Acoustic spectrum of vortex lattice in magnetic-field-induced superconducting phase of QCD

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Plan of the talk:

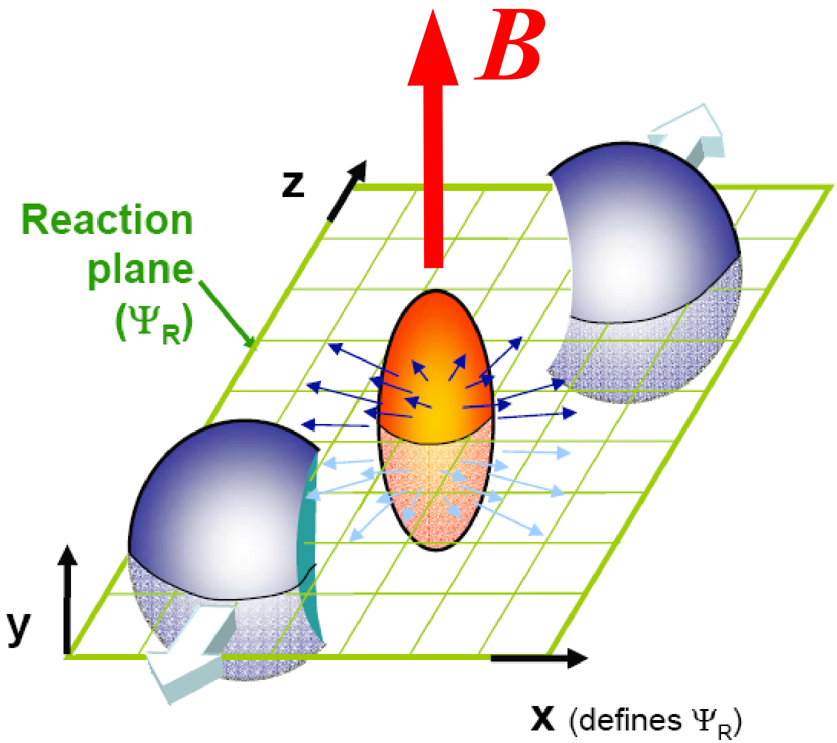
- **QCD in strong magnetic field: relevance to heavy-ion collisions**
- **Superconducting phase of QCD in strong magnetic field**
- **Acoustics of the superconducting phase**

Magnetic field in heavy-ion collisions

Noncentral heavy-ion collisions should produce a very strong magnetic field.

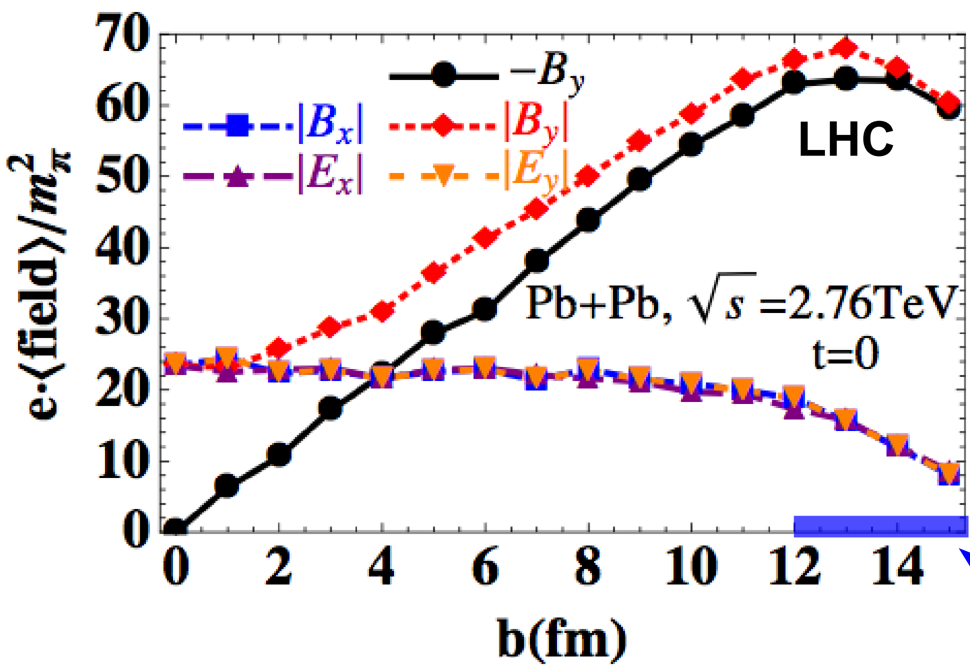
The field is directed out of the plane of the collision.

The duration of the field's pulse is very short however, (typically, 1 fm/c).

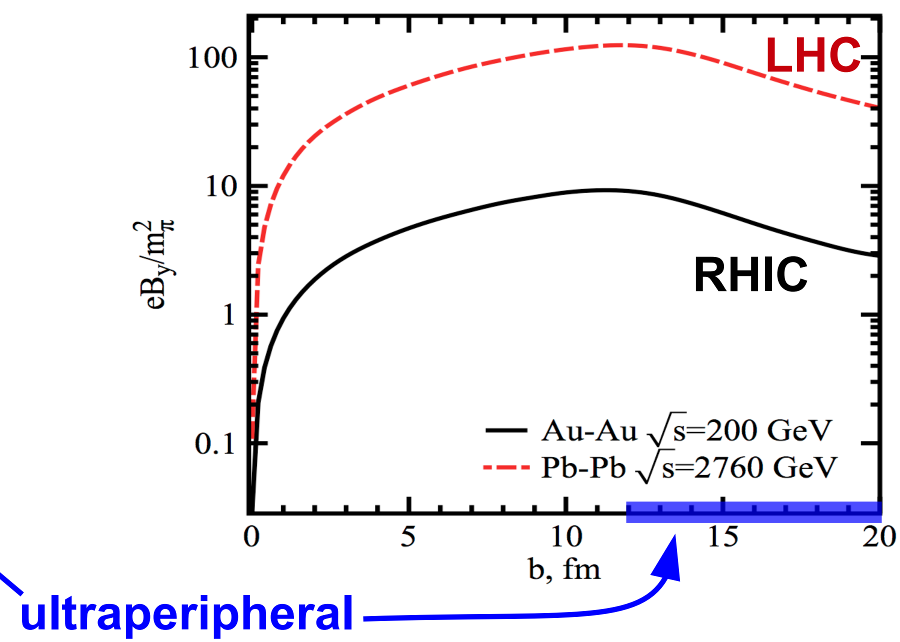


How strong field is created?

Estimations: $eB_{max} \simeq (2...3)\cdot 10^{16}$ Tesla



W. T. Deng and X. G. Huang,
Phys.Rev. C85 (2012) 044907



A. Bzdak and V. Skokov,
Phys.Lett. B710 (2012) 171
+ Vladimir Skokov,
private communication.

Conversion of units:

$$m_\pi^2 \simeq 0.02\ \text{GeV}^2 \simeq 3\cdot 10^{14}\ \text{Tesla} = 3\cdot 10^{18}\ \text{Gauss}$$

Is it really strong field?

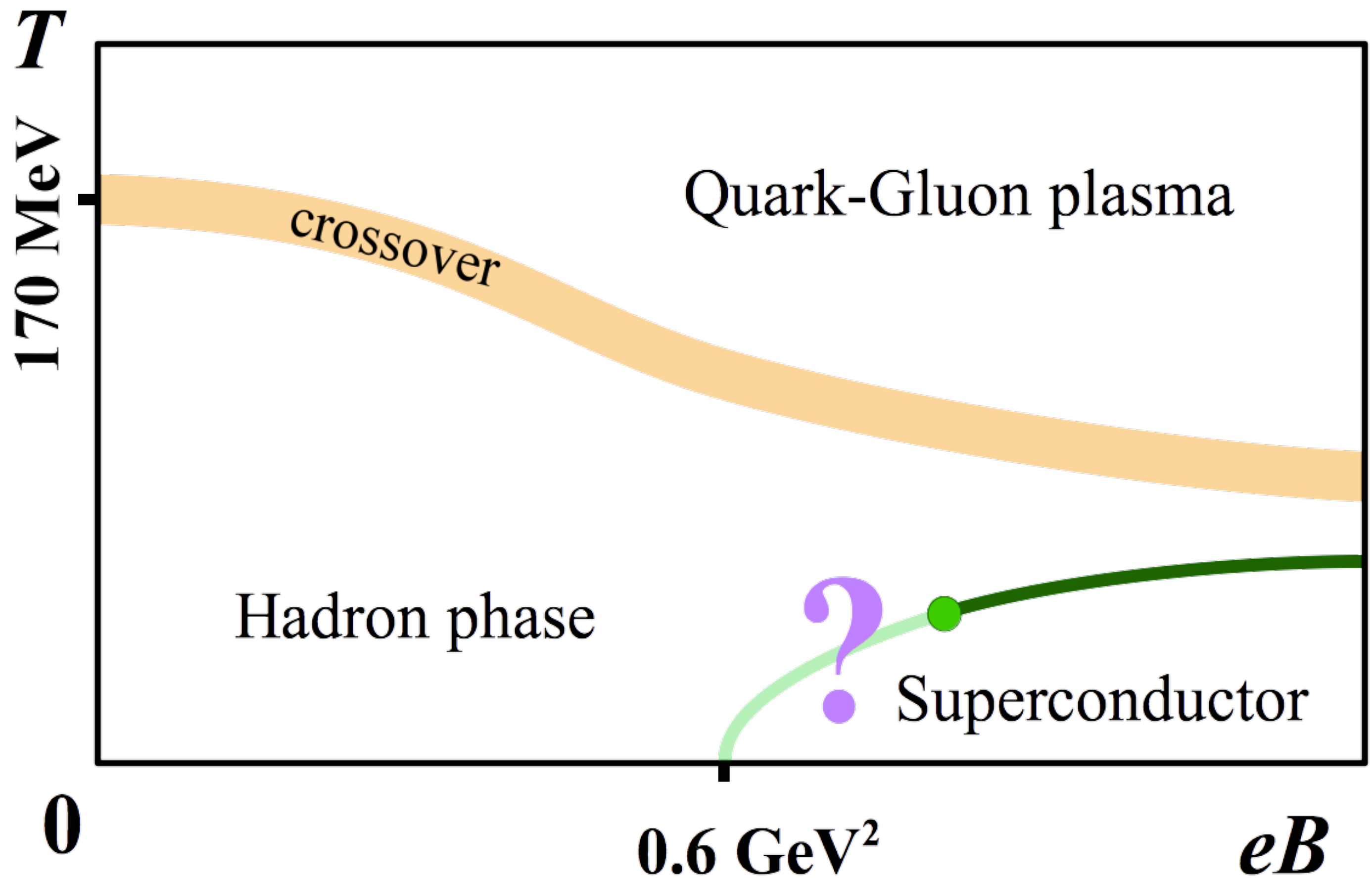
- Thinking — human brain: 10^{-12} Tesla
- Earth's magnetic field: 10^{-5} Tesla
- Loudspeaker magnet: 1 Tesla
- Strongest field in Lab: 10^{3} Tesla
- Typical neutron star: 10^{6} Tesla
- Magnetar: 10^{9} Tesla
- Heavy-ion collisions: 10^{16} Tesla

A. Bzdak, V. Skokov, Phys.Lett.B (2012)

W. T. Deng, X. G. Huang, Phys. Rev. C (2012)

Yes, the magnetic field created at the LHC is the strongest field we know.

Phase diagram – current status



Crossover and inverse magnetic catalysis:
G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP 1304, 130 (2013);

Electromagnetically superconducting phase: present talk and references therein.

Possible superconducting phase at strong field

In a background of strong enough magnetic field the vacuum may become a superconductor.

M.Ch., PRD82 (2010) 085011; PRL 106 (2011) 142003

The superconductivity emerges in empty space. Literally, “nothing becomes a superconductor”.

This claim seemingly contradicts textbooks which state that:

1. Superconductor is a material (= a form of matter, not an empty space)
2. Weak magnetic fields are suppressed by superconductivity
3. Strong magnetic fields destroy superconductivity

General features

Some features of the superconducting state of vacuum:

1. spontaneously emerges <u>above</u> the critical magnetic field

$$B_c \simeq 10^{16} \text{ Tesla} = 10^{20} \text{ Gauss}$$

or

$$eB_c \simeq m_\rho^2 \simeq 31\, m_\pi^2 \simeq 0.6 \text{ GeV}^2$$

2. usual Meissner effect does not exist

3. perfect conductor (= zero DC resistance) in one spatial dimension (along the axis of the magnetic field)

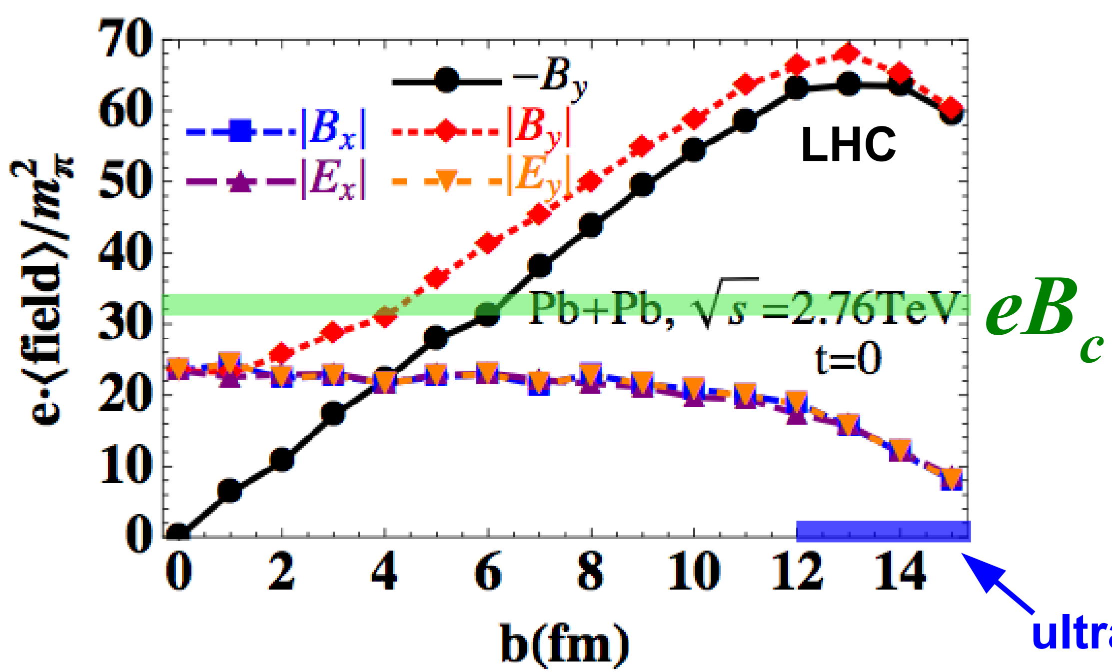
4. insulator in other (perpendicular) directions

5. Hyperbolic metamaterial (Smolyaninov, 2011): has a negative refraction index ("perfect lens").

Too strong critical magnetic field?

$$eB_c \simeq m_\rho^2 \simeq 31\, m_\pi^2 \simeq 0.6 \text{ GeV}^2$$

Over-critical magnetic fields (of the strength $B \sim 2...3\, B_c$) may be generated in ultraperipheral heavy-ion collisions
(duration is short, however – detailed calculations are required)



W. T. Deng and X. G. Huang,
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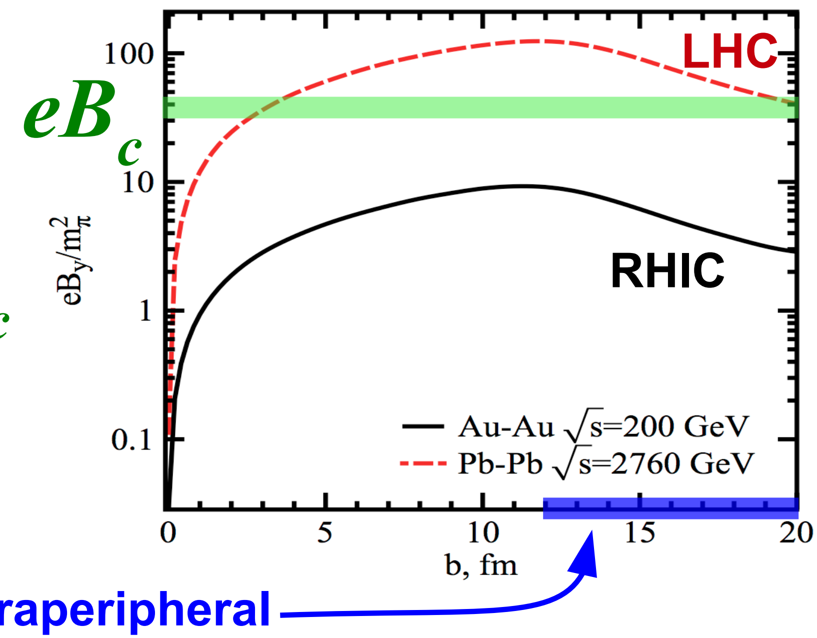


A. Bzdak and V. Skokov,
Phys.Lett. B710 (2012) 171
+ Vladimir Skokov,
private communication.

Approaches to the problem:

1. Effective bosonic model for electrodynamics of ρ mesons [M.Ch., arXiv:1008.1055]; [M.Ch., Van Doorsselaere, Verschelde, arxiv:1203.5963]

2. The Nambu-Jona-Lasinio model [M.Ch., arXiv:1101.0117]

3. Nonperturbative effective models based on gauge/gravity duality
   [Erdmenger, Kerner, Strydom (Munich, Germany), arXiv:1106.4551]
   [Callebaut, Dudal, Verschelde (Gent U., Belgium), arXiv:1105.2217]
   [Bu, Erdmenger, Shock, Strydom(Munich, Germany), arXiv:1210.6669]
   [Callebaut, Dudal, arXiv:1309.5042]

4. Closely related: vector meson condensates in strong magnetic field:
   [Rong-Gen Cai, Song He, Li Li, Li-Fang Li, arXiv:1309.2098]
   [Rong-Gen Cai, Li Li, Li-Fang Li, arXiv:1309.4877]
   [K. Wong, arXiv:1307.7839]

5. Numerical simulations of QCD vacuum [Braguta, Buividovich, M.Ch, Polikarpov, Kotov, arXiv:1104.3767 and arXiv:1104.3767]

+ Discussion [Y.Hidaka and A. Yamamoto, arXiv:1209.0007; Chuan Li, Qing Wang, arXiv:1301.7009; M. Ch., arXiv:1209.3587; arXiv:1309.4071]

The vacuum in strong magnetic field

Ingredients needed for possible superconductivity:

A. **Presence of electric charges?**
Yes, we have them: there are virtual particles which may potentially become "real" (= pop up from the vacuum) and make the vacuum (super)conducting.

B. **Reduction to 1+1 dimensions?**
Yes, we have this phenomenon: in a very strong magnetic field the dynamics of electrically charged particles (quarks, in our case) becomes effectively one-dimensional, because the particles tend to move along the magnetic field only.

C. **Attractive interaction between the like-charged particles?**
Yes, we have it: the gluons provide attractive interaction between the quarks and antiquarks ($q_u=+2\,e/3$ and $q_{\bar{d}}=+e/3$)

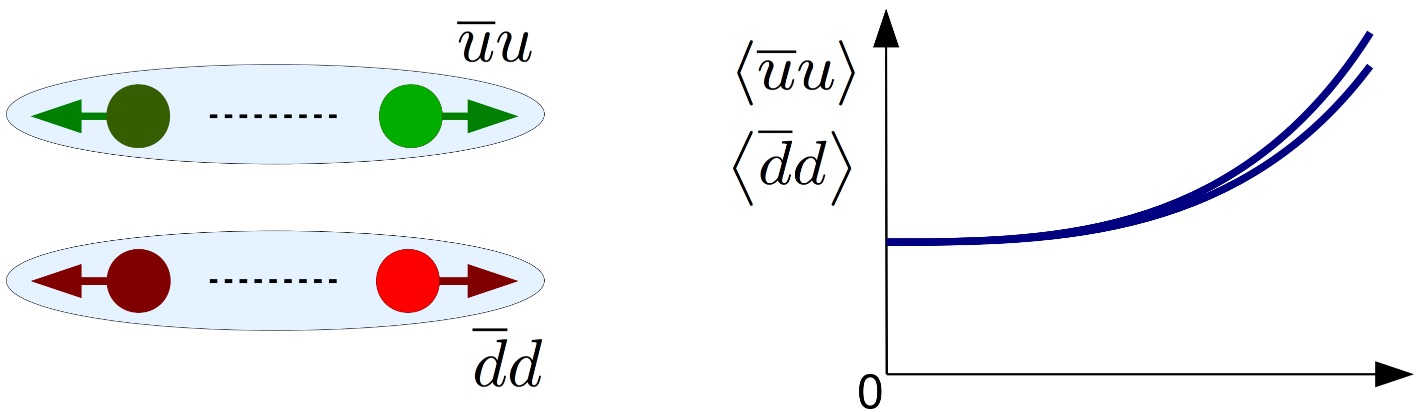
Pairing of quarks in strong magnetic field

Well-known “magnetic catalysis”:

S.P. Klevansky and R. H. Lemmer ('89); H. Suganuma and T. Tatsumi ('91) - effective models

V. P. Gusynin, V. A. Miransky and I. A. Shovkovy ('94, '95, '96,...) - real QCDxQED

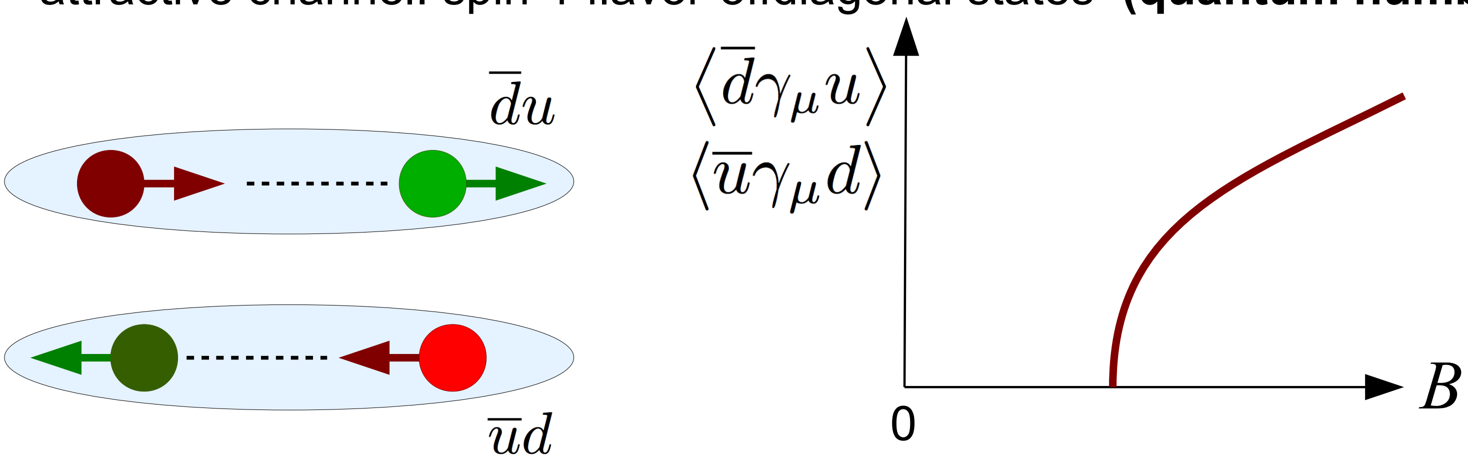
attractive channel: spin-0 flavor-diagonal states



enhances chiral symmetry breaking

This talk:

attractive channel: spin-1 flavor-offdiagonal states **(quantum numbers of $\rho^{\pm}$ mesons)**



electrically charged condensates: lead to electromagnetic superconductivity

B_c

Naïve qualitative picture of quark pairing in the electrically charged vector channel: ρ mesons

- Energy of a relativistic particle in the external magnetic field B_{ext}:

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\text{ext}} + m^2$$

momentum along the magnetic field axis — nonnegative integer number — projection of spin on the magnetic field axis

(the external magnetic field is directed along the *z*-axis)

- Masses of ρ mesons and pions in the external magnetic field

$$m^2_{\pi^\pm}(B_{\text{ext}}) = m^2_{\pi^\pm} + eB_{\text{ext}}$$ becomes heavier

$$m^2_{\rho^\pm}(B_{\text{ext}}) = m^2_{\rho^\pm} - eB_{\text{ext}}$$ becomes lighter

$$\rho^\pm \to \pi^\pm \pi^0$$

- Masses of ρ mesons and pions:

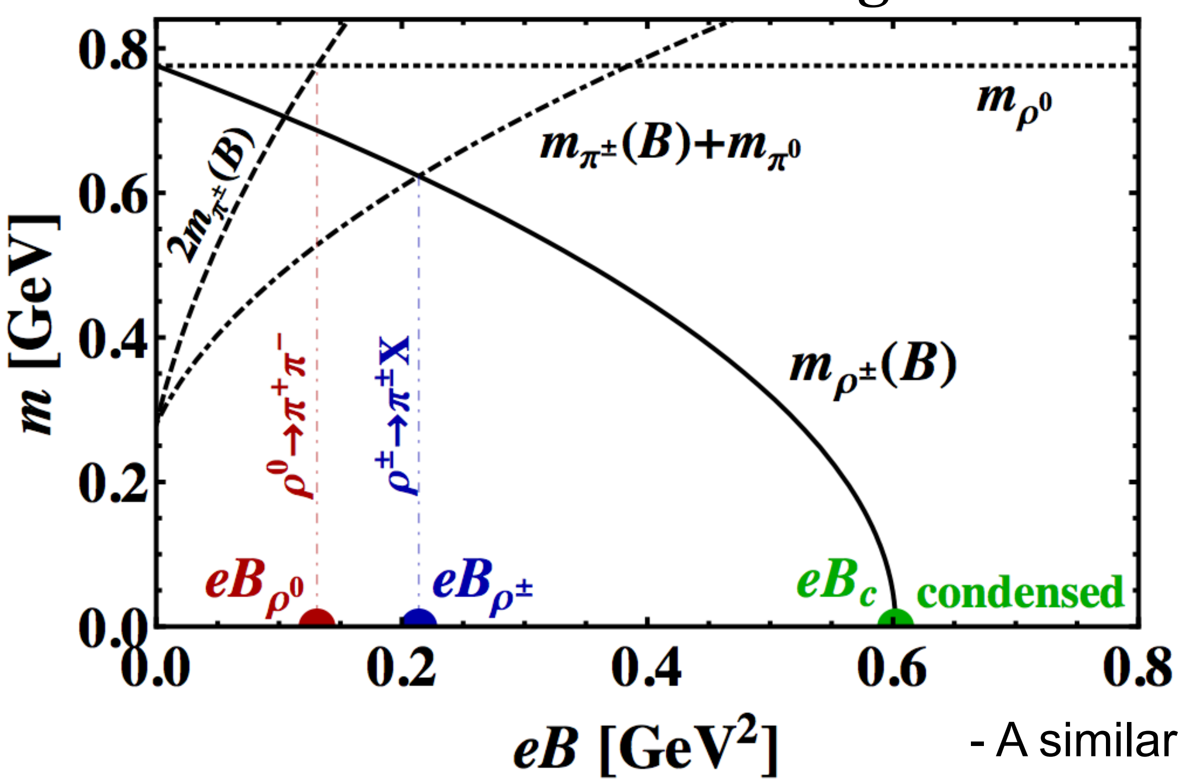
$$m_\pi = 139.6\,\text{MeV}\,, \qquad m_\rho = 775.5\,\text{MeV}$$

Condensation of ρ mesons (naïve)

The $\rho^{\pm}$ mesons become massless and condense at the critical value of the external magnetic field

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16}\,\text{Tesla}$$

masses in the external magnetic field



Kinematical impossibility of dominant decay modes

The pion becomes heavier while the ρ meson becomes lighter

- The decay $\rho^{\pm} \to \pi^{\pm}\pi^{0}$ stops at certain value of the magnetic field

$$m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}$$

- A similar statement is true for $\rho^0 \to \pi^+\pi^-$

Electrodynamics of ρ mesons

- Lagrangian (based on vector dominance models):

$$\mathcal{L} = -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\, \rho^{\dagger}_{\mu\nu}\rho^{\mu\nu} + m_\rho^2\, \rho^{\dagger}_{\mu}\rho^{\mu}$$
$$-\frac{1}{4}\, \rho^{(0)}_{\mu\nu}\rho^{(0)\mu\nu} + \frac{m_\rho^2}{2}\, \rho^{(0)}_{\mu}\rho^{(0)\mu} + \frac{e}{2g_s}\, F^{\mu\nu}\rho^{(0)}_{\mu\nu}$$

Nonminimal coupling leads to *g=2*

- Tensor quantities

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,,$$
$$f^{(0)}_{\mu\nu} = \partial_\mu \rho^{(0)}_\nu - \partial_\nu \rho^{(0)}_\mu\,,$$
$$\rho^{(0)}_{\mu\nu} = f^{(0)}_{\mu\nu} - ig_s(\rho^{\dagger}_{\mu}\rho_\nu - \rho_\mu\rho^{\dagger}_{\nu})$$
$$\rho_{\mu\nu} = D_\mu\rho_\nu - D_\nu\rho_\mu\,,$$

- Gauge invariance

$$U(1):\quad \begin{cases} \rho^{(0)}_\mu(x) & \rightarrow\ \ \rho^{(0)}_\mu(x)\,, \\ \rho_\mu(x) & \rightarrow\ \ e^{i\omega(x)}\rho_\mu(x)\,, \\ A_\mu(x) & \rightarrow\ \ A_\mu(x) + \partial_\mu\omega(x) \end{cases}$$

- Covariant derivative

$$D_\mu = \partial_\mu + ig_s\rho^{(0)}_\mu - ieA_\mu$$

- Kawarabayashi-Suzuki-Riadzuddin-Fayyazuddin relation

$$g_s \equiv g_{\rho\pi\pi} = \frac{m_\rho}{\sqrt{2}f_\pi} = 5.88$$
$$g_s \gg e \equiv \sqrt{4\pi\alpha_{\mathrm{e.m.}}} \approx 0.303$$

[D. Djukanovic, M. R. Schindler, J. Gegelia, S. Scherer, PRL (2005)]

Homogeneous approximation

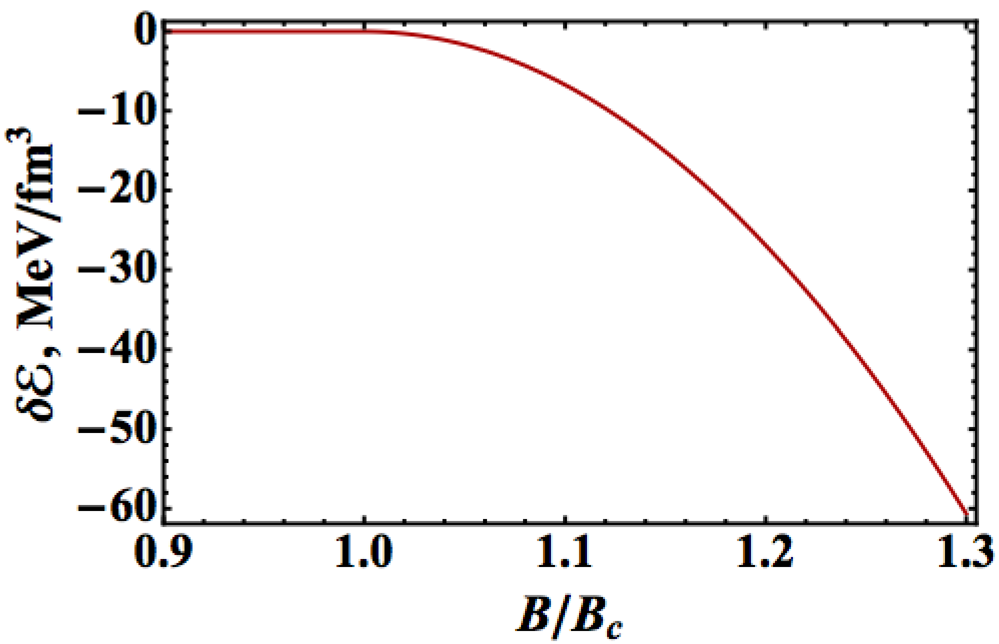
- The energy of the condensed state:

$$\epsilon_0(\rho) = \frac{1}{2} B_{\mathrm{ext}}^2 + 2(m_\rho^2 - eB_{\mathrm{ext}})\,|\rho|^2 + 2g_s^2\,|\rho|^4$$

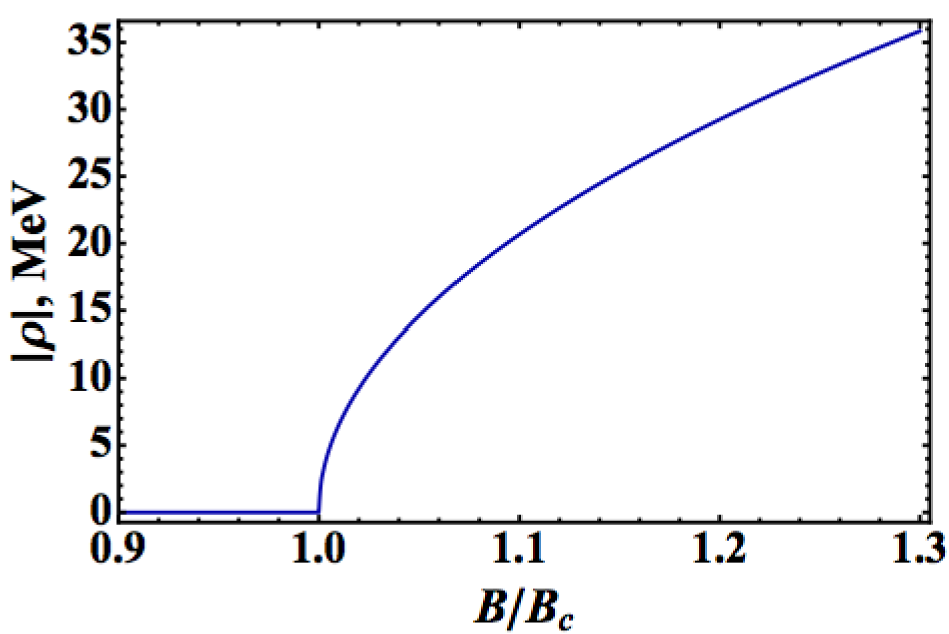
Similar to a temperature-dependent Ginzburg-Landau potential for superconductivity:

$$\epsilon(\phi) = \epsilon(0) + a\cdot(T - T_c)|\phi|^2 + b|\phi|^4$$

Condensation Energy



Condensate



Condensates of ρ mesons, solutions

Superconducting condensate (charged ρ condensate):

$$\rho(z) = \sum_{n\in\mathbb{Z}} C_n h_n\Big(\nu, \frac{z}{L_B}, \frac{\bar{z}}{L_B}\Big) \qquad L_B = \sqrt{\frac{2\pi}{eB}}$$

$$h_n(\nu, z, \bar{z}) = \exp\Big\{-\frac{\pi}{2}\big(|z|^2 + \bar{z}^2\big) - \pi\nu^2 n^2 + 2\pi\nu n\bar{z}\Big\}$$

$$z = x_1 + ix_2,\ \bar{z} = x_1 - ix_2$$

Superfluid condensate (neutral ρ condensate)

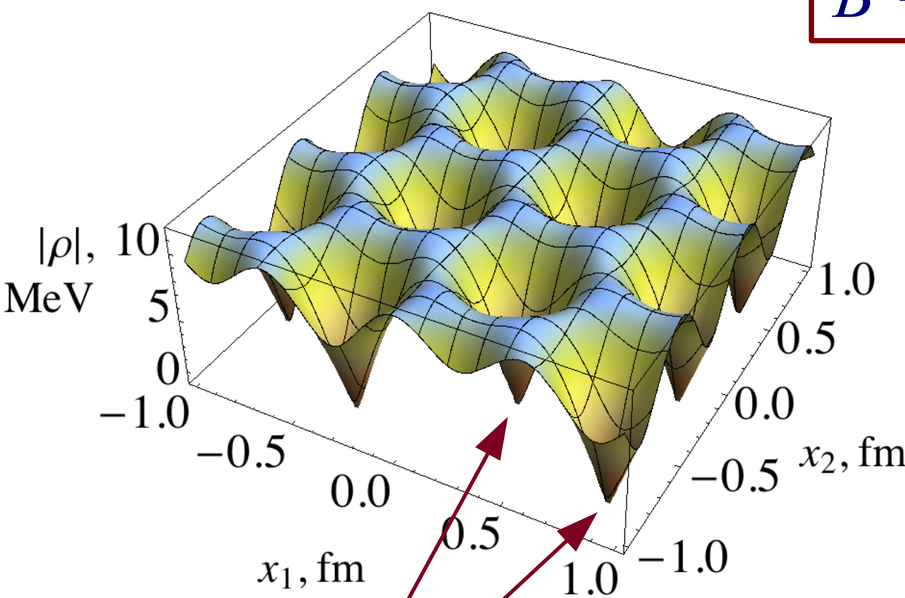
$$\rho^{(0)}(x_\perp) = 2ig_s \cdot \Big(\frac{\partial}{-\partial_\perp^2 + m_0^2}|\rho|^2\Big)(x_\perp)$$

$$\rho^{(0)} = \rho_1^{(0)} + i\rho_2^{(0)}$$

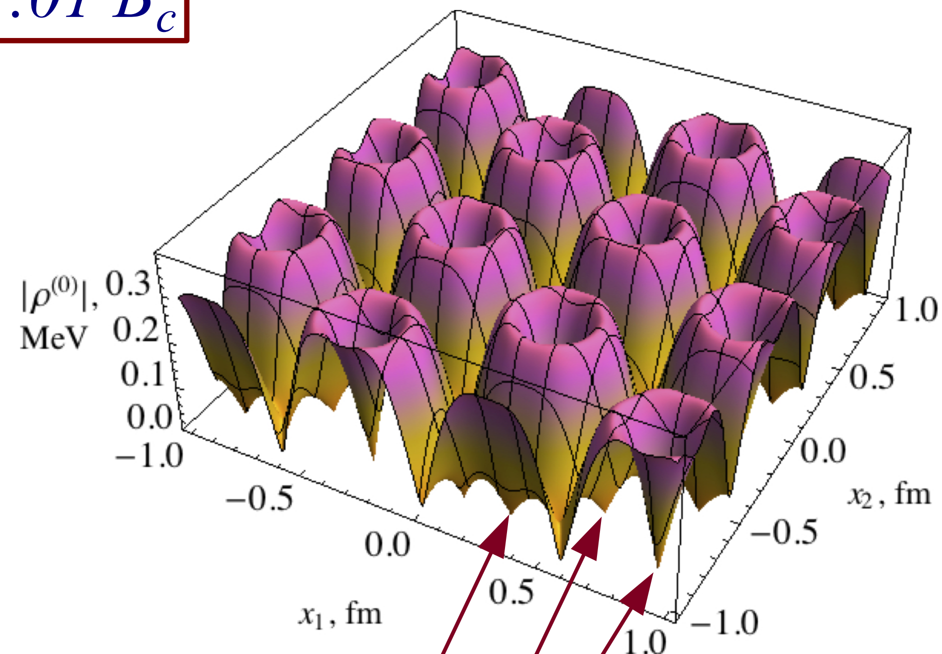
Condensates of ρ mesons, solutions

Superconducting condensate
(charged rho mesons)

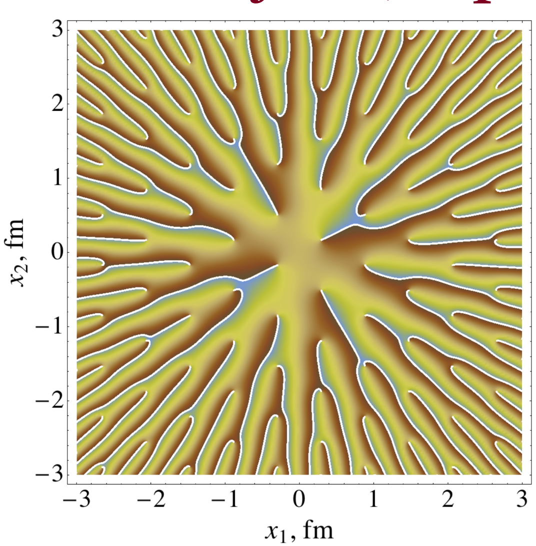
$B = 1.01\ B_c$



Superfluid condensate
(neutral rho mesons)

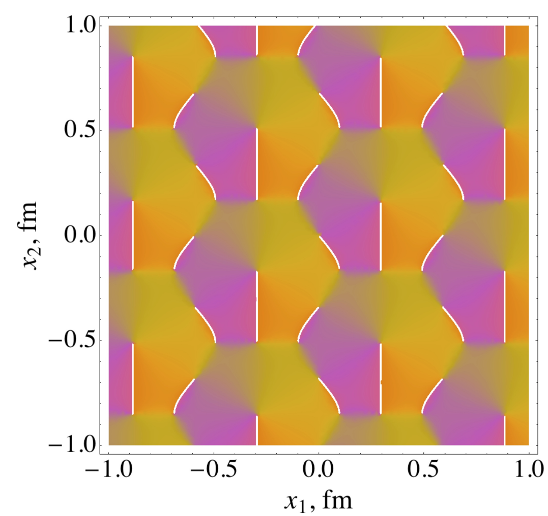


New objects, topological vortices, made of the rho-condensates



The phases of the rho-meson fields wind around vortex centers, at which the condensates vanish.

similar results in holographic approaches by M. Ammon, Y. Bu, J. Erdmenger, P. Kerner, J. Shock, M. Strydom (2012)



Anisotropic superconductivity
(via an analogue of the London equations)

- Apply a weak electric field $\boldsymbol{E}$ to an ordinary superconductor
- Then one gets accelerating electric current along the electric field:

$$\frac{\partial \vec{J}_{\rm GL}}{\partial t} = m_A^2 \vec{E}$$ [London equation]

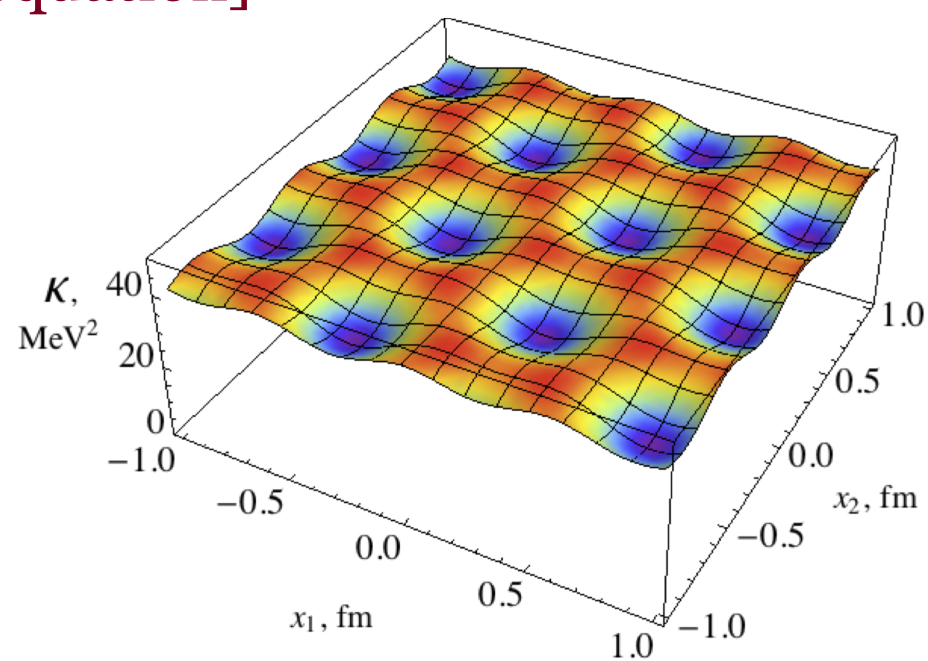
- In the QCDxQED vacuum, we get an accelerating electric current along the magnetic field $\boldsymbol{B}$:

$$\frac{\partial}{\partial t}\langle J_3 \rangle = -\frac{2e^3}{g_s^2}(B_{\rm ext} - B_c)E_3$$

$$\frac{\partial}{\partial t}\langle J_1 \rangle = \frac{\partial}{\partial t}\langle J_2 \rangle = 0$$

$$(\text{for } B \geqslant B_c)$$

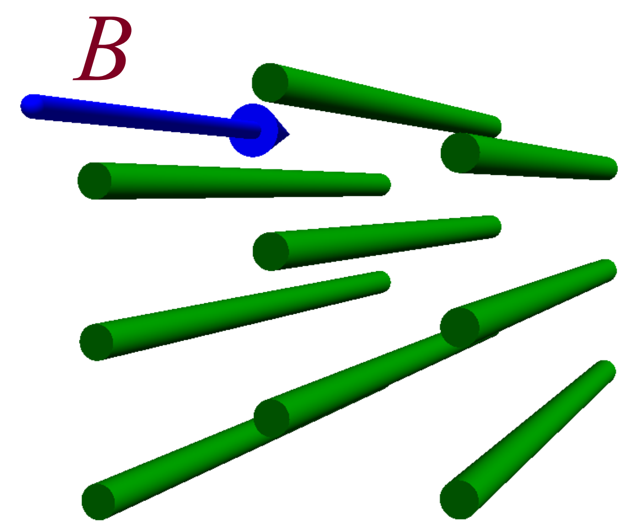
Written for an electric current averaged over one elementary (unit) rho-vortex cell



M.Ch., (2010)
similar results in NJL
M.Ch. (2011)

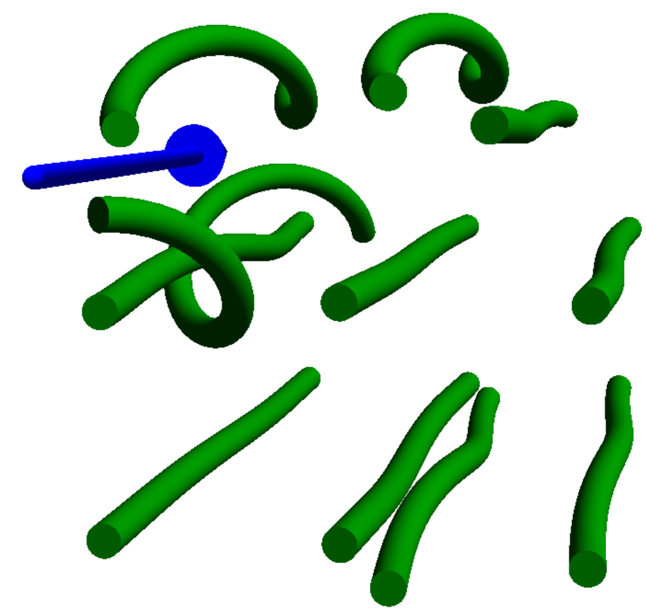
Mean-field vs. theory with fluctuations

J. Van Doorsselaere, H. Verschelde, M. Ch, PRD (2012) [arXiv:1111.4401]



Ground state: hexagonal lattice arrangement of the vortices along the magnetic field B **(naïve)**

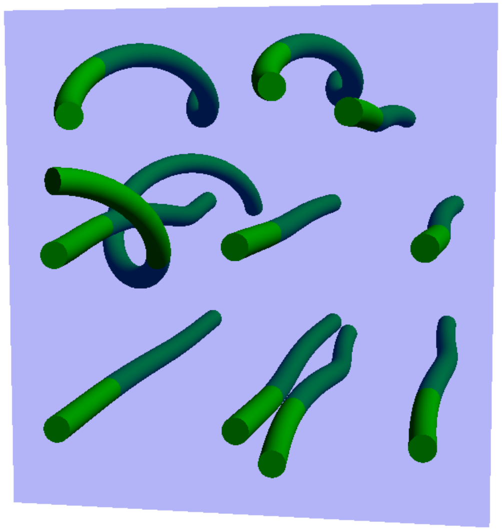
However: other lattice arrangements (square, rhombic, etc) are very close (0.5%) in energy the hexagonal order may be destroyed by fluctuations.

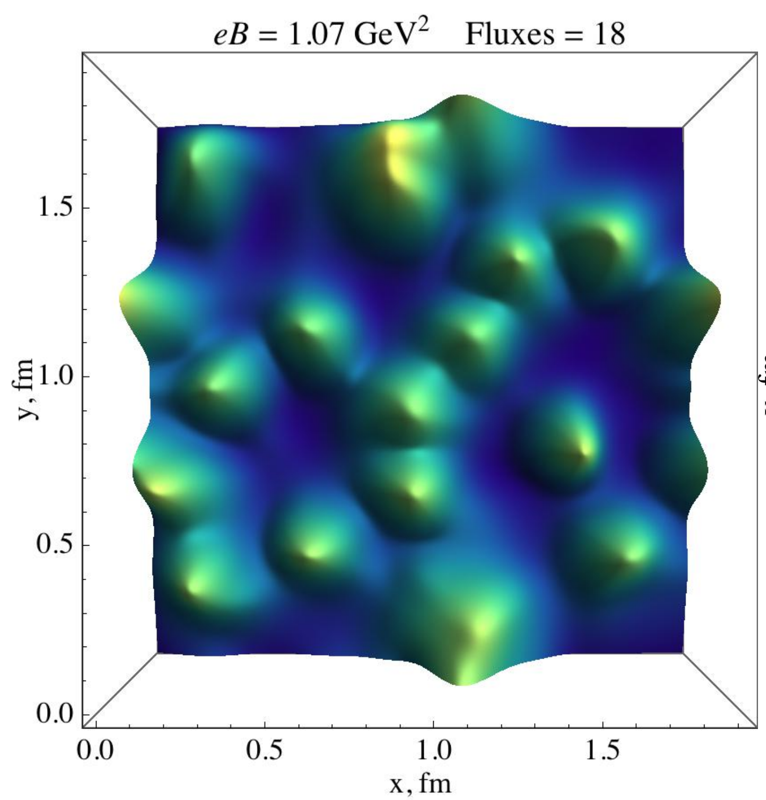


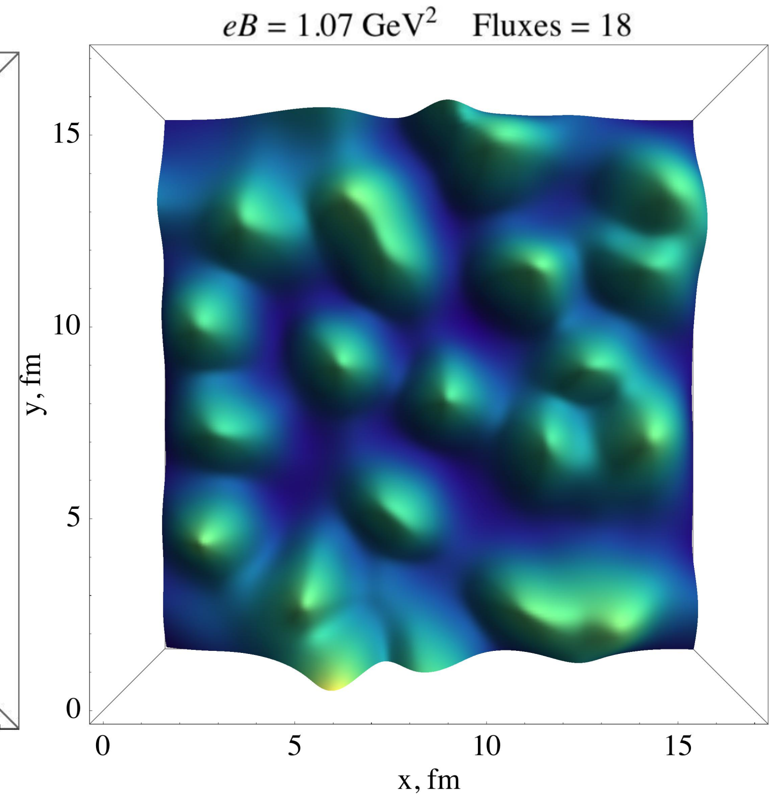
Expected reality: quantum/thermal fluctuations may move and disturb vortices. They mat not be straightly parallel and/or static.

Normalized energy of the ρ meson condensate in the transverse plane.

Check x-y slice at fixed time t and distance z





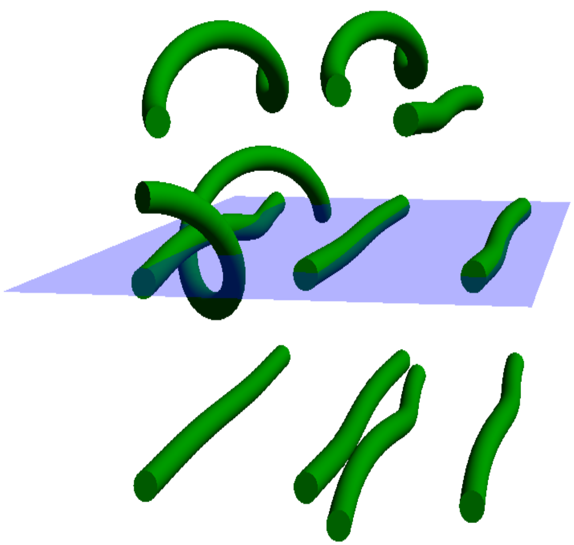


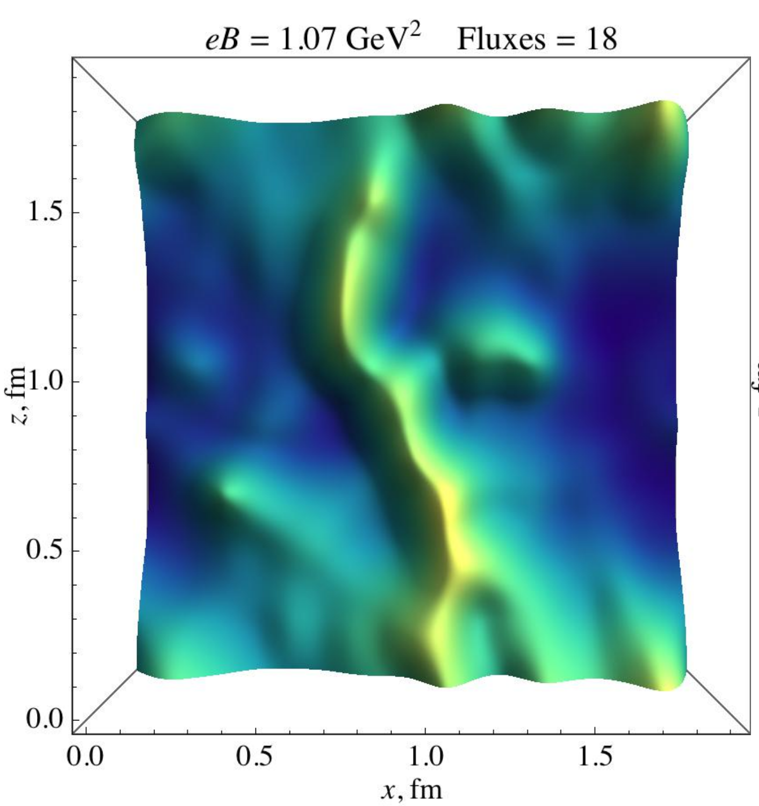
V.Braguta, P. Buividovich, A. Kotov, M. Polikarpov, M.Ch., 2013

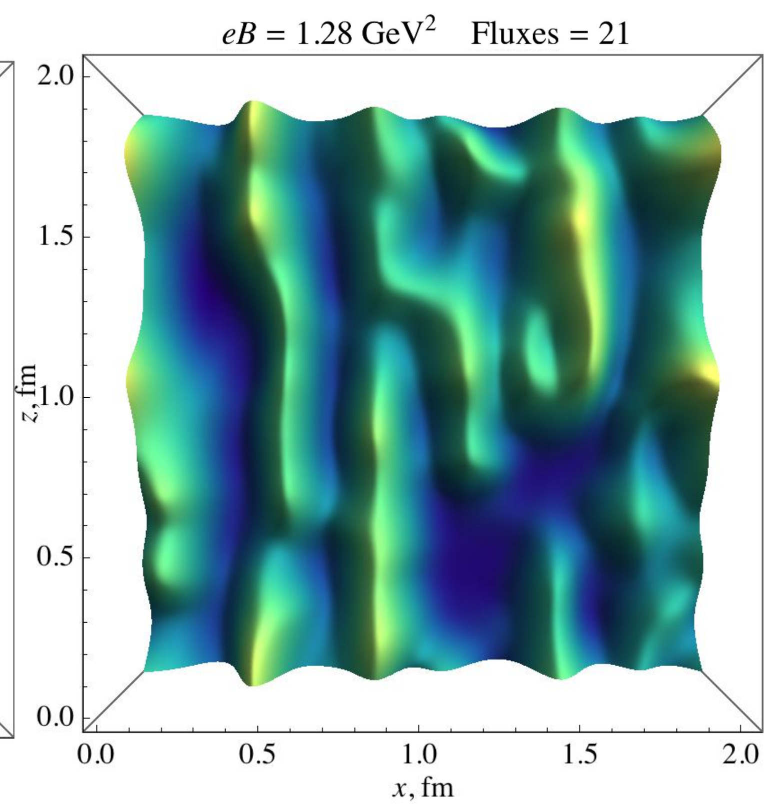
Instead of a regular lattice structure we see an irregular vortex pattern (liquid/glass?) The vortices move as we move the slice.

Normalized energy of the ρ meson condensate along the magnetic field.

Check x-z slice at fixed time t and coordinate y



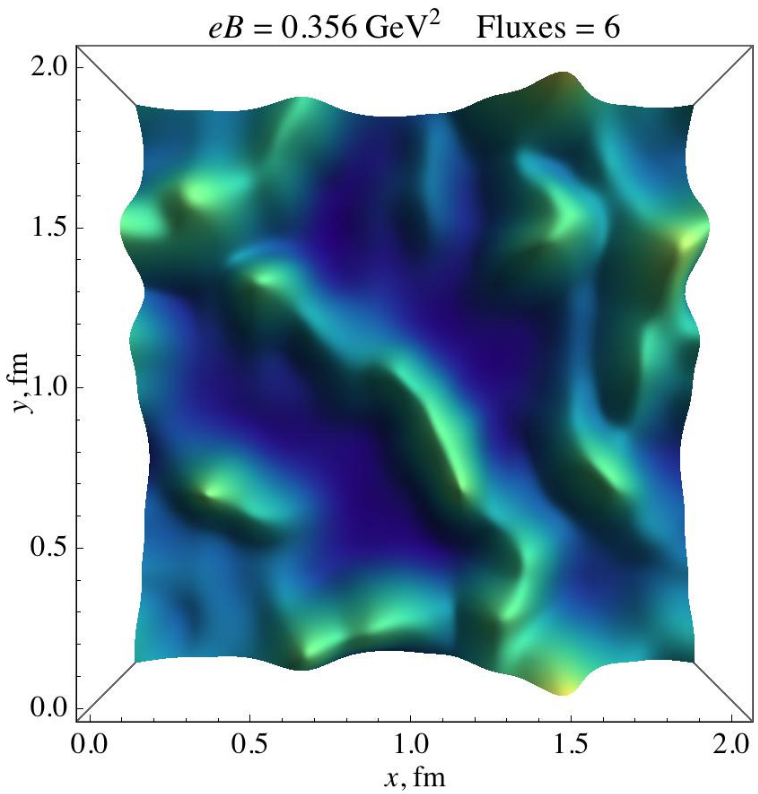


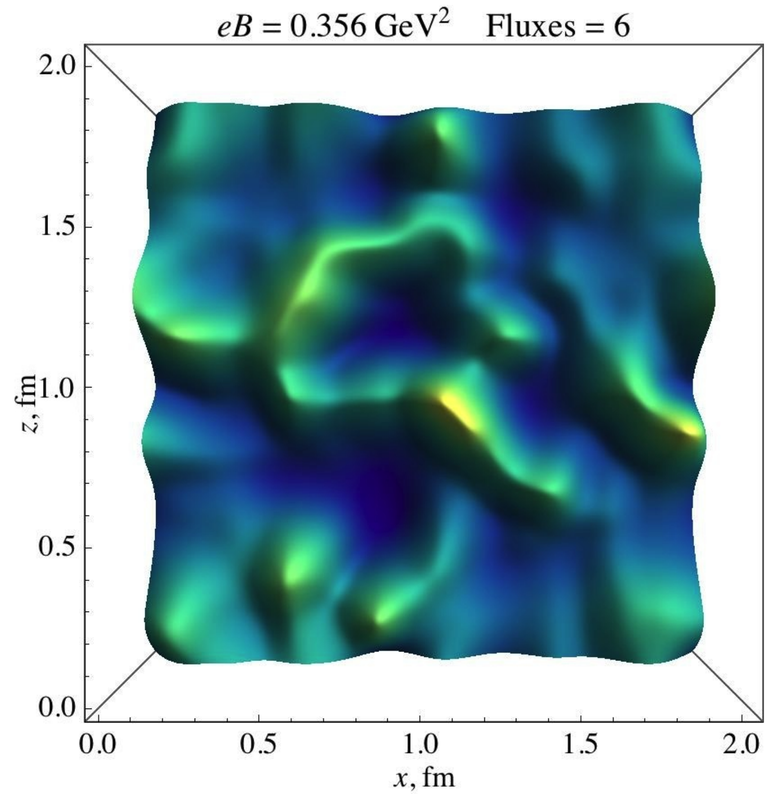


Vortices are not straight and static: they are curvy moving one-dimensional (in 3d) structures.

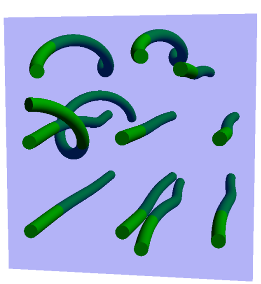
For comparison: low magnetic field

$B < B_c$

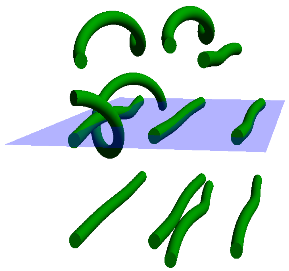




x-y slice
perp. ***B***



x-*z* slice
along ***B***



No order is seen!

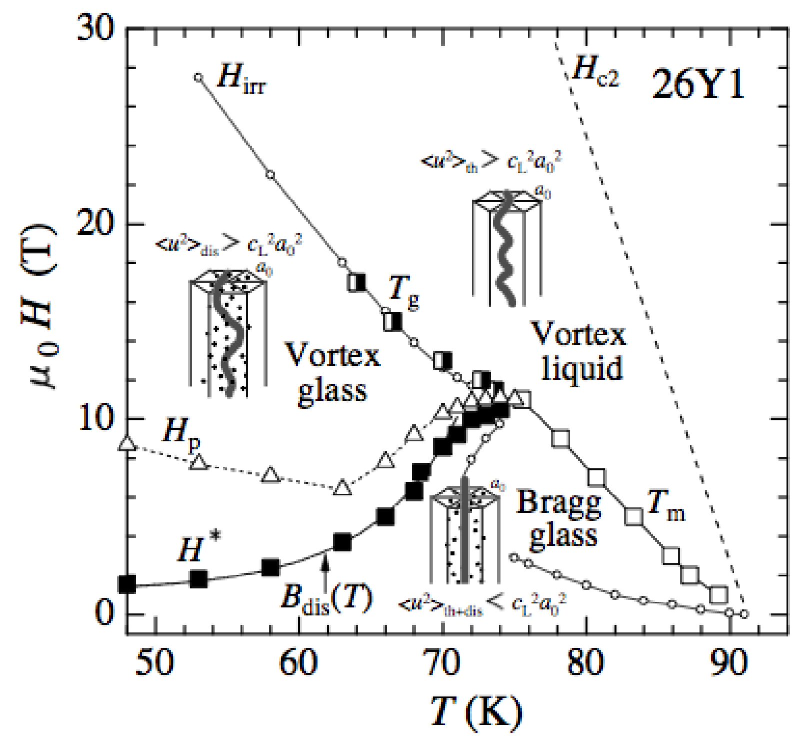
The ground state is a vortex liquid/glass?

Insight from solid state physics:

Real superconductors generally have complicated vortex phase diagrams

Example: **Vortex-matter phase diagram in $YBa_2Cu_3O_y$**

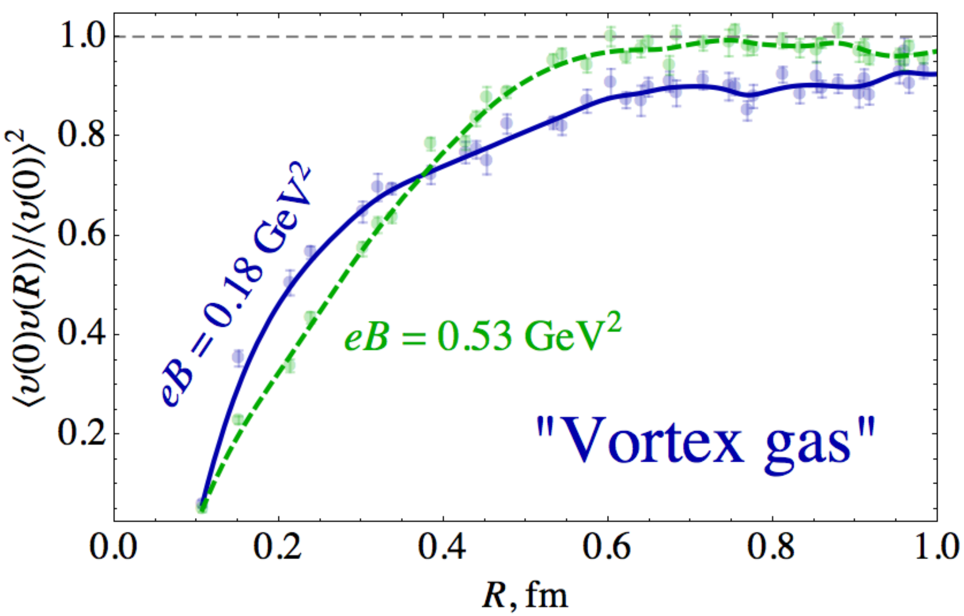
Nishizaki, Kobayashi,
Supercond. Sci. Technol. 13 (2000) 1

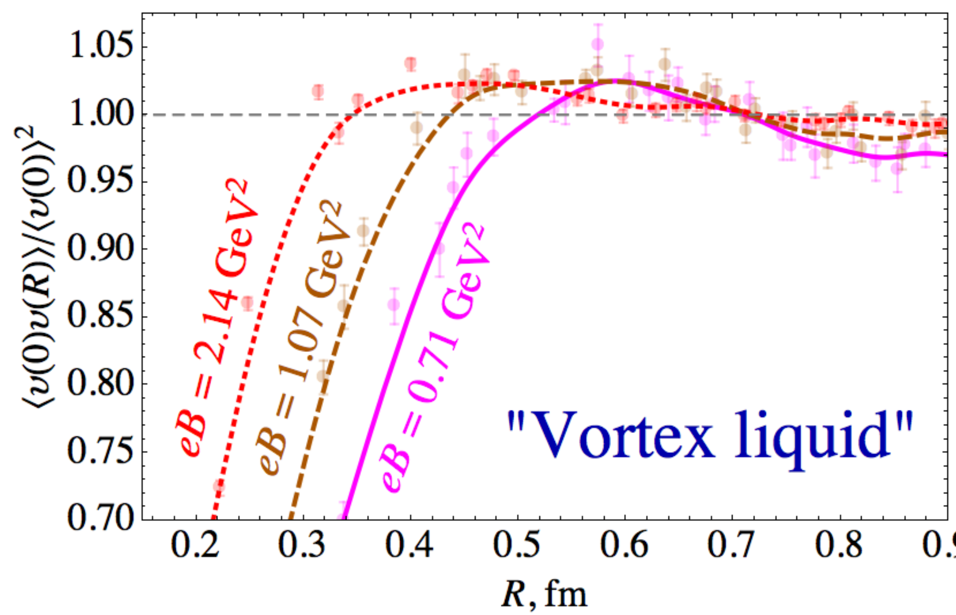


Question: how to distinguish a vortex gas from a vortex liquid?

Answer: Check ~~breathability/drinkability of the stuff~~
vortex density-density correlation functions

Vortex-vortex correlation functions





Insulating phase,
$B < B_c$, "vortex gas"

Superconducting phase,
$B > B_c$, "vortex liquid"
or "vortex glass"

Warning: quenched QCD, no dynamical quarks

Why lattice is melting? Phonons!

Let us consider spectrum of perturbations around the mean-field state.

Superconducting condensate (charged ρ condensate):

$$\rho(z) = \sum_{n\in\mathbb{Z}} C_n h_n\Big(\nu, \frac{z}{L_B}, \frac{\bar{z}}{L_B}\Big) \qquad L_B = \sqrt{\frac{2\pi}{eB}}$$

$$h_n(\nu, z, \bar{z}) = \exp\Big\{-\frac{\pi}{2}\big(|z|^2 + \bar{z}^2\big) - \pi\nu^2 n^2 + 2\pi\nu n \bar{z}\Big\}$$

$$z = x_1 + i x_2, \; \bar{z} = x_1 - i x_2$$

Equations of motion are satisfied for any constants C_n. But the true energy minimum (hexagonal lattice) is reached at

$$C_n = C_0\, \alpha_n, \quad \alpha_{2\mathbb{Z}} = 1, \quad \alpha_{2\mathbb{Z}+1} = i$$

Basically, perturbations in C_n corresponds to fluctuations of the vortex lattice.

How to describe lattice fluctuations?

1) Phonons are quanta of lattice fluctuations/oscillations.
2) Oscillations of the vortex lattice are similar to the oscillations of a usual crystal lattice.
3) One should only be careful with the condensate wavefunction.

Longitudinal momentum along the magnetic field axis (= along vortices): k_z

Transverse quasi-momentum perpendicular to the vortices: $\boldsymbol{k} = (k_x, k_y)$

Bloch oscillations:

$$\rho_k(\boldsymbol{x}, z) = e^{ik_z z} \rho_{\boldsymbol{k}}(\boldsymbol{x})$$
$$\rho_{\boldsymbol{k}}(\boldsymbol{x}) = e^{i\boldsymbol{k}\boldsymbol{x}} \psi_{\boldsymbol{k}}(\boldsymbol{x})$$

"Magnetic basis" of wavefunctions:

$$\psi_{\boldsymbol{k}}(\boldsymbol{x}) = \rho\Big(\boldsymbol{x} + \frac{2\tilde{\boldsymbol{k}}}{eB}\Big) \equiv \rho\Big(\boldsymbol{x} + \frac{L_B^2}{\pi}\tilde{\boldsymbol{k}}\Big) \qquad \tilde{k}_i = \epsilon_{ij} k_j$$

Why do we need "magnetic" basis?

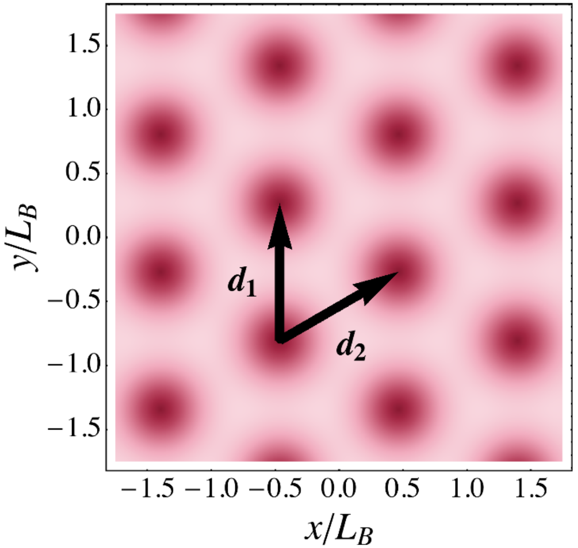
"Magnetic basis" of wavefunctions:

$$\tilde{k}_i = \epsilon_{ij} k_j$$

$$\rho_{\boldsymbol{k}}(\boldsymbol{x}) \;=\; e^{i\boldsymbol{k}\boldsymbol{x}}\psi_{\boldsymbol{k}}(\boldsymbol{x})\;;\;\; \psi_{\boldsymbol{k}}(\boldsymbol{x}) = \rho\left(\boldsymbol{x} + \frac{2\tilde{\boldsymbol{k}}}{eB}\right) \equiv \rho\left(\boldsymbol{x} + \frac{L_B^2}{\pi}\tilde{\boldsymbol{k}}\right)$$

Useful property of the magnetic basis:
Under translations it has same features
as the ground state condensate.

Condensate density



Condensate under a shift by the lattice period:

$$\rho(\boldsymbol{x} + \boldsymbol{d}_a) = e^{i\pi \boldsymbol{x}\times \boldsymbol{d}_a / L_B^2}\rho(\boldsymbol{x})\,; \qquad a = 1, 2$$

Basic lattice vectors:

$$\boldsymbol{d}_1 = \frac{L_B}{\nu}(0,1)\;;\;\; \boldsymbol{d}_2 = \frac{L_B}{\nu}\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)\;; \qquad \nu = \frac{\sqrt[4]{3}}{\sqrt{2}} = 0.9306\ldots\;; \quad L_B = \sqrt{\frac{2\pi}{eB}}$$

"Magnetic basis" of wavefunctions:

$$\rho_{\boldsymbol{k}}(\boldsymbol{x} + \boldsymbol{d}_a) = e^{i\pi \boldsymbol{x}\times \boldsymbol{d}_a / L_B^2}\rho_{\boldsymbol{k}}(\boldsymbol{x})$$

Calculation of the phonon spectrum

Perturbation of the condensate by phonons:

$$\rho_{\text{ph}}(\boldsymbol{x}) = \sum_{\boldsymbol{k}} c_{\boldsymbol{k}} \rho_{\boldsymbol{k}}(\boldsymbol{x})$$

with

$$c_{|\boldsymbol{k}|\neq 0} \ll c_0 = 1$$

Phonons ($c_{|\boldsymbol{k}|\neq 0}$) — Condensate ($c_0$)

Free transverse energy (neutral mesons and photons are integrated out):

$$\langle \mathcal{E}_\perp[\rho] \rangle = \frac{e^2}{8} \langle |\bar{\rho}|^2 \rangle^2 + \frac{1}{2}(m_\rho^2 - eB)\langle |\bar{\rho}|^2 \rangle + \frac{g_s^2}{8} m_\rho^2 \Big\langle |\bar{\rho}|^2 \frac{1}{-\Delta + m_0^2} |\bar{\rho}|^2 \Big\rangle$$

with

$$\langle \mathcal{O} \rangle = \frac{1}{\text{Area}_\perp} \int dx dy \, \mathcal{O}(x, y)$$

Expand over c_k

$$\mathcal{E}_\perp[\rho_{\text{ph}}] = \mathcal{E}_\perp^{(0)} + \mathcal{E}_\perp^{(2)} + O(c_{\boldsymbol{k}}^4)$$

Ground state energy: $\mathcal{E}_\perp^{(0)} \equiv \mathcal{E}_\perp[\rho_0]$

Phonon energy: $\mathcal{E}_\perp^{(2)} \sim c_{\boldsymbol{k}} c_{\boldsymbol{k}'}$

Mixing of modes

Four-point term in the free energy mixes vibrational modes:

$$\rho_{\rm ph}(\boldsymbol{x}) = \sum_{\boldsymbol{k}} c_{\boldsymbol{k}} \rho_{\boldsymbol{k}}(\boldsymbol{x})$$

Define: $v_{\boldsymbol{k}} = (c_{\boldsymbol{k}}, c^*_{-\boldsymbol{k}})^T$

In the leading order:

$$\left\langle \rho^* \rho \frac{M^2}{-\Delta + M^2} \rho^* \rho \right\rangle = \frac{1}{2} \sum_{\boldsymbol{k}} v^\dagger_{\boldsymbol{k}} \cdot \hat{Q} \cdot v_{\boldsymbol{k}}$$

$$M = m_0 L_B$$

with

$$\hat{Q} = 2 \begin{pmatrix} Q_{\boldsymbol{k},\boldsymbol{k},0,0} + Q_{\boldsymbol{k},0,0,\boldsymbol{k}} & Q^*_{\boldsymbol{k},0,-\boldsymbol{k},0} \\ Q_{\boldsymbol{k},0,-\boldsymbol{k},0} & Q_{\boldsymbol{k},\boldsymbol{k},0,0} + Q_{\boldsymbol{k},0,0,\boldsymbol{k}} \end{pmatrix}$$

$$Q_{\boldsymbol{l}_2,\boldsymbol{k}_2,\boldsymbol{l}_1,\boldsymbol{k}_1} = \frac{1}{|C_0|^4} \left\langle \rho^*_{\boldsymbol{l}_2} \rho_{\boldsymbol{k}_2} \frac{M^2}{-\Delta + M^2} \rho^*_{\boldsymbol{l}_1} \rho_{\boldsymbol{k}_1} \right\rangle$$

← Complicated, but calculable

Diagonalization

Eigenvalues:

$$\lambda_{\boldsymbol{k},\pm} = 2\left(Q_{\boldsymbol{k},\boldsymbol{k},0,0} + Q_{\boldsymbol{k},0,0,\boldsymbol{k}} \pm |Q_{\boldsymbol{k},0,-\boldsymbol{k},0}|\right)$$

Eigenvectors:

$$o_{\boldsymbol{k}} = \frac{c_{\boldsymbol{k}} + c^*_{-\boldsymbol{k}}}{2}, \qquad a_{\boldsymbol{k}} = \frac{c_{\boldsymbol{k}} - c^*_{-\boldsymbol{k}}}{2i}$$

Optical (massive) modes — Acoustic (massless) modes

Energy: $\mathcal{E}_{\perp}^{(2)} = \sum_{\boldsymbol{k}} \mathcal{E}_{\perp}^{(2)}(\boldsymbol{k})$

$$C_0(B) = \sqrt{\frac{2\sqrt{2}(eB - m_\rho^2)\nu}{e^2 + (g_s^2 - e^2)\beta_\rho}} \simeq 0.2733\sqrt{eB - eB_c}$$

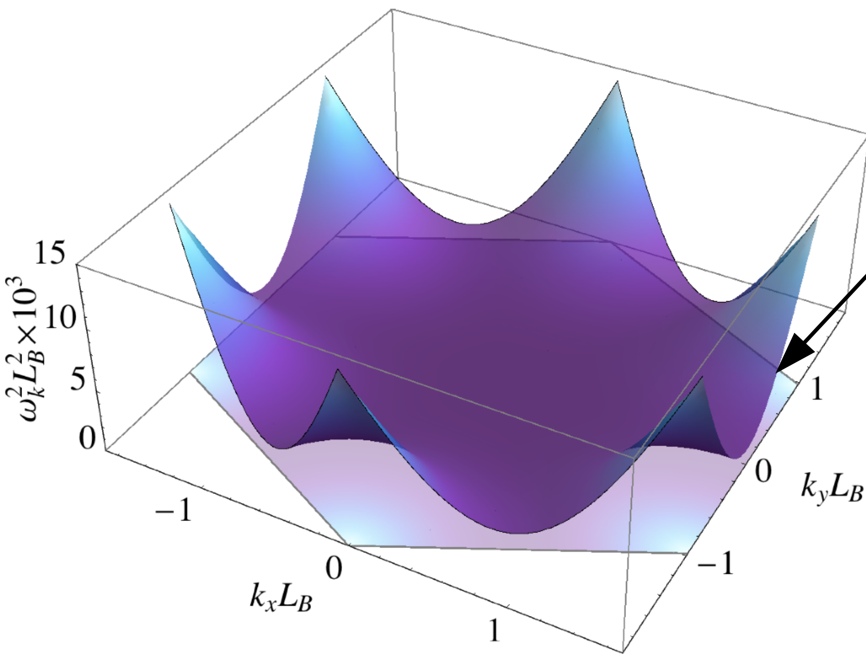
with

$$\mathcal{E}_{\perp}^{(2)}(\boldsymbol{k}) = \mathcal{E}_{\perp,O}^{(2)}(\boldsymbol{k}) + \mathcal{E}_{\perp,A}^{(2)}(\boldsymbol{k}) = \frac{g_s^2 - e^2}{8}|C_0|^2 \left[o_{\boldsymbol{k}}^2(\lambda_{\boldsymbol{k},+} - \lambda_{0,-}) + a_{\boldsymbol{k}}^2(\lambda_{\boldsymbol{k},-} - \lambda_{0,-})\right]$$

Acoustic phonon spectrum

$$\omega_k^2 = \frac{2e(g_s^2 - e^2)(B - B_c)}{e^2 + (g_s^2 - e^2)\beta_\rho} \left[\nu^2 \lambda_-\left(\frac{L_B \boldsymbol{k}}{\pi} \right) - \beta_\rho \right] + k_z^2$$

Transverse phonons   Longitudinal phonons



Low-energy phonon spectrum:

!!!

$$\omega_k^2 = k_z^2 + f(B)\left(\boldsymbol{k}^2\right)^2 + \dots$$

$$f(B) = \frac{C_f}{|eB|}\left(1 - \frac{B_c}{B}\right) + \dots$$

$$C_f \simeq 0.455$$

$$\nu = \frac{\sqrt[4]{3}}{\sqrt{2}} = 0.9306\dots$$

$$g_s \equiv g_{\rho\pi\pi} = \frac{m_\rho}{\sqrt{2} f_\pi} = 5.88$$

$$g_s \gg e \equiv \sqrt{4\pi\alpha_{\text{e.m.}}} \approx 0.303$$

$$\beta_\rho = \left\langle \frac{|\rho^2|}{\langle|\rho^2|\rangle} \frac{m_0^2}{-\Delta + m_0^2} \frac{|\rho^2|}{\langle|\rho^2|\rangle} \right\rangle$$

$$\beta_\rho(B) = 1.01937 - 0.01702\left(\frac{B}{B_c} - 1\right) + \dots$$

Low energy phonons

Spectrum:

$$\omega_k^2 = k_z^2 + f(B)\left(\boldsymbol{k}^2\right)^2$$

Longitudinal phonon:
Propagates with
speed of light

Transverse phonon:
Propagates with the speed

$$v_\perp(\boldsymbol{k}, k_z = 0) = 2\sqrt{f(B)}|\boldsymbol{k}|$$

At the magnetic field $B = 1.01 B_c$ the transverse phonon of the energy $\omega_{\boldsymbol{k},0} = 1\,\mathrm{MeV}$ should propagate with the velocity equal to 2% of speed of light.

The presence of the "supersoft" transverse phonons may lead to instabilities of the vortex lattice. It may melt at certain temperature!

Conclusions

- In a sufficiently strong magnetic field condensates with $r^{\pm}$ meson quantum numbers are formed spontaneously.

- The vacuum (= no matter present, = empty space, = nothing) becomes electromagnetically superconducting.

- The superconductivity is anisotropic: the vacuum behaves as a perfect conductor only along the axis of the magnetic field.

- New type of topological defects,"r vortices", emerge.

- Mean field: the ground state of r vortices is the Abrikosov-type lattice in transverse (w.r.t. the axis of magnetic field) directions.

- The vortex lattice possesses acoustic phonons (massless excitations of the vortex lattice). The “supersoft” transverse phonons may lead to lattice instabilities driven by temperature (or, quantum?) fluctuations. The lattice may melt and become a liquid (new phase?)

Anisotropic superconductivity
(Lorentz-covariant form of the London equations)

We are working in the vacuum, thus the transport equations may be rewritten in a Lorentz-covariant form:

$$\partial_{[\mu,} j_{\nu]} = \kappa \frac{(F \cdot \widetilde{F})}{(F \cdot F)} \widetilde{F}_{\mu\nu}$$

Electric current averaged over one elementary rho-vortex cell

A scalar function of Lorentz invariants.
In this particular model:

$$\kappa = (e^3/g_s^2)(\sqrt{(F \cdot F)/2} - B_c)$$

(slightly different form of k function in NJL)

Lorentz invariants:

$$(F \cdot \widetilde{F}) = F^{\mu\nu} \widetilde{F}_{\mu\nu} \equiv 4(\vec{B} \cdot \vec{E})$$

$$(F \cdot F) = F^{\mu\nu} F_{\mu\nu} \equiv 2(\vec{B}^2 - \vec{E}^2)$$

$$\widetilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$$

If B is along x_3 axis, then we come back to

$$\frac{\partial}{\partial t} \langle J_3 \rangle = -\frac{2e^3}{g_s^2} (B_{\text{ext}} - B_c) E_3 \quad \text{and} \quad \frac{\partial}{\partial t} \langle J_1 \rangle = \frac{\partial}{\partial t} \langle J_2 \rangle = 0$$

It is … a metamaterial!

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Vacuum in a Strong Magnetic Field as a Hyperbolic Metamaterial

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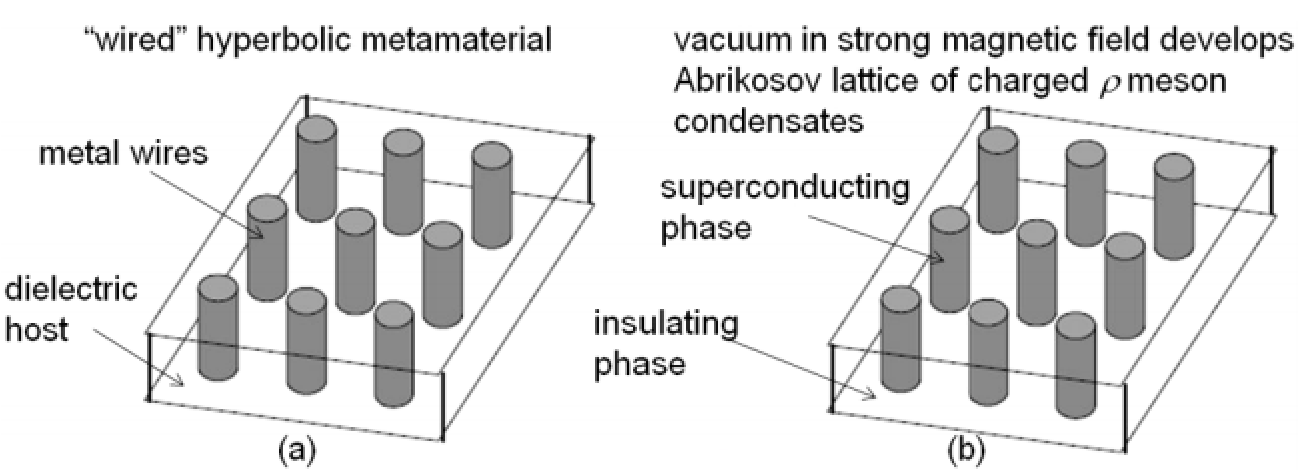
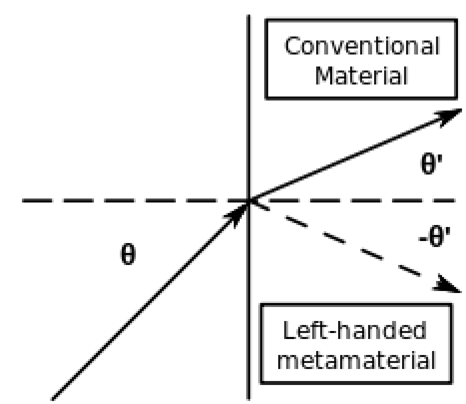
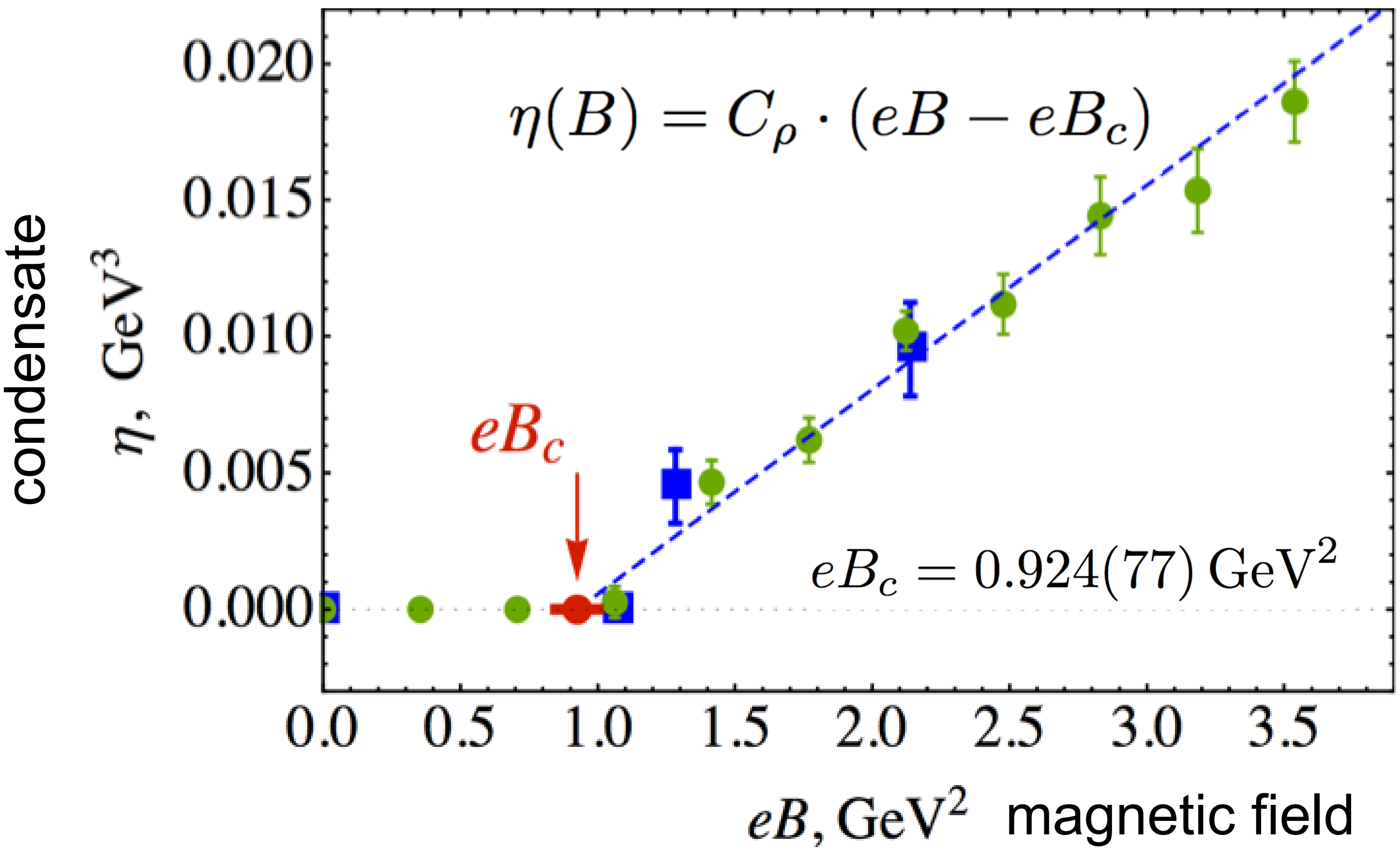
As demonstrated by Chernodub, vacuum in a strong magnetic field behaves as Abrikosov vortex lattice in a type-II superconductor. We investigate electromagnetic behavior of vacuum in this state and demonstrate that vacuum behaves as a hyperbolic metamaterial. If the magnetic field is constant, low frequency extraordinary photons experience this medium as a (3 + 1) Minkowski spacetime in which the role of time is played by the spatial z coordinate. Variations of the magnetic field curve this spacetime, and may lead to formation of "electromagnetic black holes." Since hyperbolic metamaterials behave as diffractionless "perfect lenses," and large enough magnetic fields probably existed in the early Universe, the demonstrated hyperbolic behavior of early vacuum may have imprints in the large scale structure of the present-day Universe.





Numerical simulations of quenched two-color QCD

V.Braguta, P. Buividovich, A. Kotov, M. Polikarpov, M.Ch., arXiv:1104.3767



Quenched SU(2) + overlap fermions.
Spacings: a ~ 0.1 fm
Lattices: $L^4 \sim (2\ \mathrm{fm})^4$

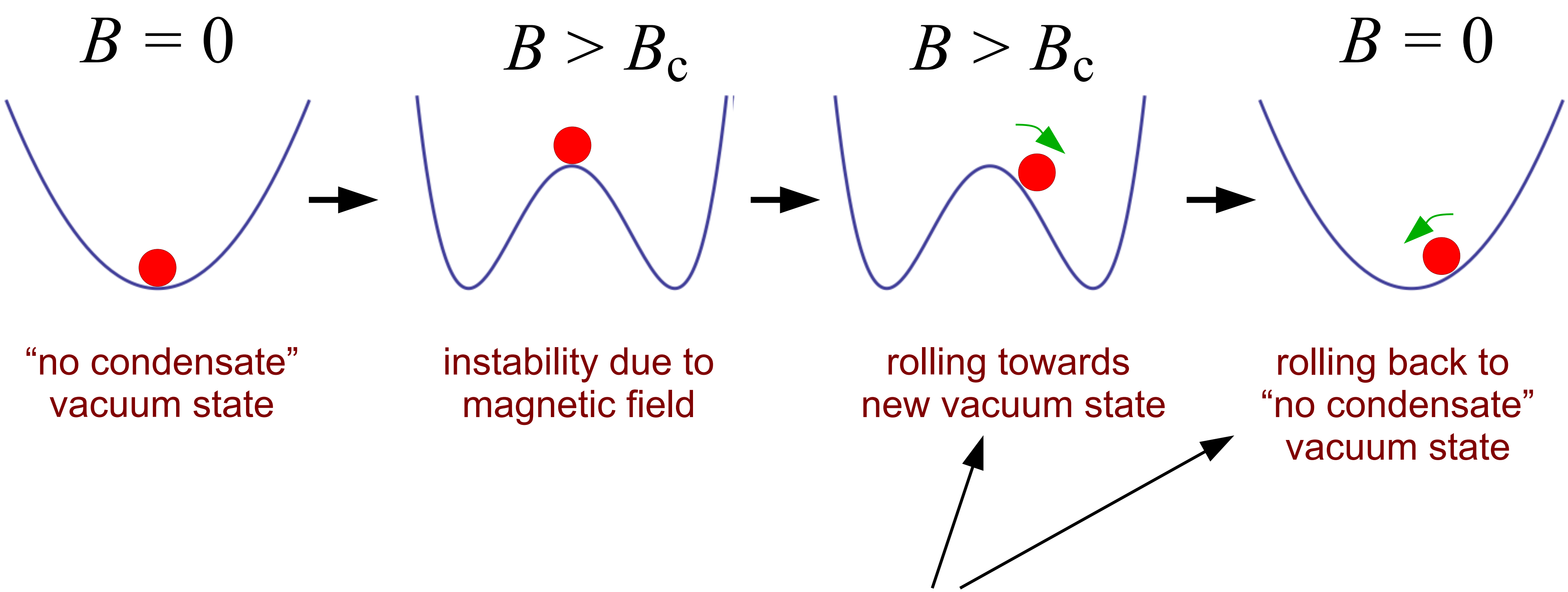
Theory:
$\eta \sim \sqrt{B - B_c}$
for $B \geqslant B_c$

[qualitatively realistic vacuum, quantitative results may receive corrections (20%-50% typically)]

Ongoing discussion:
1. results by Y. Hidaka, A. Yamamoto, arXiv:1209.0007 – Vafa-Witten theorem;
2. (counter)arguments by M. Ch., PRD (2012) [arXiv:1209.3587] – $U(1)_{em}$ was forgotten in “1”
3. quenched QCD (overlap fermions vs. Wilson fermions) – vortex liquid phase?

Collisions are too quick for the condensate to be developed, but signatures may be seen as the instability of the vacuum state

ρ-meson vacuum state between the ions in ultraperipheral collisions



A future project.

Emission of ρ mesons