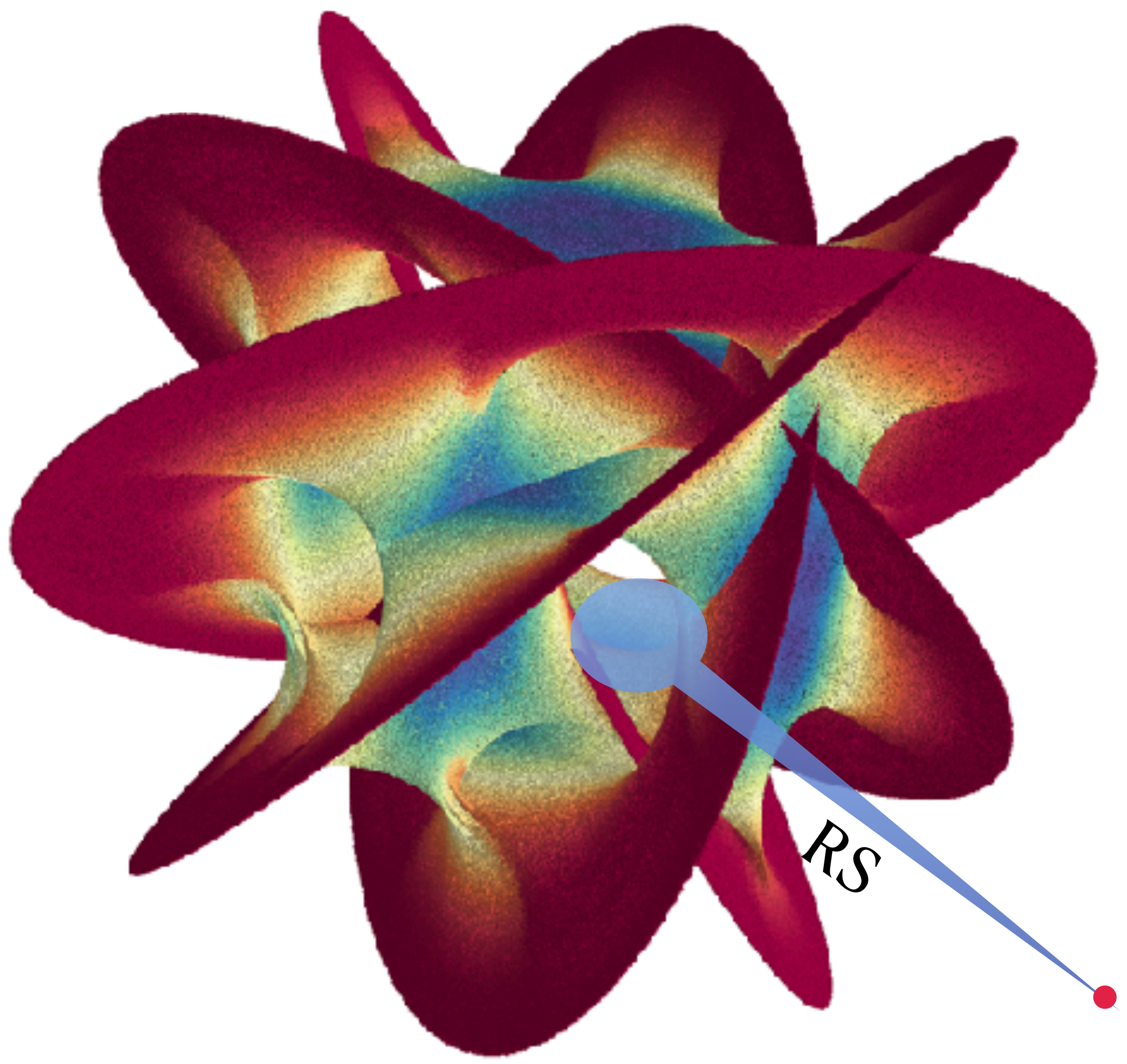
Candidate De Sitter Vacua



based on arXiv:2406.13751 with Liam McAllister, Richard Nally and Andreas Schachner

and previous works with Mehmet Demirtas, Manki Kim and Andres Rios-Tascon

Jakob Moritz

CERN

10/03/2024 at Rencontres Theoriciennes

upshot of this talk:

First concrete candidates for de Sitter vacua
as envisioned by Kachru, Kallosh, Linde and Trivedi (KKLT)

with an important caveat: our candidates come with finite control parameters, such as the string coupling, and are potentially vulnerable to unknown corrections.

Plan:

1. Some Motivation & Introduction
2. Anti de Sitter Vacua with small superpotential
3. Warped throats and "Uplift" to de Sitter: an example
4. Control over corrections
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The laws of physics in our Universe display shocking hierarchies

$v_{Higgs} \sim 10^{-16} M_{pl}$

$$\rho_{vac} \sim 10^{-120} M_{pl}^4$$

$\frac{\delta\rho_{CMB}}{\rho_{CMB}} \sim 10^{-4}$

$\theta_{QCD} < 10^{-10}$

$y_e \sim 10^{-6}$

JW Telescope ‘21

Let us go through important length scales in physics, and collect hierarchy problems along the way…

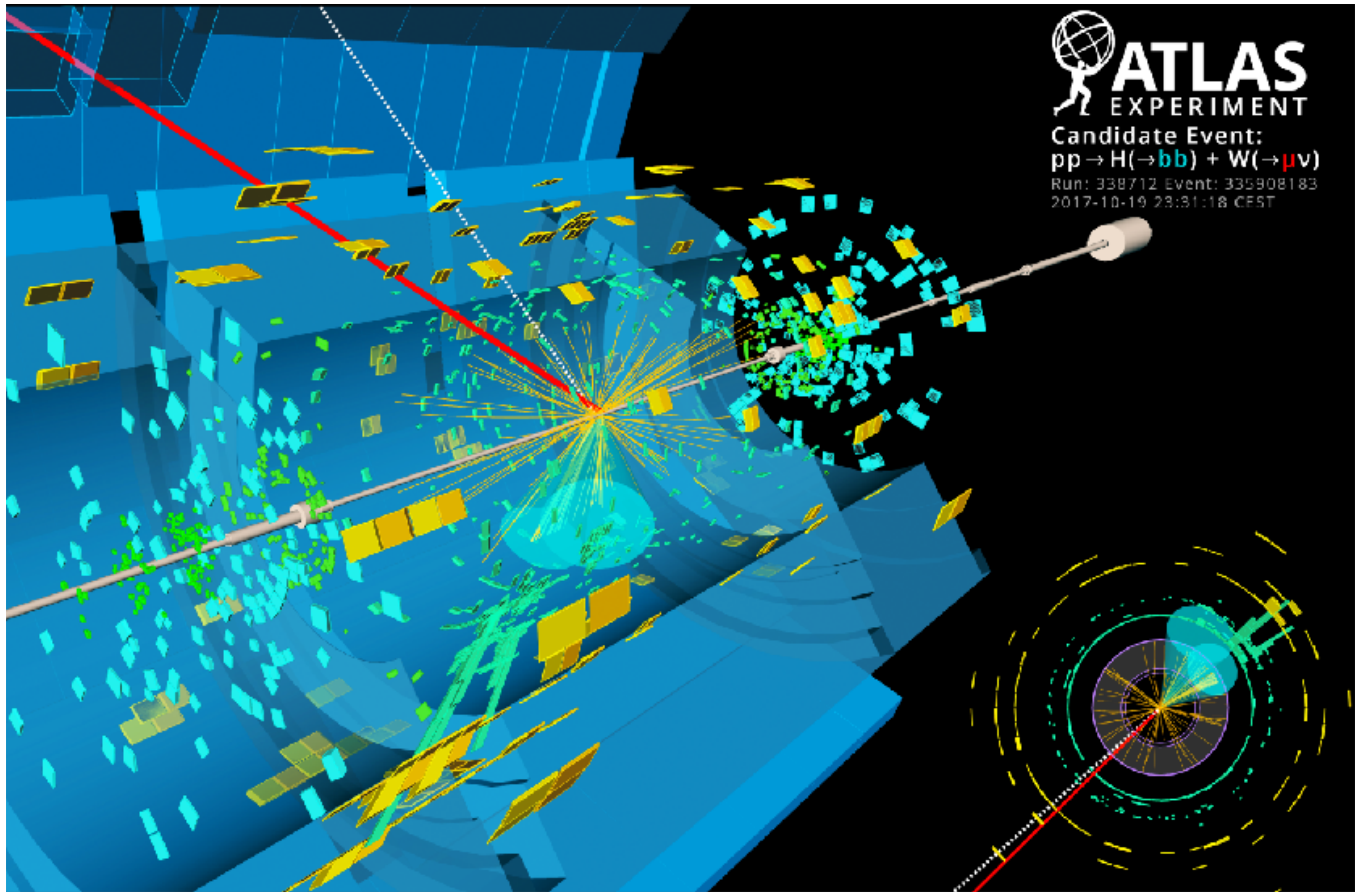
We begin with the smallest conceivable scale:

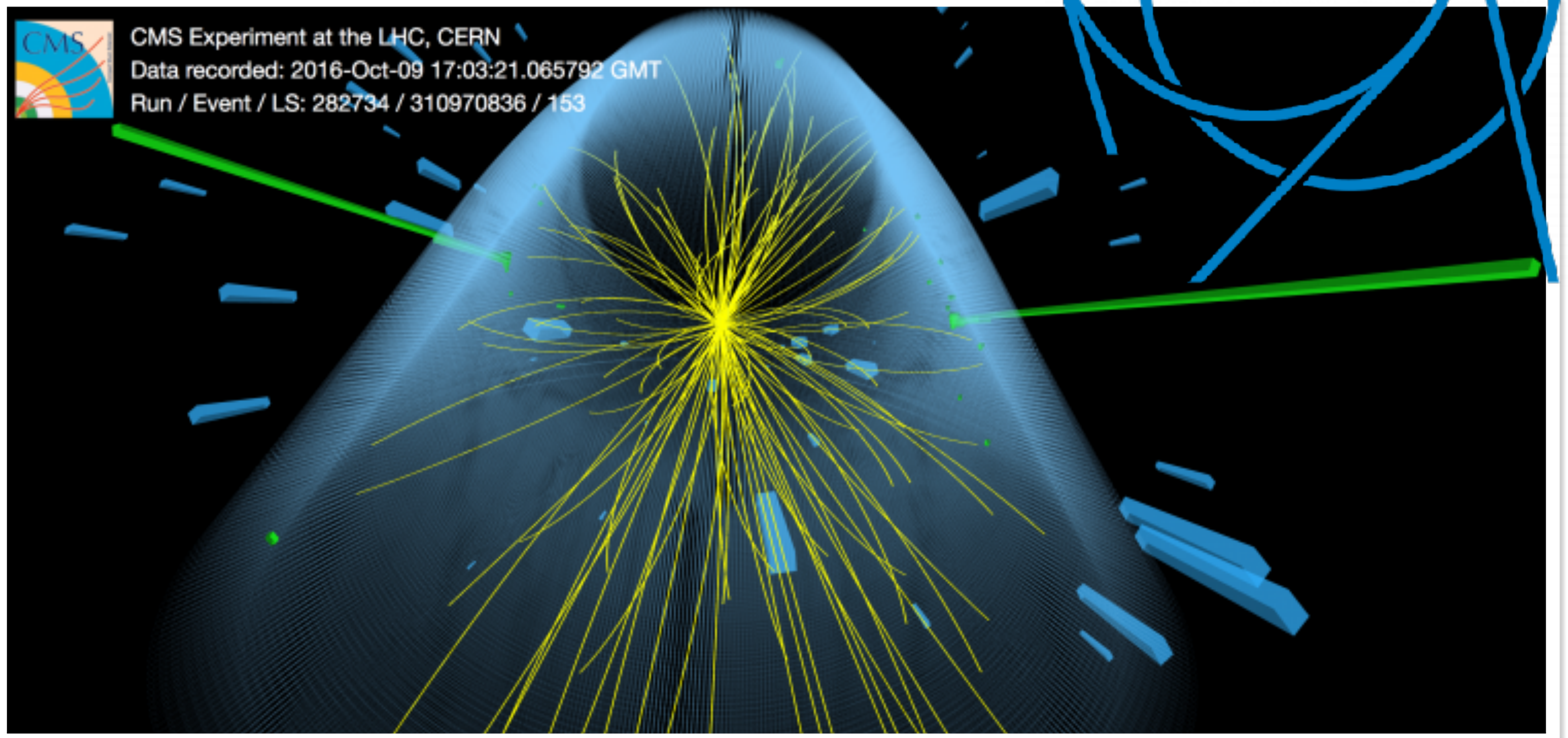
The length scale of quantum gravity could lie near the Planck scale

$$10^{15} \times M_{pl}^{-1} \gtrsim \ell_{Quantum\ Gravity} \gtrsim 1 \times M_{pl}^{-1}$$

$$M_{pl} \approx 2.4 \times 10^{18}\,\text{GeV}$$

Electroweak scale



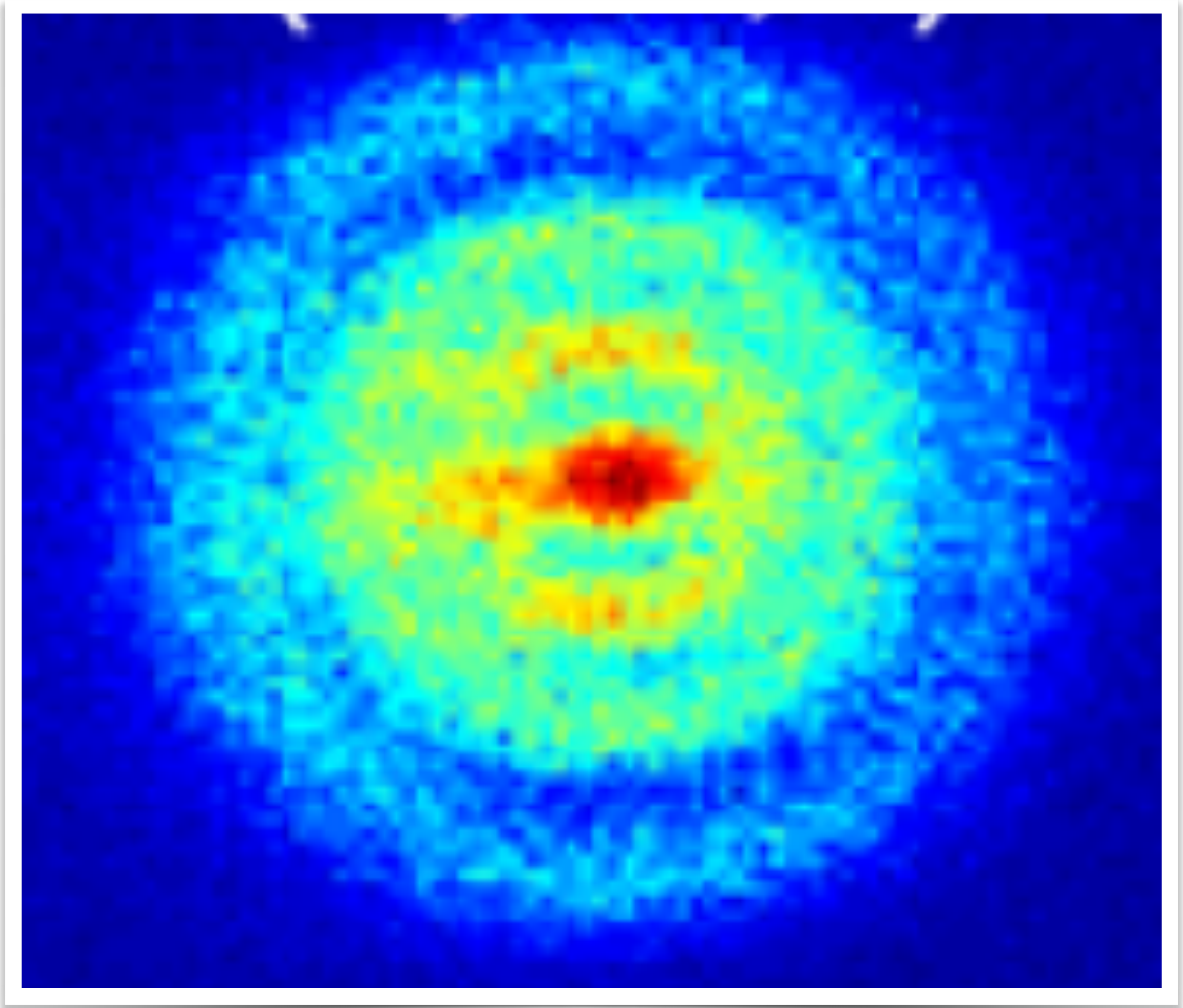


The electroweak hierarchy problem:

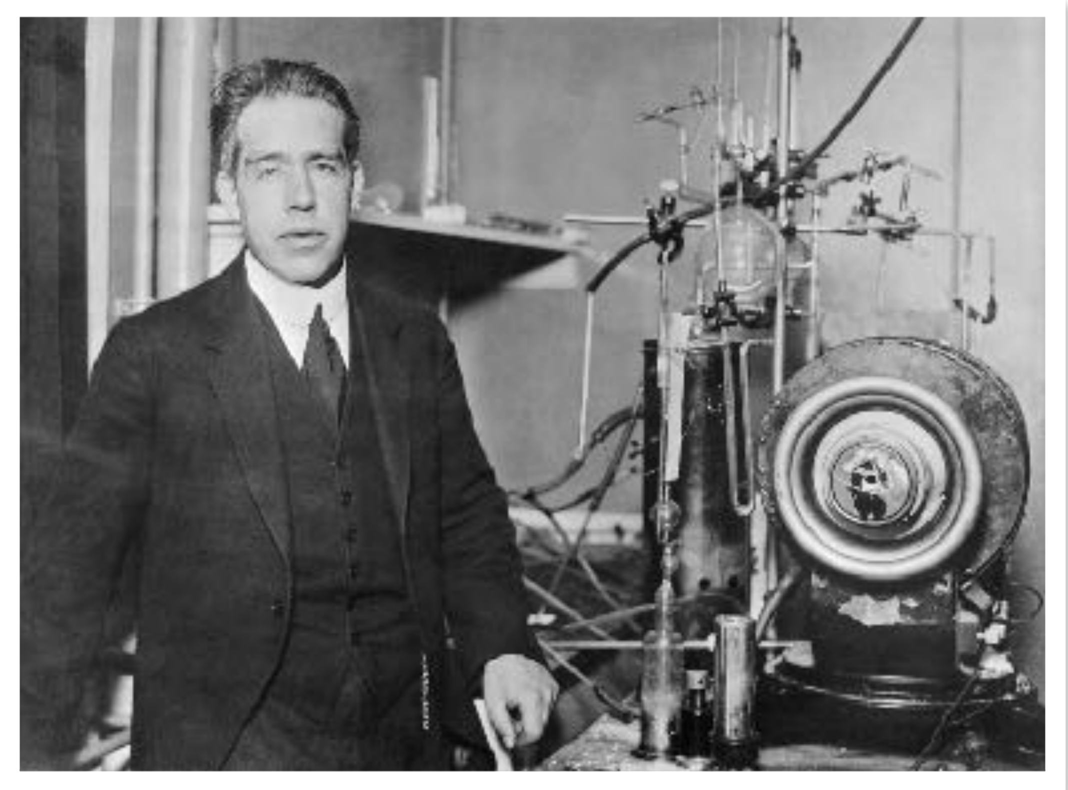
Why is the weak scale so long?

$$1/v_{Higgs} \approx 1/(250\, GeV) \sim 10^{16} M_{pl}^{-1}$$

Hydrogen Atom



Stodolna et al '13



Bohr '13

Bohr radius $\ell_{Bohr} \sim 10^{23} \times M_{pl}^{-1}$

$$\ell_{Bohr} = \frac{1}{\alpha_{EM} m_e} \sim \frac{1}{\alpha_{EM} \times y_e \times v_{Higgs}}$$

Why are Yukawa's small?

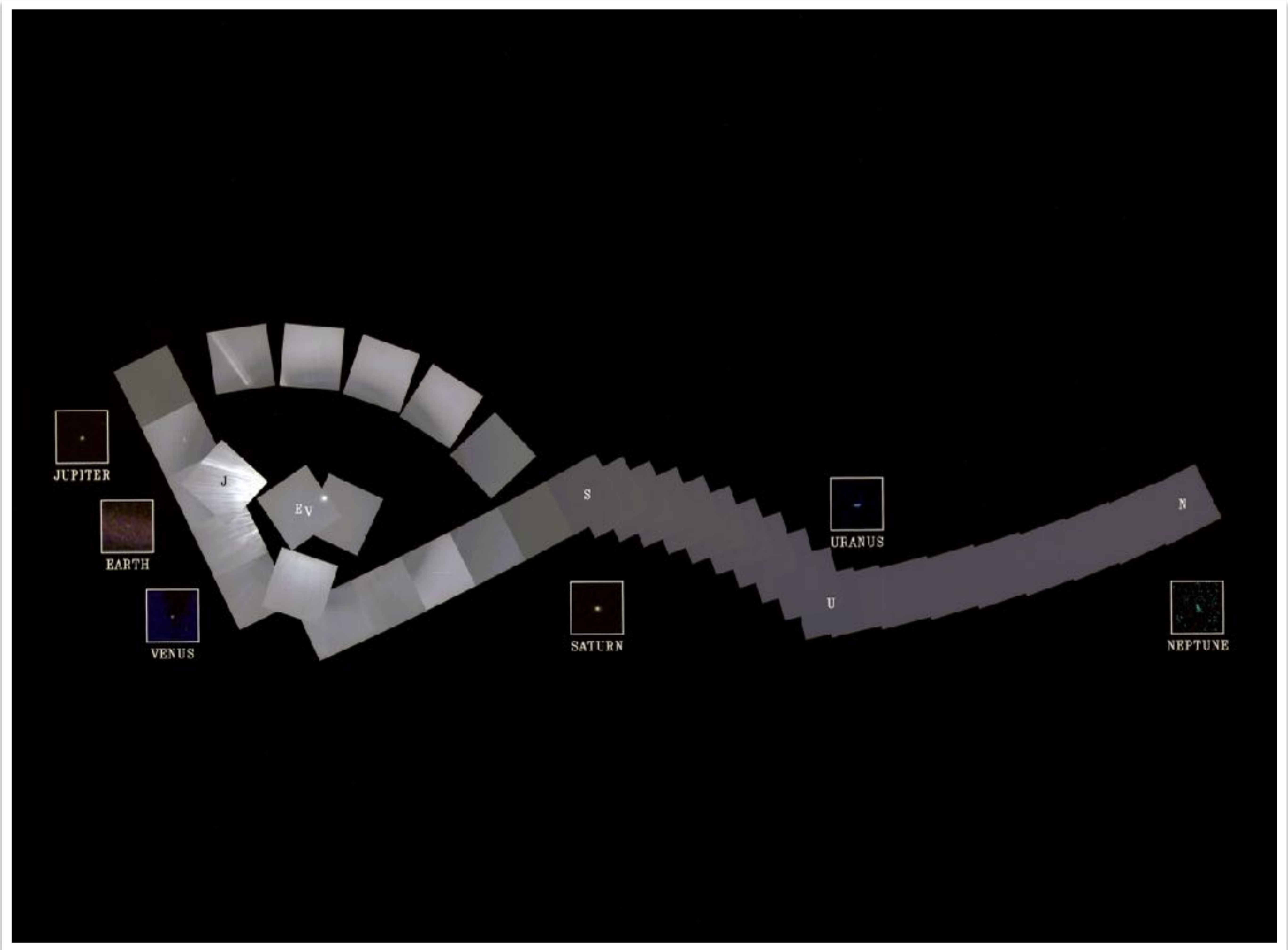
$y_e \sim 10^{-6}$

Earth

$\ell_{Earth} \approx 10^{42} \times M_{pl}^{-1}$

Apollo 8

The Solar System



Voyager 1

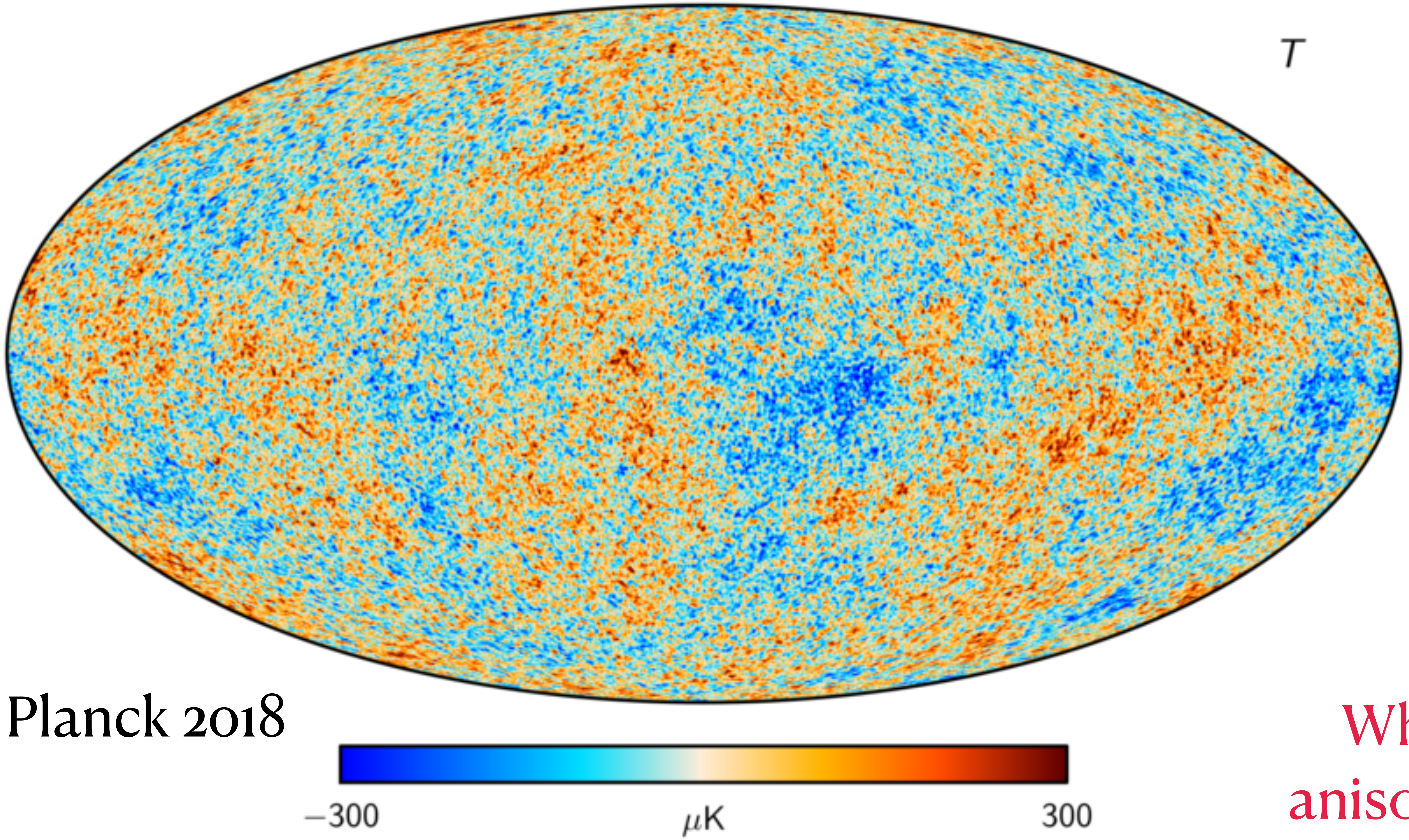
$$\ell_{Solar\ system} \approx 10^{51} \times M_{pl}^{-1}$$

The Milky Way

Paranal Observatory

$$\ell_{Milky\ way} \approx 10^{55} \times M_{pl}^{-1}$$

The observable Universe



Planck 2018

Seed for structure formation

$$\frac{\delta\rho_{CMB}}{\rho_{CMB}} \sim 10^{-4}$$

Why are the CMB anisotropies so small?

$$\ell_{CMB} \approx 10^{61} \times M_{pl}^{-1}$$

Size of "observable" universe

But towering above all we have

the cosmological constant problem:

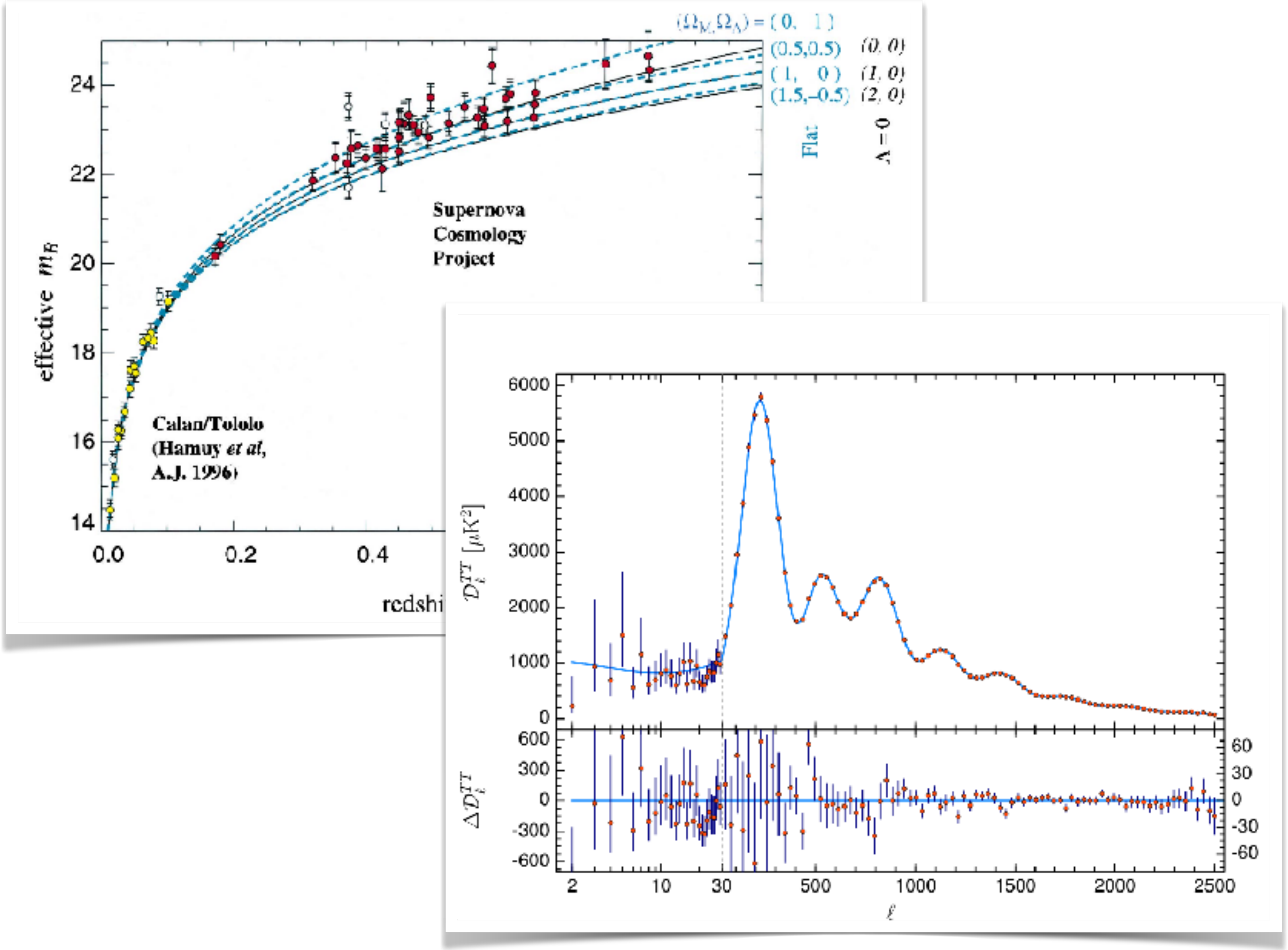
Why is the vacuum energy so small?

$$\rho_{vac} \sim 10^{-120} M_{pl}^4$$

or

Why is our universe so large?

$$\ell_{Horizon} \approx 10^{60} \times M_{pl}^{-1}$$



Famously, both the electroweak hierachy problem, as well as the cosmological constant problem are fine tuning problems, and are thus sensitive to the physics of the deep UV.

It is therefore plausible to me that much can be learned about these problems of fundamental physics by trying to reproduce (aspects of) them in string theory!

(for example, one might gain insight into the microscopic meaning of the de Sitter entropy?)

Concretely, this talk is about addressing the cosmological constant problem,

$$\rho_{cc} \approx 10^{-120} M_{\rm pl}^4$$

in string theory.

Solving the true cosmological constant problem seems impossible

Vacuum energy receives divergent loop corrections from all particles → $\frac{\mu^4_{UV-cutoff}}{\rho_{vacuum}} \gtrsim 10^{56}$ fine tuning

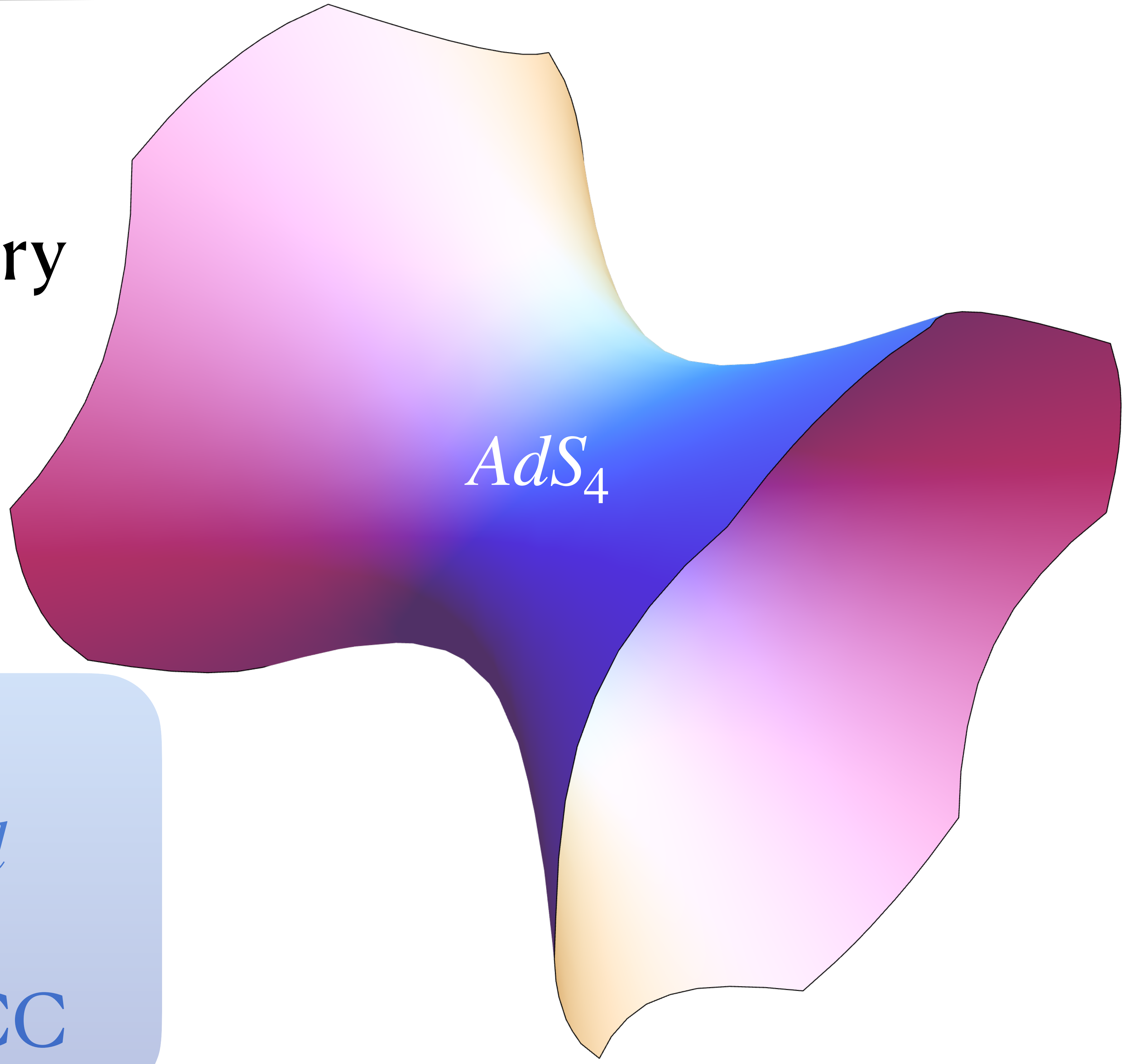
Fine tuned cancellation spoiled by loop corrections

e.g. with $\alpha = \alpha_{QCD} \sim 0.1$ one would need to compute to $\gtrsim 56$ order in loops...

But a version with unbroken supersymmetry can be solved:

Task: Identify

- weakly coupled supersymmetric EFT in string theory
- isolated supersymmetric vacuum
- small vev of *superpotential* $W_0 := \langle |W| \rangle \ll 1$



$$\longrightarrow \quad \rho_{vac} = -3|W_0|^2 M_{pl}^4 \ll M_{pl}^4$$

i.e. Anti-de Sitter (AdS) vacuum with small CC

key fact: superpotential can be computed (more or less) exactly in string theory!

this general idea has been around for ~20 years, but was believed to be exponentially hard to achieve!

Kachru, Kallosh, Linde, Trivedi '03

More generally, one can study a supersymmetric version of the cosmological constant problem by finding vacua of stringy F-term potentials:

$$V_F = e^K (g^{a\bar{b}} D_a W \overline{D_b W} - 3|W|^2)$$

with small superpotential: $\langle W \rangle \ll 1$

but without any fine tuning: $g^{a\bar{b}} D_a W \overline{D_b W} \sim |W|^2$

In this way one can hope to even construct controlled de Sitter vacua in string theory!

Self-consistency at quantum level: string theory has ten dimensions of spacetime

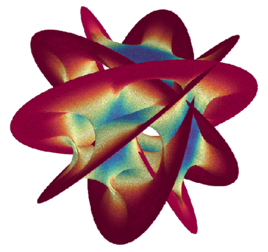
Therefore:

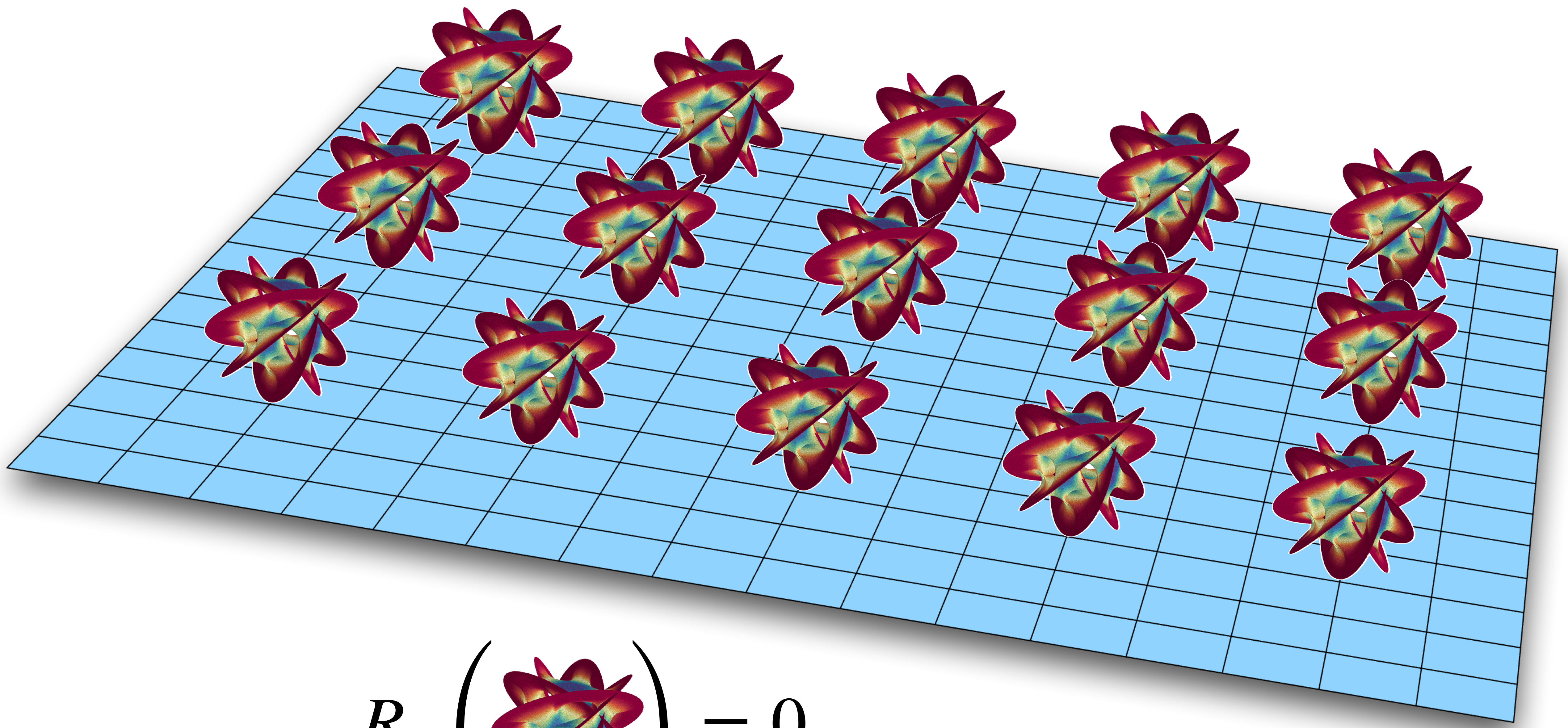
Weakly coupled string theory solutions

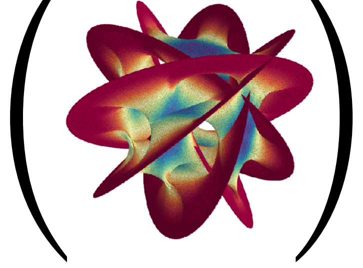
=

compactifications of ten-dimensional string theories down to four dimensions

cf Kaluza ’1921, and Klein ‘1926

$$M_{1,9} \simeq (\mathbb{R}^{1,3},\ AdS_4,\ \text{or}\ \ dS_4) \times$$ 



$$R_{ij}\Big($$  $$\Big) = 0$$

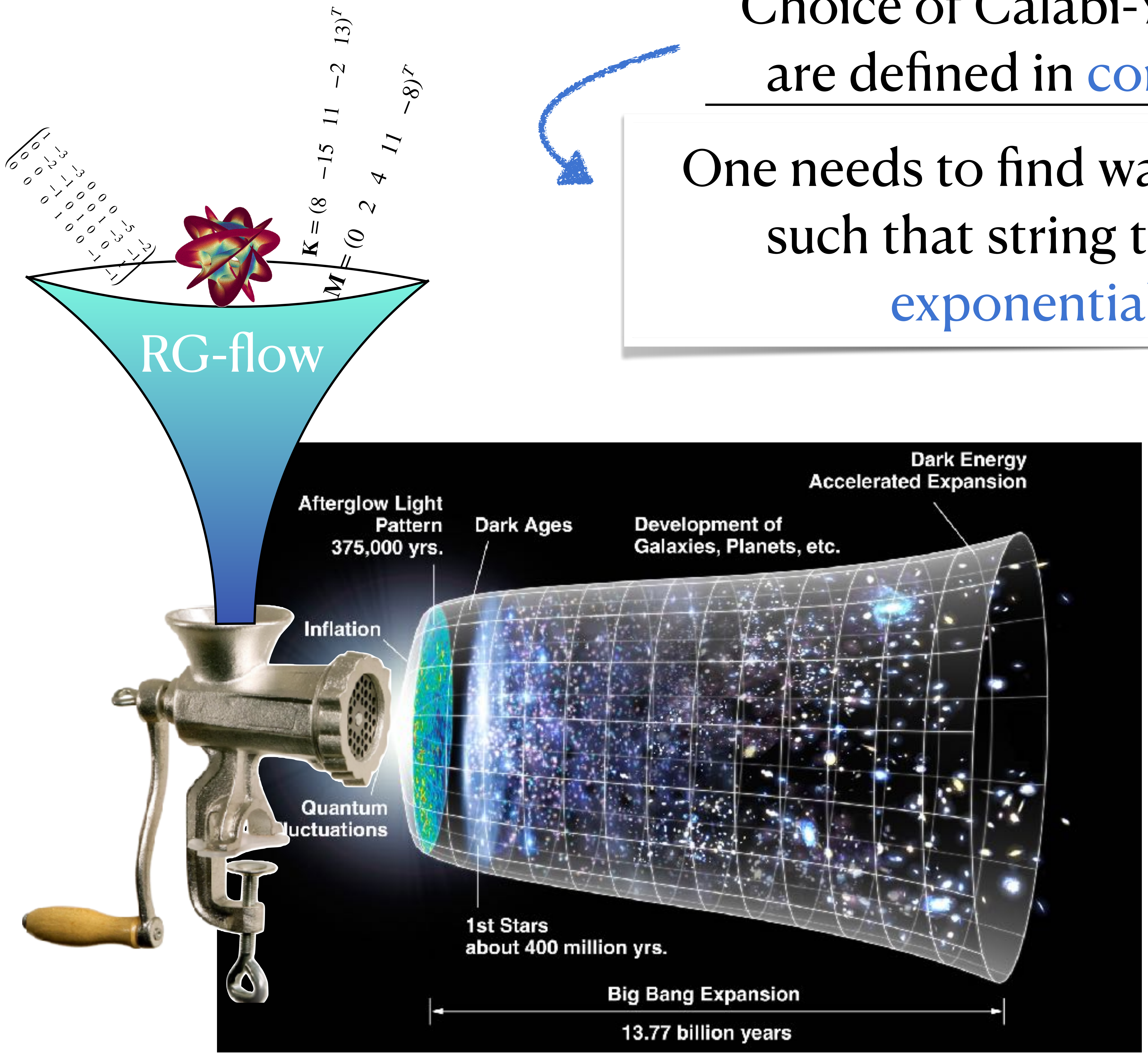
A real Calabi-Yau!
Geoffrey Fatin

Compact space is called Calabi-Yau threefold*

*equipped with other sources of stress-energy

Choice of Calabi-Yau threefold (and other sources) are defined in combinatorial terms (will see later)

One needs to find ways to choose integer data in the UV such that string theory generates a Universe with exponential hierarchies in the deep IR



NASA / WMAP Science Team

Let's see how this works in practice!

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Roadmap: Constructing minimally supersymmetric EFTs

Type IIB string theory

- highly supersymmetric
- pair of 3-form field strengths (F_3, H_3)

"compactification"

Type IIB on CY threefold X

- extended supersymmetry, only $\mathbb{R}^{1,3}$
- many exactly massless scalars (moduli)

orientifold projection

def. in terms of a reflection symmetry of the Calabi-Yau

Type IIB on CY Orientifold X/Z_2

- Minimally supersymmetric (like e.g. MSSM)
- Light scalar fields can become massive through superpotential
- enough structure for AdS_4 and dS_4

$$W(\vec{z}, \tau) = \left(\vec{f} - \tau \vec{h}\right) \cdot \vec{\Pi}(\vec{z})$$

Gukov, Vafa, Witten '99

String coupling $g_s \equiv 1/\text{Im}(\tau)$

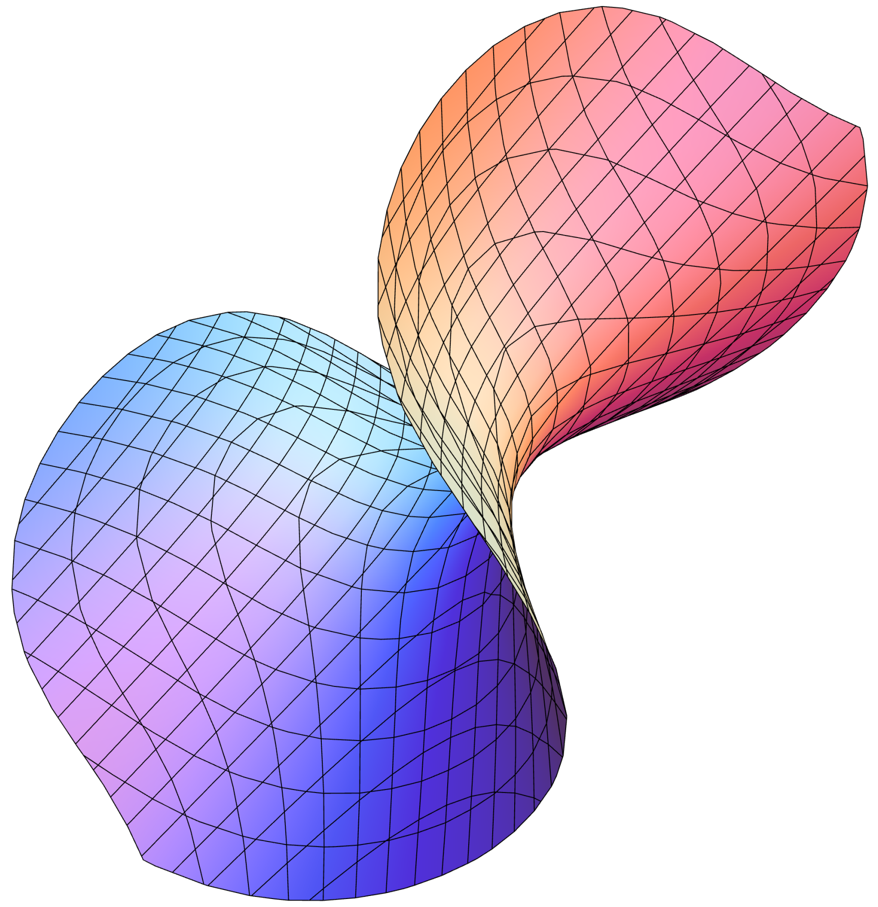
Coordinates on metric space

Dirac-quantized fluxes

$$\int_{3d} F_3 \,, \qquad \int_{3d} H_3$$

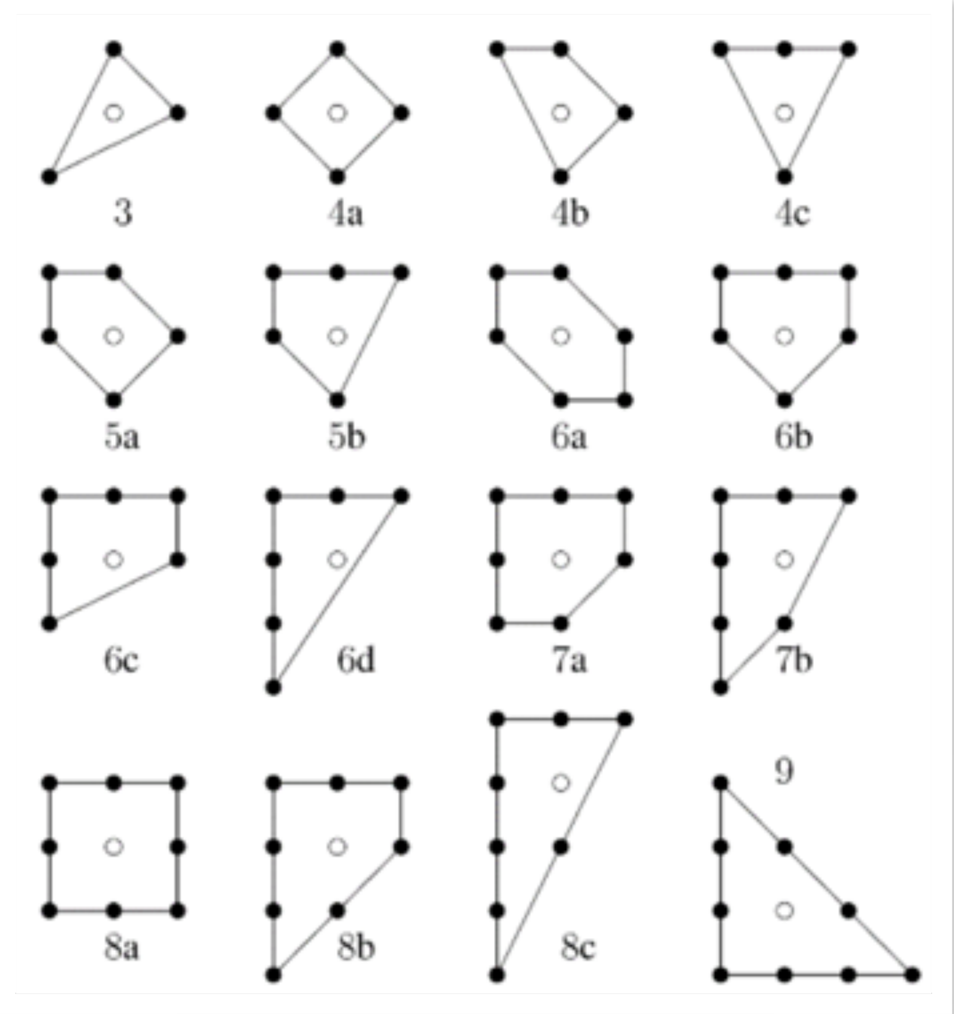
Step 1: How to construct EFTs

Hypersurfaces in "projective space"



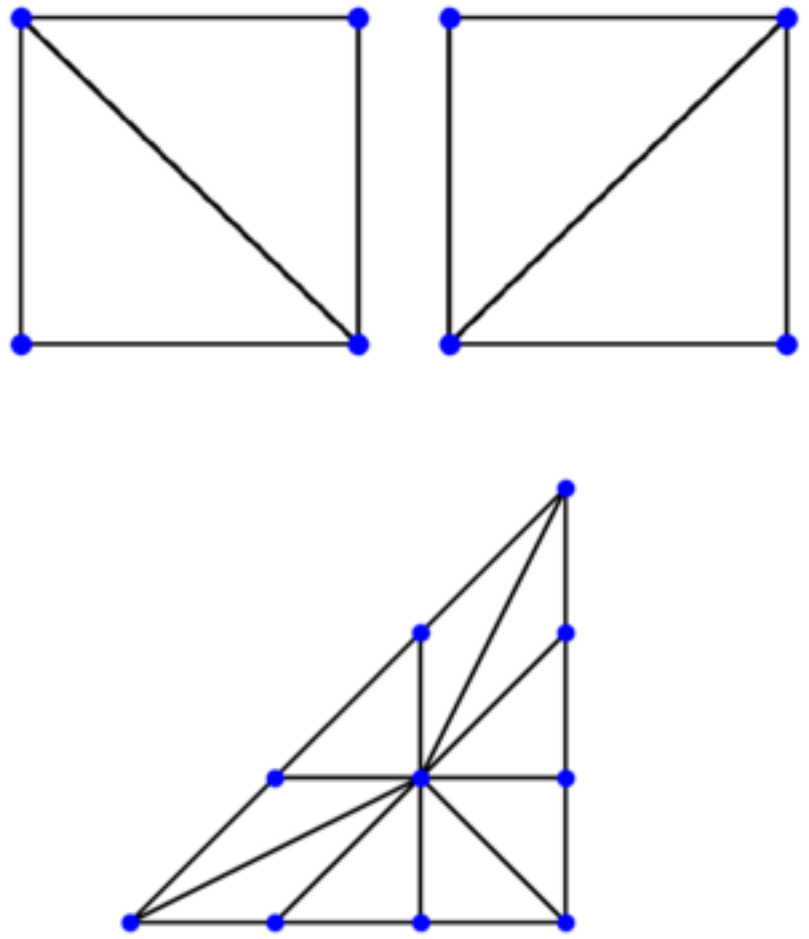
$y^2 = x^3 + fx + g$

"projective spaces" (toric varieties) constructed combinatorially: Batyrev '94



473,800,776 (reflexive) polytopes in 4d
Kreuzer, Skarke '00

triangulate 4d lattice polytopes



$\lesssim 10^{428}$ Calabi-Yau threefolds
Demirtas, McAllister, Rios-Tascon '20

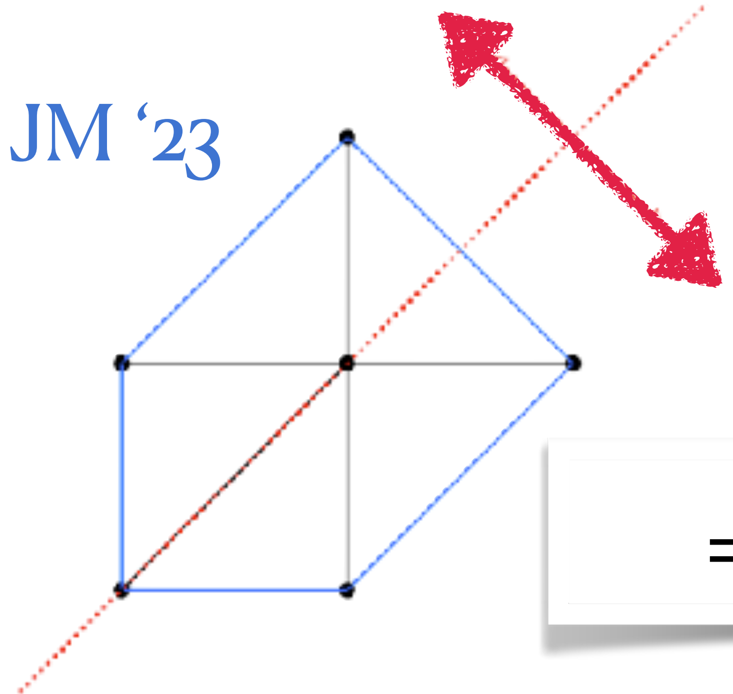
Demirtas, Rios-Tascon, McAllister '22

Ever growing open source library of algorithms to treat and analyze Calabi-Yau hypersurfaces

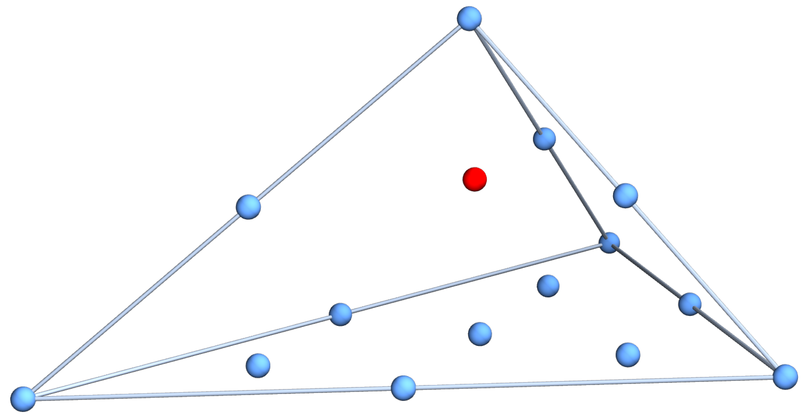
e.g. Demirtas, Kim, McAllister, JM, Rios-Tascon '23

To define minimally supersymmetric EFTs: reflection-symmetries of Calabi-Yau's

JM '23



allows studying previously inaccessible regime of generic Calabi-Yau's



$\Rightarrow$ construct explicit models in $\mathcal{O}(ms)$

Step 2: How to compute the superpotential

Key task: evaluate superpotential $W(z,\tau) = (\vec{f} - \tau\vec{h}) \cdot \vec{\Pi}(\vec{z})$ ⟷ Compute periods $\vec{\Pi}(\vec{z})$ of Calabi-Yau threefold

Famous result of the 1990s:

Candelas, de la Ossa '90

$$\vec{\Pi}(\vec{z}) = \vec{\Pi}_{classical}(\vec{z}) + \vec{\Pi}_{instanton}(\vec{z})$$

determined by "classical" integers

determined by "quantum" integers

computation (in principle; simplest examples): Hosono, Klemm, Theisen, Yau '94

computation (in practice; general case): Demirtas, Kim, McAllister, JM, Rios-Tascon '23

Step 3: Vacua with small superpotential

Demirtas, Kim, McAllister, JM '19

A restricted ansatz of fluxes yields quadratic superpotential:

$$W(z,\tau) = \frac{1}{2}(\tau \quad \vec{z}) \cdot \mathbf{N} \cdot \begin{pmatrix} \tau \\ \vec{z} \end{pmatrix} + \mathcal{O}\left(e^{-S_{instanton}}\right)$$

integer matrix computed from pair of integer flux vectors $\vec{M}, \vec{K}$

Because of Dirac quantization!

Vacuum solutions $W = dW = 0$ ⟷ Solutions of Diophantine equation $\det \mathbf{N} = 0$

One-dimensional solution space

$$\vec{z} = \vec{p}\,\tau\,, \quad \vec{p} \in \mathbb{Q}^N$$

"Perturbatively Flat Vacua" (PFVs)

$$W_{eff} = \mathcal{O}(e^{-S_{instanton}})$$

PFVs = EFTs of single light modulus τ and superpotential generated by instantons

Key feature:

All perturbative contributions to W are cancelled dynamically.

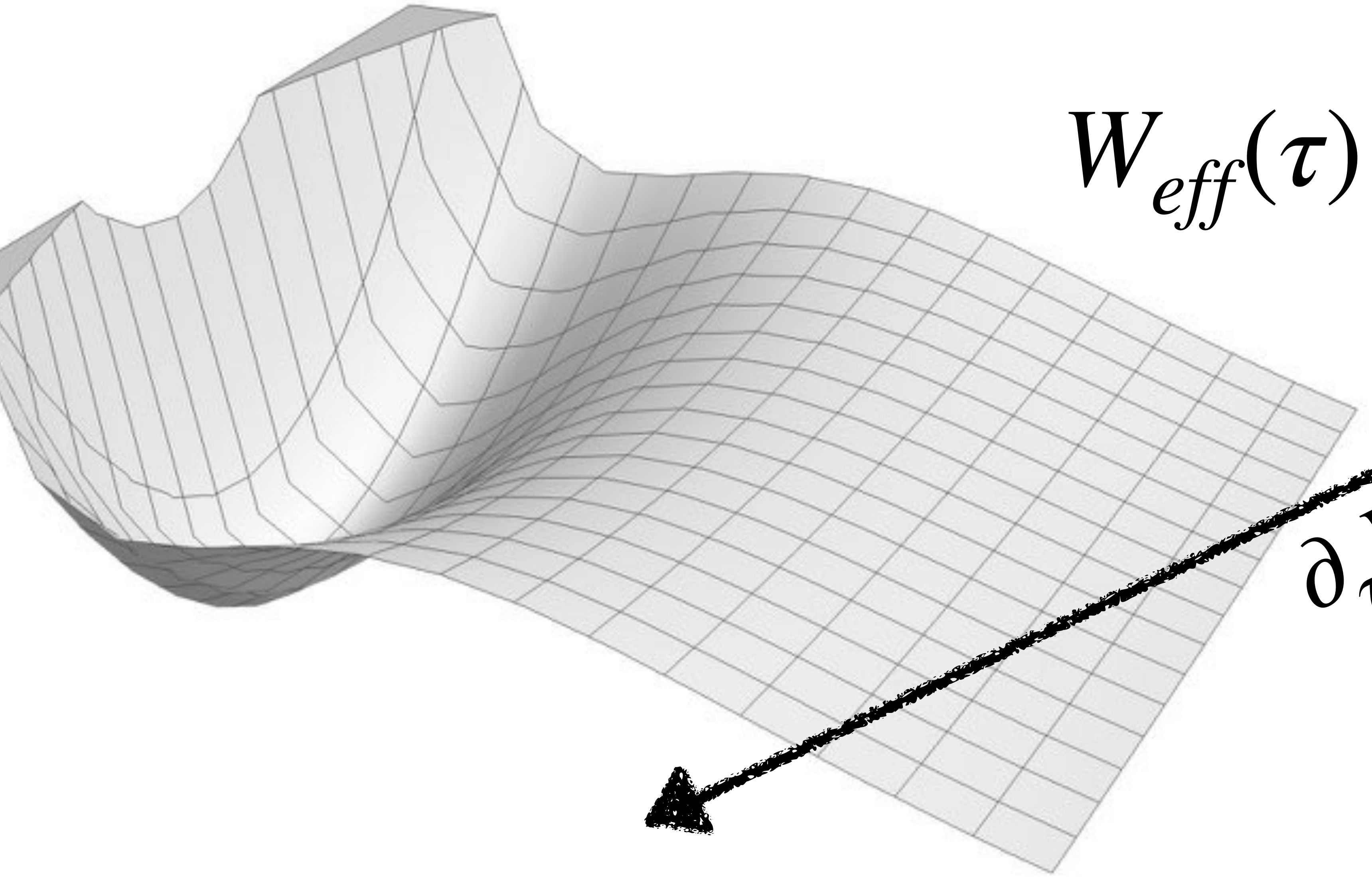
↓

Scale of vacuum energy set entirely by non-perturbative physics!

Stabilizing the flat direction:

"Racetrack" superpotential

$$W_{eff}(\tau) = Ae^{2\pi i\alpha\,\tau} - Be^{2\pi i\beta\,\tau} + \dots$$



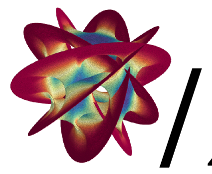
$\partial_\tau W_{eff} = 0$

$$\langle e^{2\pi i\tau}\rangle = \left(\frac{A}{B}\right)^{\frac{1}{\beta-\alpha}}$$

exponentially small

exponential hierarchy problem reduced to polynomial tuning

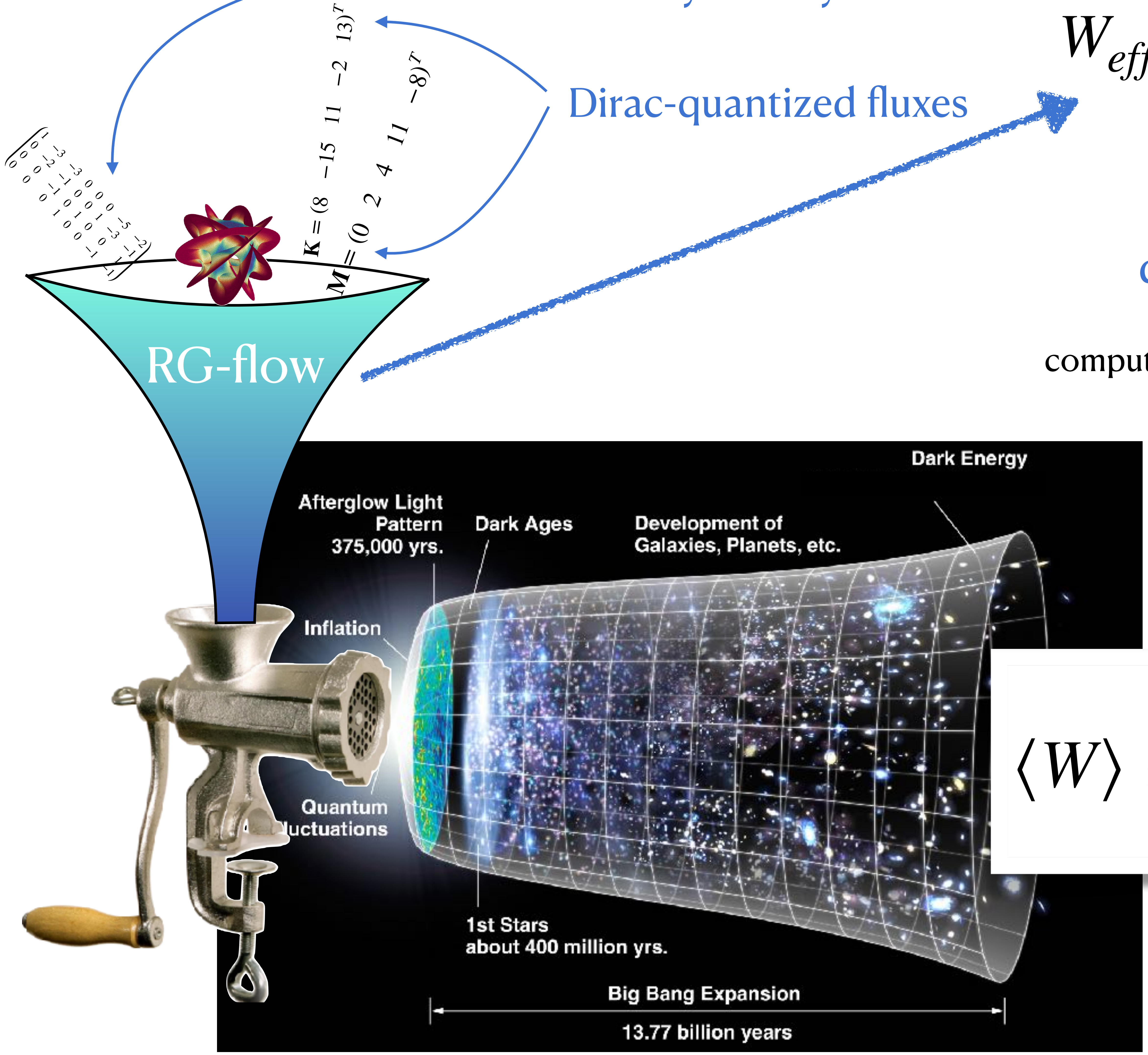
$A \ll B \quad \& \quad \alpha \approx \beta$

RG-flow: $\{$  $/Z_2\,, \vec{M}\,, \vec{K}\}$ → Racetrack model (A, B, α, β)

A real example:

Calabi-Yau and reflection symmetry

Dirac-quantized fluxes



$$W_{eff}(\tau) \approx 0.16 \times \left(-2 e^{2\pi i \tau \cdot \frac{7}{29}} + 252 e^{2\pi i \tau \cdot \frac{7}{28}} \right) + \mathcal{O}\left(e^{2\pi i \tau \cdot \frac{43}{116}}\right)$$

quantum integers

Aligned exponents

computed using Demirtas, Kim, McAllister, JM, Rios-Tascon '23

$$\langle W \rangle \approx 0.53 \times \left(\frac{2}{252} \right)^{29} \approx 6.5 \times 10^{-62}$$

Demirtas, Kim, McAllister, JM, Rios-Tascon '21

We have closely followed a proposal for de Sitter vacua in string theory made in 2003 by Kachru, Kallosh, Linde and Trivedi:

1. a Calabi-Yau threefold X ✓
2. a holomorphic O3/O7 orientifold projection $(-1)^{F_L} \circ \Omega \circ (z^\alpha \mapsto f^\alpha(z))$ ✓
3. a choice of threeform fluxes yielding a very small flux superpotential:

$$W_0 \ll 1$$ ✓

4. sufficiently generic non-perturbative corrections to the superpotential. ✓ I didn't show you
5. an F-term vacuum for Kähler moduli. ✓ I didn't show you
6. a warped throat region with redshift of scales of order $|W_0|$, hosting a supersymmetry breaking anti-D3 brane state. Not yet!

Let's see how the final ingredient can be realized!

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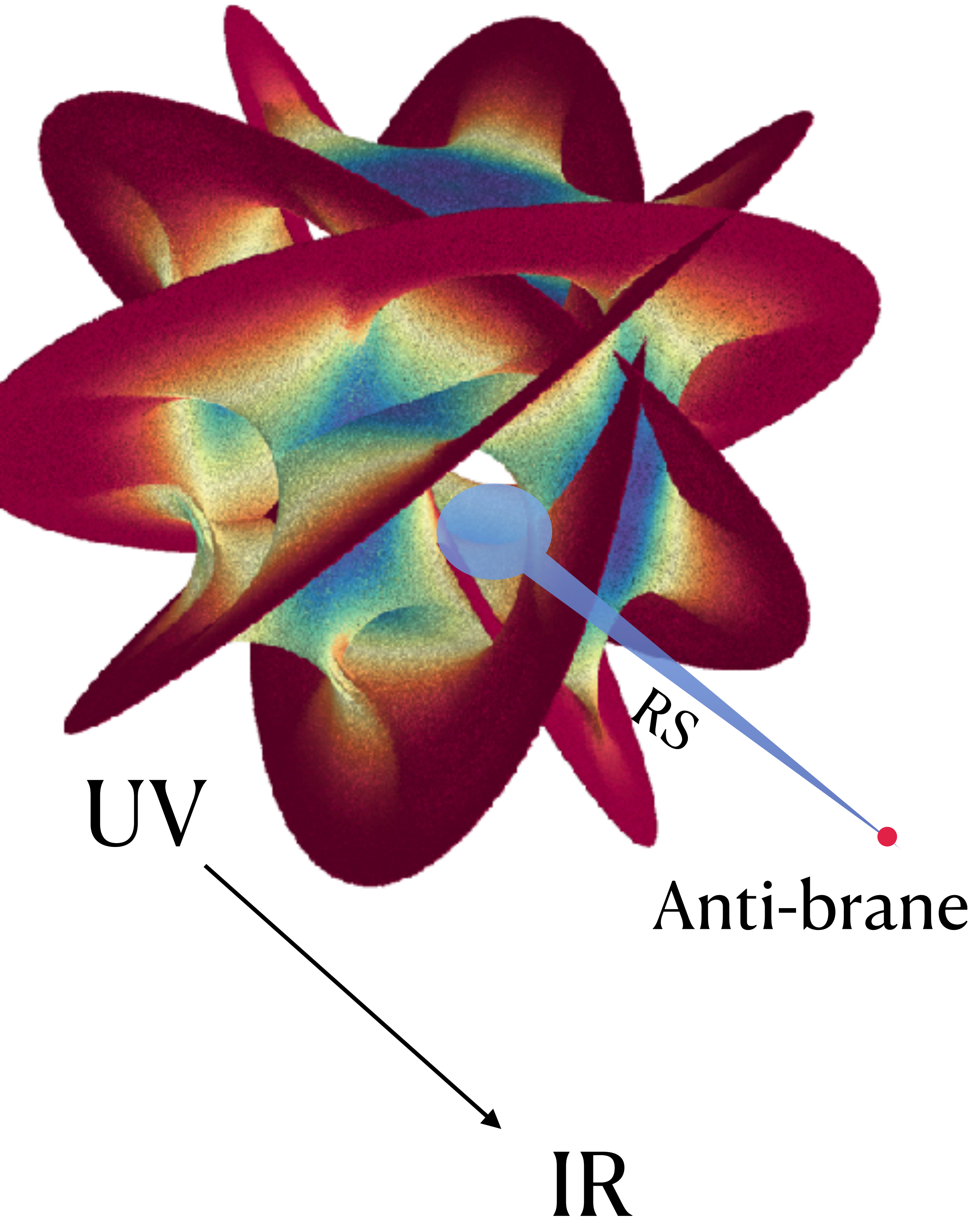
Engineering Warped Throats

For an "Uplift" to de Sitter we have to change our setup in some regards.

First, instead of stabilizing at large complex structure, we need to stabilize them near a conifold singularity in moduli space.



$$ds^2 = e^{2\mathcal{A}(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2\mathcal{A}(y)}g_{mn}dy^m dy^n$$

$$e^{2\mathcal{A}_{IR}} \sim |z|^{\frac{2}{3}}$$

Klebanov, Strassler '00
Giddings, Kachru, Polchinski '01

distance from conifold locus in moduli space

For a single Anti-D3 brane to raise the vacuum energy to positive values, without causing a decompactification instability, we need

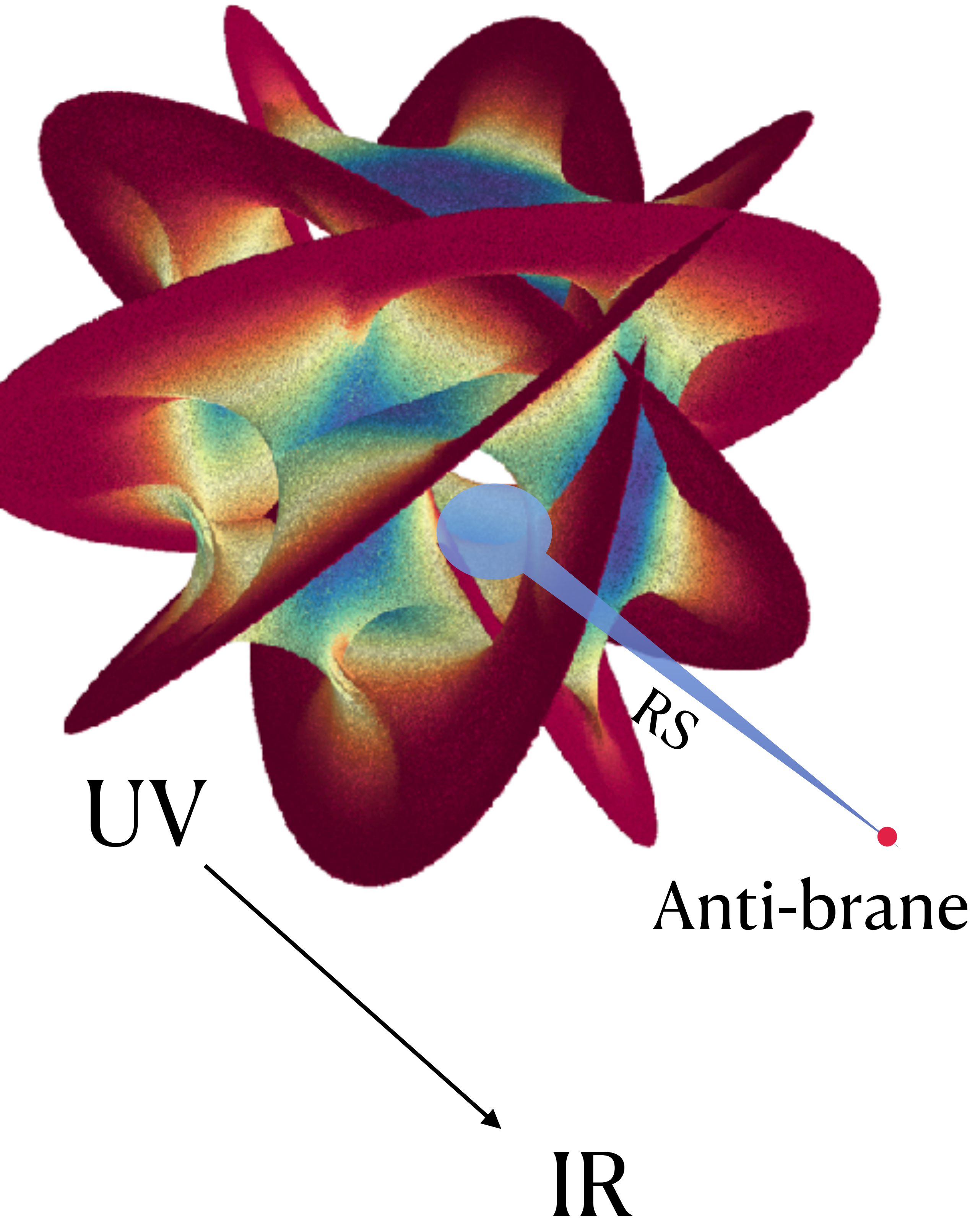
$$\frac{|z|^{\frac{4}{3}}}{(g_s M)^2} \approx \underbrace{5.5 \times 10^{-3}}_{\text{from KS solution}} \times \frac{|W_0|^2}{\mathcal{V}_E^{\frac{2}{3}}\tilde{\mathcal{V}}_s^{\frac{1}{3}}} \ll 1 \qquad M := \int_{\text{Conifold } S^3} F_3$$

Therefore we need to stabilize moduli such that both z and W_0 are small!

Engineering Warped Throats (actually doing it)

One can compute the superpotential systematically, order by order in z :

Álvarez-García, Blumenhagen, Brinkmann, Schlechter'20
Demirtas, Kim, McAllister, JM '20



$$W_{GVW}(z, z^{\alpha}, \tau) = W_{\rm bulk}(z^{\alpha}, \tau) + z\, W^{(1)}(z, z^{\alpha}, \tau) + \mathcal{O}(z^2)$$

The conifold F-term is solved for

$$\langle |z| \rangle = \frac{1}{2\pi} \exp\left(-\frac{2\pi}{g_s M^2} Q_{D3}^{\rm throat} \right)$$

$$Q_{D3}^{\rm throat} := -\frac{1}{2}\vec{\mathbb{M}} \cdot \vec{\mathbb{K}} - \langle \vec{\mathbb{M}}, \vec{\mathbb{M}} \rangle$$

Meta-stable Anti-D3 brane

In addition to constructing a strongly warped throat, one needs to ensure meta-stability of an Anti-D3 brane uplift.

At leading order in α' this requires $M > 12$, Kachru, Pearson, Verlinde '01

and controlling α'-corrections to KS requires $g_s M \overset{!}{\gtrsim} 1$

Requiring an uplift to de Sitter then severely limits our computational control:

$$\frac{1}{(2\pi)^{4/3}(g_s M)^2} \exp\left(-\frac{8\pi}{3 g_s M^2} Q_{D3}\right) < \frac{|z|^{\frac{4}{3}}}{(g_s M)^2} \approx \underbrace{5.5 \times 10^{-3}}_{\text{from KS solution}} \times \frac{|W_0|^2}{\mathcal{V}_E^{\frac{2}{3}} \tilde{\mathcal{V}}_s^{\frac{1}{3}}} \ll 1$$

E.g., for the largest D3-charge possible in known Calabi-Yau threefolds, $Q_{D3} = 252$ and control parameters $1/(g_s M) = g_s = 0.2$, typical values for volumes, this bound is saturated for $W_0 = 10^{-2}$ …

cf. Bena, Dudas, Graña, Lüst '18

Gao, Hebecker, Schreyer, Venken '22

McAllister, JM, Nally, Schachner '24

Everything, Everywhere, All at Once

Kwan, Scheinert'22

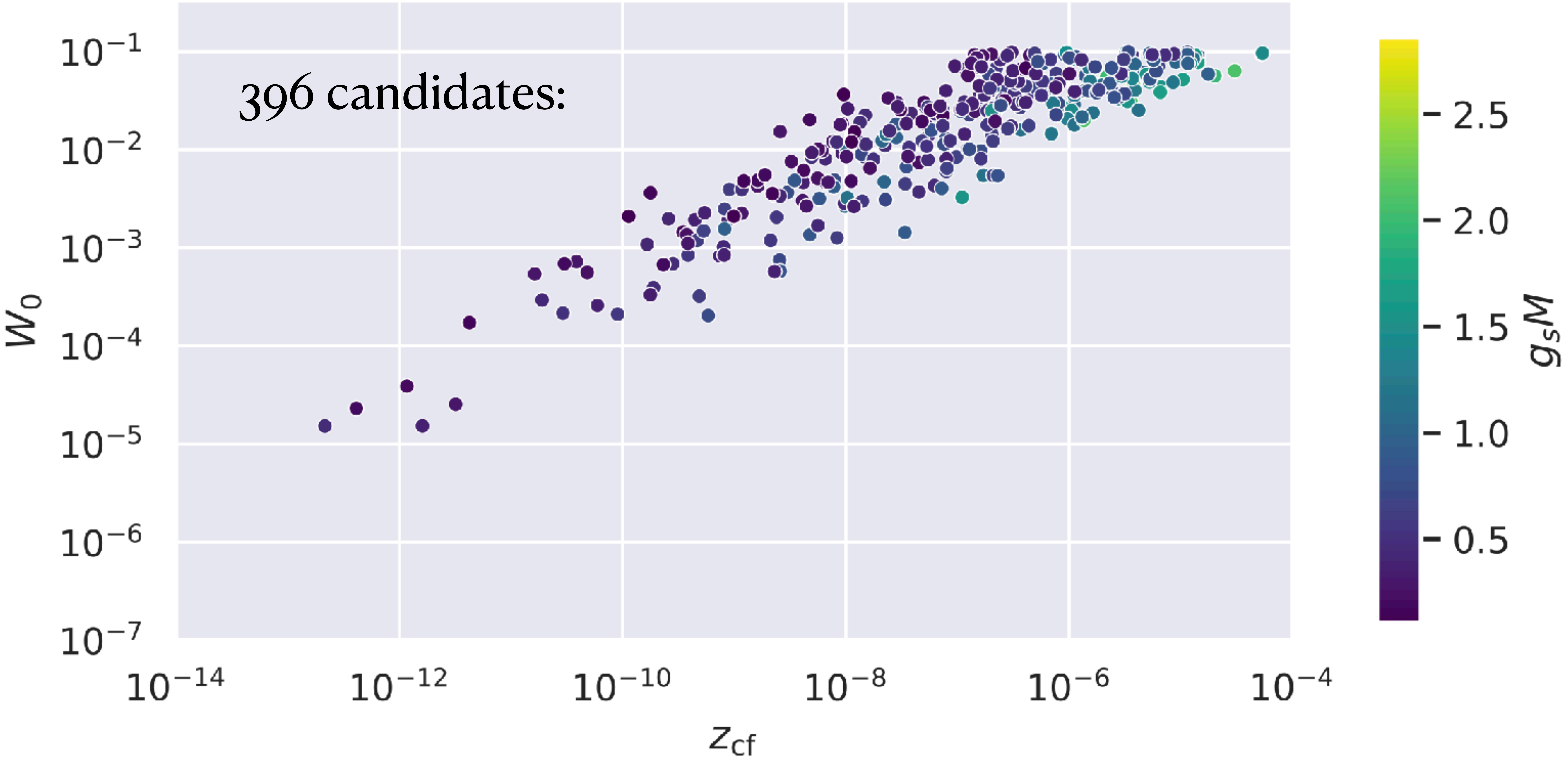
So far, we have understood all components of the KKLT proposal separately.

But, finding fully concrete solutions that feature them all, has required sifting through a substantial set of candidates:

- 202,703 polytopes in Kreuzer-Skarke in range $3 \leq h^{2,1} \leq 8$
- 3,187 favorable polytopes admitting an orientifold with $h^{1,1}_{-} = h^{2,1}_{+} = 0$
- 322 polytopes yielding large D3-charges $Q_{D3} \geq 100$, and hosting enough rigid divisors.
- 416 Calabi-Yau orientifolds with suitable conifold limits (i.e., that arise away from O-planes).
- 240,480,253 vacua with conifolds.
- 33,371 vacua with $Q_{D3}^{\mathrm{flux}} = Q_{D3} + 1$ and $M > 12$

McAllister, JM, Nally, Schachner '24

In the remaining set of "only" 33,371 vacua one still has to select those in which the generically unrelated scales of the warped throat, and the bulk superpotential, match:



McAllister, JM, Nally, Schachner '24

Even so, 30 good examples make it through to the end, and here is one of them:

$$g_s = 0.0657\,,$$
$$W_0 = 0.0115\,,$$
$$z_{\rm cf} = 2.822\times 10^{-8}\,,$$
$$g_s M = 1.051\,.$$

$$\mathcal{V}_{\rm E} = g_s^{-3/2}\mathcal{V} \approx 3.646\times 10^4$$

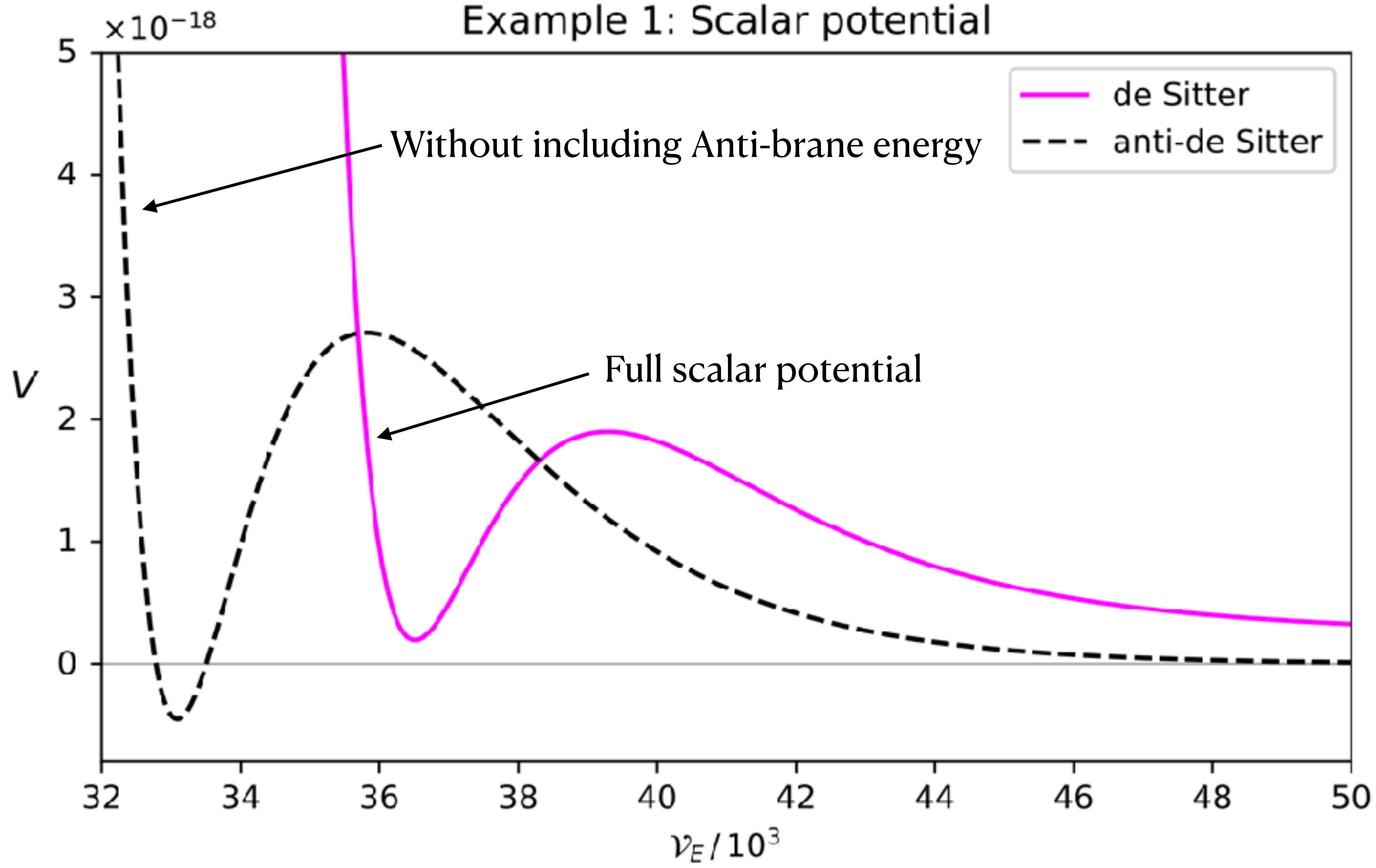
Including the contribution of the anti-D3 brane, the vacuum energy is positive:

$$\rho_{\rm vacuum} \approx +1.937\times 10^{-19} M_{\rm pl}^4$$

$$h^{1,1} = 150 \qquad h^{2,1} = 8$$

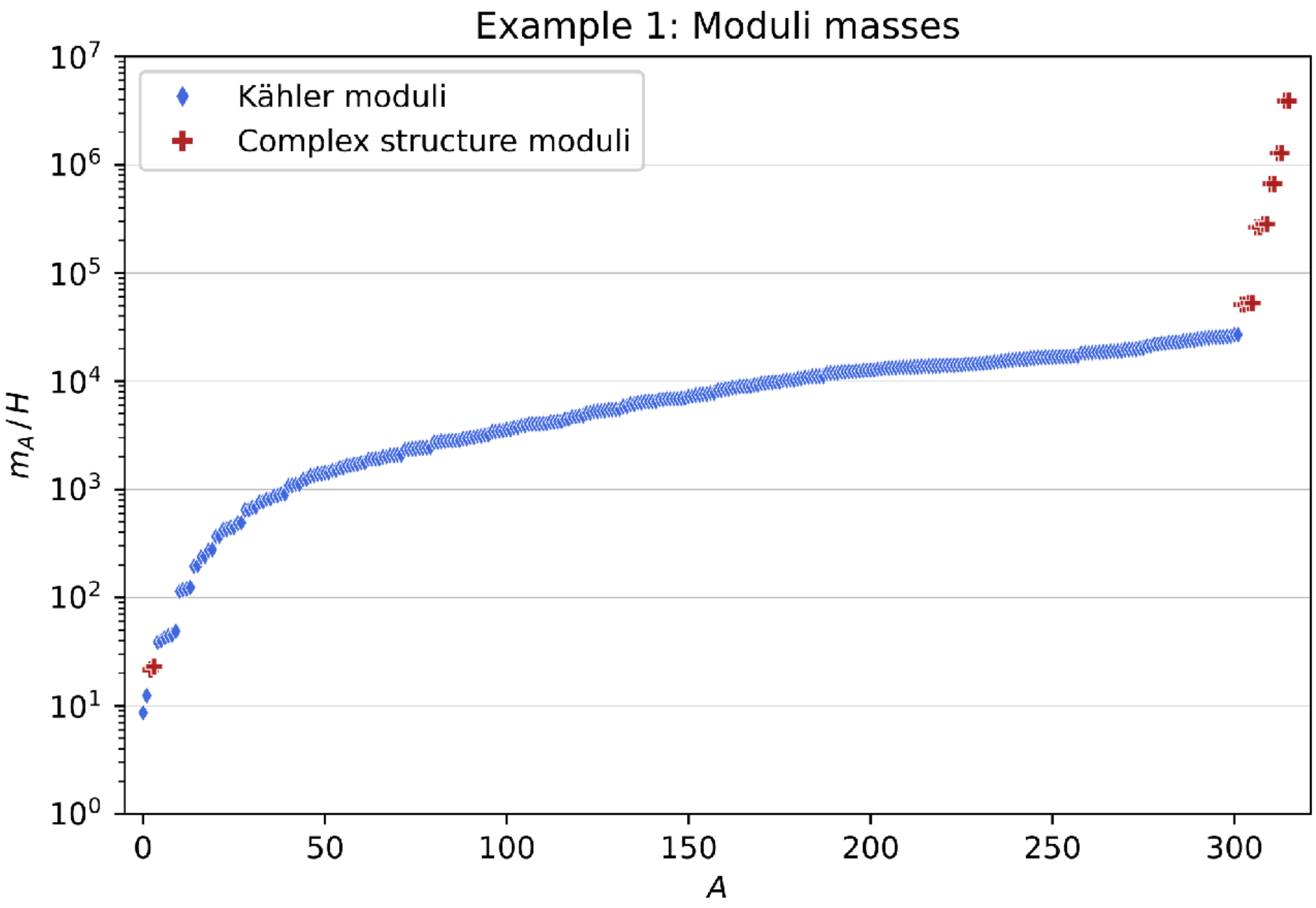
$$\mathbb{M} = \begin{pmatrix} 16 & 10 & -26 & 8 & 32 & 30 & 18 & 28 \end{pmatrix}$$

$$\mathbb{K} = \begin{pmatrix} -6 & -1 & 0 & 1 & -3 & 2 & 0 & -1 \end{pmatrix}$$



McAllister, JM, Nally, Schachner '24

… and the vacuum is free of tachyons:



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At first sight, the perhaps most serious issue with our solutions is that there is no parametric control over α' corrections, whatsoever!

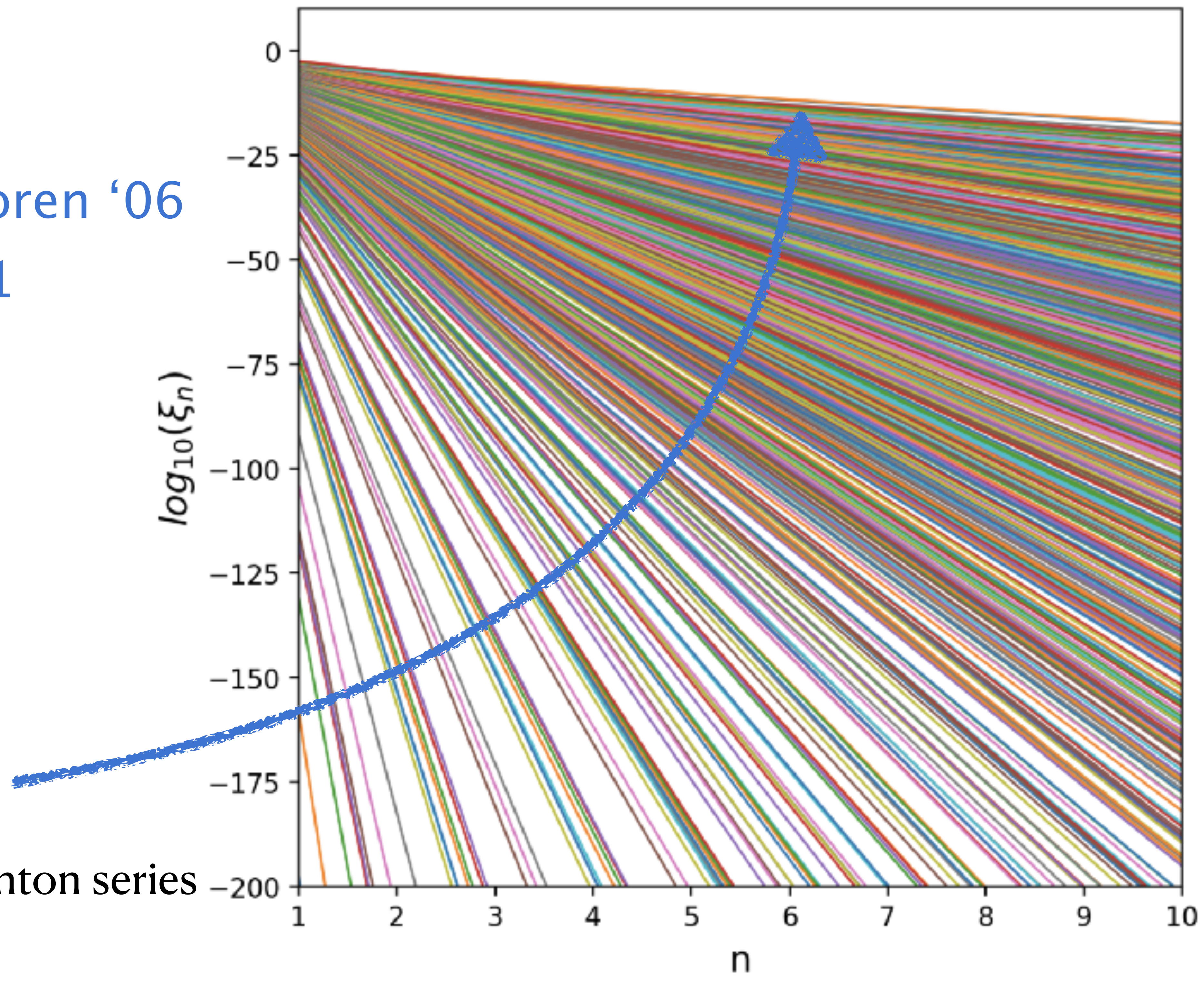
For small superpotential, Einstein-frame volumes become large, but simultaneously the string coupling becomes small...

Fortunately, to leading order in the string coupling, all α' corrections to the Kähler potential are inherited from the N=2 parent compactification, and are thus computable using mirror symmetry:

Becker, Becker, Haack, Louis '02
Robles-Llana, Rocek, Saueressig, Theis, Vandoren '06
Demirtas, Kim, McAllister, JM, Rios-Tascon '21

$$\delta K|_{\mathcal{O}(1/g_s^2)} = \delta K_{\alpha'^3} + \delta K_{\text{worldsheet instantons}}$$

Actually computing them was a serious undertaking, but we were able to this, and consistently incorporate them in evaluating the F-term potential.



Negative slopes
↕
Converging worldsheet instanton series

The control parameters in these solutions are the best we could do in 2024, but can conceivably be improved.

The perhaps most vulnerable aspect of these constructions is the question of meta-stability of the warped anti-D3 state. At tree level in α' we satisfy all constraints ...

... but recent computations of α' corrections to the anti-D3 brane imply that our throat radii are not large enough to safely ignore them.

Hebecker, Schreyer, Venken '22
Schreyer, Venken '22
Gao, Hebecker, Schreyer, Venken '22
Schreyer '24

The question of meta-stability of the uplift in the regime $g_s M \sim 1$ remains an important open problem!

Similarly, the string coupling is not extremely small, and Einstein-frame cycle volumes are not impressively large. Simple models of loop and warping corrections to the Kähler potential suggest $\mathcal{O}(20-30\%)$ corrections.

Further, while relevant perturbations to the KS-throat are parametrically negligible when $|z|^{2/3} \sim |W_0| \to 0$, one needs to check numerically how it turns out in our example(s). This requires knowing the CY-metric…

Finally, in orientifolds of tori, odd integer quantized fluxes lead to exotic O3 planes, related to the existence of "twisted cycles" ~ $T^3/\mathbb{Z}_2$ Frey, Polchinski '02

Whether odd fluxes are allowed in our Calabi-Yau orientifolds, or if one has to adapt the search to find all even fluxes remains to be understood.

Plan:

1. Some Motivation & Introduction
2. Anti de Sitter Vacua with small superpotential
3. Warped throats and “Uplift” to de Sitter: an example
4. Control over corrections
5. Conclusions

Main takeaway: we have constructed the first explicit de Sitter solutions in type IIB string theory along the lines anticipated by Kachru, Kallosh, Linde and Trivedi in '03, by

1. computing superpotentials from fluxes and D3-instantons using toric geometry, and enumerative invariants,
2. finding vacua by solving Diophantine equations in flux quanta, and identifying F-term solutions in low energy effective theory featuring explicit racetrack superpotential,
3. explicitly constructing warped throat regions suitable for anti-D3 uplift to de Sitter,
4. identifying the F-term minima in Kähler moduli (via following a discretized BPS attractor flow in the extended Kähler cone).

This is not the last word on this subject...

... within constraints set by D3-tadpole, one should be able to find better values for the control parameters.

Furthermore, one can improve control by better understanding the structure of corrections along lines of recent work

Alexandrov, Firat, Kim, Sen, Stefanski '22
Gendler, Kim, McAllister, JM, Stillman '22
Liu, Minasian, Savelli, Schachner '22
Hebecker, Schreyer, Venken '22
Schreyer, Venken '22
Gao, Hebecker, Schreyer, Venken '22
3x Kim '23
Cho, Kim '23
Schreyer '24 ...
Kim '24

Thank you!

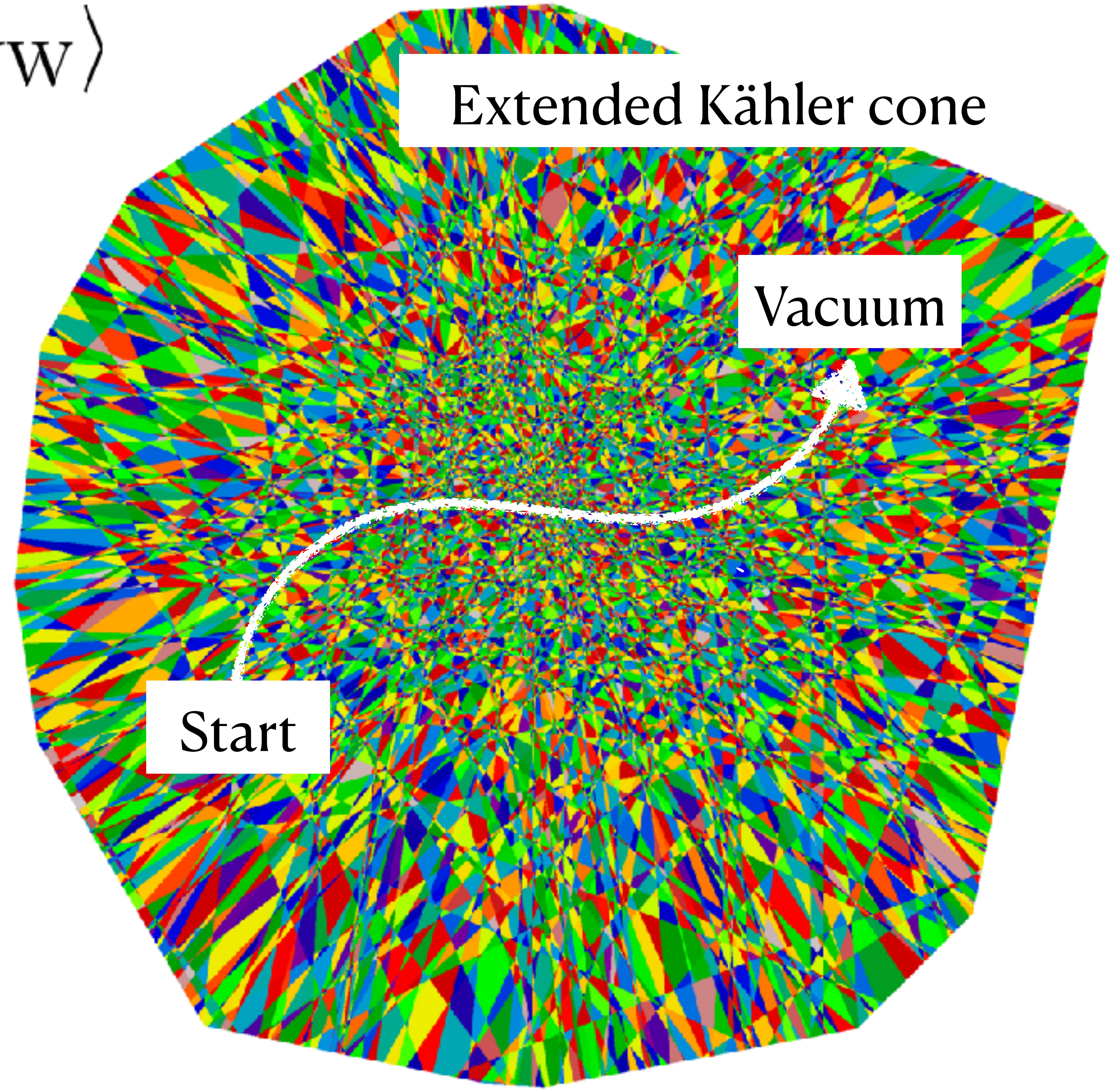
Kähler moduli stabilization

Given non-perturbative contributions to superpotential (of full rank) one expects Kähler moduli to be stabilized near

$$\langle \mathrm{Re}(T_i) \rangle \sim \frac{\log(|W_0|^{-1})}{2\pi} \quad \text{with} \quad W_0 := \langle W_{\mathrm{GVW}} \rangle$$

It is useful to first find this point, by following a BPS attractor flow of sorts, starting from any point in Kähler moduli space.

Once one arrives at this point, one typically is close enough to the minimum, such that straightforward methods such Newton's method can be successfully implemented to find the vacuum solution numerically.



An Anti de Sitter vacuum with even fluxes

Here is an example of a supersymmetric flux vacuum in which all fluxes are even:

A Calabi-Yau hypersurface with Hodge numbers $h^{1,1} = 85$ and $h^{2,1} = 5$

leads to a "PFV" with $\vec{z} = \frac{1}{22}\begin{pmatrix}21 & 1 & 44 & 50 & 32\end{pmatrix}\tau$

For flux choice: $\mathbb{M} = 2\begin{pmatrix}10 & -11 & 1 & -4 & 0\end{pmatrix}$ $\mathbb{K} = 2\begin{pmatrix}7 & 9 & -2 & -2 & 1\end{pmatrix}$

The resulting effective superpotential reads

$$W_{\text{eff}}(\tau) = \xi \cdot \left(-2e^{2\pi i \frac{21}{22}\tau} - 200e^{2\pi i\tau} - 20e^{2\pi i \frac{23}{22}\tau} + \dots\right), \quad \xi = \frac{\sqrt{2/\pi}}{(2\pi)^2}$$

And leads to a vacuum with

$g_s \approx 0.06$ $W_0 \approx 7 \times 10^{-46}$

After stabilizing Kähler moduli:

$\rho_{\text{vacuum}} \approx -1.34 \times 10^{-108} M_{\text{pl}}^4$ $\mathcal{V}_E \approx 1.2 \times 10^6\, \ell_s^6$

Distribution of conifold fluxes: