Trace Anomalies of Boundaries and Defects

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December 4, 2025

Credits

1309.4523 O'B. + Jensen
1509.02160 O'B. + Jensen
1812.00923 O'B. + Estes, Krym, Robinson, Rodgers
1812.08745 O'B. + Jensen, Robinson, Rodgers
2003.02857 O'B. + Chalabi, Robinson, Sisti
2111.14713 O'B. + Chalabi, Herzog, Robinson, Sisti

1205.1573 Nozaki, Takayanagi, Ugajin
1305.2334 Fursaev
1510.00021 Herzog, Huang, Jensen
1510.01427 Fursaev
1510.04566 Solodukhin
1511.06713 Bianchi, Meineri, Myers, Smolkin
1601.06418 Fursaev + Solodukhin
1601.02883 Billò, Gonçalves, Lauria, Meineri
1604.07571 Bethiere + Solodukhin
1702.00566 Astaneh + Solodukhin
1703.04186 Astaneh, Berthiere, Fursaev, Solodukhin
1707.06224 Herzog + Huang
1709.07431 Herzog, Huang, Jensen
1804.01974 Prochazka
1810.06995 Kobayashi, Nishioka, Sato, Watanabe
1812.08183 Casini, Landea, Torroba
1906.11281 Herzog + Shamir
1907.04952 Herzog + Shamir
1907.06193 Bianchi
1911.05082 Bianchi + Lemos
2010.04995 Herzog + Shrestha
2012.06574 Wang
2101.12648 Wang
2102.07661 Astaneh + Solodukhin
2303.16935 Casini, Landea, Torroba
2406.04561 Kravchuk, Radcliffe, Sinha

$\boxed{\text{Trace Anomaly}}$

$\boxed{\text{Conformal Field Theory}}$

$$g_{\mu\nu} = \delta_{\mu\nu}$$

Conformal invariance

$$T_{\mu}{}^{\mu} = 0$$

Trace Anomaly

Conformal Field Theory

$$g_{\mu\nu} \neq \delta_{\mu\nu}$$

Quantum Effects
Break Conformal Invariance

$$T_\mu{}^\mu \neq 0$$

Trace Anomaly

$d = 2$ CFT

$$T_\mu{}^\mu = \frac{c}{24\pi} R$$

central charge c

Stress tensor 2-pt. function

$$\langle T_{zz}(z) T_{zz}(0) \rangle = \frac{c/2}{z^4}$$

Entanglement Entropy

$$S_{\mathrm{EE}} = \frac{c}{3} \ln(\ell/\varepsilon) + \ldots$$

Thermodynamic entropy

$$S_{\mathrm{thermo}} = \frac{\pi}{3} c L T + \ldots$$

Trace Anomaly

$d = 2$ CFT

central charge c

Counts the number of degrees of freedom

Reflection positivity (unitarity)
vacuum state normalizability

$$c \geq 0$$

Add a single, free, massless, real scalar field
or single, free, massless Dirac fermion

$$c \to c + 1$$

Trace Anomaly

$d = 2$ CFT

central charge c

Counts the number of degrees of freedom

The c-theorem

A.B. Zamolodchikov
JETP Vol. 43 No. 12 p. 565, 1986

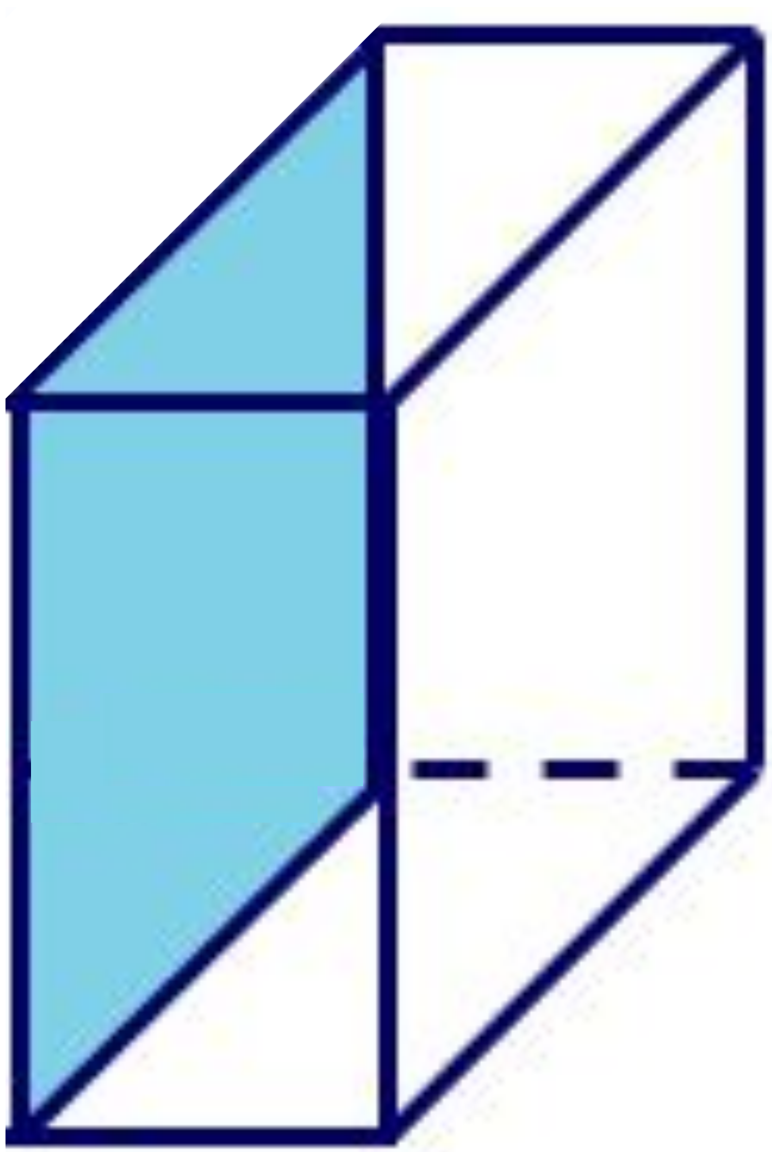
RG flow from UV CFT to IR CFT

$$c_{\rm UV} \geq c_{\rm IR}$$

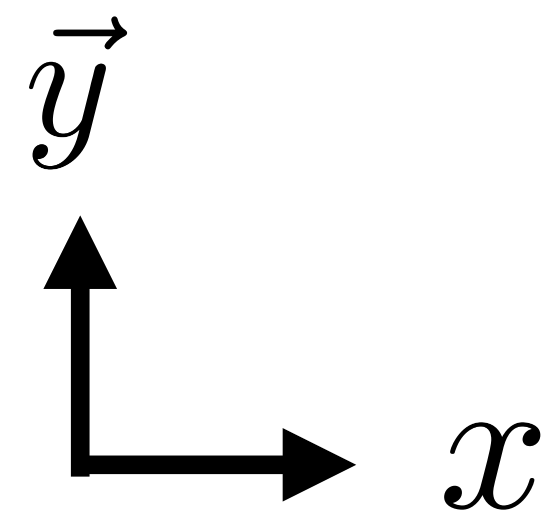
Reflection positivity (Unitarity)
Euclidean symmetry (Poincaré symmetry)
Locality

conformal field theory (CFT) with a boundary
+ conformally-invariant boundary conditions
possibly + massless degrees of freedom at the boundary

Boundary Conformal Field Theory (BCFT)



CFT dimension	d
Boundary dimension	$d-1$
Co-dimension	1



Examples

Experiments
graphene with a boundary

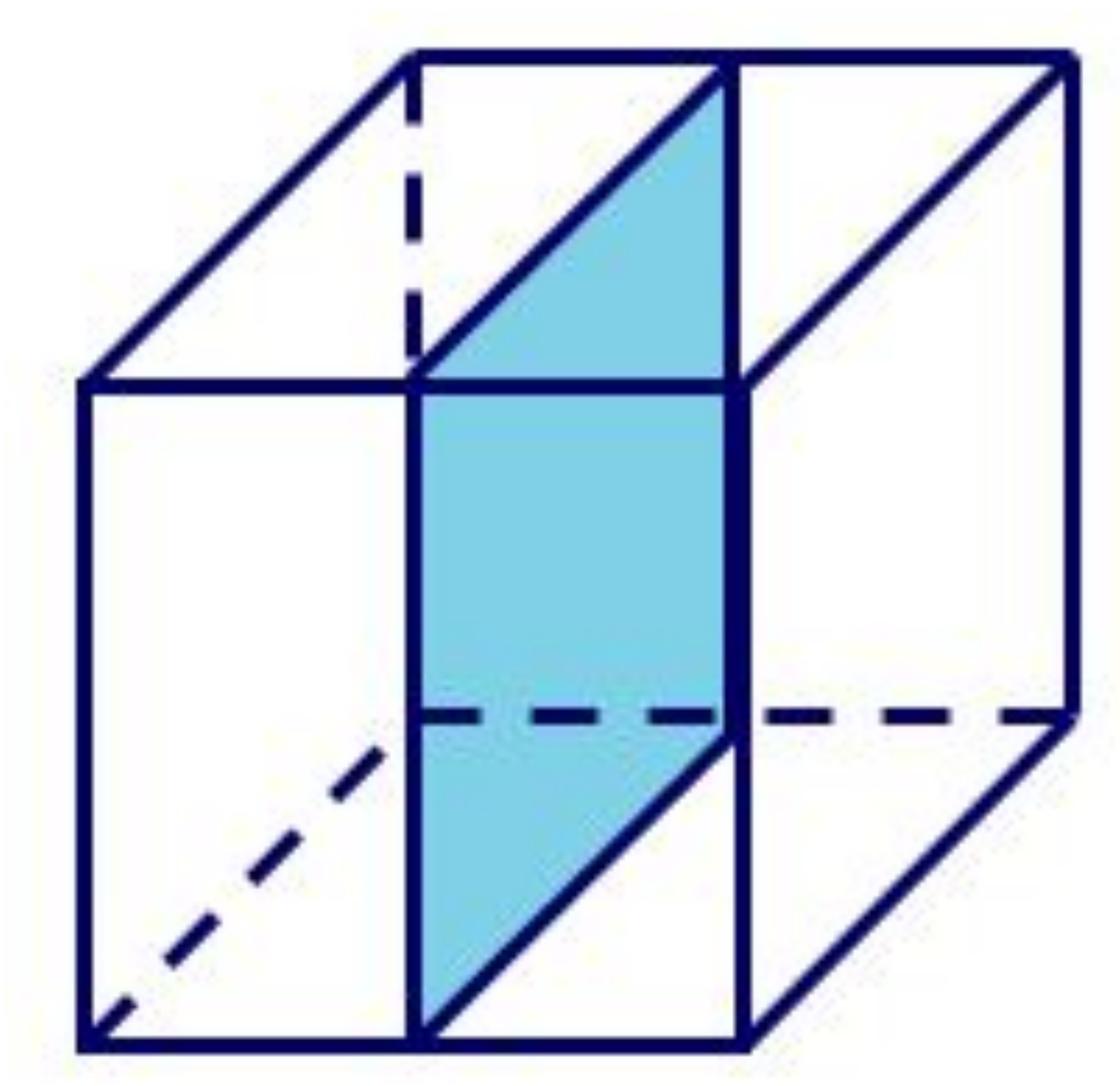
Quantum Field Theory
free fields, Ising model, Wilson-Fisher, etc.
with a boundary

String theory + M-theory
various branes ending on branes

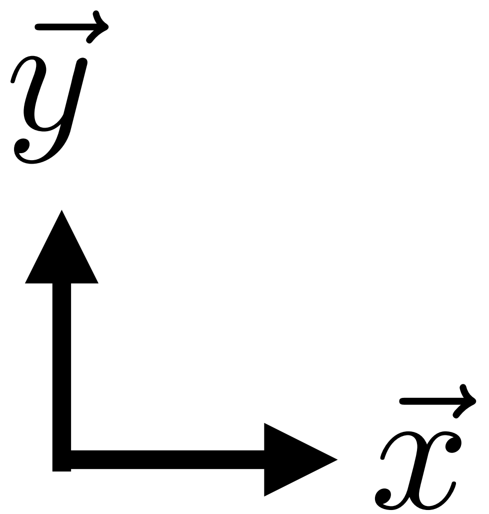
Holography
many examples

CFT with a conformally-invariant defect:
conformally-invariant boundary conditions along a submanifold
and/or massless degrees of freedom supported along a submanifold

Defect Conformal Field Theory (DCFT)



CFT dimension	d
Defect dimension	p
Co-dimension	$d-p$



Examples

Experiments
graphene with an impurity, line defect, etc.

Quantum Field Theory
Wilson lines, ’t Hooft lines, surface operators, etc.

String theory + M-theory
various branes intersecting branes

Holography
many examples

Questions

How do boundaries and defects contribute to the trace anomaly?

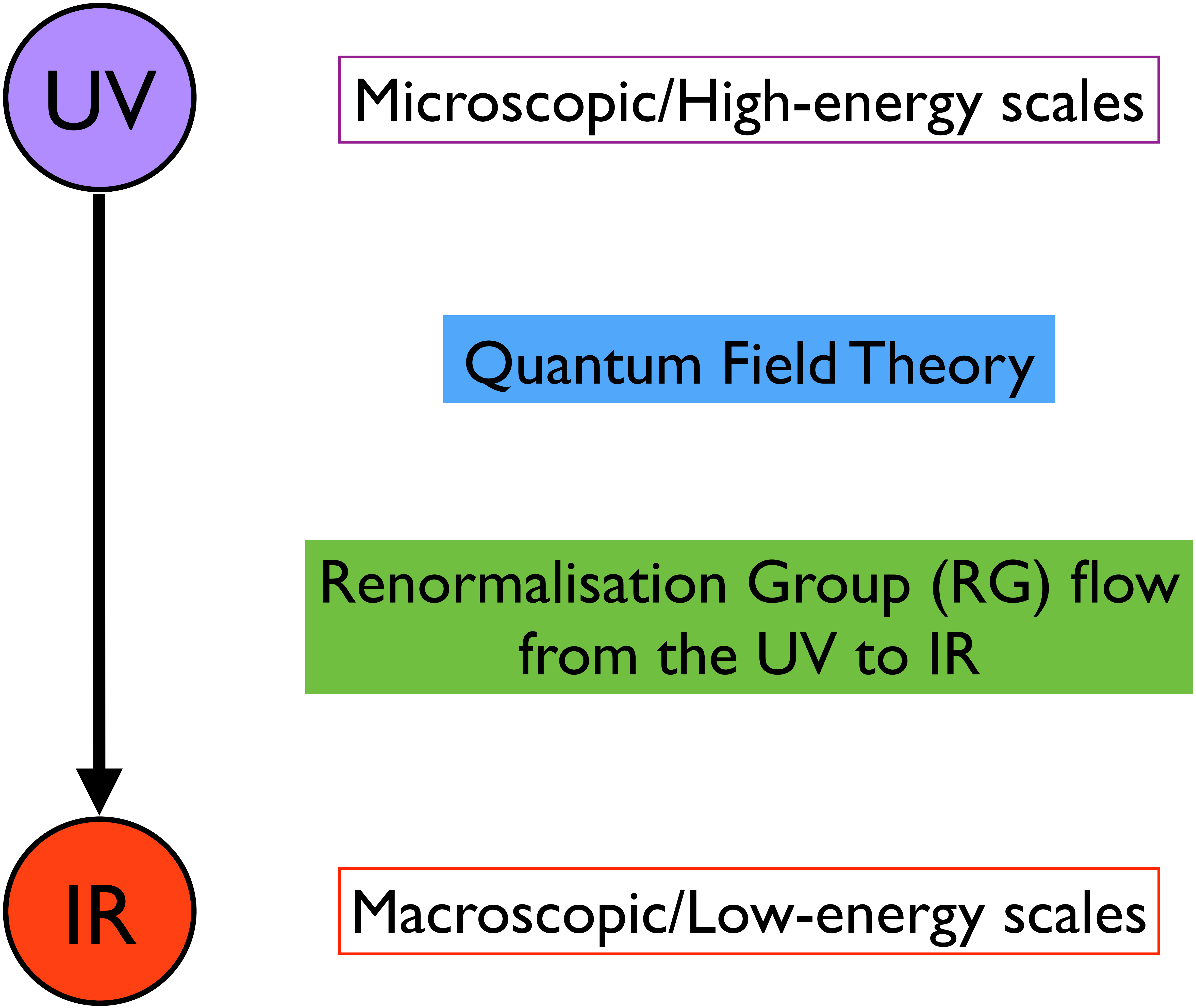
Can we define boundary and defect central charges?

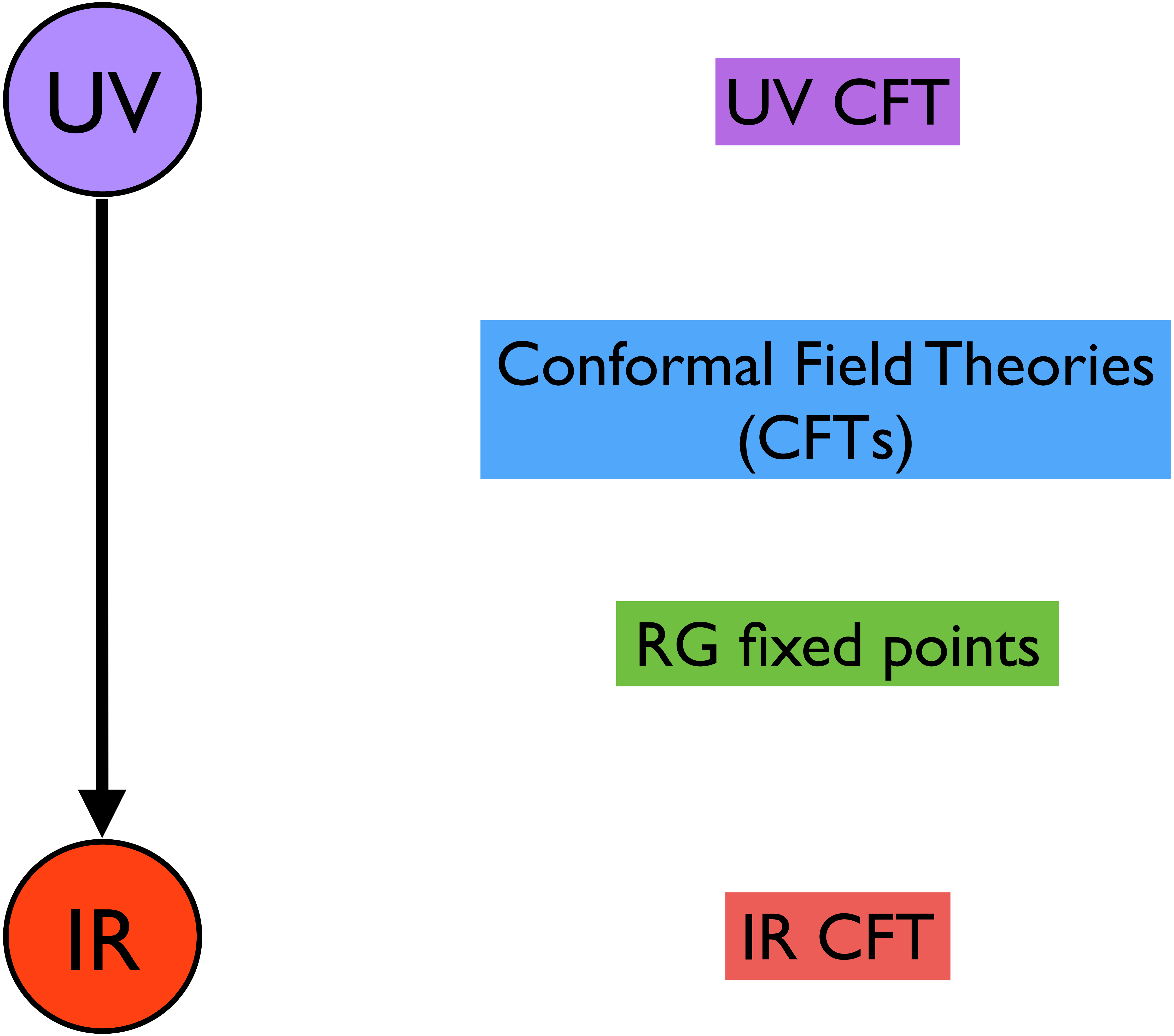
How do they appear in physical observables?

Do they obey constraints? c-theorems?

Outline:

- Review: Trace Anomalies
- Review: Boundaries and Defects
- 1d Boundary or Defect
- 2d Boundary or Defect
- 3d Boundary or Defect
- 4d Boundary or Defect
- Summary and Outlook


UV
Microscopic/High-energy scales
Quantum Field Theory
Renormalisation Group (RG) flow
from the UV to IR
IR
Macroscopic/Low-energy scales


UV
IR
UV CFT
Conformal Field Theories (CFTs)
RG fixed points
IR CFT

Quantum Field Theory

Generating functional Z

Non-dynamical background metric $g_{\mu\nu}(x)$

Stress-Energy Tensor

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} \ln Z[g_{\mu\nu}]$$

Quantum Field Theory

Generating functional Z

Non-dynamical background metric $g_{\mu\nu}(x)$

Stress-Energy Tensor

$$g_{\mu\nu} \to \delta_{\mu\nu}$$

Translational and Rotational Symmetry

$$\partial_\mu T_{\mu\nu} = 0$$

Conformal Field Theory

Generating functional Z

Non-dynamical background metric $g_{\mu\nu}(x)$

Conformal Transformation

Diffeomorphism

$$x^\mu \to x'^\mu(x)$$

such that

$$g_{\mu\nu}(x) \to e^{2\Omega(x)} g_{\mu\nu}(x)$$

Conformal Field Theory

$$g_{\mu\nu}(x) \to e^{2\Omega(x)} g_{\mu\nu}(x)$$

$$T_\mu{}^\mu = -\frac{1}{\sqrt{g}} \frac{\delta}{\delta\Omega} \ln Z[g_{\mu\nu}]$$

Conformal invariance

$$T_\mu{}^\mu = 0$$

Conformal Field Theory

flat space $\mathbb{R}^d$

Rotations $\quad x^\mu \to \Lambda^\mu{}_\nu \, x^\nu$

Translations $\quad x^\mu \to x^\mu + c^\mu$

Dilatations $\quad x^\mu \to \lambda \, x^\mu$

Special Conformal $\quad x^\mu \to \dfrac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$

$$SO(d+1,1)$$

$\boxed{\text{Trace Anomaly}}$

$\boxed{\text{Conformal Field Theory}}$

$$g_{\mu\nu} = \delta_{\mu\nu}$$

Conformal invariance

$$T_{\mu}{}^{\mu} = 0$$

Trace Anomaly

Conformal Field Theory

$$g_{\mu\nu} \neq \delta_{\mu\nu}$$

Quantum Effects
Break Conformal Invariance

$$T_\mu{}^\mu \neq 0$$

Trace Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #1

Write down all curvature invariants built from $g_{\mu\nu}$ with the correct dimension

$d = 4$ CFT

$$T_\mu{}^\mu = c_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R^2 + c_4 \Box R$$

Trace Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #2

Wess-Zumino consistency

$$g_{\mu\nu} \to e^{2\Omega_1} e^{2\Omega_2} g_{\mu\nu} \quad = \quad g_{\mu\nu} \to e^{2\Omega_2} e^{2\Omega_1} g_{\mu\nu}$$

$$R_{\mu\nu\rho}{}^{\sigma} \to R_{\mu\nu\rho}{}^{\sigma} + 2\delta^{\sigma}_{[\mu} \left(\nabla_{\nu]} + \partial_{\nu]}\Omega\right) \partial_\rho \Omega - 2g_{\rho[\mu} \left(\nabla_{\nu]} + \partial_{\nu]}\Omega\right) \partial^\sigma \Omega - 2g_{\rho[\mu}\delta^{\sigma}_{\nu]}\partial_\alpha\Omega\partial^\alpha\Omega$$

Trace Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #2

Wess-Zumino consistency

$$g_{\mu\nu} \to e^{2\Omega_1} e^{2\Omega_2} g_{\mu\nu} \quad = \quad g_{\mu\nu} \to e^{2\Omega_2} e^{2\Omega_1} g_{\mu\nu}$$

gives constraints on the coefficients

$$T_\mu{}^\mu = c_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R^2 + c_4 \Box R$$

Trace Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #3

Add local counterterms

Determine how they enter $T_\mu{}^\mu$

gives more constraints on the coefficients

$$0$$

$$T_\mu{}^\mu = c_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R^2 + c_4 \Box R$$

Trace Anomaly

Conformal Field Theory

What is the general form of $T_\mu{}^\mu$?

d odd $\qquad T_\mu{}^\mu = 0$

d even $\qquad T_\mu{}^\mu \neq 0$

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\right)$$

$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c_{6d}^{(1)}\, I_1 + c_{6d}^{(2)}\, I_2 + c_{6d}^{(3)}\, I_3\right)$$

Deser, Duff, Isham NPB 111 (1976) 45

Bonora, Pasti, Bregola CQG 3 (1986) 635

Duff hep-th/9308075

Bastianelli, Frolov, Tseytlin hep-th/0001041

Trace Anomaly

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Euler density

$$E_d = \frac{1}{2^{d/2}}\, \delta^{\mu_1\nu_1\ldots\mu_{d/2}\nu_{d/2}}_{\rho_1\sigma_1\ldots\rho_{d/2}\sigma_{d/2}}\, R^{\rho_1\sigma_1}{}_{\mu_1\nu_1}\, R^{\rho_2\sigma_2}{}_{\mu_2\nu_2} \ldots R^{\rho_{d/2}\sigma_{d/2}}{}_{\mu_{d/2}\nu_{d/2}}$$

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right)$$

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Weyl tensor

$$W_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} + \frac{2}{d-2}\left(g_{\nu[\rho}R_{\sigma]\mu} - g_{\mu[\rho}R_{\sigma]\nu}\right) + \frac{2}{(d-1)(d-2)}\, g_{\mu[\rho}g_{\sigma]\nu}\, R$$

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right)$$

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$$I_1 = W_{\mu\lambda\rho\nu}W^{\lambda\sigma\tau\rho}W_\sigma{}^{\mu\nu}{}_\tau$$

$$I_2 = W_{\mu\nu}{}^{\lambda\rho}W_{\lambda\rho}{}^{\sigma\tau}W_{\sigma\tau}{}^{\mu\nu}$$

$$I_3 = W_{\mu\nu\lambda\rho}\left(D^2\,\delta^\nu{}_\sigma - \frac{6}{5}R\,\delta^\nu{}_\sigma + 4R^\nu{}_\sigma\right)W^{\sigma\nu\lambda\rho}$$

Wess-Zumino consistency

$\int d^d x \sqrt{g}\, T_\mu{}^\mu$ is conformally invariant

$$d = 4 \quad T_\mu{}^\mu = \frac{1}{(4\pi)^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\right)$$

Type A

$$\sqrt{g}\, E$$

Changes by a total derivative

Type B

$$\sqrt{g}\, W^2$$

Invariant

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right)$$

$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c_{6d}^{(1)}\, I_1 + c_{6d}^{(2)}\, I_2 + c_{6d}^{(3)}\, I_3\right)$$

Type A

Type B

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right)$$

$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c^{(1)}_{6d}\, I_1 + c^{(2)}_{6d}\, I_2 + c^{(3)}_{6d}\, I_3\right)$$

“central charges”

“Weyl anomaly coefficients”

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right)$$

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How do they appear in physical observables?

Do they obey constraints? c-theorems?

Trace Anomaly

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fix normalisation of $\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle$

$$d = 2 \qquad \langle T_{zz}(z) T_{zz}(0)\rangle = \frac{c_{2d}/2}{z^4}$$

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

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fix normalisation of
$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle$$

reflection positivity (unitarity)

$$c_{2d} \geq 0 \qquad c_{4d} \geq 0 \qquad c_{6d}^{(3)} \geq 0$$

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

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$$d = 4$$

reflection positivity (unitarity)

$$\frac{31}{18} \geq \frac{a_{4d}}{c_{4d}} \geq \frac{1}{3} \qquad\qquad \Rightarrow \qquad a_{4d} \geq 0$$

Hofman + Maldacena 0803.1467, Hofman et al. 1603.03771

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

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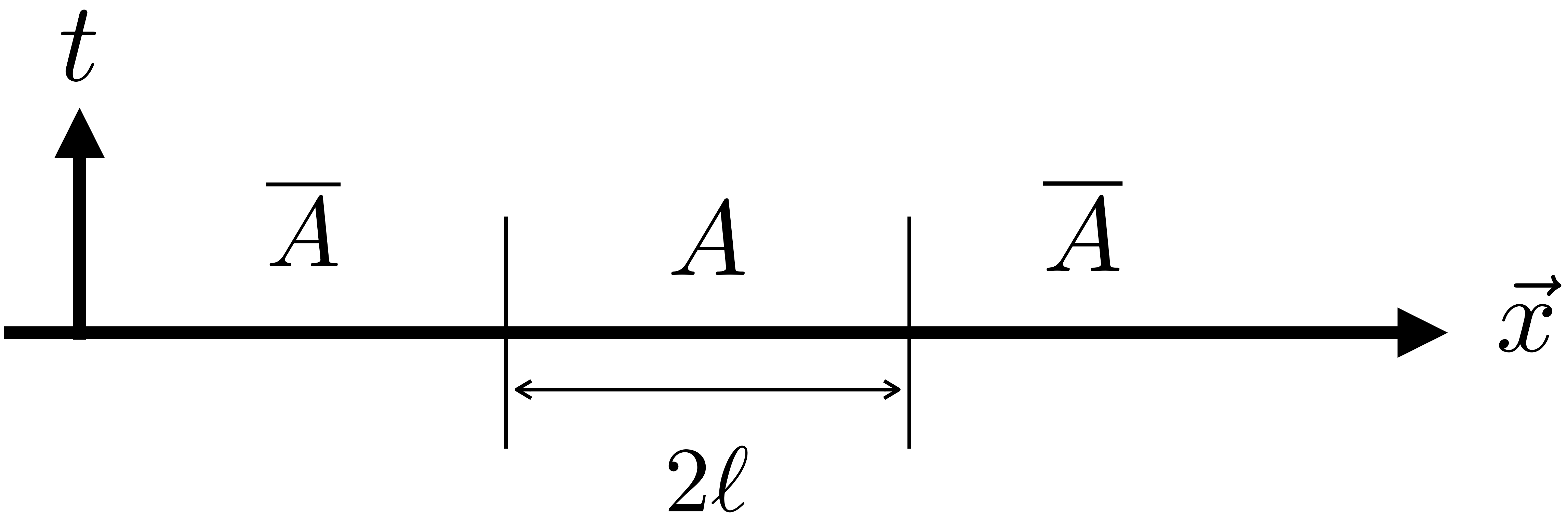
Entanglement Entropy (EE)

spherical region of radius ℓ

$$S_{\mathrm{EE}} = \#\frac{\mathrm{Area}}{\varepsilon^{d-2}} + \frac{\#}{\varepsilon^{d-4}} + \ldots + \#\, a\, \ln\left(\ell/\varepsilon\right) + \ldots$$

Entanglement Entropy



$$\rho_{\rm A} \equiv {\rm tr}_{\overline{A}}\, \rho$$

$$S_{\rm EE} = -{\rm tr} \rho_{\rm A} \ln \rho_{\rm A}$$

Trace Anomaly

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Entanglement Entropy (EE)

spherical region of radius ℓ

$$S_{\rm EE} = \#\frac{\text{Area}}{\varepsilon^{d-2}} + \frac{\#}{\varepsilon^{d-4}} + \ldots + \#\, a\, \ln\left(\ell/\varepsilon\right) + \ldots$$

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Type A

c-theorem

$$c_{2d}^{\mathrm{UV}} \geq c_{2d}^{\mathrm{IR}}$$

A.B. Zamolodchikov
JETP Vol. 43 No. 12 p. 565, 1986

a-theorem

$$a_{4d}^{\mathrm{UV}} \geq a_{4d}^{\mathrm{IR}}$$

Cardy PLB 215 (1988) 749
Komargodski + Schwimmer 1107.3987

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

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Type A

c-theorem

$$c_{2d}^{\mathrm{UV}} \geq c_{2d}^{\mathrm{IR}}$$

a-theorem

$$a_{4d}^{\mathrm{UV}} \geq a_{4d}^{\mathrm{IR}}$$

Counts the number of degrees of freedom

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

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$d = 2$ CFT

Theory	c_{2d}
free massless scalar	1
free massless Dirac fermion	1
Kac-Moody currents $\mathfrak{g}_k$	$\frac{\mathrm{Dim}(\mathfrak{g})k}{C_2(\mathfrak{g})+k}$

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

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$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c_{6d}^{(1)}\, I_1 + c_{6d}^{(2)}\, I_2 + c_{6d}^{(3)}\, I_3\right)$$

$d = 4$ CFT

Theory	a_{4d}	c_{4d}
free massless scalar	$1/360$	$1/120$
free massless Dirac fermion	$11/360$	$1/20$
Maxwell	$31/180$	$1/10$
$\mathcal{N}=4$ SYM with gauge algebra $\mathfrak{g}$	$\dim(\mathfrak{g})/4$	$\dim(\mathfrak{g})/4$

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right)$$

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$d = 6$ CFT

$\mathcal{N} = (2,0)$ SUSY

Lie algebra $\mathfrak{g}$

$$c^{(1)}_{6d} = c^{(2)}_{6d} = c^{(3)}_{6d}$$

Trace Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{4\pi}\frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right)$$

$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c^{(1)}_{6d}\, I_1 + c^{(2)}_{6d}\, I_2 + c^{(3)}_{6d}\, I_3\right)$$

$d = 6$ CFT

$\mathcal{N} = (2,0)$ SUSY

$$a_{6d} = \mathrm{rank}_\mathfrak{g} + \frac{16}{7}\mathrm{dim}_\mathfrak{g}\, h^\vee_\mathfrak{g}$$

Cordova, Dumitrescu, Yin 1505.03850

$$c_{6d} = \mathrm{rank}_\mathfrak{g} + 4\mathrm{dim}_\mathfrak{g}\, h^\vee_\mathfrak{g}$$

Beem, Rastelli, van Rees 1404.1079

$h^\vee_\mathfrak{g}$ = dual Coxeter number

Outline:

- Review: Trace Anomalies
- Review: Boundaries and Defects
- 1d Boundary or Defect
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CFT

flat space $\mathbb{R}^d$

Rotations $$x^\mu \to \Lambda^\mu{}_\nu \, x^\nu$$

Translations $$x^\mu \to x^\mu + c^\mu$$

Dilatations $$x^\mu \to \lambda \, x^\mu$$

Special Conformal $$x^\mu \to \frac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$$

$$SO(d+1,1)$$

BCFT + DCFT

boundary or defect in flat space

Rotations $\qquad x^\mu \to \Lambda^\mu{}_\nu\, x^\nu$

Translations $\qquad x^\mu \to x^\mu + c^\mu$

Dilatations $\qquad x^\mu \to \lambda\, x^\mu$

Special Conformal $\qquad x^\mu \to \dfrac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$

$$SO(d+1,1)$$

Broken to subgroup
that preserves the boundary or defect

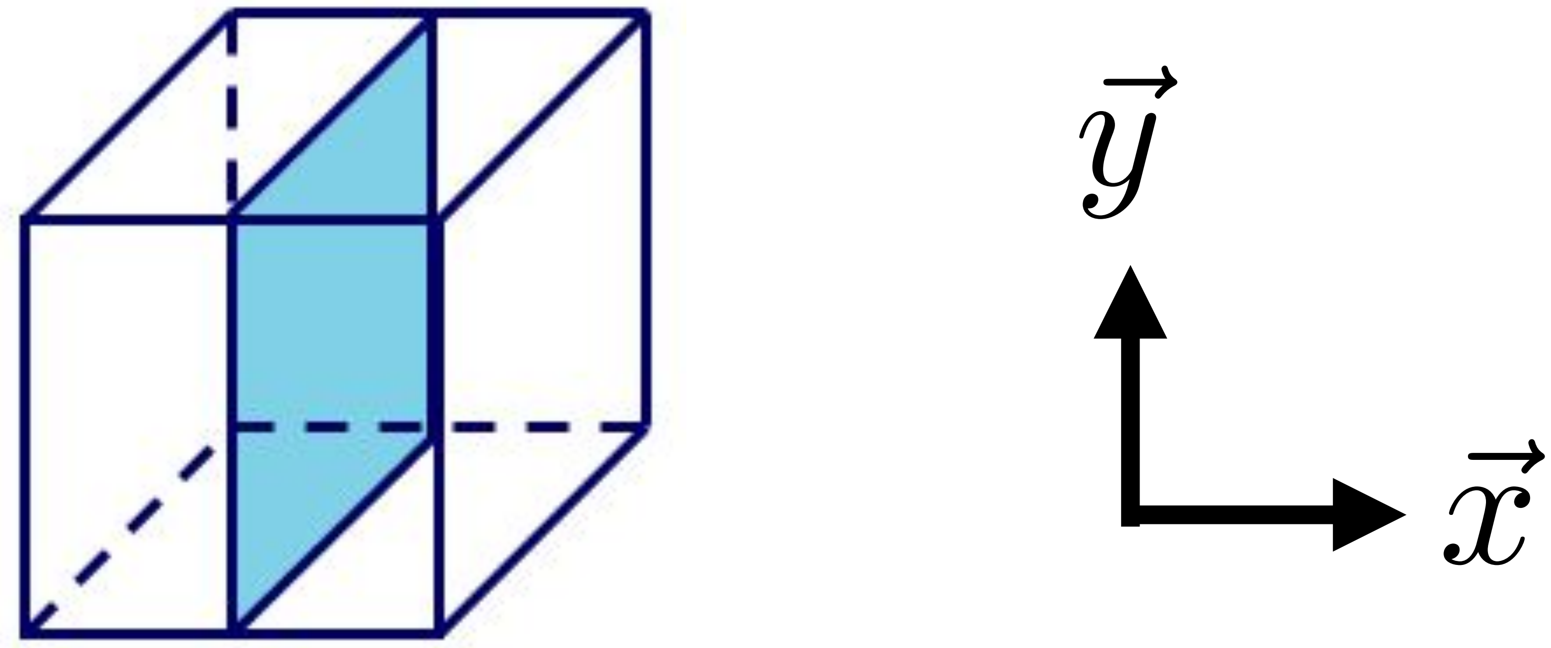
BCFT + DCFT

boundary or defect in flat space

Rotations

$$x^\mu \to \Lambda^\mu{}_\nu \, x^\nu$$

Broken to rotations in $\vec{y}$ + rotations in $\vec{x}$

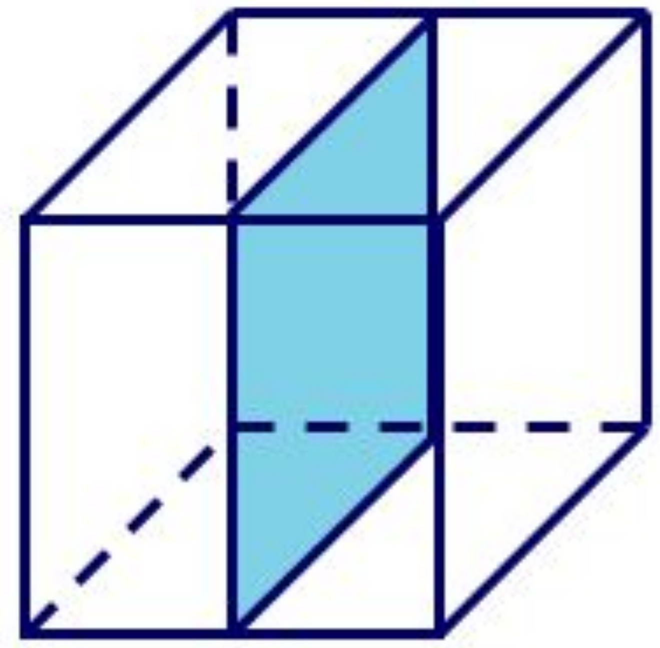


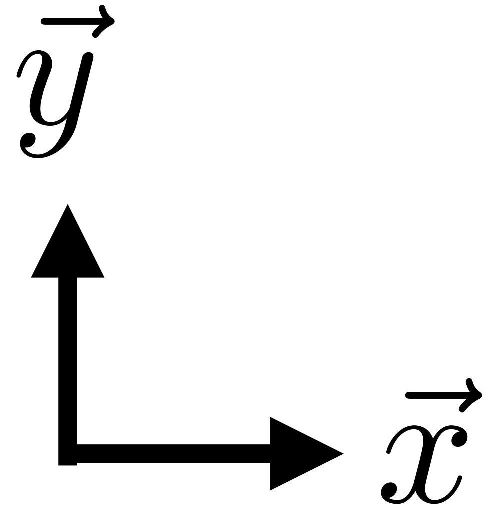
BCFT + DCFT

boundary or defect in flat space

Translations $x^\mu \to x^\mu + c^\mu$

Broken to translations along $\vec{y}$



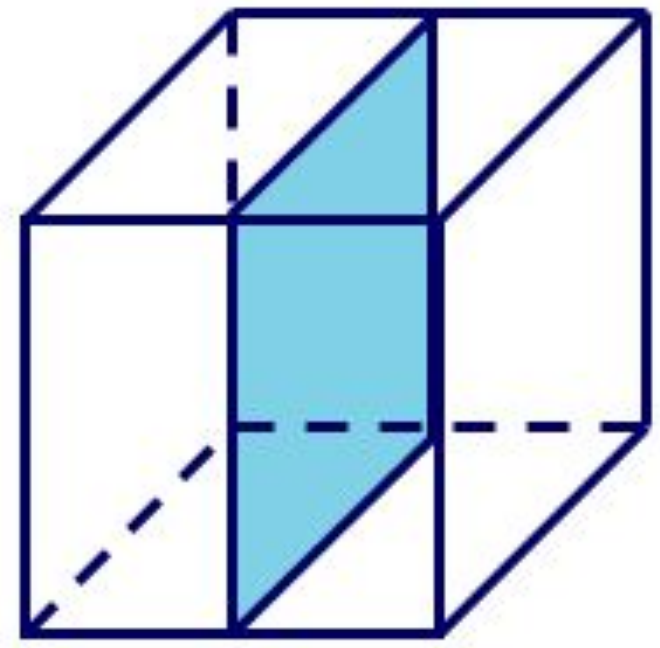


BCFT + DCFT

boundary or defect in flat space

Dilatations $\qquad x^\mu \to \lambda\, x^\mu$

Unbroken



$\vec{y}$

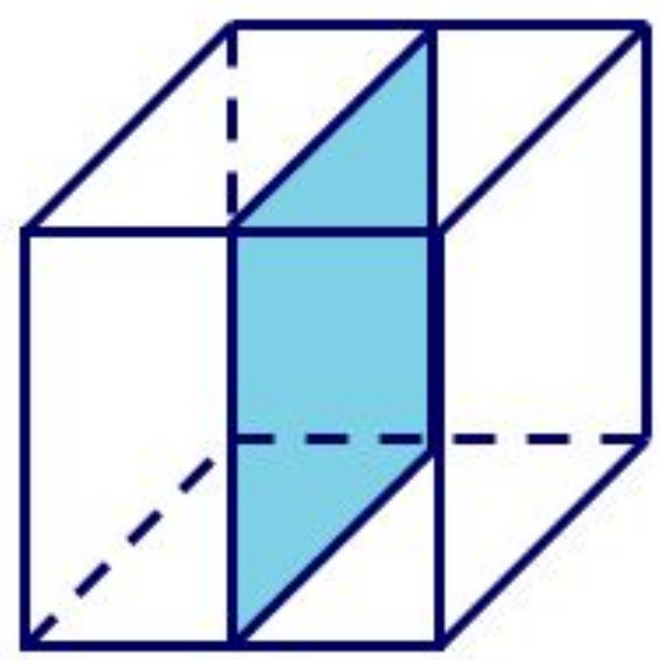
$\vec{x}$

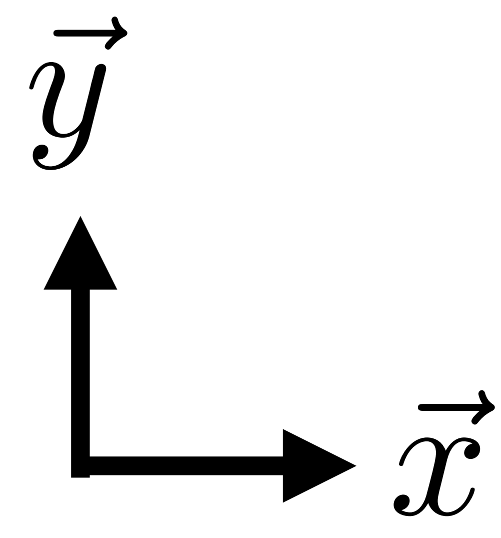
BCFT + DCFT

boundary or defect in flat space

Special Conformal $\quad x^\mu \to \dfrac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$

Broken to $b^\perp = 0$





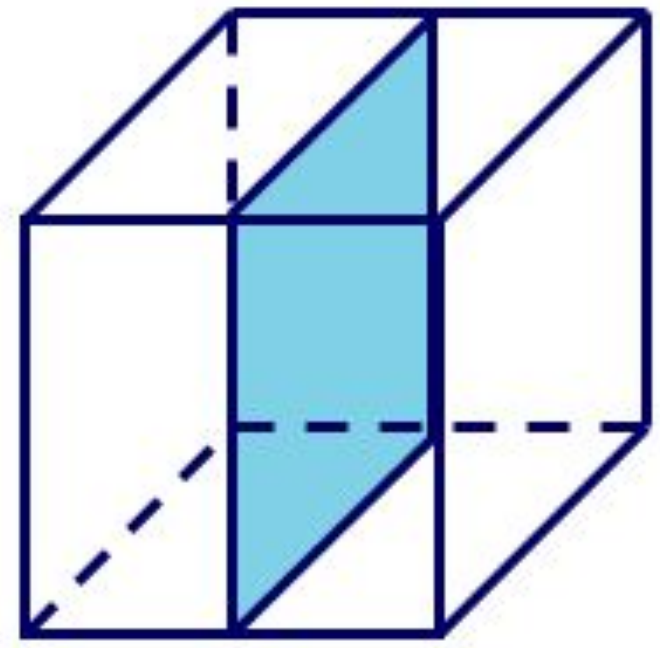
BCFT + DCFT

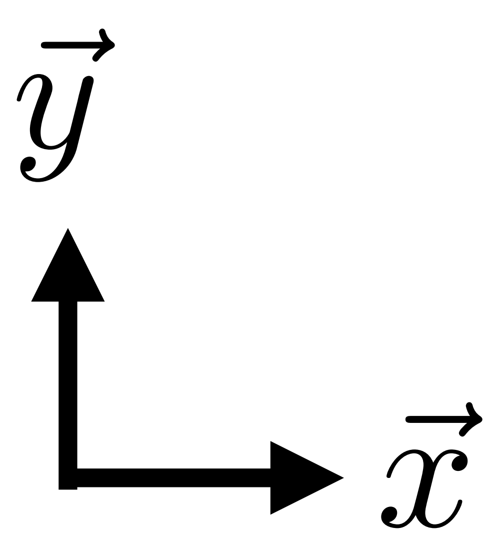
boundary or defect in flat space

$$SO(d+1,1) \to SO(p+1,1) \times SO(d-p)$$

conformal transformations preserving the boundary/defect

rotations around the defect





BCFT + DCFT

boundary or defect in flat space

x^μ with $\mu = 1, 2, 3, \ldots, d$

along the boundary/defect

y^a with $a = 1, 2, \ldots, p$

transverse to the boundary/defect

x^i with $i = p+1, p+2, \ldots, d$

BCFT + DCFT

boundary or defect in flat space

$$\partial_\mu T^{\mu i} = \delta^{d-p}(\vec{x}) D^i$$

Displacement operator D^i

$$T^{\mu\nu} = [T^{\mu\nu}]_{\text{bulk}} + \delta^{d-p}(\vec{x})\, [T^{\mu\nu}]_p$$

Gaussian pillbox integration around boundary/defect

$$D \propto \left[T^{\perp\perp}\right]^{\text{defect}}_{\text{bulk}}$$

$$\boxed{p = d-1}$$

$$\partial_a \left[T^{ab}\right]_{p=d-1} \propto \left[T^{b\perp}\right]^{\text{defect}}_{\text{bulk}}$$

BCFT + DCFT

boundary or defect in flat space

$$\partial_\mu T^{\mu i} = \delta^{d-p}(\vec{x}) D^i$$

Displacement operator D^i

$$T^{\mu\nu} = \left[T^{\mu\nu}\right]_{\text{bulk}} + \delta^{d-p}(\vec{x}) \left[T^{\mu\nu}\right]_p$$

Conformal invariance

$$T_\mu{}^\mu = 0 \qquad \Rightarrow \qquad \left[T_\mu{}^\mu\right]_{\text{bulk}} = 0 \qquad \left[T_\mu{}^\mu\right]_p = 0$$

BCFT + DCFT

boundary or defect in flat space

$$\partial_\mu T^{\mu i} = \delta^{d-p}(\vec{x}) D^i$$

Displacement operator D^i

$$T^{\mu\nu} = [T^{\mu\nu}]_{\mathrm{bulk}} + \delta^{d-p}(\vec{x})\, [T^{\mu\nu}]_p$$

Conformal invariance

$$\langle D^i(y^a) D^j(0) \rangle \propto \frac{\delta^{ij}}{|y^a|^{2(p+1)}}$$

Questions

How do boundaries and defects contribute to the trace anomaly?

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\text{bulk}} + \delta^{d-p}(\vec{x})\left[T_\mu{}^\mu\right]_p$$

What is the general form of $\left[T_\mu{}^\mu\right]_p$?

Depends on d and p

Geometry of Submanifolds

boundary/defect	σ^a
"target space"	x^μ
embedding	$x^\mu(\sigma^a)$
induced metric	$\hat{g}_{ab} = g_{\mu\nu} \dfrac{\partial x^\mu}{\partial \sigma^a} \dfrac{\partial x^\nu}{\partial \sigma^b}$
Intrinsic curvature	$\hat{R}_{abcd} \Rightarrow \hat{R}_{ab} \Rightarrow \hat{R}$
second fundamental form	$\mathrm{I\!I}^\mu_{ab}$
trace	$\mathrm{I\!I}^\mu \equiv \hat{g}^{ab} \mathrm{I\!I}^\mu_{ab}$
traceless	$\mathring{\mathrm{I\!I}}^\mu_{ab} \equiv \mathrm{I\!I}^\mu_{ab} - \hat{g}_{ab} \mathrm{I\!I}^\mu / p$

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1d Boundary or Defect

CFT

$d \geq 2$

Boundary/Defect

$p = 1$

What is the general form of $\left[T_\mu{}^\mu\right]_{p=1}$?

Non-zero only when $d = 2$

(codimension $d - p = 1$)

$$\left[T_\mu{}^\mu\right]_{p=1} = -\frac{1}{12\pi}\, c_{2d}\, \mathrm{II}^{\perp}$$

Boundary contribution to Euler density E_2

No new central charge(s)

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2d Boundary or Defect

CFT

$d \geq 3$

Boundary/Defect

$p = 2$

What is the general form of $\left[T_\mu{}^\mu\right]_{p=2}$?

Berenstein, Corrado, Fischler, Maldacena hep-th/9809188

Graham + Witten hep-th/9901021

Henningson + Skenderis hep-th/9905163

Gustavsson hep-th/0310037, 0404150

Asnin 0801.1469

Schwimmer + Theisen 0802.1017

$2\mathrm{d}$ Boundary or Defect

$$[T_\mu{}^\mu]_{p=2} = \frac{b}{24\pi}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{I\!I}}^\mu{}_{ab}\mathring{\mathrm{I\!I}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

boundary/defect central charges

$b \qquad d_1 \qquad d_2$

2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{24\pi}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{I\!I}}^\mu{}_{ab}\mathring{\mathrm{I\!I}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Type A

$$\sqrt{\hat{g}}\,\hat{R}$$

Changes by a total derivative

Type B

$$\sqrt{\hat{g}}\,\mathring{\mathrm{I\!I}}^2 \qquad \sqrt{\hat{g}}\,W_{ab}{}^{ab}$$

Invariant

2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{24\pi}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{II}}^\mu{}_{ab}\mathring{\mathrm{II}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

b-theorem

Jensen and O'Bannon 1509.02160
Casini, Landea, Torroba 1812.08183, 2303.16935

RG flow on the boundary/defect

$$b_{\mathrm{UV}} \geq b_{\mathrm{IR}}$$

Reflection positivity (Unitarity)
Euclidean symmetry (Poincaré symmetry) along the boundary/defect
Locality

Counts the number of degrees of freedom

2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{24\pi}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{II}}^\mu{}_{ab}\mathring{\mathrm{II}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Bianchi, Meineri, Myers, Smolkin 1511.06713
Herzog + Huang 1707.06224
Herzog, Huang, Jensen 1709.07431

Displacement operator D^i

$$\langle D^i(y^a)D^j(0)\rangle \propto \frac{d_1\,\delta^{ij}}{|y^a|^4}$$

Reflection positivity (unitarity)

$$d_1 \geq 0$$

2d Boundary or Defect

$$[T_\mu{}^\mu]_{p=2} = \frac{b}{24\pi}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{II}}^\mu{}_{ab}\mathring{\mathrm{II}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Weyl tensor

$$d = 3 \;\;\Rightarrow\;\; W_{\mu\nu\rho\sigma} \equiv 0$$

d_2 is only defined for $d > 3$

(codimension $d - p > 1$)

BCFT + DCFT

boundary or defect in flat space

Co-dimension $d - p > 1$

$$SO(d+1,1) \to SO(p+1,1) \times SO(d-p)$$

symmetry allows

$$\langle [T^{ab}]_{\rm bulk} \rangle = -h(d-p-1)\frac{\delta^{ab}}{|x_\perp|^d} \qquad \langle [T^{ai}]_{\rm bulk} \rangle = 0$$

$$\langle [T^{ij}]_{\rm bulk} \rangle = h\frac{(p+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

2d Boundary or Defect

$$[T_\mu{}^\mu]_{p=2} = \frac{b}{24\pi}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{I\!I}}^\mu{}_{ab}\mathring{\mathrm{I\!I}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

$$\langle [T^{ab}]_{\text{bulk}}\rangle = -h(d-p-1)\frac{\delta^{ab}}{|x_\perp|^d} \qquad \langle [T^{ai}]_{\text{bulk}}\rangle = 0$$

$$\langle [T^{ij}]_{\text{bulk}}\rangle = h\frac{(p+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

$$h = \frac{d-3}{3(d-1)\mathrm{vol}(\mathbb{S}^d)}\,d_2$$

Bianchi, Meineri, Myers, Smolkin 1511.06713

Jensen, O'Bannon, Robinson, Rodgers 1812.08745

CFT

Average Null Energy Condition (ANEC)

Faulkner, Leigh, Parrikar, Wang 1605.08072

Hartman, Kundu, Tajdini 1610.05308

Lorentzian CFT

excited state

along any null direction u

$$\int_{-\infty}^{\infty} du \langle T_{uu} \rangle \geq 0$$

BCFT + DCFT

Average Null Energy Condition (ANEC)

Jensen, O'Bannon, Robinson, Rodgers 1812.08745

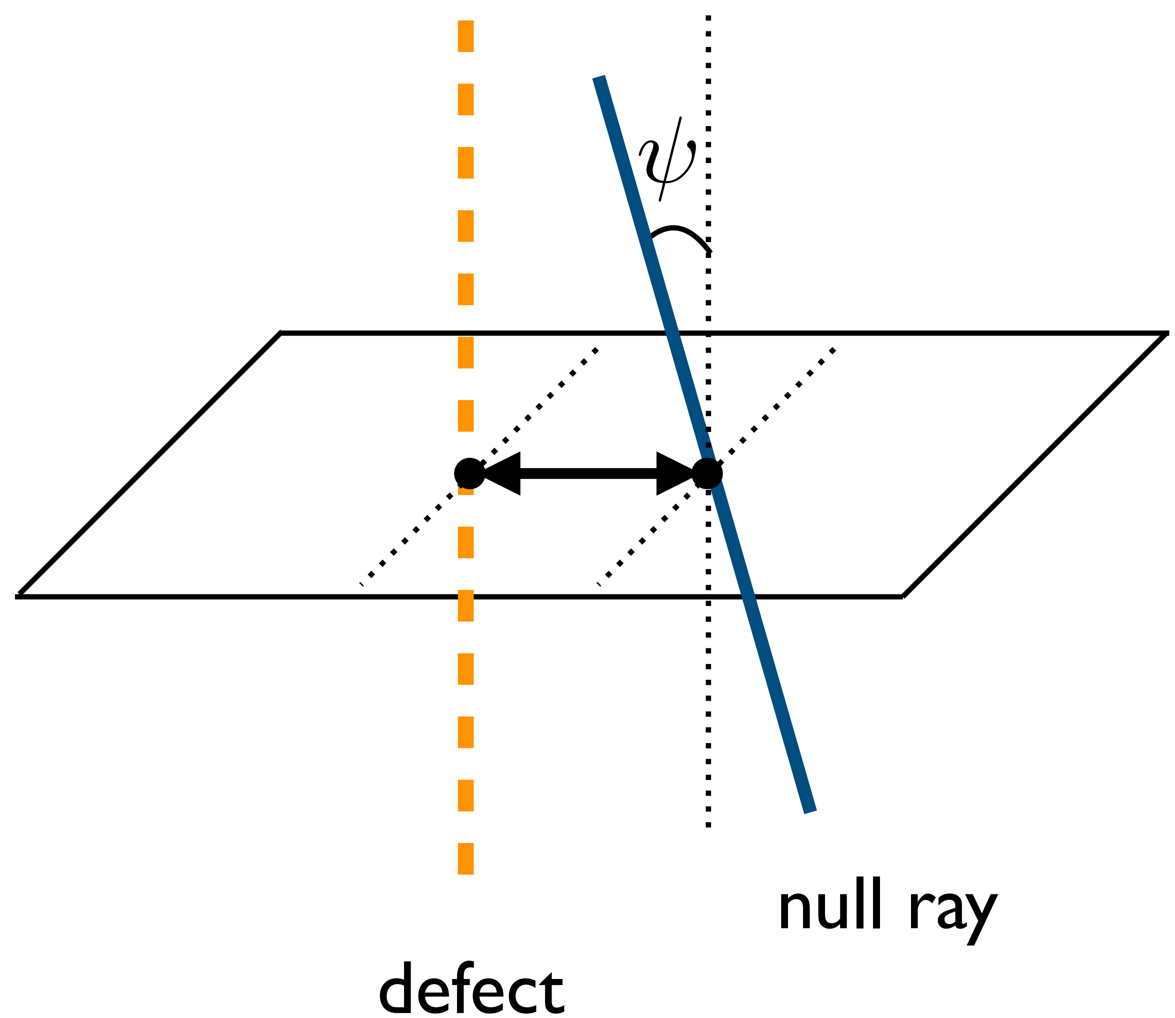
Casini, Landea, Torroba 2303.16935

Lorentzian BCFT/DCFT

along any null direction u

$$\int_{-\infty}^{\infty} du \langle T_{uu} \rangle \geq 0$$

$h \geq 0 \quad \Rightarrow \quad d_2 \geq 0$



2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{24\pi}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{II}}^\mu{}_{ab}\mathring{\mathrm{II}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Entanglement Entropy (EE)

Kobayashi, Nishioka, Sato, Watanabe 1810.06995
Jensen, O'Bannon, Robinson, Rodgers 1812.08745

$$S_{\text{EE}} = S_{\text{EE}}^{\text{CFT}} + S_{\text{EE}}^{p=2}$$

$$S_{\text{EE}}^{\text{CFT}} = \#\frac{\text{Area}}{\varepsilon^{d-2}} + \frac{\#}{\varepsilon^{d-4}} + \ldots + \#a\ln\left(\ell/\varepsilon\right) + \# + \ldots$$

$$S_{\text{EE}}^{p=2} = \frac{1}{3}\left(b - \frac{d-3}{d-1}d_2\right)\ln\left(\ell/\varepsilon\right) + \# + \ldots$$

Examples

$d = 3$ BCFTs

free massless real scalar or Dirac fermion

Theory	BC	b	d_1	d_2
Scalar	Dirichlet	$-1/16$	$3/32$	N/A
Scalar	Neumann	$1/16$	$3/32$	N/A
Fermion	Mixed	0	$3/16$	N/A

Nozaki, Takayanagi, Ugajin 1205.1573

Jensen and O'Bannon 1509.02160

Fursaev and Solodukhin 1601.06418

Examples

$d = 3$ BCFTs

free massless real scalar or Dirac fermion

Theory	BC	b	d_1	d_2
Scalar	Dirichlet	$-1/16$	$3/32$	N/A
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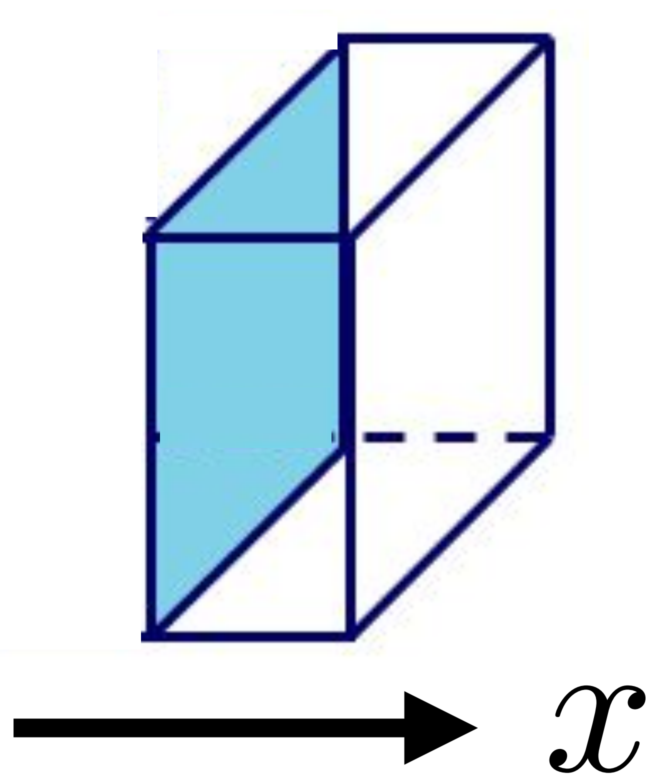


"mixed"

$$\Psi_\pm \equiv \frac{1}{2}\left(1 \pm \gamma^x\right)\Psi$$

$$\text{Dirichlet for } \Psi_+ \text{ or } \Psi_-$$

$$+ \text{ Neumann for } \Psi_- \text{ or } \Psi_+$$

Examples

$d = 3$ BCFTs

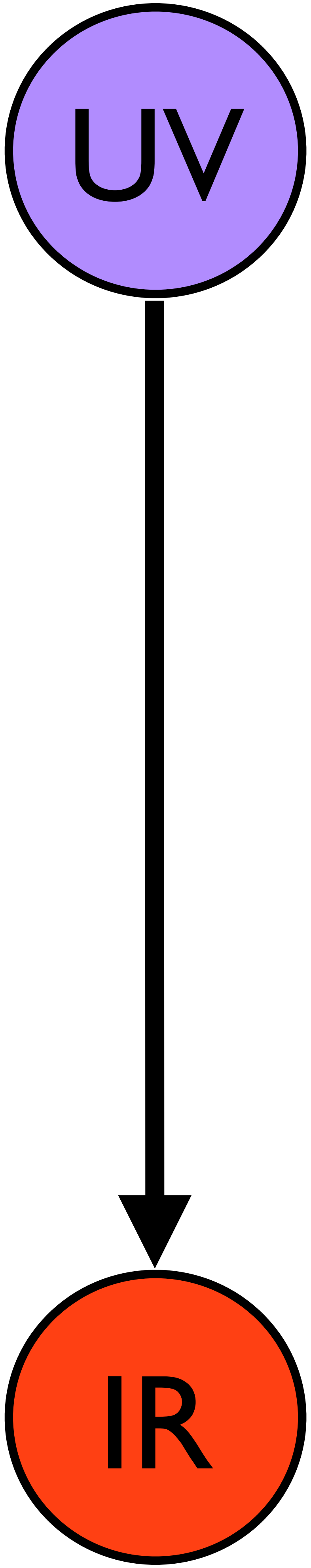
free massless real scalar or Dirac fermion

Theory	BC	b	d_1	d_2
Scalar	Dirichlet	$-1/16$	$3/32$	N/A
Scalar	Neumann	$1/16$	$3/32$	N/A
Fermion	Mixed	0	$3/16$	N/A

$b < 0$ in reflection-positive theory

Does b have a lower bound?

$d_1 \geq 0$



UV BCFT

Free, massless, real scalar

Neumann B.C.

Boundary RG Flow

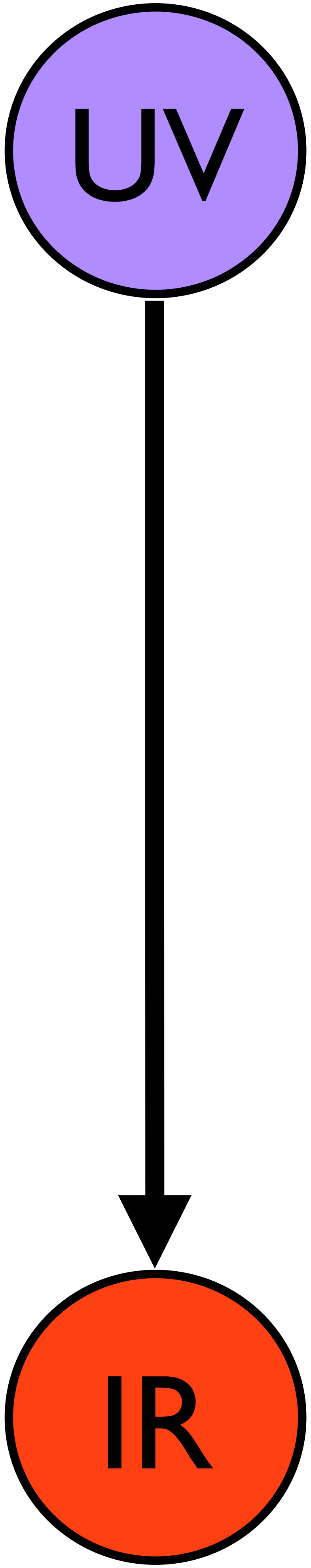
$$S^{\mathrm{UV}}_{\mathrm{BCFT}} \to S^{\mathrm{UV}}_{\mathrm{BCFT}} + \int d^3x\, \delta(x)\, m^2 \Phi^2(\vec{x})$$

$$\Delta_{\Phi^2} = 1 < 2$$

IR BCFT

Free, massless, real scalar

Dirichlet B.C.



$$\boxed{b_{\mathrm{UV}} = \frac{1}{16}}$$

Neumann B.C.

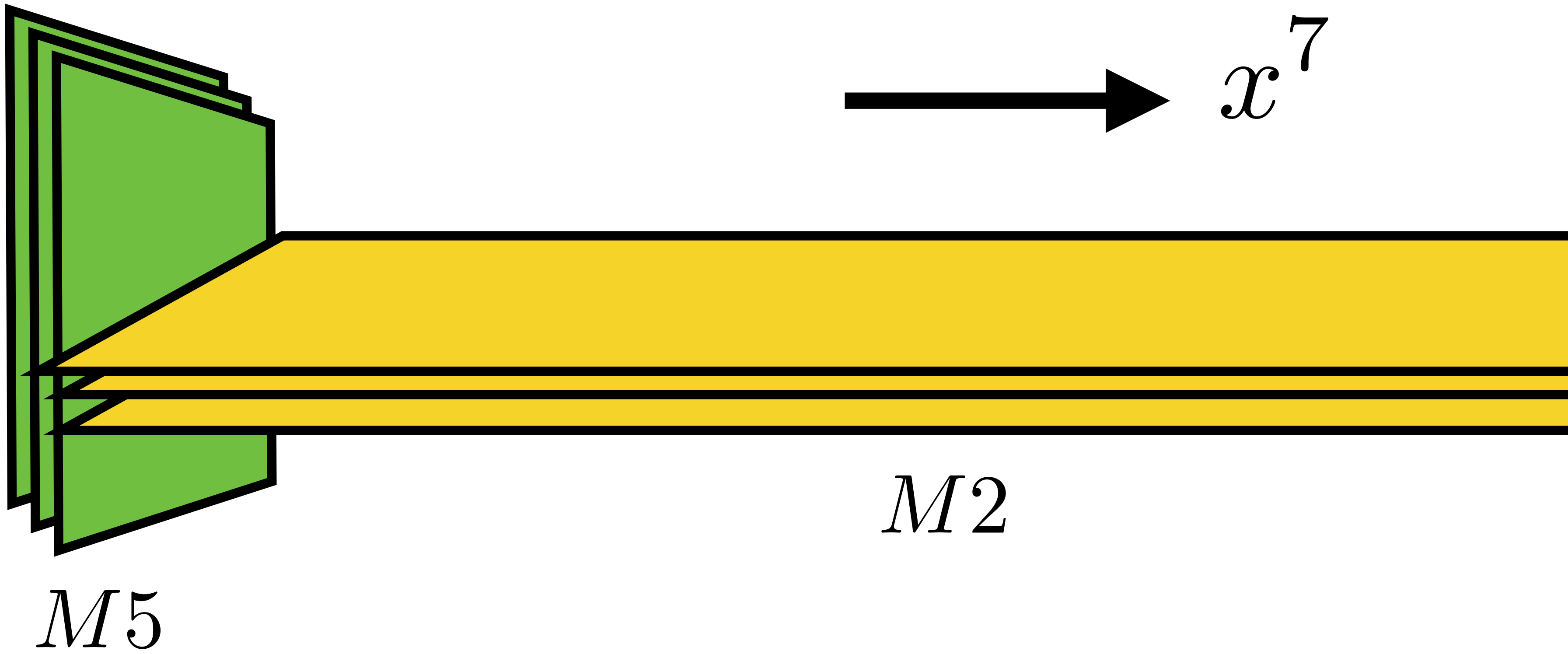
$$b_{\mathrm{UV}} \geq b_{\mathrm{IR}}$$

$$\boxed{b_{\mathrm{IR}} = -\frac{1}{16}}$$

Dirichlet B.C.

Examples

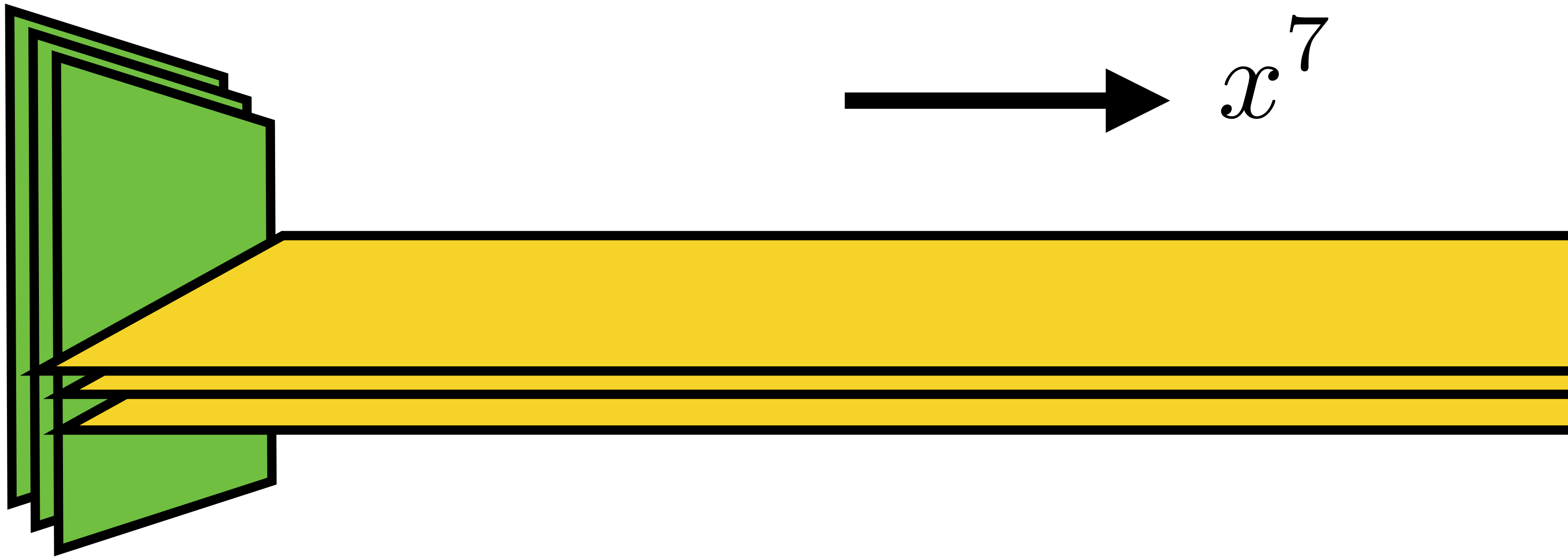
M2-branes ending on M5-branes



	1	2	3	4	5	6	7	8	9	10	11
M5	●	●	●	●	●	●	—	—	—	—	—
M2	●	●	—	—	—	—	●	—	—	—	—

Examples

M2-branes ending on M5-branes



N coincident M5-branes' worldvolume theory

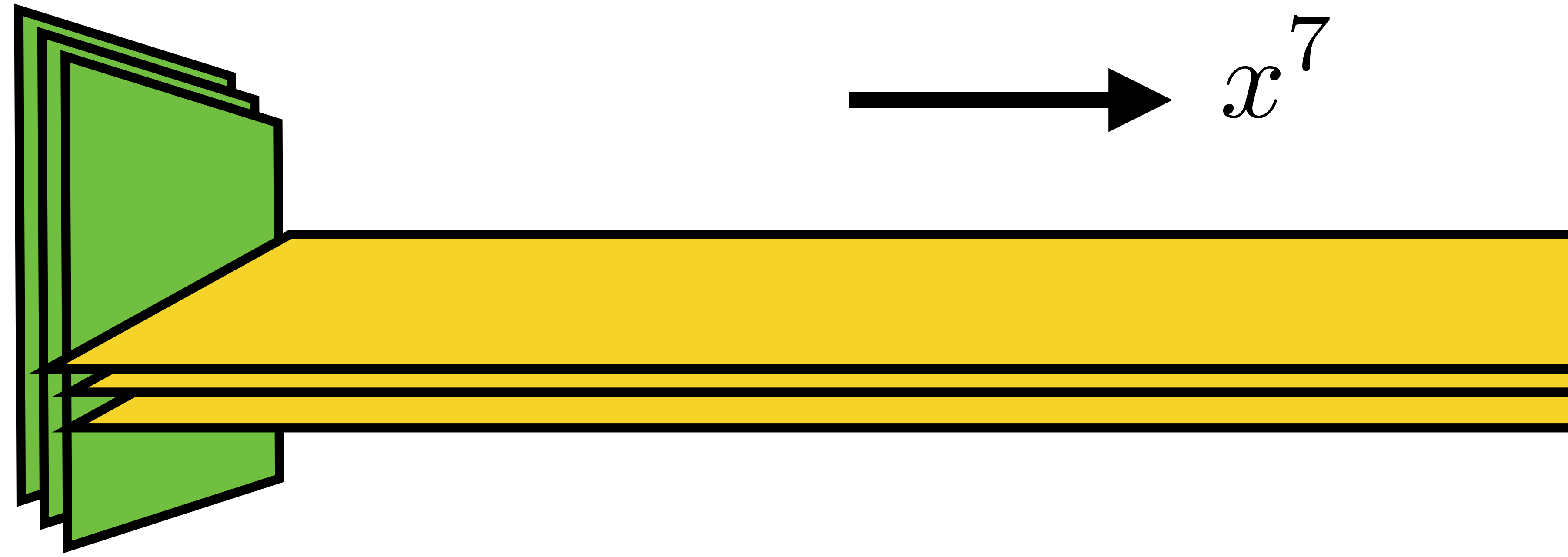
$d=6$ $\mathcal{N}=(2,0)$ SUSY CFT

Gauge algebra $U(N)$

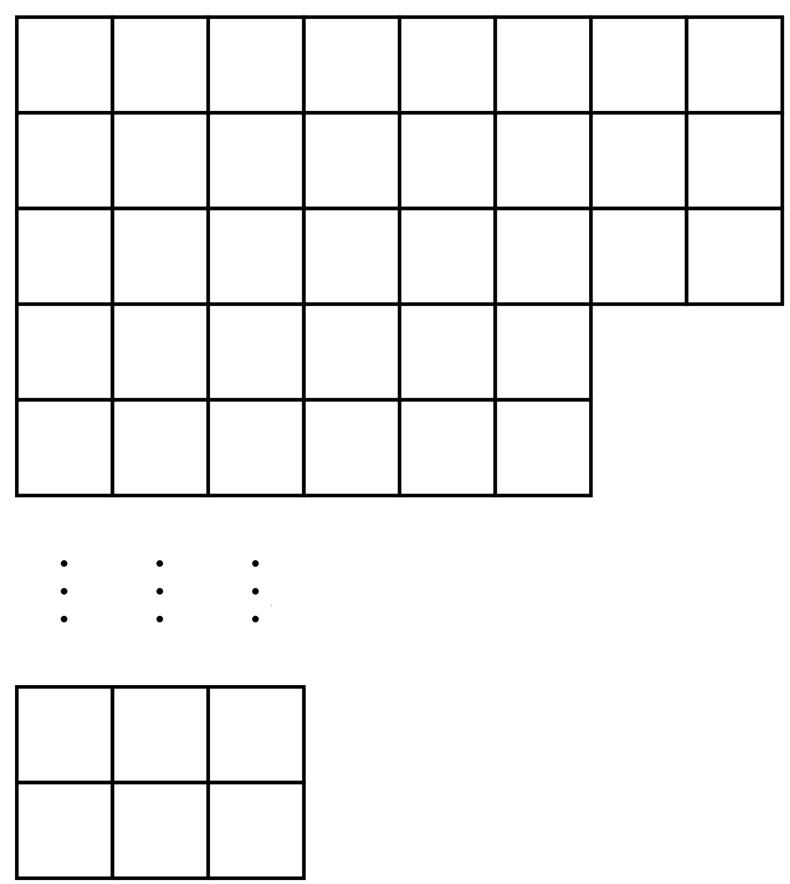
M2-branes = "Wilson surface"

Examples

M2-branes ending on M5-branes



$\mathcal{R} \leftrightarrow$



Examples

M2-branes ending on M5-branes

Estes, Krym, O'Bannon, Robinson, Rodgers 1812.00923
Jensen, O'Bannon, Robinson, Rodgers 1812.08745
Bianchi + Lemos 1911.05082
Chalabi, O'Bannon, Robinson, Sisti 2003.02857
Drukker, Probst, Trépanier 2003.12372, 2009.10732, 2012.11087
Drukker, Giombi, Tseytlin, Zhou 2004.04562
Wang 2012.06574
Bachas, Bianchi, Chen 2501.13197

$$b = 24(\lambda, \rho) + 3(\lambda, \lambda)$$

$$d_1 = 24(\lambda, \rho) + 6(\lambda, \lambda)$$

$$d_2 = 24(\lambda, \rho) + 6(\lambda, \lambda)$$

$\lambda \equiv \text{highest weight}$ $\qquad$ $\rho \equiv \text{Weyl vector}$

Examples

M2-branes ending on M5-branes

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Bachas, Bianchi, Chen 2501.13197

$$b = 24(\lambda, \rho) + 3(\lambda, \lambda)$$

$$d_1 = 24(\lambda, \rho) + 6(\lambda, \lambda)$$

$$d_2 = 24(\lambda, \rho) + 6(\lambda, \lambda)$$

supersymmetry $\Rightarrow \quad d_1 = d_2$

Examples

M2-branes ending on M5-branes

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Bachas, Bianchi, Chen 2501.13197

$$b = 24(\lambda, \rho) + 3(\lambda, \lambda)$$

$$d_1 = 24(\lambda, \rho) + 6(\lambda, \lambda)$$

$$d_2 = 24(\lambda, \rho) + 6(\lambda, \lambda)$$

$$b \geq 0 \qquad d_1 \geq 0 \qquad d_2 \geq 0$$

Examples

M2-branes ending on M5-branes

Estes, Krym, O'Bannon, Robinson, Rodgers 1812.00923
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Bianchi + Lemos 1911.05082
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Drukker, Giombi, Tseytlin, Zhou 2004.04562
Wang 2012.06574
Bachas, Bianchi, Chen 2501.13197

$$b = 24(\lambda, \rho) + 3(\lambda, \lambda)$$

$$d_1 = 24(\lambda, \rho) + 6(\lambda, \lambda)$$

$$d_2 = 24(\lambda, \rho) + 6(\lambda, \lambda)$$

All three are invariant under the Weyl group
and complex conjugation of the representation $\mathcal{R} \to \mathcal{R}^*$

Examples

AdS_{d+1} radius of curvature L

AdS_3 probe brane

$$S_{\rm brane} = \mathcal{T} \int d^3\xi \sqrt{\det P[G_{MN}]}$$

$$b = d_1 = d_2 = 6\pi \, L^3 \, \mathcal{T}$$

Graham + Witten hep-th/9901021

Jensen, O'Bannon, Robinson, Rodgers 1812.08745

Similar to holographic CFTs in $d = 4$, which have $a_{4d} = c_{4d}$

Henningson + Skenderis hep-th/9806087,9812032

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3d Boundary or Defect

CFT	Boundary/Defect
$d \geq 4$	$p = 3$

What is the general form of $\left[T_\mu{}^\mu\right]_{p=3}$?

Non-zero only when $d = 4$

(codimension $d - p = 1$)

Herzog, Huang, Jensen 1510.00021, 1709.07431

3d Boundary

$$\left[T_\mu{}^\mu\right]_{p=3} = \frac{1}{(4\pi)^2}\left(a_{4d}\, E_4\big|_\partial - d_1\, \mathring{\mathrm{I\!I}}^3 - d_2\, \mathring{\mathrm{I\!I}}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

Fursaev 1510.01427

Solodukhin 1510.04566

3d Boundary

$$[T_\mu{}^\mu]_{p=3} = \frac{1}{(4\pi)^2}\left(a_{4d}\, E_4|_\partial - d_1\, \mathring{\mathrm{I\!I}}^3 - d_2\, \mathring{\mathrm{I\!I}}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

Fursaev 1510.01427   Solodukhin 1510.04566

Boundary contribution to Euler density E_4

No new central charge

3d Boundary

$$\left[T_\mu{}^\mu\right]_{p=3} = \frac{1}{(4\pi)^2}\left(a_{4d}\, E_4|_\partial - d_1\, \mathring{\mathrm{I\!I}}^3 - d_2\, \mathring{\mathrm{I\!I}}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

Fursaev 1510.01427

Solodukhin 1510.04566

boundary central charges

$d_1 \qquad d_2$

Type B

3d Boundary

$$[T_\mu{}^\mu]_{p=3} = \frac{1}{(4\pi)^2}\left(a_{4d}\, E_4|_\partial - d_1\, \mathring{\mathrm{II}}^3 - d_2\, \mathring{\mathrm{II}}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

Fursaev 1510.01427 Solodukhin 1510.04566

Displacement operator D

$$\langle D(\mathbf{y}_1) D(\mathbf{y}_2) D(0)\rangle \propto \frac{d_1}{|\mathbf{y}_1|^4 |\mathbf{y}_2|^4 |\mathbf{y}_1 - \mathbf{y}_2|^4}$$

3d Boundary

$$\left[T_\mu{}^\mu\right]_{p=3} = \frac{1}{(4\pi)^2}\left(a_{4d}\, E_4|_\partial - d_1\, \mathring{\mathrm{I\!I}}^3 - d_2\, \mathring{\mathrm{I\!I}}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

Fursaev 1510.01427

Solodukhin 1510.04566

Displacement operator D

$$\langle D(y^a) D(0)\rangle \propto \frac{d_2}{|y^a|^8}$$

Reflection positivity (unitarity)

$d_2 \geq 0$

Examples

$d=4$ BCFTs

free massless real scalar, Dirac fermion, Maxwell field

Herzog, Huang, Jensen 1510.00021, 1709.07431

Fursaev 1510.01427 Solodukhin 1510.04566

Theory	a_{4d}	c_{4d}	BC	d_1	d_2
Scalar	$1/360$	$1/120$	Dirichlet	$2/35$	$1/15$
Scalar	$1/360$	$1/120$	Neumann	$2/45$	$1/15$
Fermion	$11/360$	$1/20$	Mixed	$2/7$	$2/5$
Maxwell	$31/180$	$1/10$	Electric	$16/35$	$4/5$
Maxwell	$31/180$	$1/10$	Magnetic	$16/35$	$4/5$

$d_2 = 8\,c_{4d}$ (with interactions, no longer true)

Outline:

- Review: Trace Anomalies
- Review: Boundaries and Defects
- 1d Boundary or Defect
- 2d Boundary or Defect
- 3d Boundary or Defect
- 4d Boundary or Defect
- Summary and Outlook

4d Boundary or Defect

CFT

$d \geq 5$

Boundary/Defect

$p = 4$

What is the general form of $\left[T_\mu{}^\mu\right]_{p=4}$?

Astaneh + Solodukhin 2102.07661

Chalabi, Herzog, O'Bannon, Robinson, Sisti 2111.14713

4d Boundary or Defect

$$
\begin{aligned}
\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(&- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2 \\
&+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk} \\
&+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj} \\
&+ d_{13} W_{ab}{}^{ab}\mathring{\Pi}^i_{cd}\mathring{\Pi}^{cd}_i + d_{14} W^a{}_{bij}\mathring{\Pi}^i_{ac}\mathring{\Pi}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\Pi}^i{}_{ac}\mathring{\Pi}^{jbc} \\
&+ d_{16} W^{abcd}\mathring{\Pi}^i_{ac}\mathring{\Pi}_{ibd} + d_{17} W_a{}^{bac}\mathring{\Pi}^i_{bd}\mathring{\Pi}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\Pi}^i_{ab}\mathring{\Pi}^{jab} \\
&+ d_{19}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i\mathring{\Pi}^j\mathring{\Pi}_j + d_{20}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j\mathring{\Pi}_i\mathring{\Pi}_j + d_{21}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j)(\mathrm{Tr}\,\mathring{\Pi}_i\mathring{\Pi}_j)
\end{aligned}
$$

boundary/defect central charges

$$b \qquad d_1, d_2, \ldots, d_{22}$$

Type A

Type B

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$

W^2

$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$

$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$

$W\mathrm{II}^2$

$$+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{II}}^i_{cd}\mathring{\mathrm{II}}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\mathrm{II}}^i_{ac}\mathring{\mathrm{II}}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\mathrm{II}}^i{}_{ac}\mathring{\mathrm{II}}^{jbc}$$

$$+ d_{16} W^{abcd}\mathring{\mathrm{II}}^i_{ac}\mathring{\mathrm{II}}_{ibd} + d_{17} W_a{}^{bac}\mathring{\mathrm{II}}^i_{bd}\mathring{\mathrm{II}}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\mathrm{II}}^i_{ab}\mathring{\mathrm{II}}^{jab}$$

II^4

$$+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}_i\mathring{\mathrm{II}}^j\mathring{\mathrm{II}}_j + d_{20}\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}^j\mathring{\mathrm{II}}_i\mathring{\mathrm{II}}_j + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}^j)(\mathrm{Tr}\,\mathring{\mathrm{II}}_i\mathring{\mathrm{II}}_j)$$

boundary/defect central charges

$$b \qquad d_1, d_2, \ldots, d_{22}$$

4d Boundary or Defect

$$[T_\mu{}^\mu]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$

W^2

$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$

$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$

$W\mathrm{I\!I}^2$

$$+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{I\!I}}{}^i_{cd}\mathring{\mathrm{I\!I}}{}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\mathrm{I\!I}}{}^i_{ac}\mathring{\mathrm{I\!I}}{}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\mathrm{I\!I}}{}^i{}_{ac}\mathring{\mathrm{I\!I}}{}^{jbc}$$

$$+ d_{16} W^{abcd}\mathring{\mathrm{I\!I}}{}^i_{ac}\mathring{\mathrm{I\!I}}{}_{ibd} + d_{17} W_a{}^{bac}\mathring{\mathrm{I\!I}}{}^i_{bd}\mathring{\mathrm{I\!I}}{}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\mathrm{I\!I}}{}^i_{ab}\mathring{\mathrm{I\!I}}{}^{jab}$$

$\mathrm{I\!I}^4$

$$+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}_i\mathring{\mathrm{I\!I}}{}^j\mathring{\mathrm{I\!I}}{}_j + d_{20}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}^j\mathring{\mathrm{I\!I}}{}_i\mathring{\mathrm{I\!I}}{}_j + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}^j)(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}_i\mathring{\mathrm{I\!I}}{}_j)$$

$$\mathcal{J}_1 = \frac{1}{d-1}R\mathring{\mathrm{I\!I}}{}^i_{ab}\mathring{\mathrm{I\!I}}{}^{ab}_i - \frac{1}{d-2}N^{\mu\nu}R_{\mu\nu}\mathring{\mathrm{I\!I}}{}^i_{ab}\mathring{\mathrm{I\!I}}{}^{ab}_i - \frac{2}{d-2}R^a{}_b\mathring{\mathrm{I\!I}}{}^i_{ac}\mathring{\mathrm{I\!I}}{}^{bc}_i - \frac{1}{2}W^c{}_{acb}\mathrm{I\!I}_i\mathring{\mathrm{I\!I}}{}^{iab}$$
$$+ \frac{4}{9}W^c{}_{ica}\overline{D}{}^b\mathring{\mathrm{I\!I}}{}^i_{ab} + \mathring{\mathrm{I\!I}}{}^{iab}D_iW^c{}_{acb} - \frac{1}{2}\mathrm{I\!I}^i\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}_i\mathring{\mathrm{I\!I}}{}^j\mathring{\mathrm{I\!I}}{}_j + \frac{1}{16}\mathrm{I\!I}^i\mathrm{I\!I}_i\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^j\mathring{\mathrm{I\!I}}{}_j$$
$$+ \frac{2}{9}\overline{D}{}^b\mathring{\mathrm{I\!I}}{}^i_{ab}\overline{D}{}^c\mathring{\mathrm{I\!I}}{}_{ic}{}^a\,,$$

$$\mathcal{J}_2 = \frac{d-4}{d-2}W_{ab}{}^{ab}N^{\mu\nu}R_{\mu\nu} - \frac{d-4}{d-1}RW_{ab}{}^{ab} + \frac{4(d-5)}{3(d-2)}R_{ab}W_c{}^{acb}$$
$$- \frac{5(d-4)}{48}W_{ab}{}^{ab}\mathrm{I\!I}^i\mathrm{I\!I}_i + \frac{2(d-5)}{3}W^c{}_{ica}\overline{D}{}^b\mathring{\mathrm{I\!I}}{}^i_{ab} + \frac{4(d+1)}{9}\mathring{\mathrm{I\!I}}{}^{iab}D_iW^c{}_{acb}$$
$$- \frac{1}{3}W_{ic}{}^{ac}\overline{D}_a\mathrm{I\!I}^i - \frac{2(d-5)}{3}\mathrm{I\!I}^i\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}_i\mathring{\mathrm{I\!I}}{}^j\mathring{\mathrm{I\!I}}{}_j + \frac{(d-10)}{12}\mathrm{I\!I}^iD_iW_{ab}{}^{ab} + \frac{1}{3}D^iD_iW_{ab}{}^{ab}$$

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(-b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{II}}^i_{cd}\mathring{\mathrm{II}}^{cd}_i + d_{14} W^a{}_{bij}\mathring{\mathrm{II}}^i_{ac}\mathring{\mathrm{II}}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\mathrm{II}}^i{}_{ac}\mathring{\mathrm{II}}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\mathrm{II}}^i_{ac}\mathring{\mathrm{II}}_{ibd} + d_{17} W_a{}^{bac}\mathring{\mathrm{II}}^i_{bd}\mathring{\mathrm{II}}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\mathrm{II}}^i_{ab}\mathring{\mathrm{II}}^{jab}$$
$$+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}_i\mathring{\mathrm{II}}^j\mathring{\mathrm{II}}_j + d_{20}\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}^j\mathring{\mathrm{II}}_i\mathring{\mathrm{II}}_j + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}^j)(\mathrm{Tr}\,\mathring{\mathrm{II}}_i\mathring{\mathrm{II}}_j)\Big)$$

b-theorem

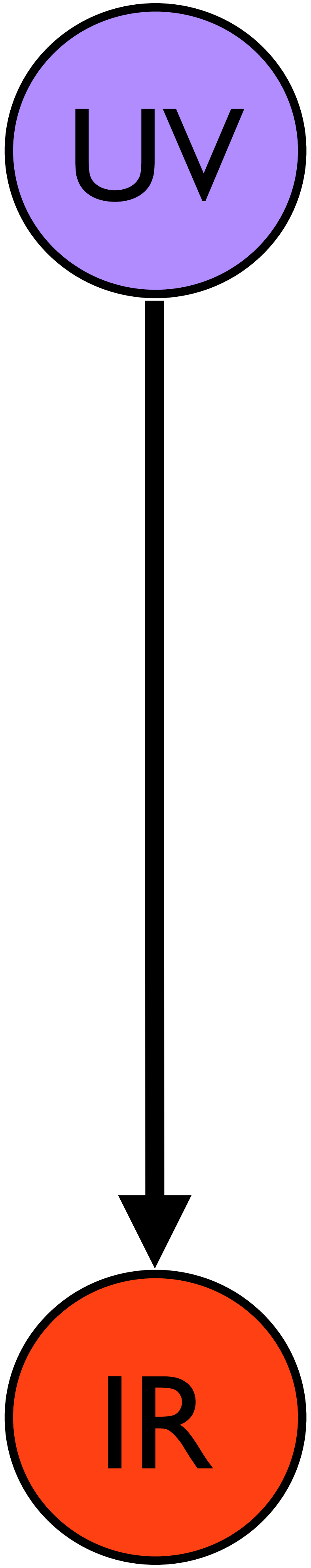
Wang 2101.12648

Casini, Landea, Torroba 2303.16935

RG flow on the boundary/defect

$$b_{\mathrm{UV}} \geq b_{\mathrm{IR}}$$

Counts the number of degrees of freedom



UV BCFT

Free, massless, real scalar

Neumann B.C.

Boundary RG Flow

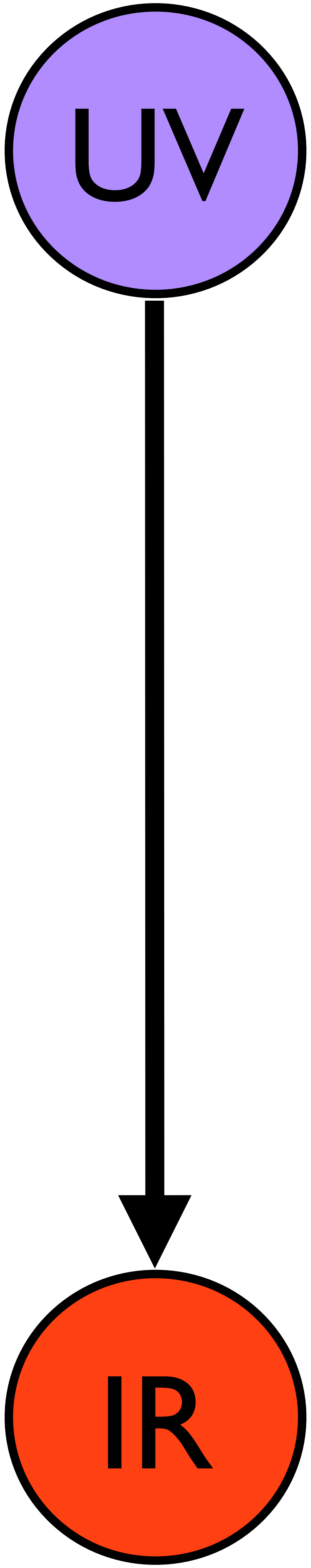
$$S_{\text{BCFT}}^{\text{UV}} \to S_{\text{BCFT}}^{\text{UV}} + \int d^5x\, \delta(x)\, m^2\, \Phi^2(\vec{x})$$

$$\Delta_{\Phi^2} = 3 < 4$$

IR BCFT

Free, massless, real scalar

Dirichlet B.C.



$$b_{\mathrm{UV}} = \frac{17}{46080}$$

Neumann B.C.

$$b_{\mathrm{UV}} \geq b_{\mathrm{IR}}$$

$$b_{\mathrm{IR}} = -\frac{17}{46080}$$

Dirichlet B.C.

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1 \mathcal{J}_1 + d_2 \mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\Pi}^i_{cd}\mathring{\Pi}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\Pi}^i_{ac}\mathring{\Pi}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\Pi}^i{}_{ac}\mathring{\Pi}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\Pi}^i_{ac}\mathring{\Pi}_{ibd} + d_{17} W_a{}^{bac}\mathring{\Pi}^i_{bd}\mathring{\Pi}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\Pi}^i_{ab}\mathring{\Pi}^{jab}$$
$$+ d_{19}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i\mathring{\Pi}^j\mathring{\Pi}_j + d_{20}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j\mathring{\Pi}_i\mathring{\Pi}_j + d_{21}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j)(\mathrm{Tr}\,\mathring{\Pi}_i\mathring{\Pi}_j)\Big)$$

Displacement operator D^i

$$\langle D^i(y^a)D^j(0)\rangle \propto \frac{d_1\,\delta^{ij}}{|y^a|^{10}}$$

Reflection positivity (unitarity)

$$d_1 \geq 0$$

Chalabi, Herzog, O'Bannon, Robinson, Sisti 2111.14713

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4(W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{I\!I}}{}^i_{cd}\mathring{\mathrm{I\!I}}{}^{cd}_i + d_{14} W^a{}_{bij}\mathring{\mathrm{I\!I}}{}^i_{ac}\mathring{\mathrm{I\!I}}{}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\mathrm{I\!I}}{}^i{}_{ac}\mathring{\mathrm{I\!I}}{}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\mathrm{I\!I}}{}^i_{ac}\mathring{\mathrm{I\!I}}_{ibd} + d_{17} W_a{}^{bac}\mathring{\mathrm{I\!I}}{}^i_{bd}\mathring{\mathrm{I\!I}}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\mathrm{I\!I}}{}^i_{ab}\mathring{\mathrm{I\!I}}{}^{jab}$$
$$+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}{}^j\mathring{\mathrm{I\!I}}_j + d_{20}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}^j\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}_j + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}^j)(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}_j)$$

$$\langle [T^{ab}]_{\mathrm{bulk}}\rangle = -h(d-p-1)\frac{\delta^{ab}}{|x_\perp|^d} \qquad \langle [T^{ai}]_{\mathrm{bulk}}\rangle = 0$$

$$\langle [T^{ij}]_{\mathrm{bulk}}\rangle = h\frac{(p+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

4d Boundary or Defect

$$
\begin{aligned}
\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(&- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2 \\
&+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk} \\
&+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj} \\
&+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{I\!I}}^i_{cd}\mathring{\mathrm{I\!I}}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\mathrm{I\!I}}^i_{ac}\mathring{\mathrm{I\!I}}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\mathrm{I\!I}}^i{}_{ac}\mathring{\mathrm{I\!I}}^{jbc} \\
&+ d_{16} W^{abcd}\mathring{\mathrm{I\!I}}^i_{ac}\mathring{\mathrm{I\!I}}_{ibd} + d_{17} W_a{}^{bac}\mathring{\mathrm{I\!I}}^i_{bd}\mathring{\mathrm{I\!I}}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\mathrm{I\!I}}^i_{ab}\mathring{\mathrm{I\!I}}^{jab} \\
&+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}^j\mathring{\mathrm{I\!I}}_j + d_{20}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}^j\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}_j + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}^j)(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}_j)\Big)
\end{aligned}
$$

$$h = \frac{\Gamma\left(\frac{d-p}{2}+1\right)}{\pi^{\frac{d-p}{2}+2}\,(d-p+3)}\,d_2$$

Average Null Energy Condition (ANEC)

$$h \geq 0 \quad \Rightarrow \quad d_2 \geq 0$$

Chalabi, Herzog, O'Bannon, Robinson, Sisti 2111.14713   Casini, Landea, Torroba 2303.16935

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{I\!I}}{}^i_{cd}\mathring{\mathrm{I\!I}}{}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\mathrm{I\!I}}{}^i_{ac}\mathring{\mathrm{I\!I}}{}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\mathrm{I\!I}}{}^i{}_{ac}\mathring{\mathrm{I\!I}}{}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\mathrm{I\!I}}{}^i_{ac}\mathring{\mathrm{I\!I}}{}_{ibd} + d_{17} W_a{}^{bac}\mathring{\mathrm{I\!I}}{}^i_{bd}\mathring{\mathrm{I\!I}}{}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\mathrm{I\!I}}{}^i_{ab}\mathring{\mathrm{I\!I}}{}^{jab}$$
$$+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}_i\mathring{\mathrm{I\!I}}{}^j\mathring{\mathrm{I\!I}}{}_j + d_{20}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}^j\mathring{\mathrm{I\!I}}{}_i\mathring{\mathrm{I\!I}}{}_j + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}^i\mathring{\mathrm{I\!I}}{}^j)(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}{}_i\mathring{\mathrm{I\!I}}{}_j)$$

Entanglement Entropy (EE)

$$S_{\mathrm{EE}} = S_{\mathrm{EE}}^{\mathrm{CFT}} + S_{\mathrm{EE}}^{p=4}$$

$$S_{\mathrm{EE}}^{p=4} = \ldots + \left(4\,b - \frac{(d-5)(d-4)}{d-1}\,d_2\right)\ln(\ell/\varepsilon) + \ldots$$

Chalabi, Herzog, O'Bannon, Robinson, Sisti 2111.14713

Example

AdS_{d+1} radius of curvature L

AdS_5 probe brane

$$S_{\text{brane}} = \mathcal{T} \int d^5\xi \sqrt{\det P\left[G_{MN}\right]}$$

b	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}
$\frac{1}{4}$	-1	$-\frac{1}{d-4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{3(d-4)}$	$-\frac{(3d+2)}{9(d-4)}$	0	0	$-\frac{2}{3(d-4)}$	$-\frac{(d-2)}{6(d-4)}$	$-\frac{(3d-10)}{3(d-4)}$

d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}
$-\frac{2}{3(d-4)}$	$-\frac{1}{2}$	$\frac{6-d}{3(d-4)}$	$\frac{2d}{3(d-4)}$	$\frac{2(2d-7)}{3(d-4)}$	$\frac{(7d-22)}{3(d-4)}$	$-\frac{(d+2)}{3(d-4)}$	-1	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$

in units of $\pi^2 L^5 \mathcal{T}$

Chalabi, Herzog, O'Bannon, Robinson, Sisti 2111.14713

Outline:

- Review: Trace Anomalies
- Review: Boundaries and Defects
- 1d Boundary or Defect
- 2d Boundary or Defect
- 3d Boundary or Defect
- 4d Boundary or Defect
- Summary and Outlook

Summary

How do boundaries and defects contribute to the trace anomaly?

Can we define boundary and defect central charges? ✓

(work in progress)

✓

How do they appear in physical observables?

(work in progress)

Do they obey constraints? c-theorems? ✓

(work in progress)

Outlook

stress-energy tensor correlators?

entanglement entropy?

thermal entropy?

c-theorems? bounds? constraints?

examples?

applications?