

# SEMPARIS – Séminaires en région parisienne

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**TBA**

**Jeudi 27 Octobre 2022, 11 :00**

IHES, Amphithéâtre Léon Motchane

Domaines : math

Titre : *Hodge-proper Stacks and Totaro's Conjecture, part 2*

Orateur : **Dmitry Kubrak ( IHES )**

Résumé : *I will talk about a series of works with Artem Prikhodko where we develop a version of  $p$ -adic Hodge theory in the setting of Artin stacks. One of the main motivations for our project was a conjecture by Totaro : namely, based on his concrete computations, he suggested that the dimension of mod  $p$  de Rham cohomology of the classifying stack  $BG$  for  $G$  reductive might always be bounded from below by the dimension of the  $F_p$ -singular cohomology on the classifying space  $BG(C)$  of the Lie group  $G(C)$  of complex points of  $G$ . For smooth and proper schemes such an inequality is a consequence of integral  $p$ -adic Hodge theory in the form proved by Bhatt-Morrow-Scholze ; however, their results can not be applied here directly since  $BG$  is not proper. To prove Totaro's conjecture, using the theory of prismatic cohomology, we develop integral  $p$ -adic Hodge theory in a more general setting of Hodge-proper stacks : these are stacks that only look proper from the point of view of its Hodge cohomology. However, one problem then still remains : namely, the étale comparison we get is with the étale cohomology of the Raynaud generic fiber, which a priori agrees with the algebraic generic fiber (and then complex points) only in the smooth proper setting. Nevertheless, we prove that the two étale cohomology theories still agree at least for quotient stacks  $[X/G]$  with  $X$  smooth and proper and  $G$  reductive. This then implies Totaro's conjecture by plugging  $X=pt$ . In further work we also show that after inverting  $p$  the two étale cohomology agree for any Hodge-proper stack, which sets up rational  $p$ -adic Hodge theory (with the crystalline and de Rham comparisons, and Hodge-Tate decomposition) in this setting. If time permits, I will also tell about some explicit computations of cohomology of reductive groups in characteristic  $p$  that one can perform using the above comparison in the case of  $BG$  (this is a series of joint works with Federico Scavia and Anlong Chua).*

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