Symmetries of gravitational scattering in absence of peeling

Based on 2407.07978 with M. Geiller and A. Laddha

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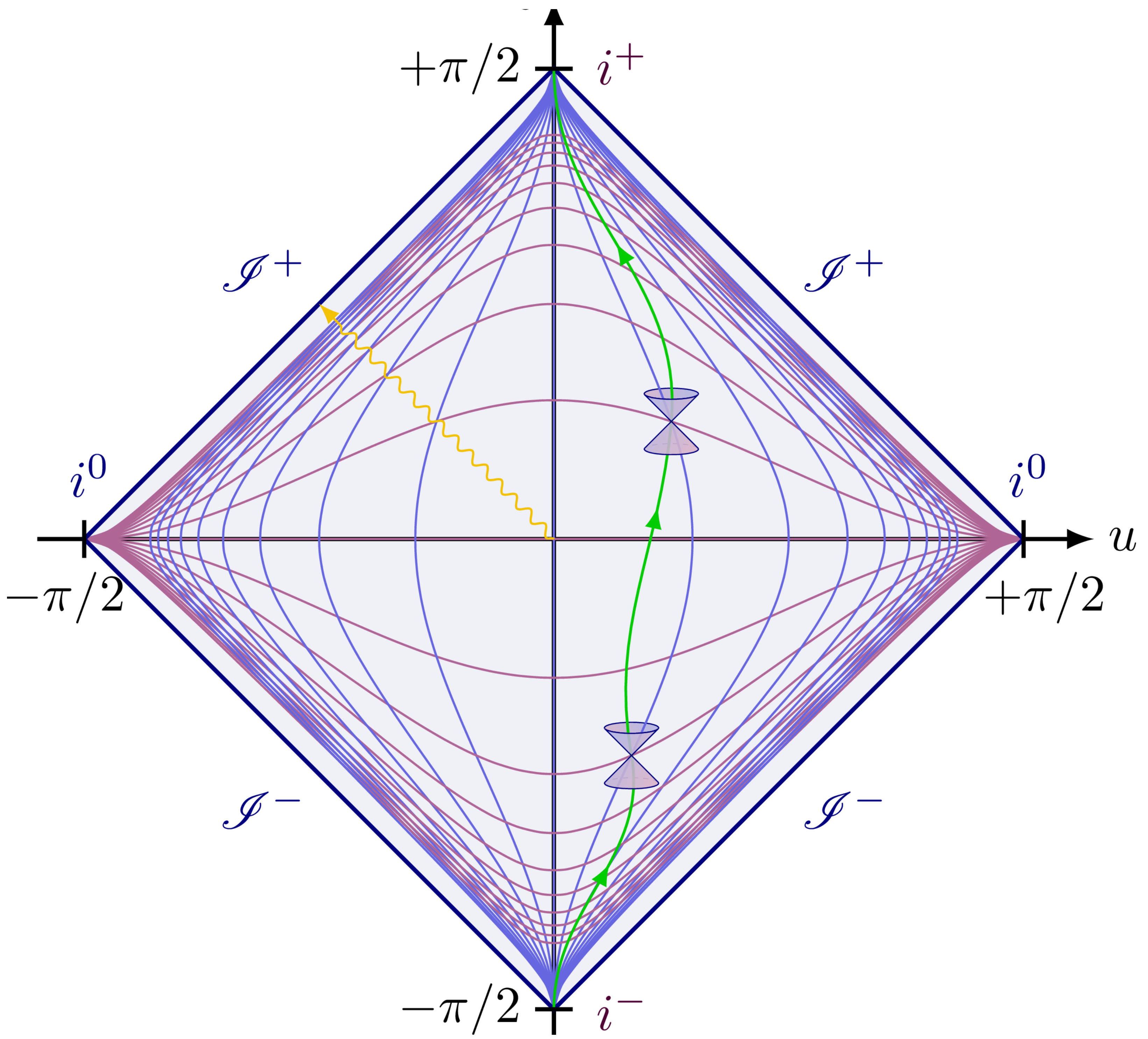
Introduction to Bondi gauge

Introduction

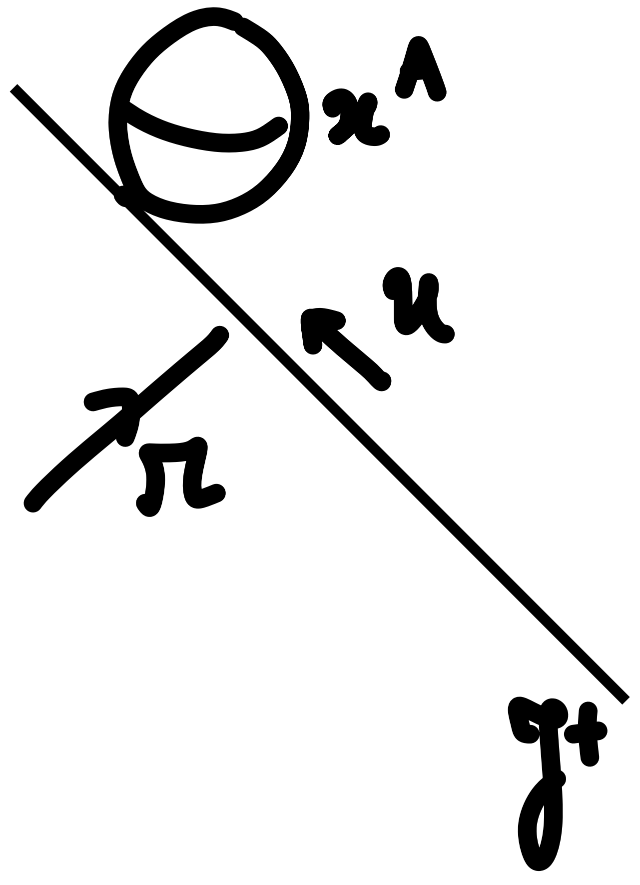
- Asymptotic states (scattering amplitudes)
- Conformal compactification: brings the infinite boundary to a finite distance by a conformal transformation
- Penrose diagram
- Focus on radiation and therefor on future null infinity
  -> adapted gauge: Bondi gauge
- Asymptotic boundaries allow to define a notion of energy (or other observables) for gravity



from I. Neutelings on TikZ.net

Bondi gauge [H. Bondi, M. G. J. van der Burg and A. W. K. Metzner ’62; Sachs ’61]

Gauge adapted to future null infinity



1. Retarded time u is a null coordinate $g^{uu} = 0$

2. Angular coordinates $x^A = (\theta, \phi)$ are transverse to the null direction $g^{uA} = 0$

$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du\,dr + \gamma_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

3. Bondi determinant condition $\partial_r\left(\frac{\sqrt{\gamma}}{r^2}\right) = 0$ and $\gamma_{AB} = \mathcal{O}(r^2)$

Solving the eom will determine the radial dependence of (V, U^A, β)

$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du\,dr + \gamma_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

Asymptotically (assuming $\beta_0 = 0 = U_0^A = \partial_u q_{AB}$),

$$\gamma_{AB} = r^2\, q_{AB} + r\, C_{AB} + o(r)$$

$$\beta = -\frac{1}{r^2}C_{AB}C^{AB} + \mathcal{O}(r^3)$$

$$U^A = -\frac{1}{2r^2}D_B C^{AB} + o(r^{-3})$$

$$V = -\frac{R}{2} + 2M + o(1)$$

C_{AB} is the shear, M the mass, R the Ricci of q_{AB}

BMS & Asymtotic symmetries

- Asymptotic symmetries preserved the choice of gauge and boundary conditions
- The generator associated to the asymptotic symmetries are **charges**, which carries physical information such as the mass of spacetimes, …
- They satisfy an algebra which is a entry of the holographic dictionary -> use in bottom-up approach of holography

BMS & Asymptotic symmetries

$\{g_{BC-Bondi} + \mathscr{L}_\xi g_{BC-Bondi}\} = \{g_{BC-Bondi}\}$

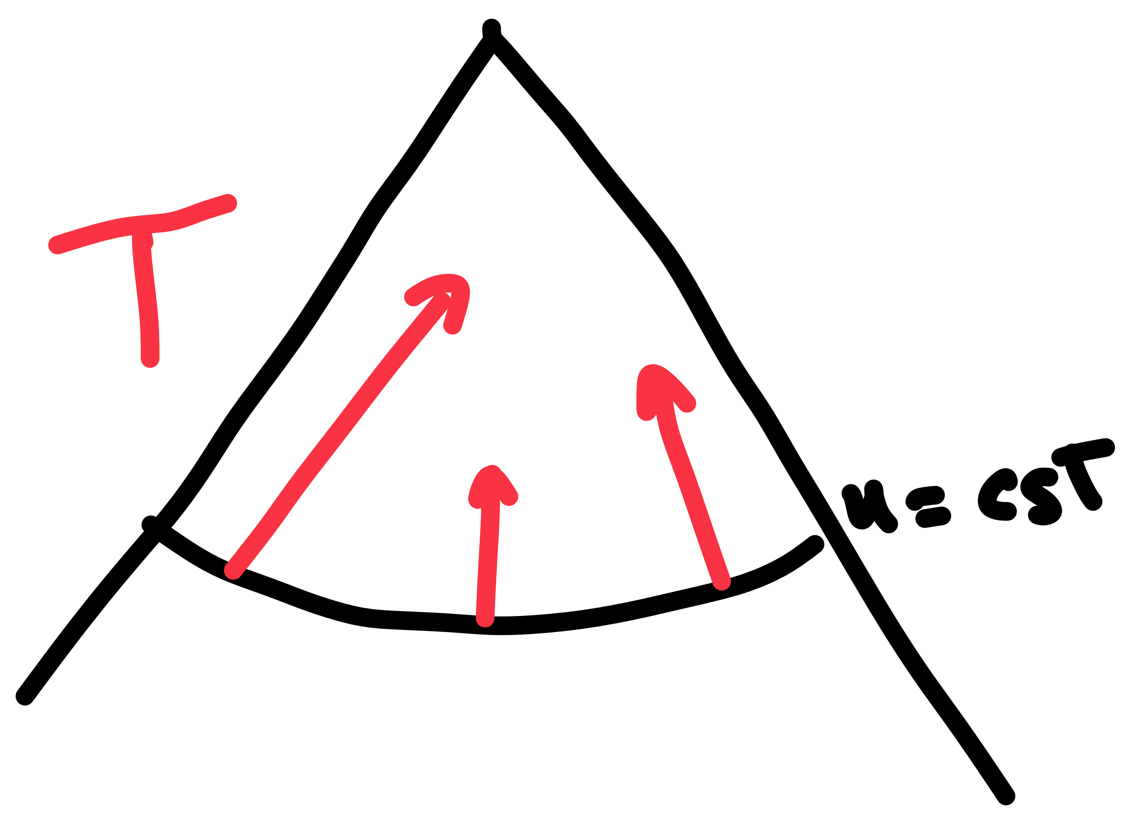
Ex: $g_{rr} = 0 \Rightarrow \mathscr{L}_\xi g_{rr} = 0 \Rightarrow \xi^u = f(u, x^A)$

Imposing the rest of gauge conditions and boundary conditions we obtain

$\xi^u = f(u, x^A) = T(x^A) + u\, W(x^A)$

$\xi^r = -\, r\, W(x^A) + \frac{r}{2}(U^A \partial_A f - D_A \xi^A) = -\, r\, W(x^A) + \frac{1}{2}\Delta f + \ldots$

$\xi^A = Y^A - \frac{1}{r}\partial^A f + \ldots$



T= supertranslation

Y^A = superrotations

W = Weyl rescaling

BMS algebra

$\xi(T(x^A), W(x^A), Y^A(x^A))$

$$[\xi(T_1, W_1, Y_1^A), \xi(T_2, W_2, Y_2^A)] = \xi(T_{12}, W_{12}, Y_{12}^A) \quad \text{with} \quad \begin{aligned} T_{12} &= T_1\, W_2 + Y_1^A \partial_A T_2 - (1 \leftrightarrow 2) \\ W_{12} &= Y_1^A \partial_A W_2 - (1 \leftrightarrow 2) \\ Y_{12}^A &= Y_1^B \partial_B Y_2^A - (1 \leftrightarrow 2) \end{aligned}$$

(Diff$(S^2)(Y^A)$ semi-direct sum $\mathbb{R}(W)$) semi-direct sum $\mathbb{R}(T)$ = BMSW

[Barnich-Troessaert ’10; Freidel, Oliveri Pranzetti, Speziale ’21]

If $\delta_\xi \sqrt{q} = 0$, then $W = \frac{1}{2} D_A Y^A$ and we talked of the generalized BMS group

[Campiglia-Laddha ’15, Compère, Fiorucci,Ruzziconi ’18]

If $\delta_\xi q_{AB} = 0$ and Y^A globally well-definied, you land on global BMS group

[H. Bondi, M. G. J. van der Burg and A. W. K. Metzner ’62; Sachs ’61]

Peeling property

Newman-Penrose formalism

- Introduction of a null tetrad $\ell^\mu = \partial_r, n^\mu = \partial_u - \frac{R}{4}\partial_r + \mathcal{O}(r^{-1})$,
  $m^\mu = \frac{1}{r} m_0^A \partial_A + \mathcal{O}(r^{-2})\,, \bar{m}^\mu = \frac{1}{r}\bar{m}_0^A \partial_A + \mathcal{O}(r^{-2})$
- $\sigma = -\frac{1}{2} C_{AB} m_0^A m_0^B$
- contraction of the Weyl tensor with the null tetrad = Weyl scalars = 5 complex numbers Ψ_n

Weyl Scalars

Ψ_4, Ψ_3 carries information about the radiation ($\Psi_4^0 \propto \partial_u^2 \sigma$, $\Psi_3^0 \propto m_0^A \partial_A \partial_u \sigma$)

Re(Ψ_2^0) the mass,

Ψ_1^0 the angular momentum

Ψ_0 the higher spin charges

Peeling property $\Psi_n = \dfrac{1}{r^{5-n}} \Psi_n^0 + \mathcal{O}(r^{n-6})$

BC compatible with the peeling = Asympt. Simple

$$\gamma_{AB} = r^2 q_{AB}\sqrt{1+\frac{\mathscr{C}_{AB}\mathscr{C}^{AB}}{2r^2}} + r\mathscr{C}_{AB}$$

$$r\mathscr{C}_{AB} = r\,C_{AB} + 0 + \sum_{n=1}^{\infty}\frac{1}{r^n}E^n_{AB}$$

$$U^A = -\frac{1}{2r^2}D_B C^{AB} + \frac{1}{r^3}N^A + \mathscr{O}(r^{-4})$$

$$V = -\frac{R}{2} + 2M + \mathscr{O}(r^{-1})$$

$$\Psi_4 = \frac{1}{r}\frac{1}{2}\partial_u^2 C_{AB}\bar{m}_0^A\bar{m}_0^B + \mathscr{O}(r^{-2})$$

$$\Psi_3 = \frac{1}{r^2}\frac{1}{2}D^B(\partial_u C_{AB})\bar{m}_0^A + \mathscr{O}(r^{-3})$$

$$\Psi_2 = \frac{1}{r^3}\left(M + \frac{1}{16}\partial_u C_{AB}C^{AB} + i\tilde{M}\right) + \mathscr{O}(r^{-4})$$

$$\Psi_1 = \frac{1}{r^4}\left(-\frac{3}{2}N_A + \frac{3}{32}D_A C_{BC}C^{BC} + \frac{3}{4}C_{AB}D_C C^{BC}\right)m_0^A + \mathscr{O}(r^{-5})$$

$$\Psi_0 = \frac{1}{r^5}\left(3E^1_{AB} - \frac{3}{16}\partial_A C_{BC}C^{BC}\right)m_0^A m_0^B + \mathscr{O}(r^{-6})$$

$$\Psi_n = \frac{1}{r^{5-n}}\Psi_n^0 + \mathscr{O}(r^{n-6})$$

Absence of the peeling

Logarithmically Asympt. Flat

$$\gamma_{AB} = r^2 q_{AB}\sqrt{1 + \frac{\mathscr{C}_{AB}\mathscr{C}^{AB}}{2r^2}} + r\mathscr{C}_{AB}$$

$$r\mathscr{C}_{AB} = r\,C_{AB} + D_{AB} + \sum_{n=1}^{\infty}\sum_{m}^{n+1}\frac{\ln(r)^m}{r^n}E^{n,m}_{AB}$$

$$U^A = -\frac{1}{2r^2}D_B C^{AB} + \frac{1}{r^3}(N^A + \ldots \ln r) + o(r^{-3})$$

$$V = -\frac{R}{2} + 2M + o(r^0)$$

$$\partial_u D_{AB} = 0$$

$$\Psi_4 = \frac{1}{r}\Psi_4^0 + \mathscr{O}(r^{-2})$$

$$\Psi_3 = \frac{1}{r^2}\Psi_3^0 + \mathscr{O}(r^{-3})$$

$$\Psi_2 = \frac{1}{r^3}\Psi_2^0 + \mathscr{O}(r^{-4})$$

$$\Psi_1 = \frac{1}{r^4}\left(\Psi_1^0 + D^B D_{AB} m_0^B \ln r\right) + o(r^{-4})$$

$$\Psi_0 = \frac{1}{r^4}D_{AB}m_0^A m_0^B + \frac{1}{r^5}\left((\Psi_0^0 + \ldots) + (3E^{1,1} + \ldots)\ln r + 3E^{1,2}_{AB}\ln r^2\right) + o(r^{-5})$$

[Winicour ’85; P. T. Chrusciel, M. A. H. MacCallum and D. B. Singleton ’93; Kroon ’98]
[L. Kehrberger and H. Masaood ‘24]

Why?

General relativity

- There has been a debate on ``Peeling or not peeling” [Friedrich ’17]

- Blanchet’s theorem: past stationarity & no incoming radiation -> No violation of peeling at future null infinity

- BUT typical radiative fields sourced by matter stress tensor yields violation of peeling (no past stationarity).
  For instance: hyperbolic encounters [Damour ‘86, Christodoulou ’02, Kehrberger ’21->’24, …]

What is the impact of violation of peeling on the symmetries, the charges, fluxes?

Why?
Soft theorems

- In a scattering process, the gravitational field has remarquable universal properties at the late/early time.

- Displacement memory effect: gravitational field is dominated to by a static mode which is fixed only by incoming and out coming momenta of the scattered particles at late/early time

  This is the same thing as Weinberg soft theorem.

$$C_{AB} = C^0_{AB}(x^A) + \frac{1}{u}C^1_{AB}(x^A) + o(u^{-1})$$

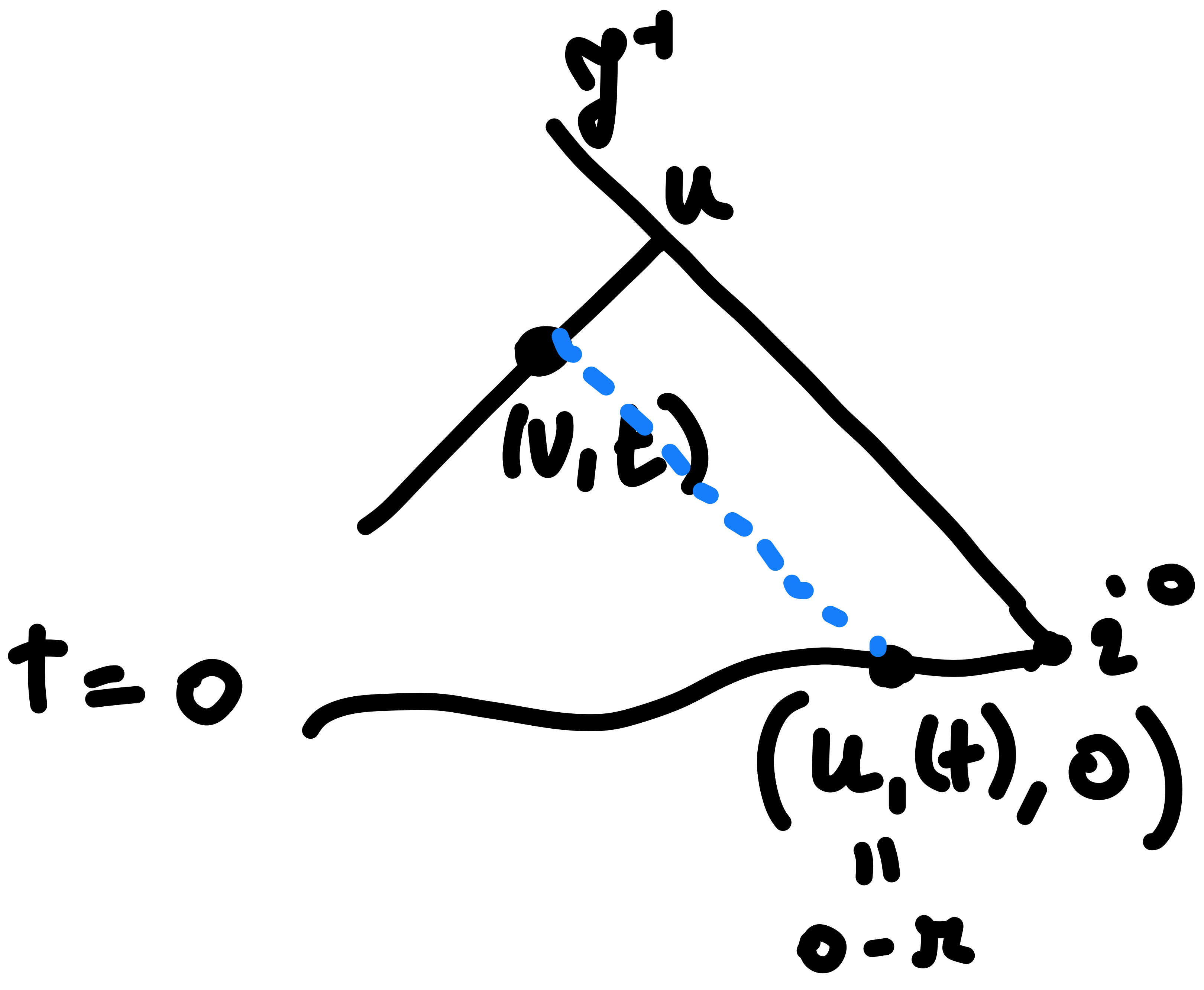
- Saha, Sahoo, Sen shows that the shear C^1_{AB} is also universal (incoming and out coming momenta of the scattered particles).

  This is the same thing as logarithmic soft theorem

Why?

Christodoulou's heuristic argument

Using the eom, Christodoulou showed that it will precisely yield a violation of peeling at the order Ψ_1



$$\sigma = \frac{C}{u} + \ldots \,,$$

$$\Psi_3^0 = -\,\eth\partial_u \sigma \,, \Psi_4^0 = -\,\partial_u^2 \sigma$$

$$\partial_u \Psi_2^0 = \eth \Psi_3^0 - \sigma \Psi_4^0 \quad \Rightarrow \quad \Psi_2^0 = -\,\frac{\eth^2 C}{u}$$

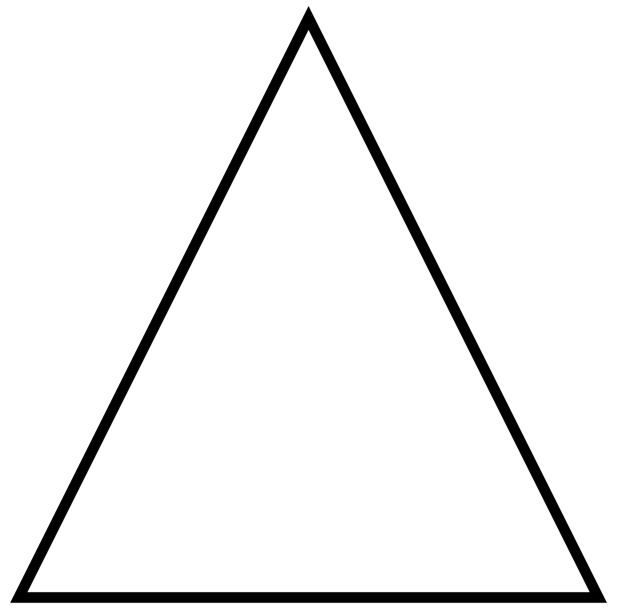
$$\partial_u \Psi_1^0 = \eth \Psi_2^0 - 2\sigma \Psi_3^0 \quad \Rightarrow \quad \partial_u \Psi_1^0 = -\,\frac{\eth^3 C}{u}$$

$$u = t - r$$

$$\int_{u_1}^{u} \Psi_1^0 = -\,\eth^3 C (\ln u - \ln r)$$

$\eth := m_0^A \partial_A$

Infrared triangle



- Shown for leading infrared triangle (supertranslations, displacement memory, leading soft theorem)

  [A. Strominger, ‘14; T. He, V. Lysov, P. Mitra and A. Strominger, ‘15; A. Strominger and A. Zhiboedov, ’16]

- Use of charges/fluxes

- Log soft theorems and $C_{AB} \sim \dfrac{1}{u}$ is shown to be related to superrotation Ward identity

  [S. Agrawal, L. Donnay, K. Nguyen and R. Ruzziconi ’23, S. Choi, A. Laddha and A. Puhm ’24]
  [L. Baulieu and T. Wetzstein ’24]

BUT based on BC & symm. preserving the peeling …

What is the impact of violation of peeling on symmetries, charges, fluxes?

Symmetries in Log. Asympt. Flat

- BMSW is still a symmetries (the derivation we used didn't depend on the expansion of γ_{AB})
- Charges (after renormalization), there is a contribution to supertranslation charges for generalized BMS

$$k_Y^{ur} = \bar{k}_Y^{ur} + 2\delta\left(\sqrt{q}Y^A D^B D_{AB}\right) - \frac{1}{2} D_C Y^C \sqrt{q} q^{AB} \delta D_{AB}$$

- Flux happens to be insensitive to the peeling!

Equations of motion

Evolution equations

With peeling

- Flux balance law for the mass and the angular momentum

$$\partial_u Re(\Psi_2^0) = m_0^A \partial_A \Psi_3^0 - \sigma \Psi_4^0$$

-> Related to leading and sub soft theorem

- Tower of flux balance laws for $E^{n,0}_{AB}$ (Ψ_0)
- Related to the tower of (sub)^#- soft theorems and $w_{1+\infty}$ algebra (in a sector)
- But not to local spacetimes symmetries (-> twistors)

Evolution equations

Log. Asympt. Flat

- Dressed eom for mass, angular momentum and $E^{n,0}_{AB}$
- Conserved logarithmic branches

$$\partial_u D_{AB} = 0\,, \qquad \partial_u E^{n,n+1}_{AB} = 0$$

- NEW flux balance law at sufficiently low order

$$\partial_u E^{3,2}_{AB} = \ldots + \frac{3}{8}\partial_u C_{AB}\, C^{CD} E^{1,2}_{CD} - \frac{1}{8} E^{1,2}_{AB} \partial_u (C_{CD} C^{CD}) + \frac{3}{16} E^{1,2}_{C\langle A} \partial_u C^{CD} C_{B\rangle D}$$

Sub log infrared triangle? (Wip)

Summary

Summary

- Some physical phenomena require a violation of peeling (Log. Asympt. Flat BC)
- Symmetries of scattering also require a violation of peeling (Log. Asympt. Flat BC)

- BMS still preserve Log. Asympt. Flat BC
- Superrotation charge is modified (for generalized BMS)
- Fluxes are NOT modified -> the connection to (leading) logarithmic soft theorem can be made assuming peeling
- New tower of conserved quantities and flux balances law in the log-sector -> Connection to sub -log soft theorems?