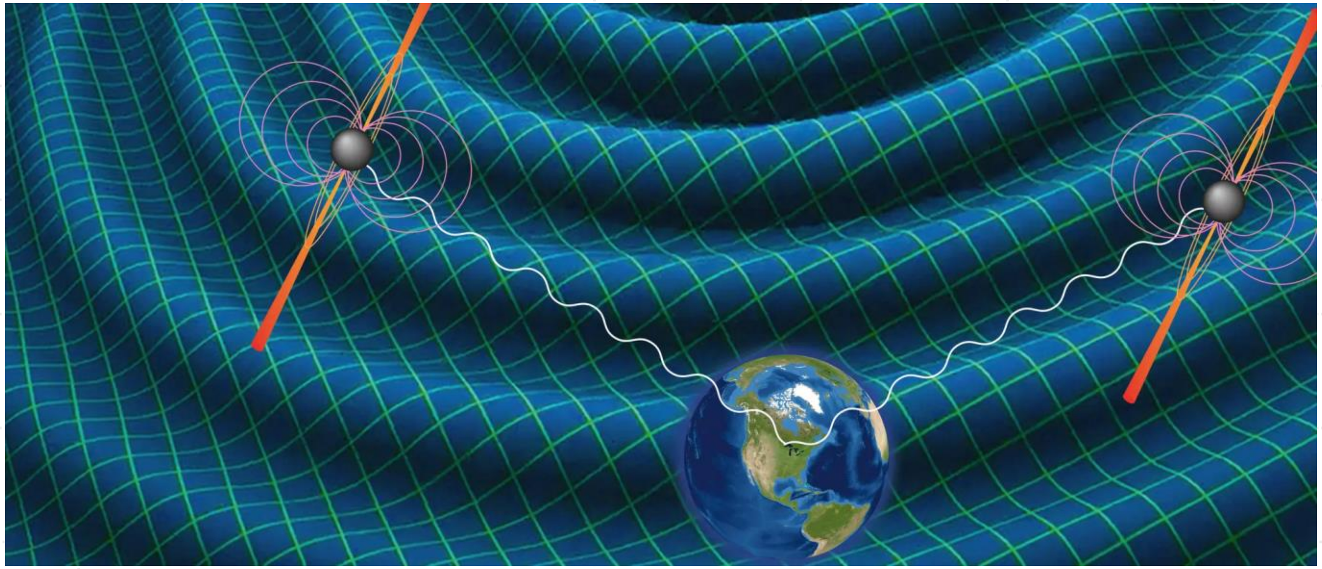


Excluded: $3 \times 10^{-24}\text{eV} < m < 10^{-20}$ eV

Constraints on self-gravitating solitons

Discovering ULDM with Gravitational Waves

Pulsar Timing Arrays (I)



Measurements of the time of arrival (TOA): pulsars as clocks

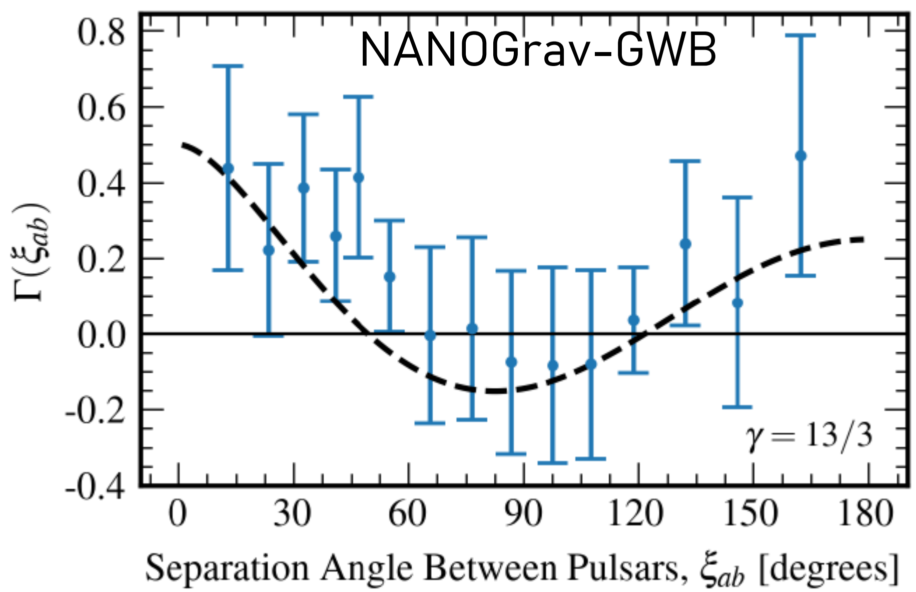
Several collaborations:

- NANOGrav
- EPTA
- PPTA
- InPTA

(… and more)

What measurement?

NANOGrav: measurements of 67 pulsars for 16 yrs



Hellings-Downs correlation

TOA residual

$$\langle r_a(t) r_b(t) \rangle \propto \int S_{ab}(f) df$$

3.5 – 4 σ evidence for a SGWB in the nHz band

$$S_{ab}(f) = \Gamma(\xi_{ab})\,\Phi(f)$$

Spatial correlation

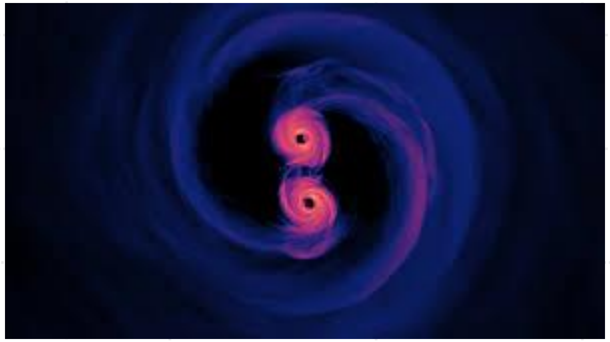
Power Spectral Density (PSD)

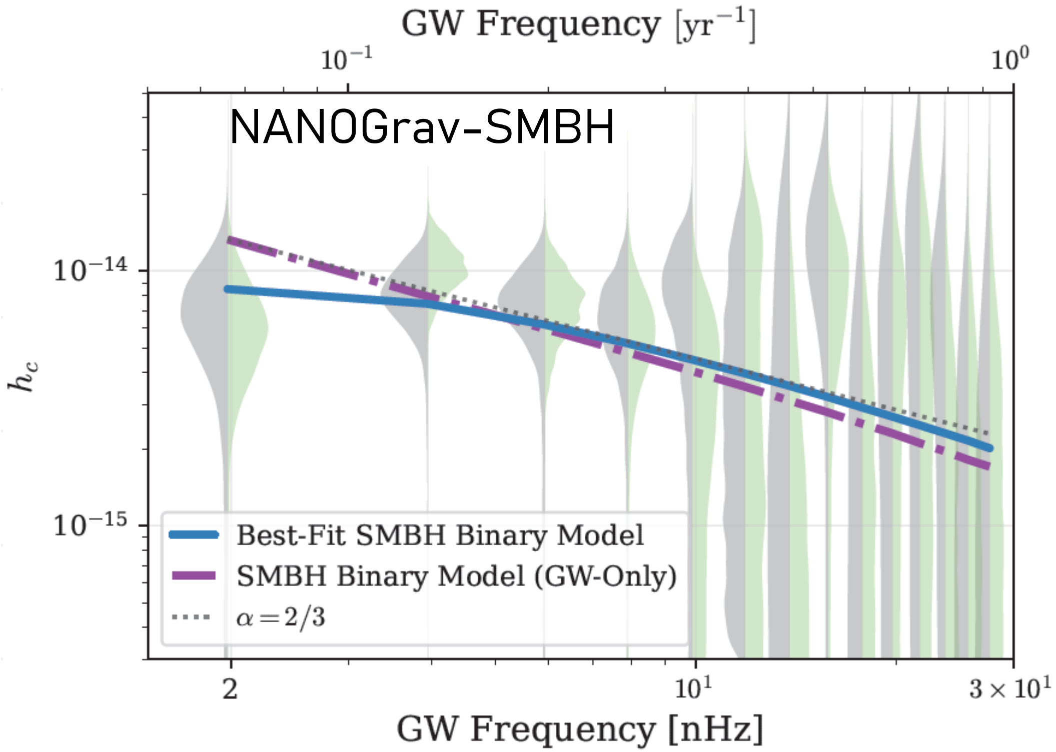
$$\Phi(f) = \frac{h_c(f)^2}{12\pi^2 f^3}$$

GW strain

Origin (I): Super-Massive BH mergers

SMBH binaries merge and eventually emit GWs in the late stage of their inspiral (r ~ pc).



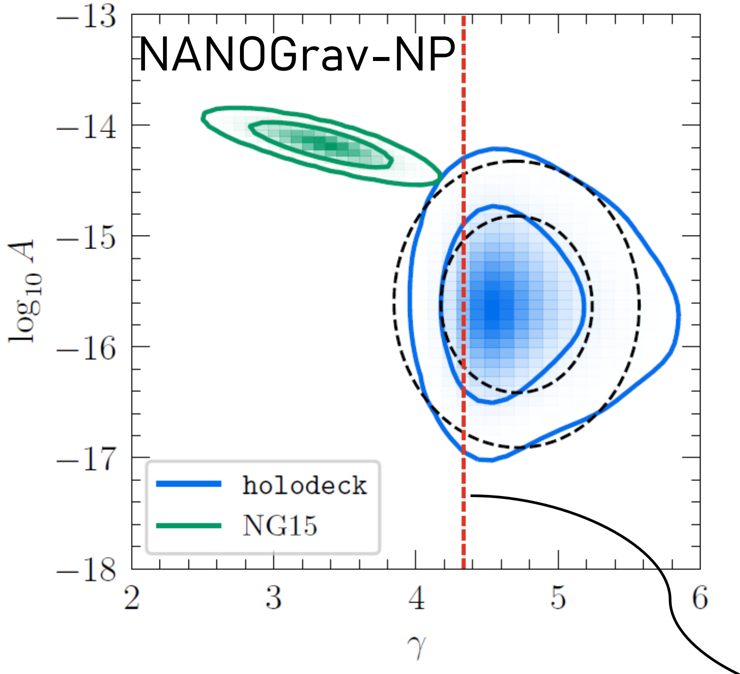


SMBH fit in the nHz band

We will see how to predict the strain!

Origin (I): Super-Massive BH mergers

Is it a «good» fit?

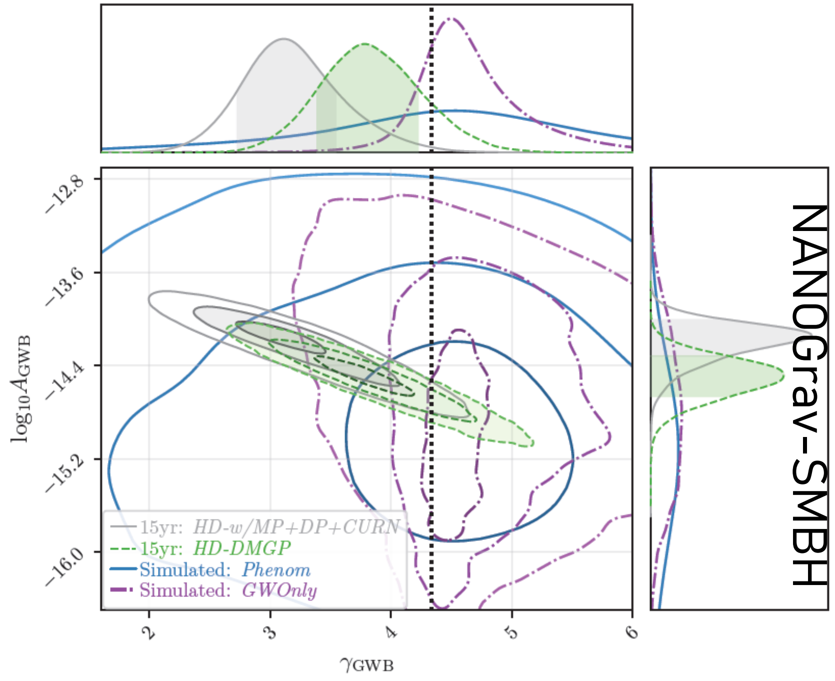


$$\Phi(f) = \frac{A^2}{12\pi^2}\left(\frac{f}{f_{\rm ref}}\right)^{-\gamma} f_{\rm ref}^{-3}$$

Standard GW spectrum parametrization

$$\gamma = 13/3$$

Agreement at 2σ contours (GW Only)

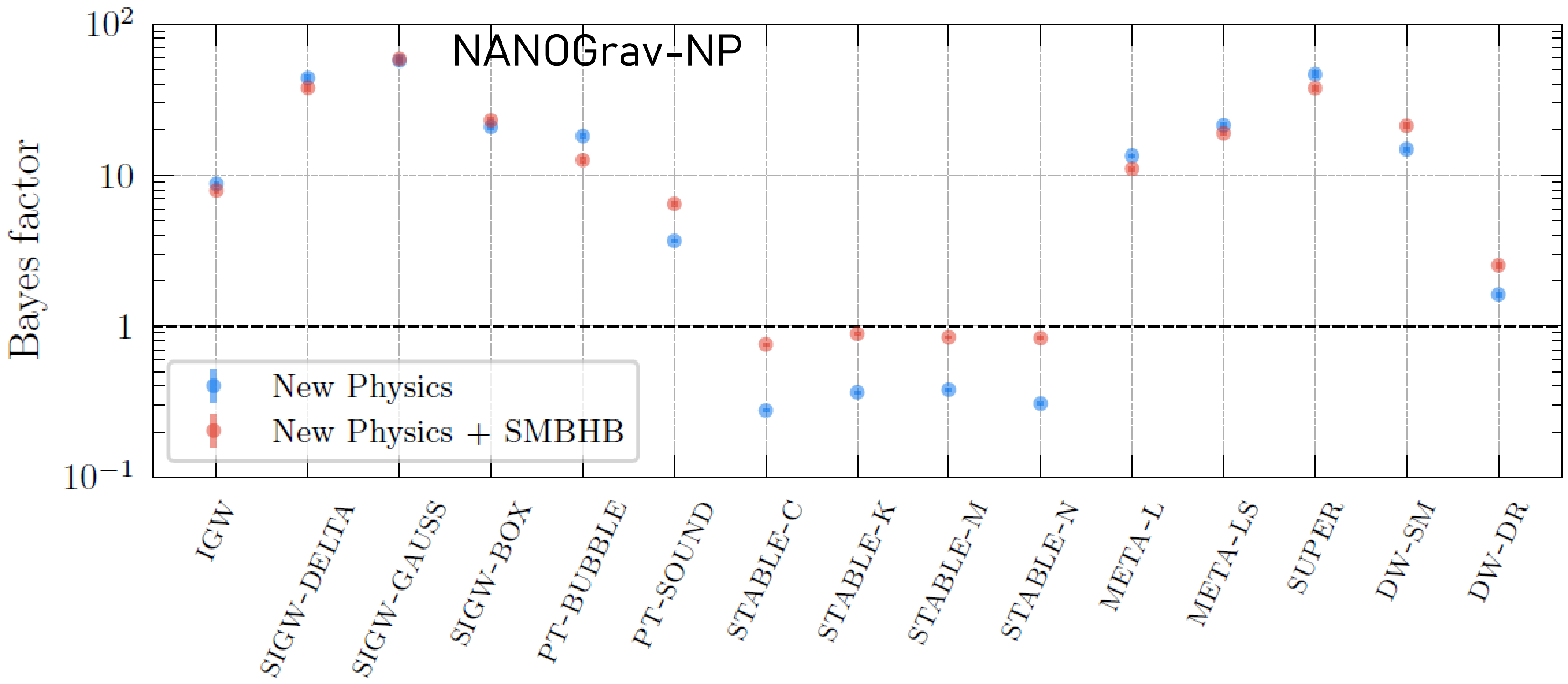


Enviromental effects can cure the tension!

Origin (II): New Physics

$$\mathcal{B}_{10}(\mathcal{D}) = \frac{\mathcal{Z}_1}{\mathcal{Z}_0} = \frac{P(\mathcal{D}|\mathcal{H}_1)}{P(\mathcal{D}|\mathcal{H}_0)}$$

Preference for New Physics?



ULDM Soliton, the Return

What happens when the soliton forms around a SMBH?

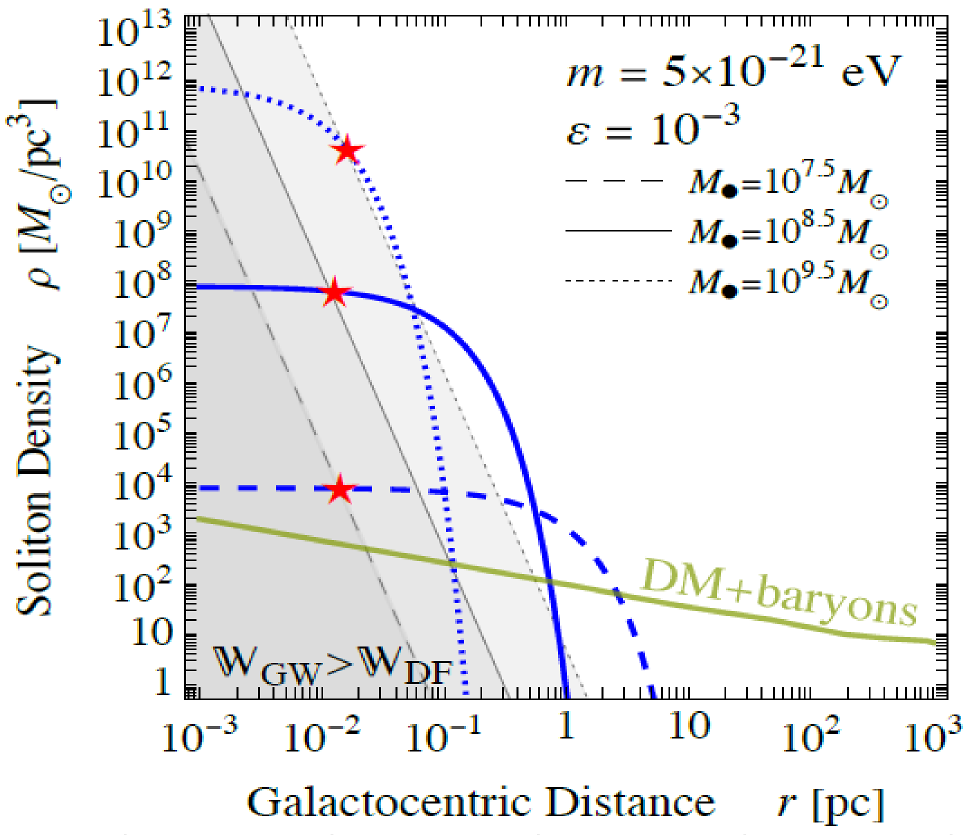
$$\rho(r;A) \approx \frac{m^2}{\pi G_N}\lambda^4 e^{-2Am\,r} \equiv \frac{m^2\varepsilon}{\pi G_N}A^4 e^{-2Am\,r}$$

$$\varepsilon = M_{sol}/M_{BH}$$

with «gravitational coupling»

$$A = G_N M_\bullet m$$

$$r_{sol}[\mathrm{pc}] \simeq \frac{2.9}{M_\bullet[10^8 M_\odot]\, m^2[10^{-21}\,\mathrm{eV}]}$$



Denser solitons overwhelm enviromental effects!!

How much mass in the soliton?

$$M_{\rm sol}^{\rm NS}[M_\odot] \simeq \frac{1.4\times 10^8}{m[10^{-21}\,{\rm eV}]} M_h^{\frac{1}{3}}[10^{12} M_\odot]$$

vs

$$M_{\rm sol}^{\rm BH}[M_\odot] \simeq \frac{5.4\times 10^6}{m^4[10^{-21}\,{\rm eV}]\, M_\bullet^3[10^8 M_\odot]} M_h^{\frac{4}{3}}[10^{12} M_\odot]$$

The SMBH «quenches» the soliton

Kinetic Equilibration is violated

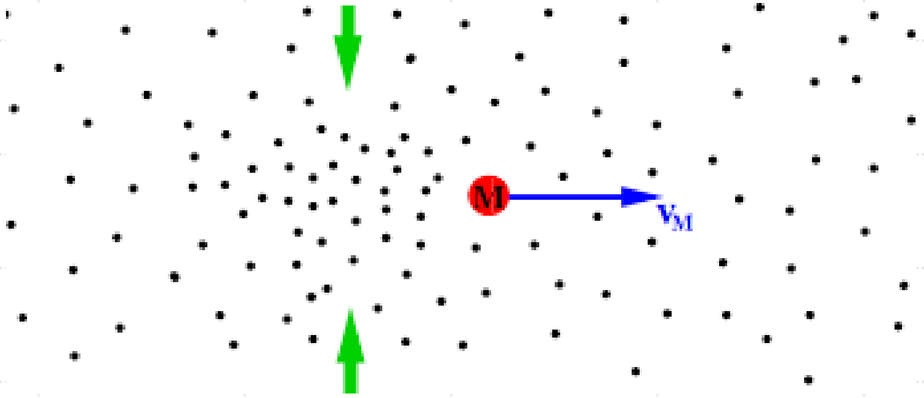
$$\left.\frac{K}{M}\right|_{\rm soliton} \approx \left.\frac{K}{M}\right|_{\rm halo}$$

$$K/M|_{\rm soliton} \;\to\; A^2/2$$

$$A^2/2 > K/M|_{\rm halo}$$

Estimate of dynamical heating by the SMBH

Dynamical friction on a SMBH



The SMBH travels through the ULDM «medium»

$$F_{\rm DF} = \frac{4\pi G_{\rm N}^2 m_{\rm cl}^2 \rho}{v^2} C_{\rm cl}(\Lambda)$$

The gravitational «wake» exerts drag on the SMBH.

with

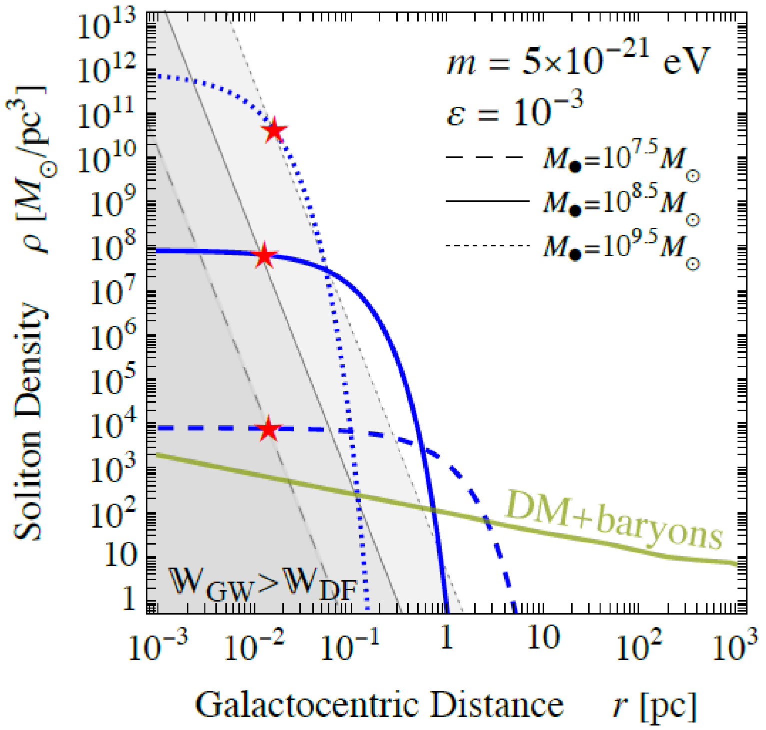
$$\Lambda = \frac{v^2 r}{G_N m_{\rm cl}}$$

$$C_{\rm cl} = \log 2\Lambda - 1 + \frac{1}{\Lambda}\log 2\Lambda + {\rm O}(\Lambda^{-2})$$

«Coulomb Log»

Dynamical friction in SMBH Binaries

The SMBH feels the dynamical friction of the soliton that is around the other SMBH in the binary system.



$$\mathcal{W}_{\rm DF} = F_{\rm DF} v_{\rm rel} = \frac{4\pi G_{\rm N}^2 \mu^2 \rho}{\omega r} C_{\rm cl}(\tilde{\Lambda})$$

$$\mathcal{W}_{\rm GW} = \frac{32}{5} G_{\rm N} \mu^2 \omega^6 r^4$$

$$r_c = \sqrt[11]{\frac{64\,(1+q_\bullet)^7}{25\,C_{\rm cl}^2(\tilde{\Lambda})} \left(\frac{1}{G_{\rm N} m^{12} M_\bullet \varepsilon^2}\right)}$$

Two regimes

$$\begin{cases} r_c > r_{\rm sol} \\ r_c < r_{\rm sol} \end{cases}$$

ULDM «visible» effects

The GW strain revisited

$$h_c^2(f) = \frac{3H_0^2}{2\pi^2\rho_c f^2}\int \mathrm{d}z\,\mathrm{d}\mathbf{X}\,\frac{\mathrm{d}n_s}{\mathrm{d}z\,\mathrm{d}\mathbf{X}}\,\frac{f_s}{1+z}\,\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}f_s}\bigg|_{\mathbf{X}}$$

(Phinney '01)

with

$$\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}f_s} = \frac{\mu}{3}\left[\pi G_{\mathrm{N}}(M_{\bullet,1}+M_{\bullet,2})\right]^{\frac{2}{3}} f_s^{-\frac{1}{3}}\,\frac{\mathcal{W}_{\mathrm{GW}}}{\mathcal{W}_{\mathrm{GW}}+\mathcal{W}_{\mathrm{DF}}}$$

For pure GW emission we recover the aforementioned power-law!

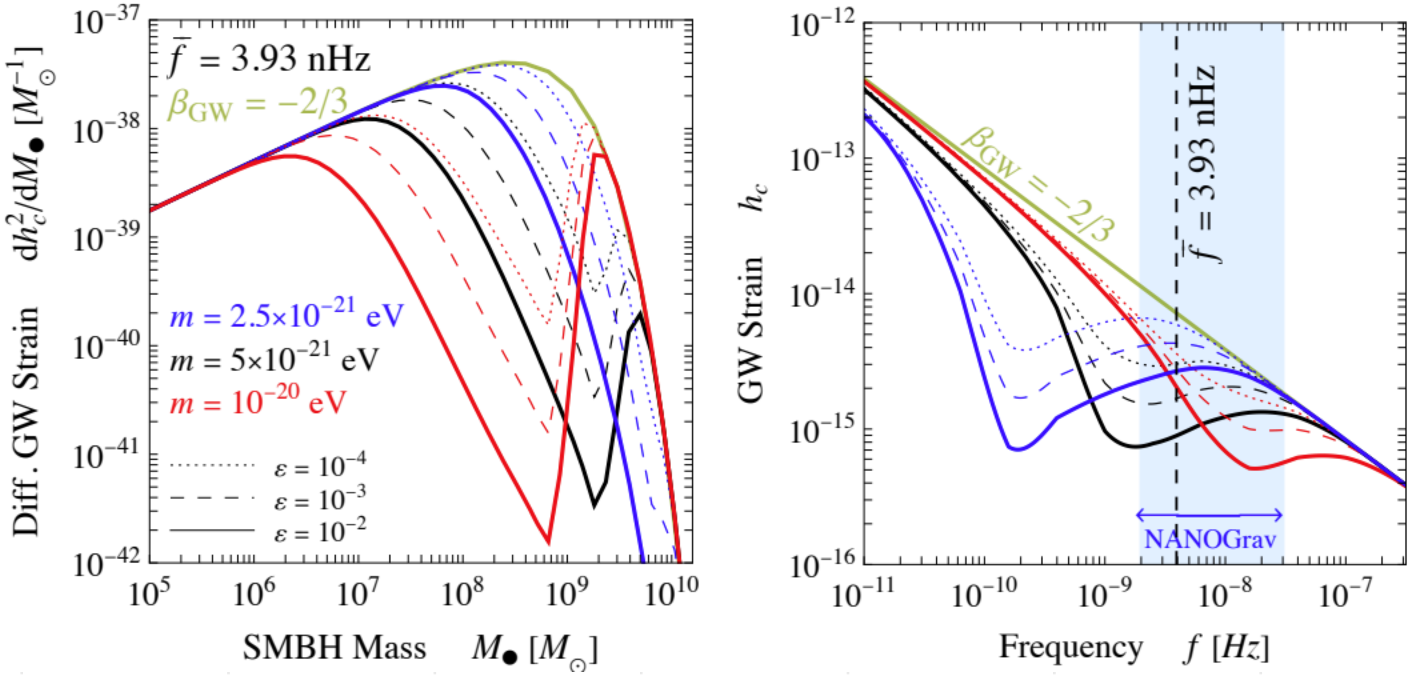
Interplay between f_{sol} and f_c

$$\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}f_s} = \frac{q_\bullet\left(\pi G_{\mathrm{N}} M_\bullet^2\right)^{\frac{2}{3}}}{3(1+q_\bullet)^{\frac{1}{3}}}\,\frac{M_\bullet^{\frac{1}{3}} f_s^{-\frac{1}{3}}}{1+\left(\frac{f_c}{f_s}\right)^{\frac{11}{3}} e^{-\left(\frac{f_{\mathrm{sol}}}{f_s}\right)^{\frac{2}{3}}}}$$

GW Strain: Results

«Pure» GW features, peaks at:

$$M_\bullet \approx 10^{8.5} M_\odot$$
$$z \approx 0.3$$



$$(f_c \sim m^{36/22}) \quad r_c \sim m^{-12/11}$$
$$r_{sol} \sim 1/M_\bullet m^2$$

Scaling of the lengths: crucial to understand the features of the strain

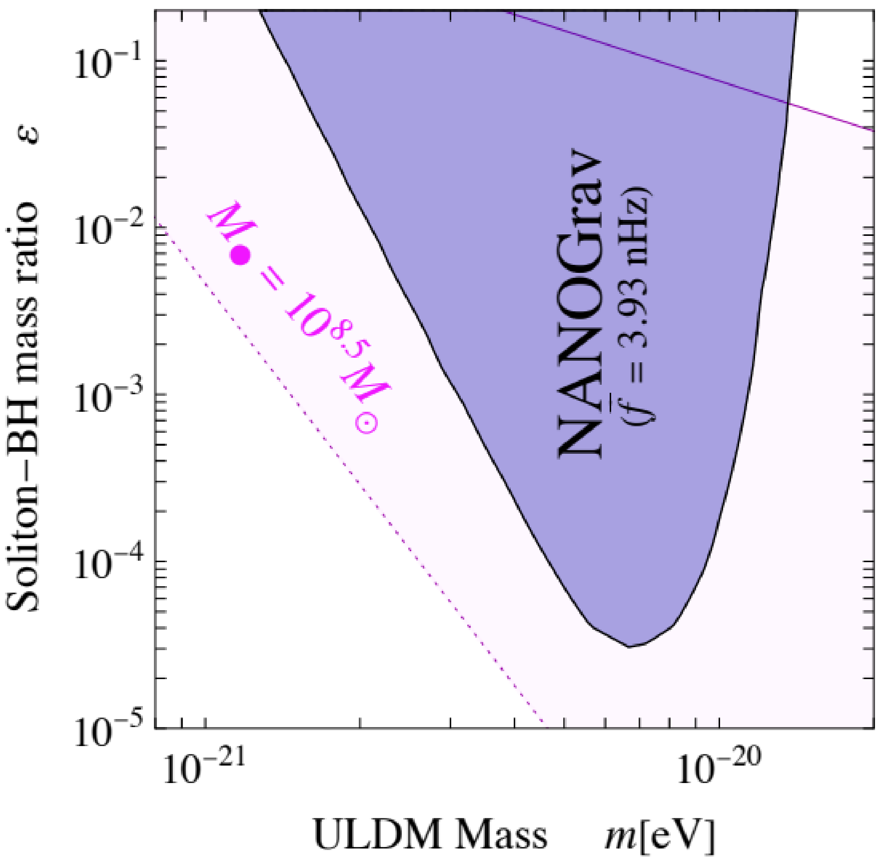
The NANOGrav Bound

Excluded mass range

$$1.3 \times 10^{-21}\,\mathrm{eV} \lesssim m \lesssim 1.4 \times 10^{-20}\,\mathrm{eV}$$

Sensitivity to BH-Soliton ratios

$$10^{-5} \lesssim \varepsilon \lesssim 0.2$$



«Robustness» of the bound:
The exclusion is consistent with the predicted soliton mass in every regime!

Final remarks

Other relevant constraints

- Absorption of the soliton by the SMBH

$$\tau_{\rm abs}[{\rm Gyr}] \simeq 5.6 \times 10^3 M_\bullet^{-5}[10^8 M_\odot] m^{-6}[10^{-21}\,{\rm eV}]$$

→ Can be comparable to the age of the Universe!

- Ly-α constraints on structure formation. Recent analyses constrain ULDM masses below $2 \times 10^{-20}\,{\rm eV}$ (Rogers & Peiris, 2020)

→ Complementary bound

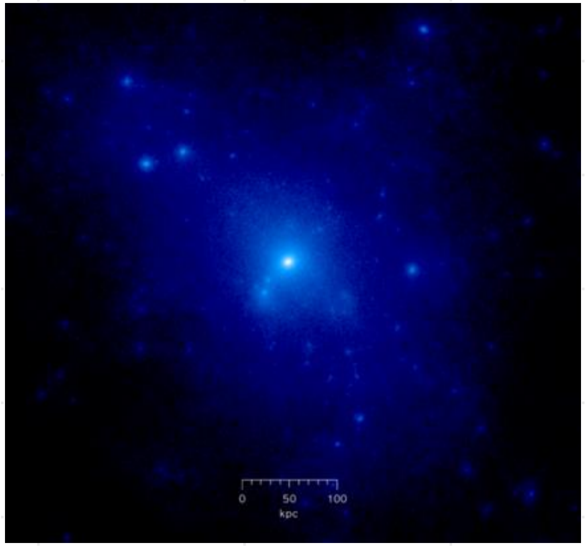
Bounds are subjected to uncertainties and caveats (computation of the absoprtion time, astrophysical uncertainties, ULDM fraction…)

Conclusions

- ULDM is an appealing alternative to CDM. Minimality : gravitational interactions only.
- Solitons are a key prediction of such paradigm.
- GW advancements allow us to test the properties of ULDM by constraining solitons around SMBH mergers.
- The road is plagued by uncertainties, both theoretical and astrophysical, however…
- …The NANOGrav bound turns out to be pretty robust
- Outlook. From exclusion to fit: is it possible to exploit the preferred low-frequency tilt of the data?

Back-up Slides

Gravitational Relaxation (I)



At large distances: It looks like a CDM halo.

Gravitational relaxation can be (analitically) simplified in a two-body scattering.

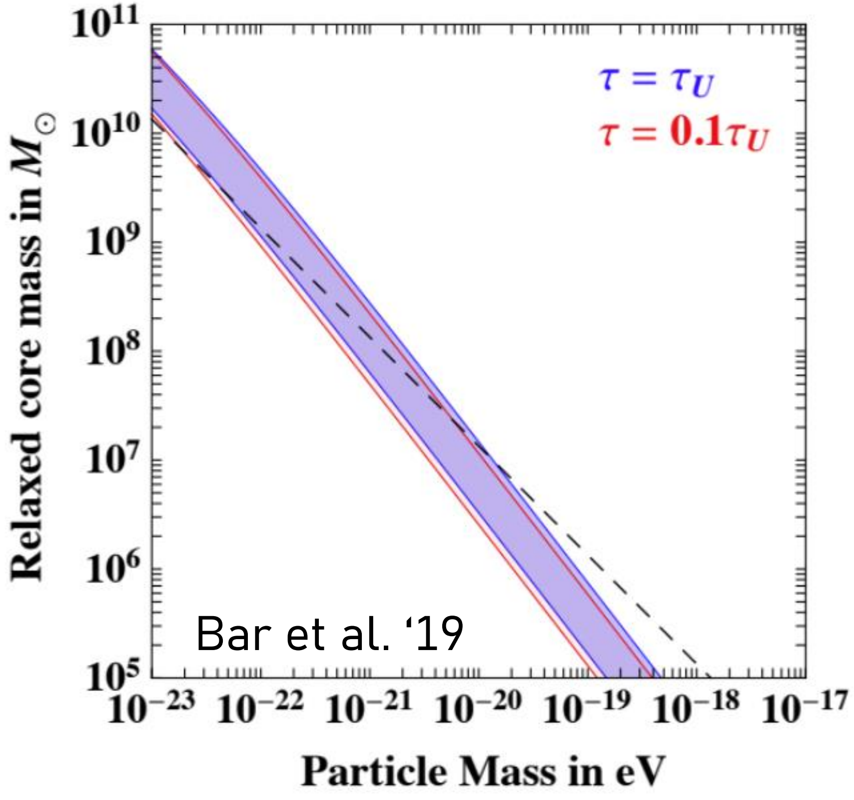
$$\Delta v^2 \equiv \int_{b_{\rm min}}^{b_{\rm max}} \sum \delta v^2 \simeq 8N \left(\frac{Gm}{Rv}\right)^2 \ln \Lambda$$

Coulomb log

$$t_{\rm relax} \simeq \frac{0.1N}{\ln N} t_{\rm cross}$$

$$t_{\rm relax} \sim 0.1 t_{\rm cr} (M/m)$$

Gravitational Relaxation (II)



Relaxation can affect the soliton-halo mass relation at large ULDM masses!!

More on the strain computation (I)

$$h_c^2(f) = \frac{3H_0^2}{2\pi^2 \rho_c f^2} \int \mathrm{d}z\, \mathrm{d}q_\star\, \mathrm{d}M_\star \frac{\mathrm{d}n_s}{\mathrm{d}z\, \mathrm{d}q_\star\, \mathrm{d}M_\star} \frac{f_s}{1+z} \frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}f_s}\bigg|_{q_\star,\, M_\star} .$$

$$\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}f_s} = \frac{q_\bullet \left(\pi G_{\mathrm{N}} M_\bullet^2\right)^{\frac{2}{3}}}{3(1+q_\bullet)^{\frac{1}{3}}} \frac{M_\bullet^{\frac{1}{3}} f_s^{-\frac{1}{3}}}{1+\left(\frac{f_c}{f_s}\right)^{\frac{11}{3}} e^{-\left(\frac{f_{\mathrm{sol}}}{f_s}\right)^{\frac{2}{3}}}}$$

The strain can be written in terms of parameters of the host galaxy

$$\log_{10}\left(\frac{M_\bullet}{M_\odot}\right) = \mu + \alpha_\mu \log_{10}\left(\frac{M_{\mathrm{bulge}}}{10^{11} M_\odot}\right) + \mathcal{N}(0, \epsilon_\mu).$$

Parametrization of the Galaxy Population

$$\frac{\partial^3 \eta_{\text{gal-gal}}}{\partial m_{\star 1}\, \partial q_\star\, \partial z} = \frac{\Psi(m_{\star 1}, z')}{m_{\star 1} \ln(10)} \frac{P(m_{\star 1}, q_\star, z')}{T_{\text{gal-gal}}(m_{\star 1}, q_\star, z')} \frac{\partial t}{\partial z'}$$

with

$$\Psi(m_{\star 1}, z) = \ln(10)\, \Psi_0 \cdot \left[\frac{m_{\star 1}}{M_\psi}\right]^{\alpha_\psi} \exp\left(-\frac{m_{\star 1}}{M_\psi}\right)$$

Galaxy Stellar Mass Function

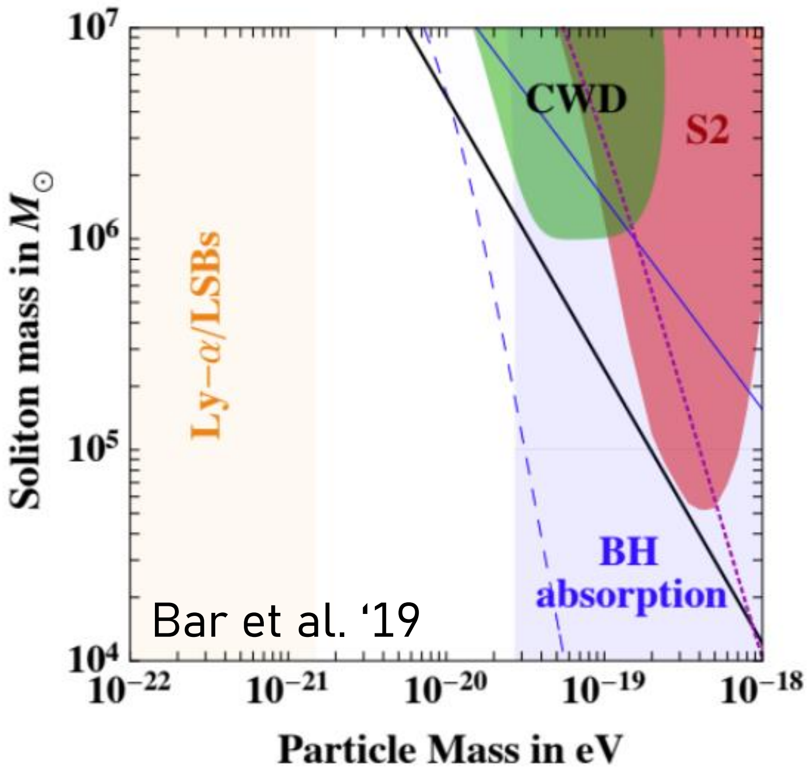
$$T_{\text{gal-gal}}(m_{\star 1}, q_\star, z') = T_0 \left(\frac{m_{\star 1}}{10^{11}\, \mathrm{M}_\odot / h}\right)^{\alpha_t} (1+z)^{\beta_t}\, q^{\gamma_t}$$

Galaxy Merger Time

$$P(m_{\star 1}, q_\star, z') = P_0 \left(\frac{m_{\star 1}}{10^{11}\, \mathrm{M}_\odot}\right)^{\alpha_p} (1+z)^{\beta_p}\, q^{\gamma_p}$$

Galaxy Pair Fraction

Other bounds on SMBH dominated Solitons



A soliton around a SMBH can influence stellar kinematics.